Arbeid - Energi

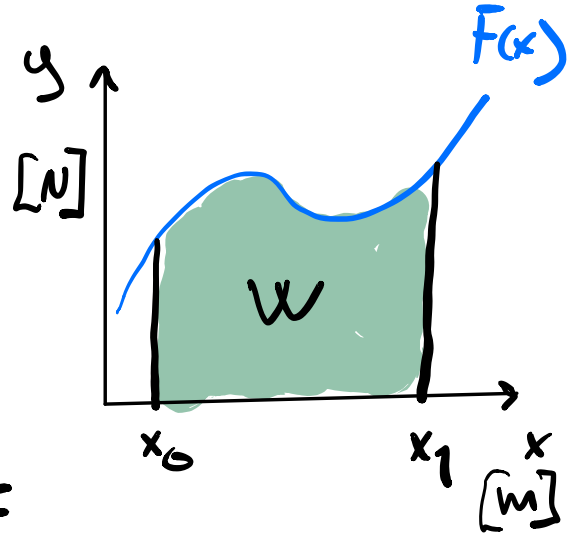
$$W = \int \vec{F} \cdot d\vec{r}$$

$W = \Delta E_K$ - forandring i Kinetisk Energi

$$\boxed{K = E_K}$$

Arbeid i 1D

$$W = \int_{x_0}^{x_1} F(x)\,dx$$



$$W = \phi(x_1) - \phi(x_0)$$

ϕ - Antiderivert av F

ϕ er et potensial.

$$W = \Delta E_K = K_1 - K_0$$

$$K_1 - K_0 = \Phi(x_1) - \Phi(x_0)$$

$$\underbrace{K_1 - \Phi(x_1)} = \underbrace{K_0 - \Phi(x_0)}$$ Konservet størrelse

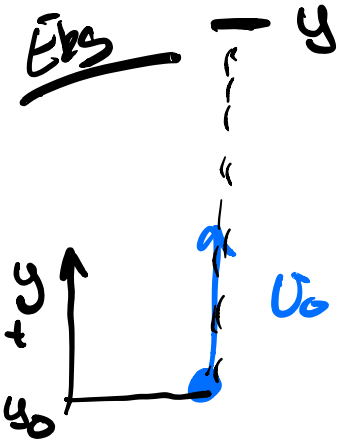
$$E_1 = E_0 = E$$

E - mekanisk energi er bevart.

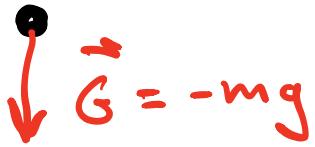
$$-\Phi(x_1) = U_1 \quad , \quad -\Phi(x_0) = U_0$$

$$K_1 + U_1 = K_0 + U_0 = E$$

Eks



Tidligere N.2.lov



$F = ma$

$$W = \int \vec{F} \cdot d\vec{r}$$

$$W = \int_{y_0}^{y_1} -mg\,dy = -mgy\Big|_{y_0}^{y_1}$$

$$W = -mgy_1 - (-mgy_0)$$

$$W = \Delta E_K = K_1 - K_0$$

$$K_1 - K_0 = -mgy_1 + mgy_0$$

$$K_1 + \underbrace{mgy_1}_{-\Phi_1 = U_1} = K_0 + \underbrace{mgy_0}_{-\Phi_0 = U_0}$$

$$\underbrace{\frac{1}{2}mv_1^2}_{=0} + mgy_1 = \frac{1}{2}mv_0^2 + \underbrace{mgy_0}_{=0}$$

$v_0 = 10\ m/s$ $\quad$ $v_1 = 0\ m/s$

$y_0 = 0\ m$ $\quad$ L Toppunkt.

$$mgy_1 = \frac{1}{2}mv_0^2$$

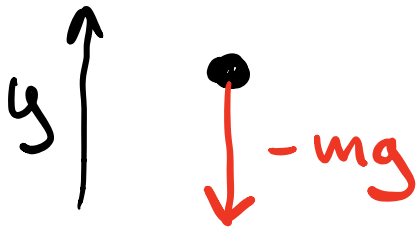
$$y_1 = \frac{v_0^2}{2g}$$

$$y_1 = \frac{(10\,m/s)^2}{2 \cdot 9{,}8\,m/s^2} = \underline{5{,}1\,m}$$

Potensial U (10)

$$F(x) = -\frac{dU}{dx}$$

Potensial i et tyngdefelt



$$F = -mg$$

$$U = mgy$$

$$F = -\frac{dU}{dy} = -mg$$

Potensial : fjær
(Hookes lov)

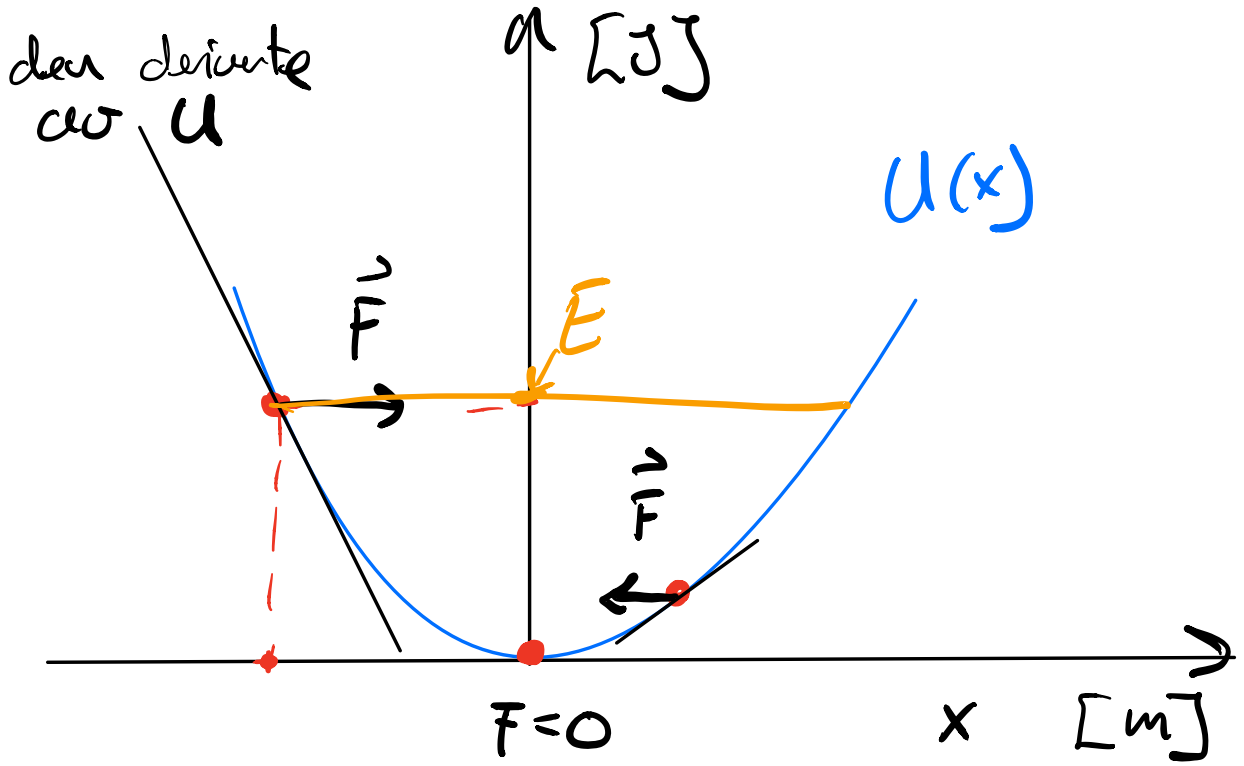
$$F = -kx$$

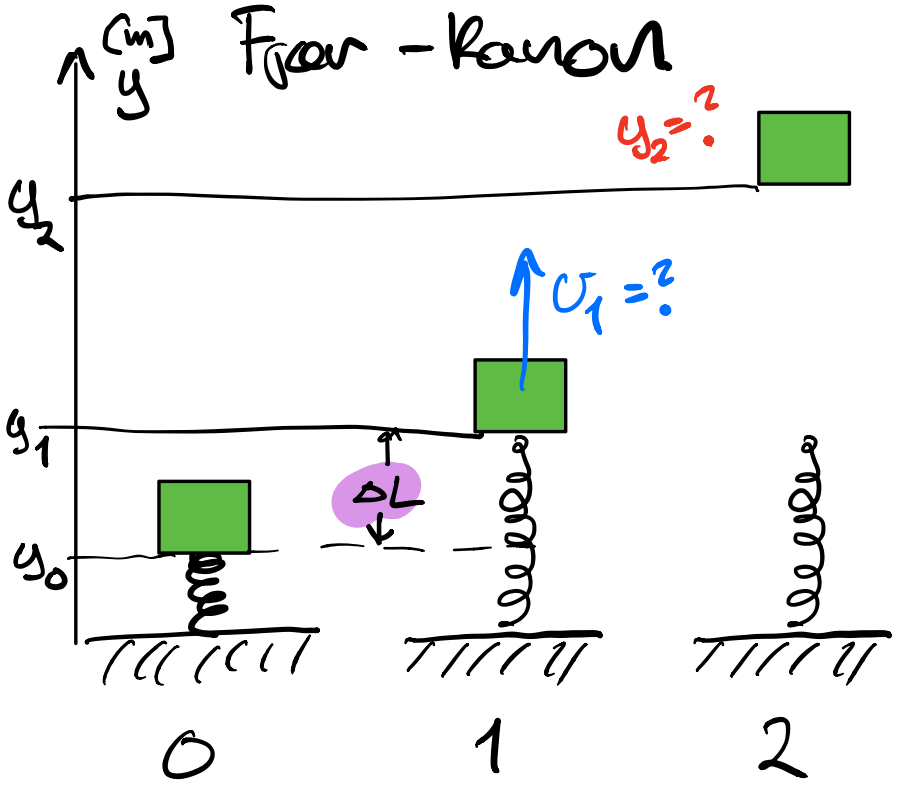
Finn U slik at $-\frac{dU}{dx} = -kx$

$$\int \frac{dU}{dx}\,dx = \int kx\,dx$$

$$U = \frac{1}{2}kx^2 + \underbrace{C}_{\text{velger } C=0}$$

$$\boxed{U(x) = \frac{1}{2}kx^2}$$





$k = 200\ N/m$

$m = 0{,}10\ kg$

$\Delta L = 0{,}10\ m$

a) Finn v_1

b) Finn y_2

$$U_F + U_G + K = \text{konstant}$$

$$U_F = \frac{1}{2}k(\Delta L)^2, \quad U_G = mgy, \quad K = \frac{1}{2}mv^2$$

(0): $U_0 = 0 \Rightarrow K_0 = 0$

$$U_F = \frac{1}{2}k(\Delta L)^2, \qquad y_1 - y_0 = \Delta L$$

$$U_{F_0} = \frac{1}{2}k(y_1 - y_0)^2$$

$$U_{G_0} = mgy_0 \qquad \text{Velger } y_0 = 0$$

(1) Klossen forlater fjæren $\Rightarrow U_{F_1} = 0$

$$U_{G_1} = mgy_1$$

$$K_1 = \frac{1}{2}mv_1^2$$

$$U_{F_0} + U_{G_0} + K_0 = U_{F_1} + U_{G_1} + K_1$$

$U_{G_0} = 0$, $K_0 = 0$, $U_{F_1} = 0$

$$U_{F_0} = U_{G_1} + K_1$$

$$\frac{1}{2} k (y_1 - \underbrace{y_0}_{=0})^2 = mg y_1 + \frac{1}{2} m v_1^2$$

$$\frac{1}{2} k y_1^2 = mg y_1 + \frac{1}{2} m v_1^2$$

$$\frac{1}{2} k y_1^2 - mg y_1 = \frac{1}{2} m v_1^2 \quad | \cdot \frac{2}{m}$$

$$v_1 = \sqrt{\frac{k}{m} y_1^2 - 2 g y_1}$$

$$v_1 = \sqrt{\frac{200 N/m}{0{,}10 kg} \cdot (0{,}10 m)^2 - 2 \cdot 9{,}8 m/s \cdot 0{,}1 m}$$

$$\underline{\underline{v_1 = 4{,}2 m/s}}$$

b) $U_{F_0} + U_{G_0} + K_0 = U_{F_2} + U_{G_2} + K_2$

$U_{G_0} = 0 \quad K_0 = 0 \quad U_{F_2} = 0 \quad K_2 = 0$

$$U_{F_0} = U_{G_2}$$

$$\frac{1}{2} k (\Delta L)^2 = mg y_2$$

$$y_2 = \frac{k}{2mg} \cdot (\Delta L)^2$$

$$y_2 = \frac{200\,N/m}{2 \cdot 0{,}10\,kg \cdot 9{,}8\,m/s^2} \cdot (0{,}1\,m)^2$$

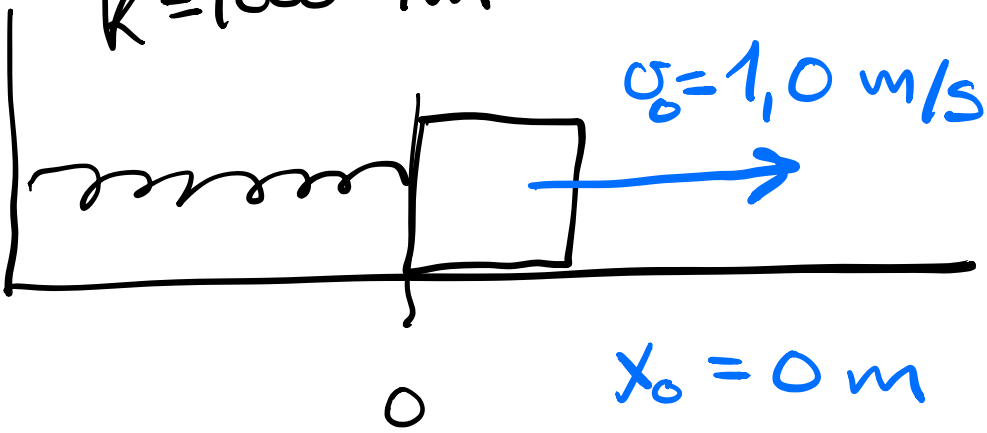
$$\underline{\underline{y_2 = 1{,}0\,m}}$$

Ikke-konservative krefter

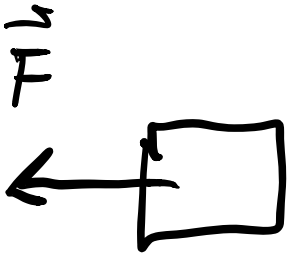
Friksjon – luftmotstand $F_D = Dv^2$

Bevaring av Energi
- numeriske metoder

$m = 0{,}10\ kg$



$x_0 = 0$ m



$F = -kx$

N. 2. lov

$-kx = ma$

$$a = -\frac{k}{m} \cdot x$$

Energi:

$K = \frac{1}{2} m v_0^2$

$K = \frac{1}{2} \cdot 0{,}1\,kg \cdot (1{,}0\,m/s)^2$

$K = 0{,}05\,Nm\ [J]$

$$U_F + K = 0{,}05\ J$$

Energien er bevart!