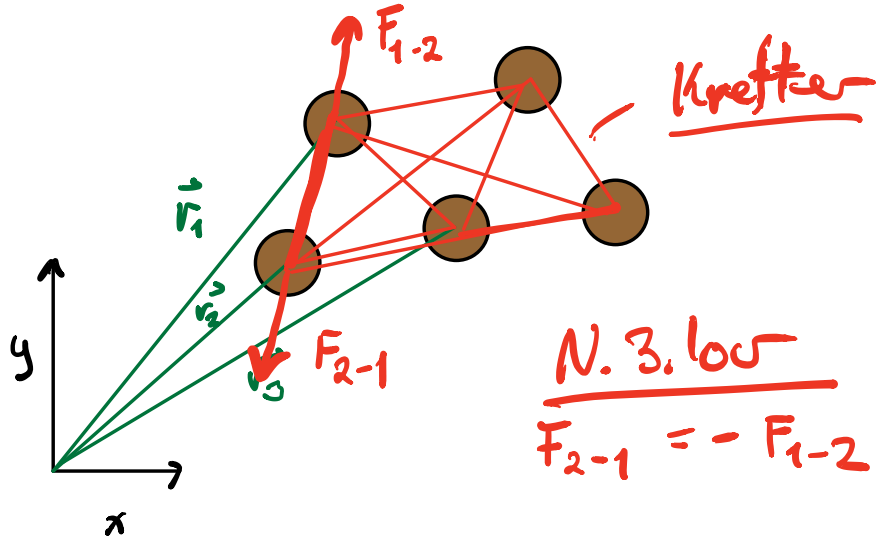
Flerpartikkelsystem

Kap 13



Summen av indre krefter er lik 0

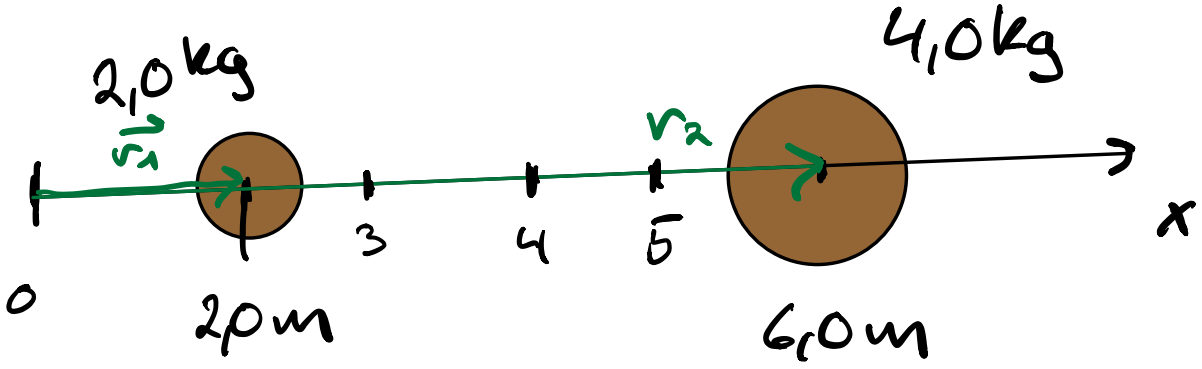
$$\sum \vec{F}_{int} = 0$$

Vi kan beskrive et flerpartikkelsystem ved å se på **Massesenteret** CM

$\vec{R}$ - posisjon til CM

$\vec{V}$ - Hastighet til CM

$\vec{A}$ - Akselerasjon til CM



$$\vec{R} = \frac{1}{M} \cdot \left(\vec{r}_1 \cdot m_1 + \vec{r}_2 \cdot m_2 + \vec{r}_3 \cdot m_3 \dots \right)$$

$$M = m_1 + m_2 + m_3 + \dots$$

$$\vec{R} = \frac{\sum \vec{r}_i m_i}{\sum m_i}$$

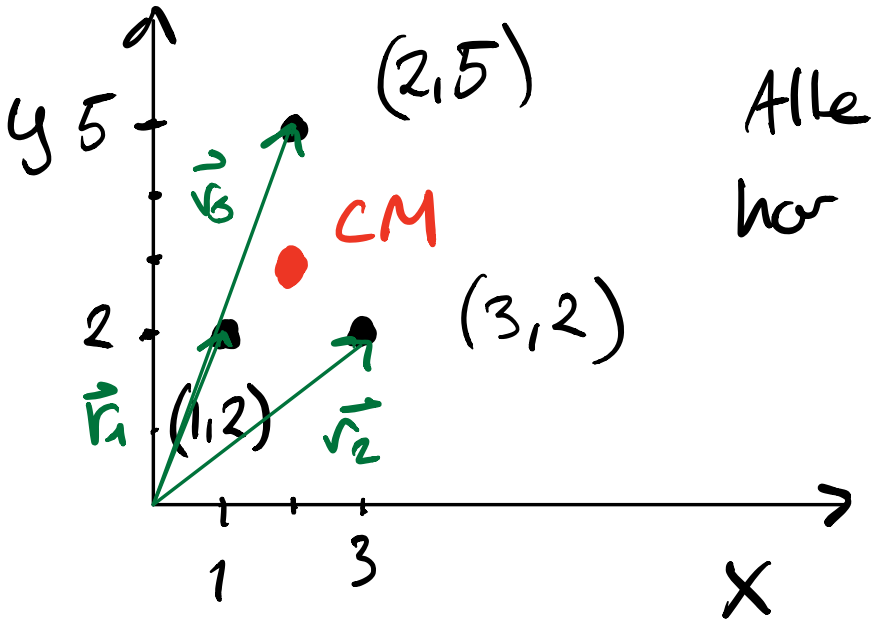
$$\vec{r}_1 = 2{,}0\ \text{m}\ \vec{i}\ , \quad m_1 = 2{,}0\ \text{kg}$$

$$\vec{r}_2 = 6{,}0\ \text{m}\ \vec{i}\ , \quad m_2 = 4{,}0\ \text{kg}$$

$$\vec{R} = \frac{2{,}0\ \text{m}\,\vec{i} \cdot 2{,}0\ \text{kg} + 6{,}0\ \text{m}\,\vec{i} \cdot 4{,}0\ \text{kg}}{2{,}0\ \text{kg} + 4{,}0\ \text{kg}}$$

$$\vec{R} = \frac{4{,}0\ \text{m kg}\,\vec{i} + 24\ \text{m kg}\,\vec{i}}{6{,}0\ \text{kg}} = \frac{28}{6}\ \text{m}\,\vec{i}$$

Massesenter i 2D



Alle partikler har masse m

$\vec{r}_1 = \vec{i} + 2\vec{j}$

$\vec{r}_2 = 3\vec{i} + 2\vec{j}$

$\vec{r}_3 = 2\vec{i} + 5\vec{j}$

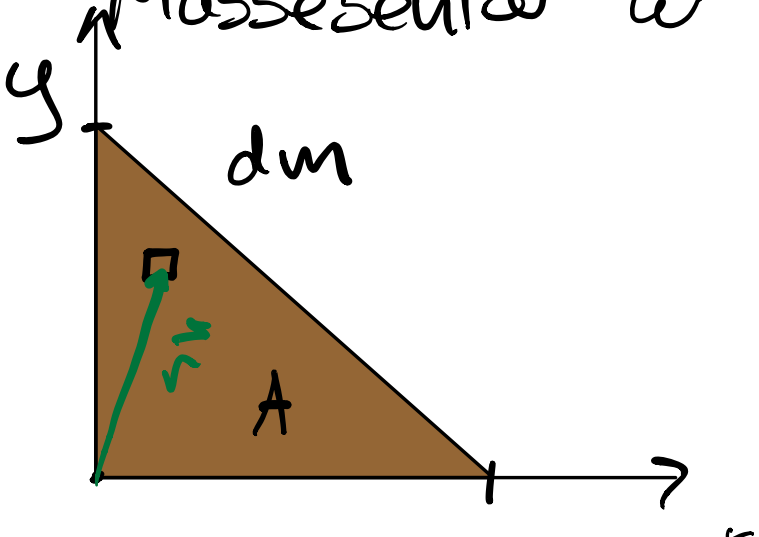
$m_1 = m_2 = m_3 = m$

$$\vec{R} = \frac{\sum \vec{r}_i \cdot m_i}{\sum m_i}$$

$$\vec{R} = \frac{(\vec{i} + 2\vec{j})\,m + (3\vec{i} + 2\vec{j})\,m + (2\vec{i} + 5\vec{j})\,m}{m + m + m}$$

$$\vec{R} = \frac{6m\vec{i} + 9m\vec{j}}{3m} = \underline{\underline{2\vec{i} + 3\vec{j}}}$$

Massesenter til kontinuerlige legemer



$$\vec{R} = \frac{\int \vec{r}\, dm}{\int dm}$$

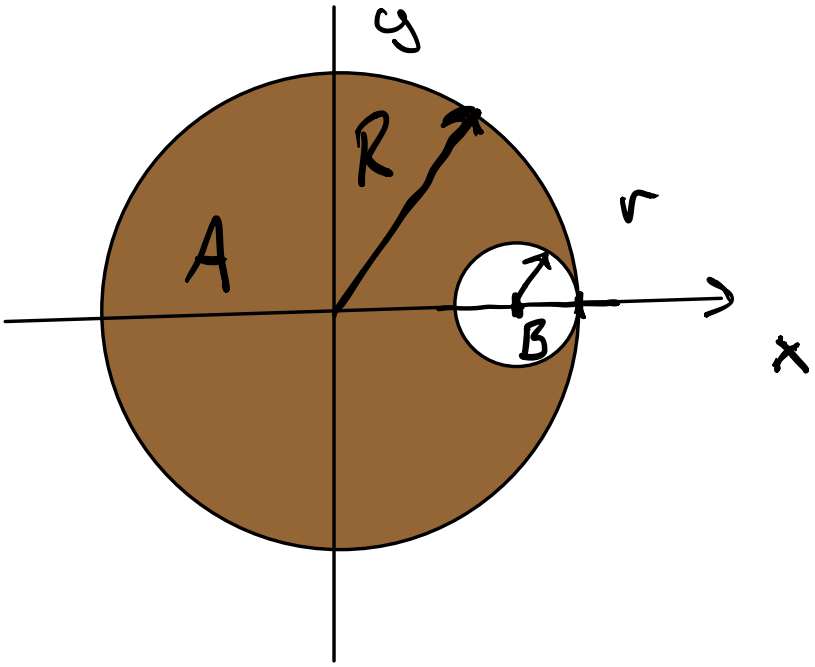
masse-tetthet

$$dm = \sigma dA$$

$$= \sigma dx dy$$

$$\vec{R} = \frac{\iint_A \vec{r}\, \sigma\, dx dy}{\iint_A \sigma dx dy}$$

Massesenter til flere legemer



$$\underbrace{M_{AB} \cdot \vec{R}}_{\text{massesenter full sirkel}} = \underbrace{M_A \vec{R}_A}_{\substack{\text{massesenter} \\ \text{Brun sirkel} \\ \text{(med hull)}}} + \underbrace{M_B \vec{R}_B}_{\substack{\text{Massesenter} \\ \text{Hullet.}}}$$

$\vec{R} = 0$

$$M_A \vec{R}_A = -M_B \vec{R}_B$$

$$\vec{R}_A = -\frac{M_B}{M_A} \vec{R}_B$$

$$\vec{R_B} = (R - r)\,\vec{i}$$

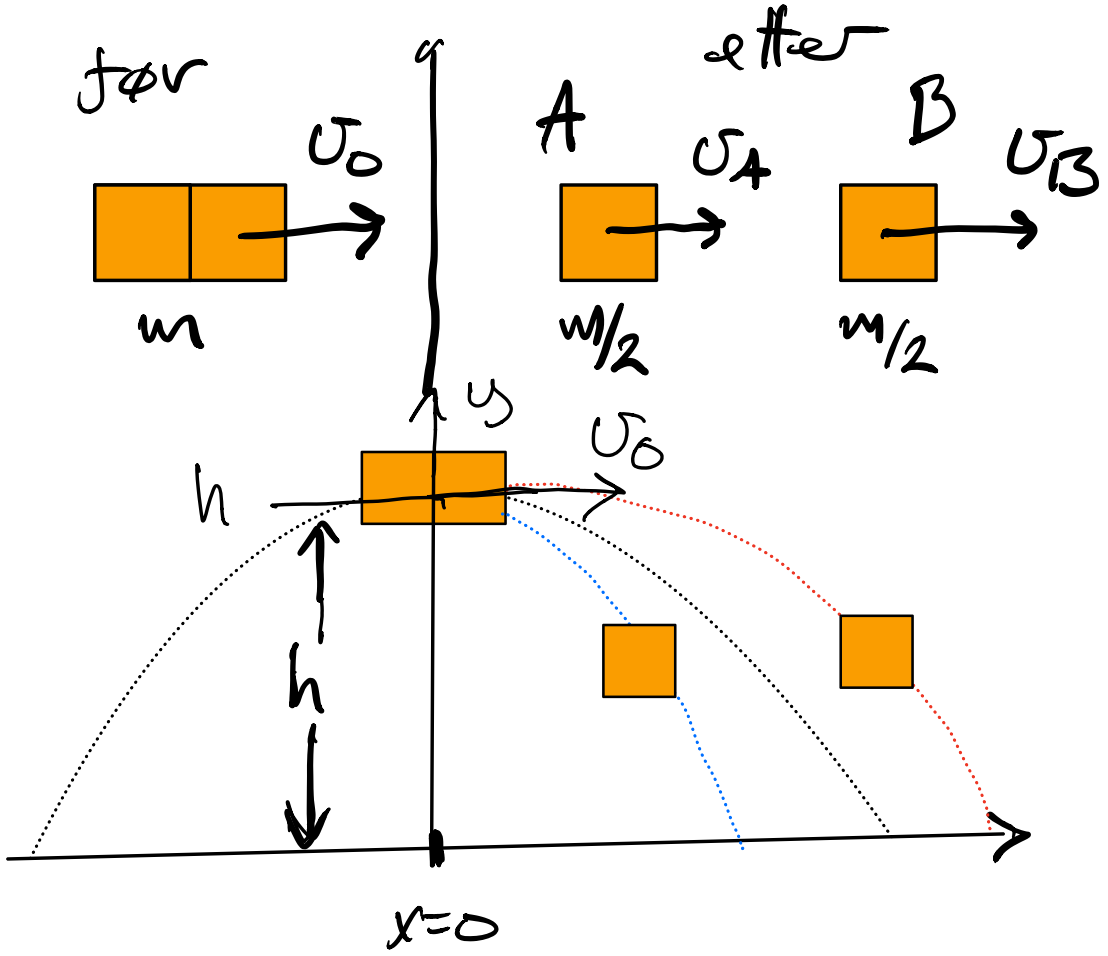
$$M_B = \sigma \cdot \pi r^2$$

Tetthet · Areal.

$$M_A = \sigma(\pi R^2 - \pi r^2)$$

$$\vec{R_A} = -\frac{\sigma \pi r^2}{\sigma \pi (R^2 - r^2)} \cdot (R - r)\,\vec{i}$$

Kast med Eksplosjon



Bevegelsesmengden er bevart.

før

$$\vec{p}_0 = m\vec{v}_0$$

$$\vec{p}_0 = \vec{p}_1$$

etter

$$\vec{p}_1 = m_A\vec{v}_A + m_B\vec{v}_B$$

$$\vec{p}_1 = \frac{m}{2}\vec{v}_A + \frac{m}{2}\vec{v}_B$$

$$m\vec{v}_0 = \frac{m}{2}\vec{v}_A + \frac{m}{2}\vec{v}_B \quad | \cdot \frac{1}{m}$$

$$\vec{v}_0 = \frac{1}{2}\vec{v}_A + \frac{1}{2}\vec{v}_B$$

$$\boxed{2\vec{v}_0 = \vec{v}_A + \vec{v}_B}$$

Beskriver bevegelsene til kloss A og kloss B.

A: $y_0 = h$ $\quad v_{0y} = 0$

$x_0 = 0$ $\quad v_{0x} = v_A$

A

N. 2. lov

$$-m_A g \vec{j} = m_A \vec{a}$$

$$\vec{a} = -g\vec{j}$$

$a_x = 0 \qquad a_y = -g$

$\therefore$ Litt regning gir:

$$x_A(t) = v_A \cdot t$$

$$y_A(t) = h - \frac{1}{2} g t^2$$

Tilsvarende for Kloss B

$$x_B(t) = v_B \cdot t$$

$$y_B(t) = h - \frac{1}{2} g t^2$$

Massesenter til Kloss A og B

$$X = \frac{x_A m_A + x_B \cdot m_B}{m_A + m_B}$$

(x-koordinat til CM)

$$x = \frac{v_A t \cdot \frac{m}{2} + v_B \cdot t \cdot \frac{m}{2}}{\frac{m}{2} + \frac{m}{2}}$$

$$x = \frac{1}{2}(v_A t + v_B t) = \frac{1}{2} \cdot t (v_A + v_B)$$

$$\boxed{2\vec{v}_0 = \vec{v}_A + \vec{v}_B} \quad = \frac{1}{2} t \cdot 2 v_0$$

$$\boxed{x = v_0 \cdot t}$$

$$\boxed{y = h - \frac{1}{2} g t^2}$$

$$\left(y = \frac{m_A y_A + m_B y_B}{m_A + m_B} \right)$$