

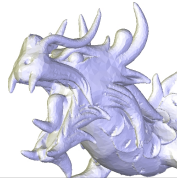


Lyon 1

## Mesh and Computational Geometry

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## Data structures

- How to store a triangular mesh so that one can easily navigate through it?

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## Data structures

- Geometric information :
  - coordinates of vertex positions
- Topological information :
  - incidence and adjacency relationships between vertices, edges and faces

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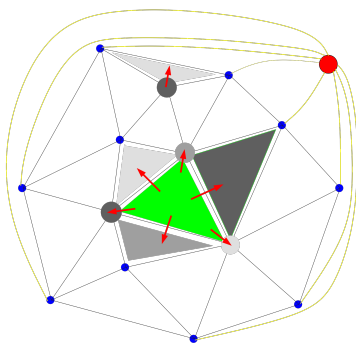
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## Data structures

- Triangle and vertex based representation (only for triangulated mesh)
  - Triangle :
    - Access to the 3 incident vertices (trigonometric order)
    - Access to the 3 adjacent triangles
 Constraint : vertex  $i$  facing adjacent triangle  $i$
  - Vertex :
    - Access to 1 incident triangle
    - Access to the underlying point (geometry)

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## Data structures

- Case of a 2D triangulation
  - Dangling pointers / indexes on the boundary of the convex hull
 OR
  - Addition of a fictitious vertex (called infinite vertex) whose incident triangles are attached to the edges of the convex envelope
- The data structure can also be used for surface triangulations
  - Use dangling pointers/indexes for each hole boundary
 OR
  - Add fictitious vertices for each hole

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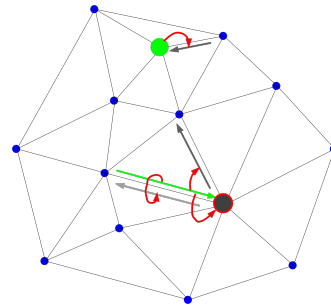
## Data structures with edges

- Representation based on  $\frac{1}{2}$  edges and vertices
  - $\frac{1}{2}$  edge :
    - Access to the coupled  $\frac{1}{2}$  edge
    - Access to the next  $\frac{1}{2}$  edge
    - Access to the target vertex
  - Vertex :
    - Access to an  $\frac{1}{2}$  edge oriented towards the vertex
    - Access to the underlying point

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## Data structures



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## Naïve and uncompressed format of a mesh in a file

- Files off
- Format description
  - Number of vertices  $s$
  - Number of faces  $c$
  - Vertices coordinates
  - Description of faces (from 0 to  $c-1$ )
    - Number of vertices in the face
    - Description of the face by the indexes of its vertices (in counter-clockwise direction in 2D, or by orienting the faces towards the outside in 3D)

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## How to load a mesh into the data structure?

- Need to find adjacencies between faces
- Using a *map* while reading faces
  - Associating to each edge encountered (key = pair of vertex indexes) the *index* of the current face (+ optionally the relative index of the vertex opposite to the edge in the face)
  - When an edge is incident to two faces, the adjacency between these two faces is reported in the structure

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## How to load a mesh into a data structure?

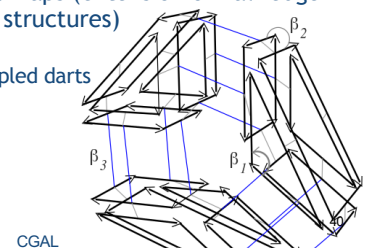
- Using a *map* while reading faces
- Complexity in  $O(s \cdot \log(s))$  where  $s$  number of vertices
- Rque : If you use an hmap instead of a map, you get an almost linear complexity.

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## Data structures

- Case of a 3D triangulation
  - Structure based on tetrahedrons and vertices
 OR
  - Combinatorial maps (extension of half edge or dart-based structures)
    - $B_1$  : next
    - $B_2$  et  $B_3$  : coupled darts



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## Data structures

- Case of nD triangulations
  - n-simplex and vertex based structures
  - Combinatorial Maps
    - $\beta_1$  : next
    - $\beta_2, \beta_3 \dots \beta_n$  : coupled darts

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