

1 Oppgave 9

$$f(t) = \left\{ 0 \text{ if } 0 \leq t < 1, 1 \text{ if } 1 \leq t \right\}$$

$$L\{f(t)\} \Rightarrow f(s)$$

$$L\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$$

$$L\{0\} = \int_0^{\infty} e^{-st} 0 dt = 0$$

$$L\{1\} = \int_0^{\infty} e^{-st} 1 dt$$

$$\lim_{A \rightarrow \infty} \int_0^A e^{-st} dt$$

$$\lim_{A \rightarrow \infty} \left[-\frac{1}{s} e^{-st} \right]_0^A = \lim_{A \rightarrow \infty} \left(-\frac{1}{s} e^{-sA} - \left(-\frac{1}{s} \right) \right)$$

$$\lim_{A \rightarrow \infty} \left[-\frac{1}{s} e^{-st} + \frac{1}{s} \right] = \underline{\underline{\frac{1}{s}}}$$

2 Oppgave 1

$$f(t) = 3t^2 + 2t + 1, t \geq 0$$

$$3L\{t^2\} + 2Lt + L1$$

$$L\{t^n\} = \frac{n!}{s^{n+1}}$$

$$L\{t^2\} = \frac{2}{s^3}$$

$$L\{t\} = \frac{1}{s^2}$$

$$L\{1\} = \frac{1}{s}$$

$$= 3 \cdot \frac{2}{s^3} + 2 \cdot \frac{1}{s^2} + \frac{1}{s} = \underline{\underline{\frac{6}{s^3} + \frac{2}{s^2} + \frac{1}{s}}}$$

3 Oppgave 13

$$F(s) = \frac{3}{s} + \frac{24}{s^2}$$

$$L^{-1}\left\{\frac{3}{s} + \frac{24}{s^2}\right\} = L^{-1}\left\{\frac{3}{s}\right\} + L^{-1}\left\{\frac{24}{s^2}\right\}$$

$$L^{-1}\left\{\frac{1}{s}\right\} = 1$$

$$3L^{-1}\left\{\frac{1}{s}\right\} = \underline{3}$$

$$L^{-1}\left\{\frac{24}{s^2}\right\} = 24L^{-1}\left\{\frac{1}{s^2}\right\}$$

$$L^{-1}\left\{\frac{n!}{s^{n+1}}\right\} = t^n$$

$$24L^{-1}\left\{\frac{1}{s^2}\right\} = \underline{24t}$$

$$L^{-1}\left\{\frac{3}{s} + \frac{24}{s^2}\right\} = \underline{\underline{3 + 24t}}$$

4 Oppave 23

$$\begin{aligned}
f(t) &= \sin(t - 2\pi)h(t - 2\pi) \\
a(b - c) &= ab - ac \\
a &= \sin(t - 2\pi)h, \quad b = t, \quad c = 2\pi \\
ht \sin(t - 2\pi) - 2\pi h \sin(t - 2\pi) \\
L\{ht \sin(t - 2\pi) - 2\pi h \sin(t - 2\pi)\} \\
L\{a \cdot f(t) + b \cdot g(t)\} &= a \cdot L\{f(t)\} + b \cdot L\{g(t)\} \\
&= hL\{t \sin(t - 2\pi)\} - 2\pi hL\{\sin(t - 2\pi)\} \\
L\{f(t)\} &= F(s), \quad L\{t^k f(t)\} = (-1)^k \frac{d}{ds^k}(F(s)) \\
L\{\sin(at - b)\} &= \frac{s \cdot \sin(b) + a \cdot \cos(b)}{s^2 + a^1} \\
L\{\sin(t - 2\pi)\} &= \frac{\sin(-2\pi)s + 1 \cdot \cos(-2\pi)}{s^2 + 1^2} = \frac{1}{s^1 + 1} \\
\frac{d}{ds}\left(\frac{1}{s^2 + 1}\right) &= \frac{d}{ds}((s^2 + 1)^{-1}) \\
\frac{df(u)}{dx} &= \frac{df}{du} \cdot \frac{du}{dx}, \quad f = u^{-1}, \quad u = (s^2 + 1) \\
\frac{d}{du}(u^{-1}) \cdot \frac{d}{ds}(s^2 + 1) \\
\frac{d}{du}(u^{-1}) &= -\frac{1}{u^2} \\
\frac{d}{ds}(s^2 + 1) &= (f \pm g)' = \underline{2s} \\
\left(-\frac{1}{u^2}\right) \cdot 2s &= \left(-\frac{1}{(s^2 + 1)^2}\right) \cdot 2s = -\frac{2s}{(s^2 + 1)^2} \\
(-1) \cdot \left(-\frac{2s}{(s^2 + 1)^2}\right) &= \underline{\frac{2s}{(s^2 + 1)^2}} \\
h \frac{2s}{(s^2 + 1)} - 2\pi h L\{\sin(t - 2\pi)\} &= \underline{\underline{\frac{2hs}{(s^2 + 1)^2} - \frac{2\pi h}{s^2 + 1}}}
\end{aligned}$$