Problem Set 4 - Solutions

Intermediate Microeconomics - Summer 2020, NYU
Instructor: Skand Goel

Rules:

- The problem set is due on **Thursday, June 25** before 4.05 PM Eastern time.
- The submission should be a PDF attachment (scanned or digital) via NYU classes. **Please don't upload multiple PDFs** - just put all your work in one PDF. An app such as CamScanner should help.
- Please **mention your NYU net ID** on your submission. e.g. my net ID is sg3992.
- Each of the three questions is for 3 points. The total score will be rescaled to 5. Even if you don't do everything right, you can still get a high mark - so try and attempt all questions, and if you hit a dead-end, try to explain how you got there and why you are stuck!
- If you feel there is an error or typo, please email me at sg3992@nyu.edu.
- You can work in groups, but must submit your own work. **If you work in groups, mention the name and net ID of students in your group.**
- **Please write legibly**. It helps the grading, *as well as your grades.* Also, when asked to plot something, please draw neat, well-labeled diagrams. Drawing larger figures increases legibility.
- Friendly reminder: Please do not circulate the problem set. **Please do not upload the problem set to the internet in any form**.

A. Industry size

There is only one good in the world. Suppose a firm has the cost function

$$C(x) = y^2 + f^2$$

where $f > 0$ is a constant. (The fixed cost is f^2 and not f. This is only for convenience as you'll see.) Denote output price by p.

(a) What is the long-run supply curve for this firm? [Hint: This is shutdown option I from the slides - fixed costs are not sunk. Also, f will show up in the expression for the supply curve.]

Answer:

If operating, the firm produces

$$p = 2y^* \implies y^* = \frac{p}{2}$$

Operating profits are

$$\begin{aligned} \pi(p) &= py^* - C(y^*) \\ &= \frac{p^2}{2} - \frac{p^2}{4} - f \\ &= \frac{p^2}{4} - f^2 \end{aligned}$$

which implies

$$\pi(p) > 0 \iff \frac{p^2}{4} > f^2 \iff p > 2f$$

The long run supply curve is given by

$$y^*(p) = \begin{cases} \frac{p}{2} & \text{if } p \geq 2f \\ 0 & \text{if } p < 2f \end{cases}$$

(b) What is the total supply curve if there are n such (identical) firms? Note that I am not calling this the industry supply, as the industry size is determined in equilibrium. Denote this by $S(p, n)$, where n refers to the number of firms in the industry. Obviously, $S(p, n)$ depends on f.

Answer:

$$S(p,n) = \begin{cases} \frac{np}{2} & \text{if } p \geq 2f \\ 0 & \text{if } p < 2f \end{cases}$$

(c) There are two types of consumers, A and B, with the following individual Marshallian demands:

$$x_A^* = 10 - 2p, \qquad x_B^* = 5 - p$$

Suppose there are 10 consumers of type A and 5 of type B. What is market demand?

Answer:

Note that the inverse demand for both types have the same $p-$intercepts, so there won't be any kink.

$$D(p) = 10(10 - 2p) + 5(5 - p) = 125 - 25p$$

(d) Find the market equilibrium if $f = 0.5$ with free entry. In particular, find (i) how many firms operate in this market, (ii) equilibrium price, (iii) equilibrium quantity. [Hint: From market clearing, p as function of n. Then, use the cutoff price condition to find n. Then, go back and find equilibrium price.]

Answer:

Equilibrium price solves

$$D(p) = S(p,n)$$

which implies

$$125 - 25p = \frac{np}{2} \implies p = \frac{125}{25 + \frac{n}{2}}.$$

For there to not be negative profits

$$\begin{aligned} p \geq 2f \implies & \frac{125}{25 + \frac{n}{2}} \geq 1 \\ \implies & 125 \geq 25 + \frac{n}{2} \\ \implies & 200 \geq n \end{aligned}$$

Therefore, there will be 200 firms in the industry. As a result, the industry supply curve is

$$S(p,200) = \begin{cases} 100p & \text{if } p \geq 1 \\ 0 & \text{if } p < 1 \end{cases}$$

Solving

$$D(p) = S(p, 200)$$

gives

$$p = \frac{125}{25 + \frac{200}{2}} = 1.$$

We knew this even without the calculation because every firm in the industry is breaking even. Equilibrium quantity is 100.

(e) Suppose the government wants to restrict production to 50. Suppose $f = 0$, but the government can impose a fixed tax T^2 on each firm, which will act like a fixed cost. What should be the tax imposed in order to ensure guarantee this level of production? [Hint: Use market demand to back out the price that would implement this level of production. Then, figure out the value of T that ensures that the industry is in long-run equilibrium (no entry or exit) at this price.]

Answer:

An equilibrium quantity of 50, implies an equilibrium price of

$$50 = 125 - 25p \implies p = 3$$

from $D(p)$.

We know firms operate as long as

$$p \geq 2T.$$

We want $p = 3$, so $T = 1.5$. You can check that at this level of T, market quantity is indeed 50.

(f) What is the total tax revenue of the government? Suppose firms are infinitely divisible i.e. fractions of firms can exist.

Answer:

To find this, we need to find the equilibrium number of firms. This can be found as the solution to

$$S(3, n) = 50 \implies \frac{3n}{2} = 50$$

which gives

$$n = 33.33$$

The total tax revenue is

$$nT^2 = 33.33 \times 1.5^2$$

B. Edgeworth box

Suppose the utility functions for consumers A and B are given by

$$u^A(x_1, x_2) = x_1 x_2 \qquad u^B(x_1, x_2) = x_1 + \log x_2$$

and their endowments are

$$\omega^A = (\omega_1^A, \omega_2^A) \qquad \omega^B = (\omega_1^B, \omega_2^B)$$

Denote market prices of good 1 and good 2 by p_1 and p_2, respectively.

(a) Find their Marshallian demands.

Answer:

$$\begin{aligned}
x_1^{A*}(p_1, p_2) &= \frac{p_1\omega_1^A + p_2\omega_2^A}{2p_1} \\
x_2^{A*}(p_1, p_2) &= \frac{p_1\omega_1^A + p_2\omega_2^A}{2p_2} \\
x_1^{B*}(p_1, p_2) &= \frac{p_1\left(\omega_1^B - 1\right) + p_2\omega_2^B}{p_2} \\
x_2^{B*}(p_1, p_2) &= \frac{p_1}{p_2}
\end{aligned}$$

(b) Find equilibrium prices in this market. [Hint: Use Walras' law and pick your choice of numeraire.]

Answer:

We need to find the solution to the excess demand system:

$$\begin{aligned}
x_1^{A*}(p_1, p_2) + x_1^{B*}(p_1, p_2) &= \omega_1^A + \omega_1^B \\
x_2^{A*}(p_1, p_2) + x_2^{B*}(p_1, p_2) &= \omega_2^A + \omega_2^B
\end{aligned}$$

Walras' Law tells us we need only solve one of these equations: we'll solve the one for good 2 (its easier). Also, we can pick our numeraire good: let's normalize $p_2 = 1$. We get

$$\frac{p_1\omega_1^A + \omega_2^A}{2} + p_1 = \omega_2^A + \omega_2^B$$

which implies

$$p_1 = \frac{0.5\omega_2^A + \omega_2^B}{0.5\omega_1^A + 1}$$

(c) How does consumer B's equilibrium consumption of good 2 change if consumer A's endowment of good 1 changed, all else equal?

Answer:

Consumer B's consumption of good 2 is given by

$$x_2^{B*}\left(\frac{0.5\omega_2^A + \omega_2^B}{0.5\omega_1^A + 1}, 1\right) = \frac{0.5\omega_2^A + \omega_2^B}{0.5\omega_1^A + 1}$$

Therefore,

$$\frac{\partial x_2^{B*}}{\partial \omega_1^A} = \frac{-0.5\left(0.5\omega_2^A + \omega_2^B\right)}{\left(0.5\omega_1^A + 1\right)^2} < 0$$

C. Edgeworth box with non-convex preferences

Do this question graphically. Suppose the utility functions for consumers A and B are given by

$$u^A(x_1, x_2) = x_1 + x_2 \qquad u^B(x_1, x_2) = \min\{x_1, x_2\}$$

and their endowments are

$$\omega^A = (3, 4) \qquad \omega^B = (4, 3)$$

(a) Is $(p_1, p_2) = (1, 1)$ an equilibrium price? If so, what is the equilibrium consumption allocation?

Answer:

Yes. At this price ratio,

$$x^{B*} = (3.5, 3.5)$$

For the markets to clear A's consumption has to be

$$x^{A*} = (3.5, 3.5)$$

We know this is (one of) the optimal bundles for A because given A's perfect substitutes preferences, he maximizes utility by consuming anywhere on the line

$$x_1^A + x_2^A = 7,$$

which is satisfied by

$$x^{A*} = (3.5, 3.5).$$

(b) Is $(p_1, p_2) = (1, 2)$ an equilibrium price? If so, what is the equilibrium consumption allocation?

Answer:

No. A must optimally consume only good 1 at these prices. B will always consume $x_1 = x_2$. So markets will not clear at this price.