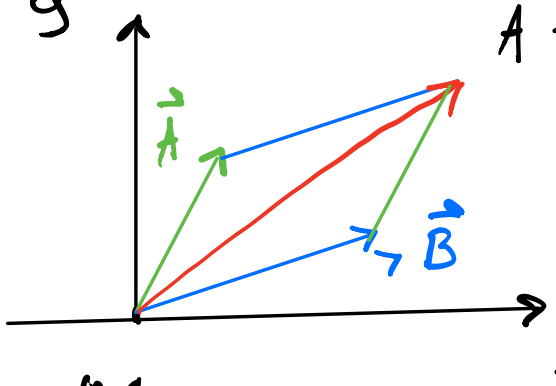
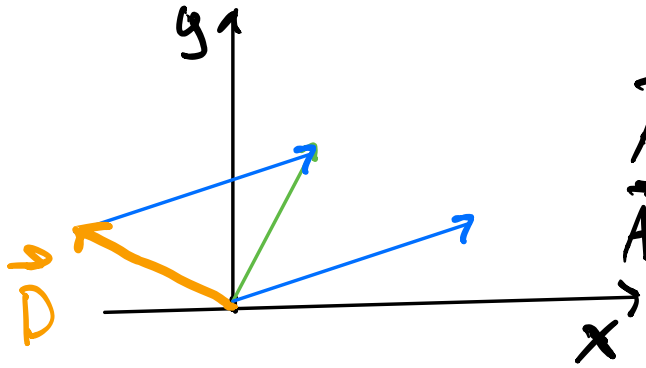
Bevegelse i 2D og 3D

Vektor



$\vec{A} + \vec{B} = \vec{C}$



$\vec{A} - \vec{B} = \vec{D}$

$\vec{A} + (-\vec{B})$

$$\vec{A} = (A_x, A_y)$$

$$\vec{B} = (B_x, B_y)$$

$$\vec{A} + \vec{B} = (A_x + B_x, A_y + B_y)$$

$$\vec{A} - \vec{B} = (A_x - B_x, A_y - B_y)$$

$$\vec{A} = (1, 2, 0) = \vec{i} + 2\vec{j}$$

$$\vec{B} = (3, 2, 1) = 3\vec{i} + 2\vec{j} + \vec{k}$$

$$\vec{A} + \vec{B} = (\vec{i} + 2\vec{j}) + (3\vec{i} + 2\vec{j} + \vec{k})$$

$$= 4\vec{i} + 4\vec{j} + \vec{k} = (4, 4, 1)$$

Python:

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a = array([1, 2, 0])
b = array([3, 1, 2])
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$a + b = [4, 3, 2]$

$a - b = [-2, 1, -2]$

$a \cdot b$ - dot(a, b) $= 1 \cdot 3 + 2 \cdot 1 + 0 \cdot 2 = 5$

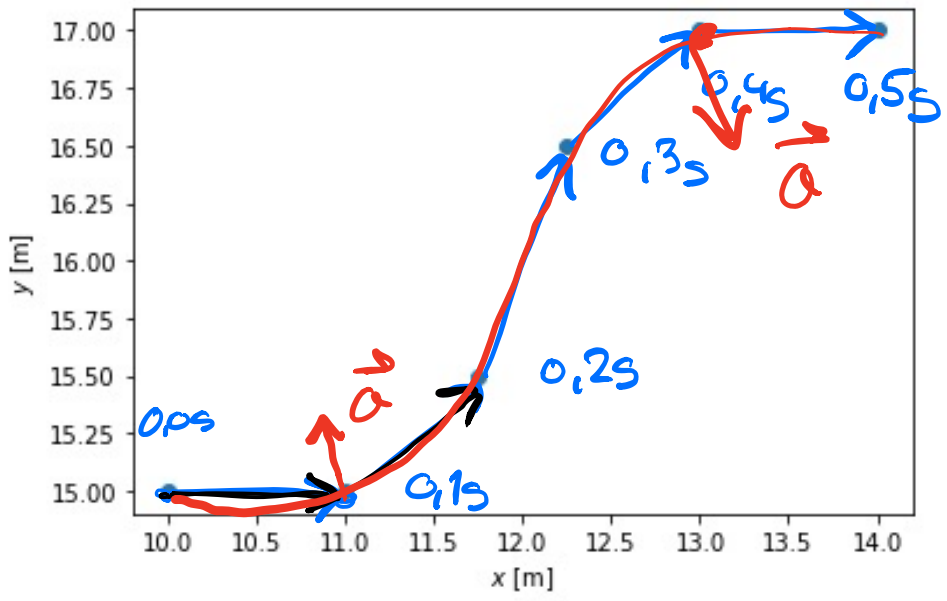
Lengden til $\vec{a}$: $|\vec{a}| = \sqrt{1^2 + 2^2 + 0^2}$

$$|\vec{a}| = \sqrt{5}$$

Oppg 6.8

Svake som flyr

t	0,0	0,1	0,2	0,3	0,4	0,5	[s]
x	10,00	11,00	11,75	12,25	13,00	14,00	[m]
y	15,00	15,00	15,50	16,50	17,00	17,00	[m]



$$\vec{v} = \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t}$$

$$\vec{r}(0) = 10{,}00\,m\,\vec{i} + 15{,}00\,m\,\vec{j}$$

$$\vec{r}(0{,}1s) = 11{,}00\,m\,\vec{i} + 15{,}00\,m\,\vec{j}$$

$$\vec{r}(0{,}1s) - \vec{r}(0) = \Delta\vec{r} = 1{,}00\,m\,\vec{i}$$

$$\Delta t = 0{,}1s$$

$$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{1{,}00\,m\,\vec{i}}{0{,}1s}$$

$$\vec{v} = 10\,m/s\,\vec{i}$$

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{r}(t) \quad , \quad \vec{v} = \frac{d\vec{r}}{dt} \quad , \quad \vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{r}(t) = \left(\cos t\,\vec{i} + \sin t\,\vec{j} + t\cdot\vec{k}\right) m$$

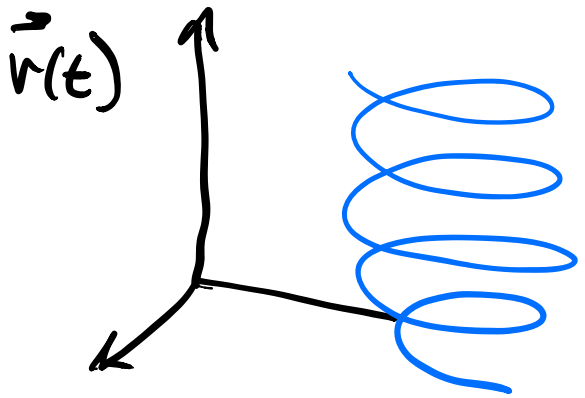
Finn hastighet og akselerasjon:

$$\frac{d\vec{r}}{dt} = \vec{v} = \underline{\underline{\left(-\sin t\,\vec{i} + \cos t\,\vec{j} + \vec{k}\right) m/s}}$$

$$\frac{d\vec{v}}{dt} = \vec{a} = \left(-\cos t\,\vec{i} - \sin t\,\vec{j}\right) m/s^2$$

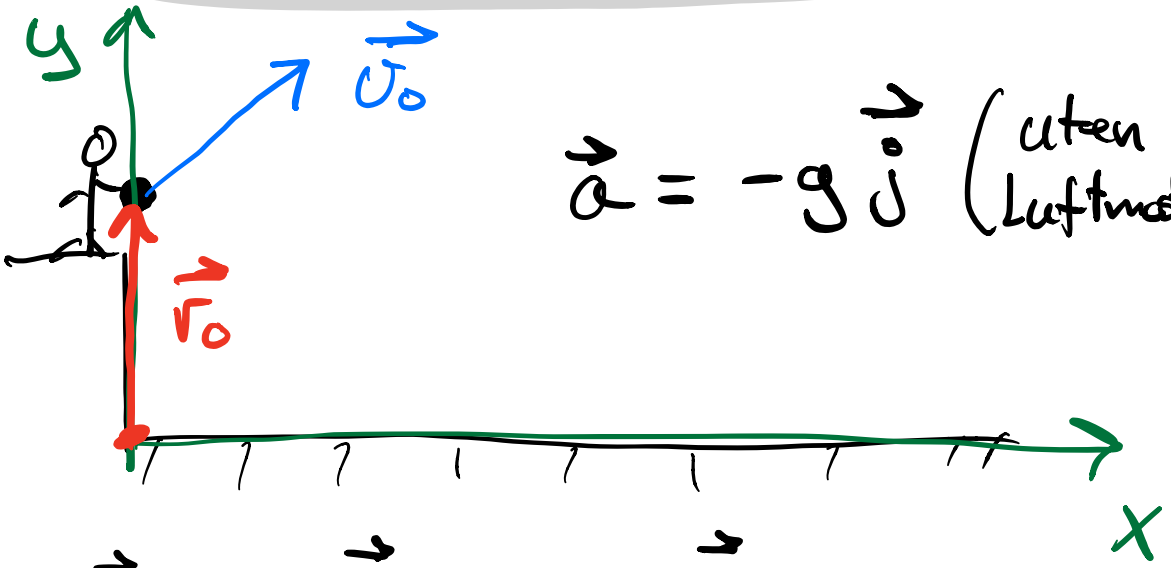
$$|\vec{a}| = \sqrt{(-\cos t)^2 + (-\sin t)^2 + 0^2}$$

$$|\vec{a}| = 1\ m/s^2$$



Graveyard spiral.

Eksempel Kast i 2D



$\vec{a} = -g\vec{j}$ (uten Luftmotstand)

$\vec{a} = -g\vec{j} = -9{,}8\,\vec{j}$ m/s²

$\vec{r}_0 = 2{,}0\,\vec{j}$ m

$\vec{v}_0 = 20\,\vec{i} + 15\,\vec{j}$ m/s

Vet at y-komponenten til $\vec{r}$ er 0 når ballen treffer bakken.

$\vec{a}$, $\vec{r}_0$, $\vec{v}_0$

Dekomponering

x-retning $\vec{i}$	y-retning $\vec{j}$
$a_x = 0$	$a_y = -9{,}8\ m/s^2 = -g$
$v_{0x} = 20\ m/s$	$v_{0y} = 15\ m/s$
$x_0 = 0$	$y_0 = 2{,}0\ m$
$v_x = \int a_x\,dt$	$v_y = \int a_y\,dt$
$v_x = v_{0x} = 20\ m/s$	$v_y = \int -g\,dt$
$x = \int v_x\,dt$	$v_y = -gt + v_{0y}$
$x = v_{0x} \cdot t + x_0$	$y = \int v_y\,dt$
$x = v_{0x} \cdot t$	$y = \int -gt + v_{0y}\,dt$
	$y = -\frac{1}{2}gt^2 + v_{0y}t + y_0$

$$\vec{r}(t) = (x, y)$$

$$\vec{v}(t) = (v_x, v_y)$$

Treffer bahnen wo $y = 0$

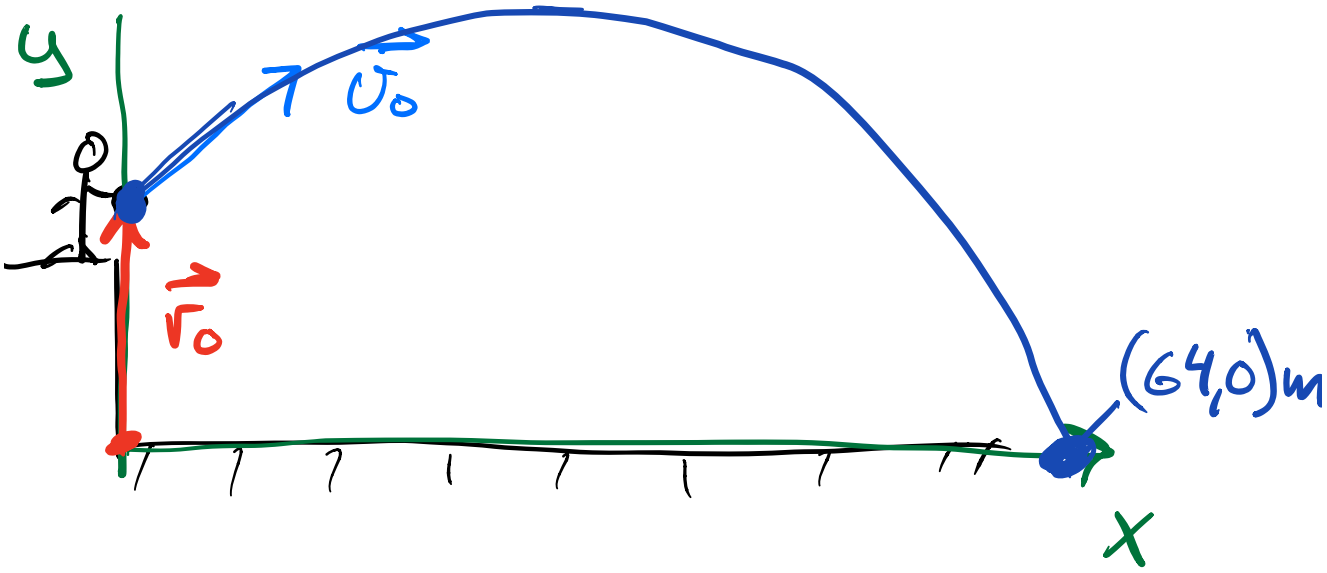
$$-\frac{1}{2} g t^2 + v_{0y} t + y_0 = 0$$

$$-\frac{1}{2} \cdot 9{,}8\,\text{m/s}^2 \cdot t^2 + 15\,\text{m/s} \cdot t + 20\,\text{m} = 0$$

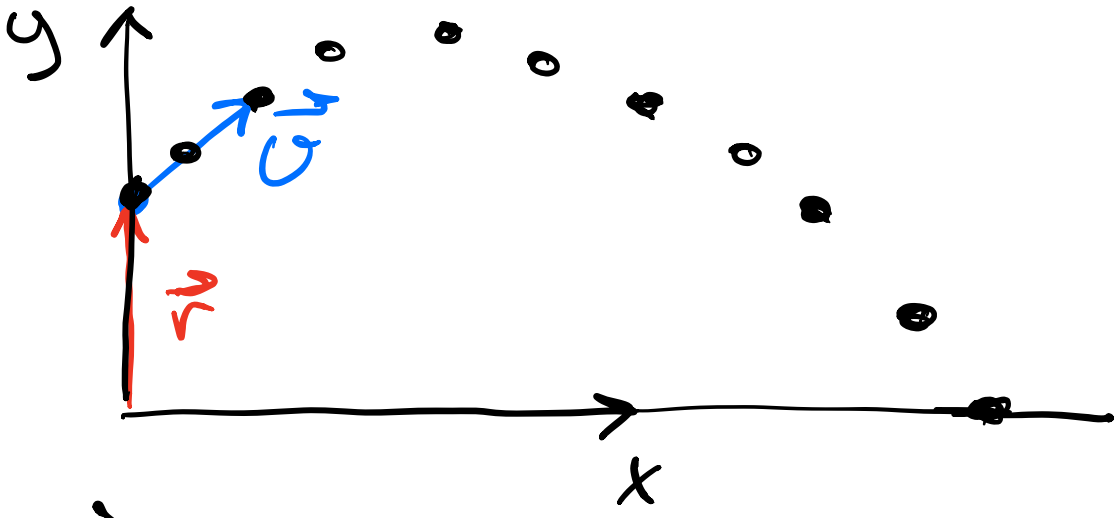
$$\boxed{t = 3{,}2\,\text{s} \quad \text{wo} \quad y = 0}$$

$$x(t) = v_{0x} \cdot t = 20\,\text{m/s} \cdot t$$

$$x(3{,}2\,\text{s}) = 20\,\text{m/s} \cdot 3{,}2\,\text{s} = \underline{64\,\text{m}}$$



Hvordan løse dette ved Euler-Cromers metode?



$\vec{r}(t)$

$$\vec{r}(t+\Delta t) = \vec{r}(t) + \vec{v}(t)\cdot\Delta t$$

$$\vec{v}(t+\Delta t) = \vec{v}(t) + \vec{a}\cdot\Delta t$$