Problem Set 5 - Solutions

Intermediate Microeconomics - Summer 2020, NYU
Instructor: Skand Goel

Rules:

- The problem set is due on **Wednesday, July 1** before 4.05 PM Eastern time.
- The submission should be a PDF attachment (scanned or digital) via NYU classes. **Please don't upload multiple PDFs** - just put all your work in one PDF. An app such as CamScanner should help.
- Please **mention your NYU net ID** on your submission. e.g. my net ID is sg3992.
- Each of the three questions is for 3 points. The total score will be rescaled to 5. Even if you don't do everything right, you can still get a high mark - so try and attempt all questions, and if you hit a dead-end, try to explain how you got there and why you are stuck!
- If you feel there is an error or typo, please email me at sg3992@nyu.edu.
- You can work in groups, but must submit your own work. **If you work in groups, mention the name and net ID of students in your group.**
- **Please write legibly**. It helps the grading, *as well as your grades.* Also, when asked to plot something, please draw neat, well-labeled diagrams. Drawing larger figures increases legibility.
- Friendly reminder: Please do not circulate the problem set. **Please do not upload the problem set to the internet in any form**.

A. A public goods problem

(The background for this problem will be discussed on Tuesday. However, I am confident that if you try it right now, you'll be able to do it!)

As discussed in class, public goods are non-rival. If I "consume" some knowledge, then you too can consume it at the same time!

We have 2 students in a class. Everytime someone asks a question in class, each student gets a benefit of b.

(a) The marginal individual benefit (to each student) is b (as stated above). A student faces a cost

$$C(x) = \frac{x^2}{2}$$

if she asks x questions. (Assume questions are infinitely divisible - you can ask a fraction of a question.) Suppose student A asks x_A number of questions, and B asks x_B number of questions. How many questions should each student ask if she/he just cares about her/his own benefit from asking a question? What is the total number of questions asked in class?

Answer:

Each student sets marginal individual benefit equal to marginal individual cost:

$$b = x_A$$
$$b = x_B$$

This implies $x_A + x_B = 2b$

(b) The total social benefit (to the class as a whole) from 1 question (from either student) is $2b$. Do you agree? What is the total social benefit from a total of x number of questions, where $x = x_A + x_B$.

Answer:

The total social benefit is $2bx$.

(c) What is the total (social) benefit <u>net of total costs</u> if A asked x_A questions and B asked x_B questions?

Answer:

$$SB(x_A, x_B) = 2b(x_A + x_B) - \frac{x_A^2}{2} - \frac{x_B^2}{2}$$

(d) What is the socially optimal number of questions? You'll have to maximize the expression in part (c) by jointly choosing x_A and x_B. (If it helps, the mathematical structure of this problem is similar to a competitive firm with 2 plants selling in 1 market - the last question in PS 3.)

Answer:

The FOCs are

$$2b = x_A^s$$
$$2b = x_B^s$$

(e) Is there a different between the total number of questions asked (if the students behaved selfishly) and the socially optimal number of questions? Why?

Answer:

The socially optimal number of questions is

$$x_A^s + x_B^s = 4b$$

B. Elasticity of demand

Suppose inverse market demand is given by

$$p(y) = Ay^{\frac{1}{\epsilon}}$$

where $\epsilon < 0$.

(a) Show that the price elasticity of demand is constant.

Answer:

$$\frac{dp}{dy} = \frac{1}{\epsilon}Ay^{\frac{1}{\epsilon}-1}$$

Therefore, elasticity is

$$\frac{dy}{dp}.\frac{p}{y} = \frac{1}{\frac{1}{\epsilon}Ay^{\frac{1}{\epsilon}-1}}.\frac{p}{y} = \frac{1}{\frac{1}{\epsilon}Ay^{\frac{1}{\epsilon}-1}}.\frac{Ay^{\frac{1}{\epsilon}}}{y} = \epsilon$$

(b) Suppose $\epsilon = -1$. Show that the total revenue for this demand function is a constant as well. Plot this demand curve for $A = 1$. What happens to the graph as A increases?

Answer:

$$TR = p(y)y = Ay^{\frac{1}{\epsilon}+1}$$

For $\epsilon = -1$, $TR = A$.

(c) What is marginal revenue for $\epsilon = -1$? What will a profit maximizing monopolist do when faced with this demand curve? Think intuitively: suppose its producing some output level y - Could it maximize profits by reducing or increasing output from this level?

Answer:

The total revenue is always the same. So profits are increased by decreasing output, as that decreases total costs. The monopolist would maximize profits by $y \to 0$.

(d) Suppose $\epsilon = -2$. Find the profit maximizing price and quantity for a monopolist with

$$C(y) = y^2 + y$$

Answer: Either by setting MC = MR, or by using the market power definition alongwith the demand curve (like I attempted in the lecture), we get

$$4y^{\frac{3}{2}} + 2y^{\frac{1}{2}} = A,$$

which we can write as

$$4x^3 + 2x = A,$$

which is a cubic, with $x = y^{\frac{1}{2}}$

I suggested in an email that you use the value of $A = 1.5$. This gives $x^* = 0.5$ as the only real-valued solution to this equation. This implies $y^* = 0.25$.

The associated price (from the demand curve) is $p^* = 3$.

C. Duopoly with different costs

Consider a Cournot duopoly. Market demand is given by

$$p(y_1, y_2) = a - y_1 - y_2$$

Firm 1 has marginal cost c_1 and firm 2 has marginal cost c_2, $c_1 < c_2$.

(a) Find the best response functions for both firms.

Answer:

$$y_1^*(y_2) = \frac{a - y_2 - c_1}{2}$$
$$y_2^*(y_1) = \frac{a - y_1 - c_2}{2}$$

(b) Find Nash equilibrium outputs. Assume $c_2 = 5c_1$.

Answer:

Solving simultaneously after substitution:

$$y_1^C = \frac{a}{3} + c_1$$
$$y_2^C = \frac{a}{3} - 3c_1$$

(c) Find equilibrium price and profits for both the firms.

Answer:

Equilibrium price is

$$p(y_1^C, y_2^C) = a - \left(\frac{a}{3} + c_1\right) - \left(\frac{a}{3} - 3c_1\right)$$
$$= \frac{a}{3} + 2c_1$$

Profit for firm 1 is

$$\pi_1^C = \left(\frac{a}{3} + c_1\right)^2$$

and for firm 2 is

$$\pi_2^C = \left(\frac{a}{3} - 3c_1\right)^2$$

(d) Suppose firm 2 moves before firm 1. Find the equilibrium using backward induction.

Answer:

By backward induction, plug in firm 1's best response into firm 2's optimization problem and solve for firm 2's equilibrium actions:

$$\max_{y_2} \pi_2(y_2) = \max_{y_2} y_2 \left(a - \left[\frac{a - y_2 - c_1}{2}\right] - y_2 - 5c_1\right)$$

which gives (from solving FOC)

$$y_2^S = \frac{a - 9c_1}{2}$$

Plugging this in the best response for firm 1, we have

$$y_1^S = \frac{a + 7c_1}{4}$$

Note: There is a fine point in this question. If $9c_1 > a$, then firm 2 will not produce anything. Firm 1 will be a monopoly and produce $\frac{a-c_1}{2}$. If we want to be proper about this, we should do the analysis for both cases. Note that this issue is present in all market power questions, even for monopolies. In this question, if $9c_1 > a$, then even in Cournot, firm 2 will not enter the market. For convenience, lets just make the assumption that $9c_1 < a$.

The equilibrium price is

$$\begin{aligned} p(y_1^S, y_2^S) &= a - \left(\frac{a + 7c_1}{4}\right) - \left(\frac{a - 9c_1}{2}\right) \\ &= \frac{a + 11c_1}{4} \end{aligned}$$

Profits are

$$\begin{aligned} \pi_1^S &= \left(\frac{a + 11c_1}{4} - c_1\right)\left(\frac{a + 7c_1}{4}\right) \\ &= \frac{a^2 + 49c_1^2 + 14ac_1}{16} \\ &= \left(\frac{a + 7c_1}{4}\right)^2 \\ \pi_2^S &= \left(\frac{a + 11c_1}{4} - 5c_1\right)\left(\frac{a - 9c_1}{2}\right) \\ &= \left(\frac{a - 9c_1}{2\sqrt{2}}\right)^2 \end{aligned}$$

(e) Compare profits of the two firms in the Stackelberg model from part (d) to the outcomes under simultaneous moves game.

Answer:

Comparing profits of firm 1: Is $\pi_1^S < \pi_1^C$? Yes, because

$$\begin{aligned}\pi_1^S < \pi_1^C &\iff \left(\frac{a+7c_1}{4}\right)^2 < \left(\frac{a}{3}+c_1\right)^2 \\ &\iff \frac{a+7c_1}{4} < \frac{a+3c_1}{3} \\ &\iff a > 9c_1\end{aligned}$$

which is true by assumption.

Comparing profits of firm 2: Is $\pi_2^S > \pi_2^C$? Yes, because

$$\begin{aligned}\pi_2^S > \pi_2^C &\iff \left(\frac{a-9c_1}{2\sqrt{2}}\right)^2 > \left(\frac{a-9c_1}{3}\right)^2 \\ &\iff \frac{a-9c_1}{2\sqrt{2}} > \frac{a-9c_1}{3} \\ &\iff 2\sqrt{2} < 3,\end{aligned}$$

which is always true.