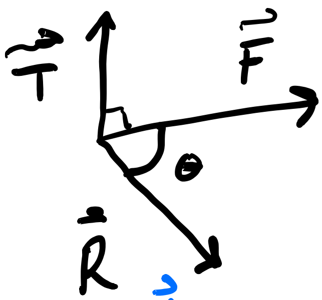
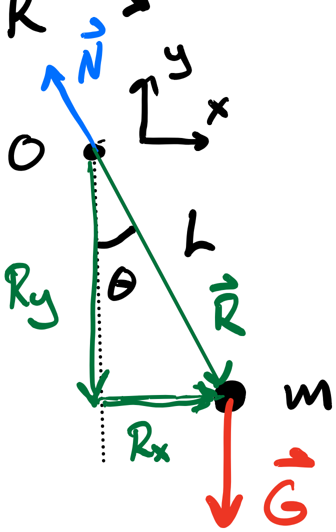
Kraftmoment

$$\vec{\tau} = \vec{R} \times \vec{F}$$



$$|\vec{\tau}| = |\vec{R}| \cdot |\vec{F}| \cdot \sin\theta$$



$$\vec{G} = -mg\,\vec{j}$$

$$\vec{R} = L \cdot \sin\theta\,\vec{i} - L\cos\theta\,\vec{j}$$

$$\vec{\tau} = \vec{R} \times \vec{G}$$

$$\vec{\tau} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ L\sin\theta & -L\cos\theta & 0 \\ 0 & -mg & 0 \end{vmatrix} = \underline{-mgL\sin\theta\,\vec{k}}$$

Bevegelsesmengde

$$\vec{p} = m\vec{v}$$

N. 2. lov: $$\vec{F} = \frac{d}{dt}\vec{p}$$

Kraftmoment $\vec{\tau}$

$$\vec{\tau} = \vec{R} \times \vec{F} = \vec{R} \times \frac{d}{dt}\vec{p}$$

$$\frac{d}{dt}(\vec{R} \times \vec{p}) = \underbrace{\underbrace{\frac{d\vec{R}}{dt} \times \vec{p}}_{\vec{v} \times m\vec{v}}}_{=0} + \underbrace{\vec{R} \times \frac{d\vec{p}}{dt}}_{\vec{\tau}}$$

$$\vec{\tau} = \frac{d}{dt}\underbrace{(\vec{R} \times \vec{p})}_{=\vec{l}}$$

$$\vec{\tau} = \frac{d}{dt}\vec{l} \,,$$

Spinn

$$\boxed{\vec{l} = \vec{R} \times \vec{p}}$$

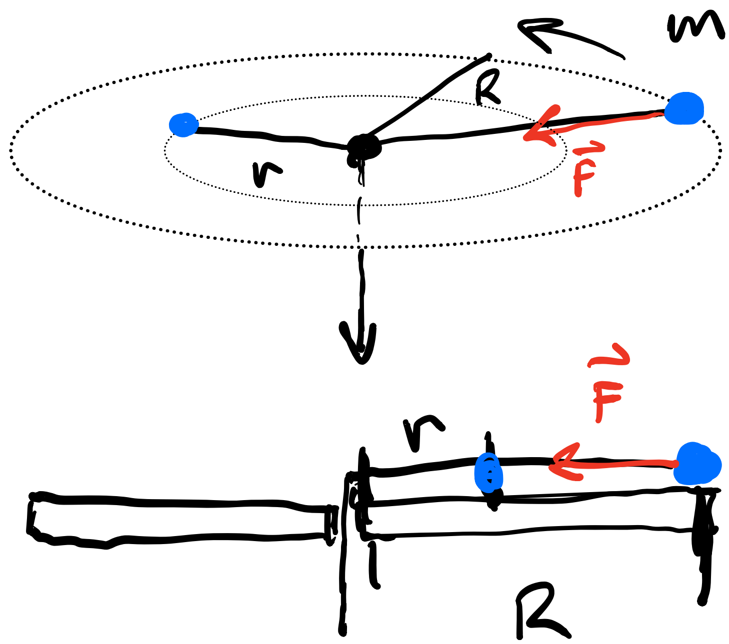
Kraftmoment = Endring i spinn

angulært moment

$$\vec{\tau} = 0 \Rightarrow \frac{d\vec{l}}{dt} = 0$$

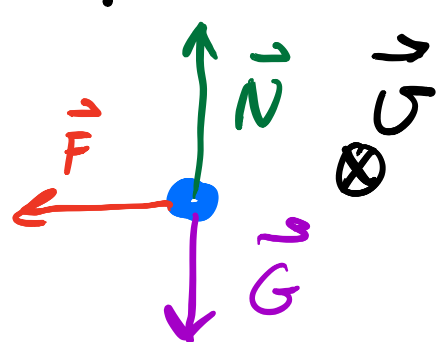
$\vec{l}$ = konstant. spinnet er bevart.

Eks



$\vec{v} = R \cdot \vec{\omega}$

Frilegemediagram



Hva er kraftmomentet?

$$\vec{\tau} = \vec{R} \times \vec{F} = 0$$

vinkel mellom $\vec{R}$ og $\vec{F}$ er $\underline{180^\circ}$

$$\vec{\tau} = \frac{d}{dt}\vec{l} = 0 \Rightarrow \underline{\vec{l} = \text{konstant}}$$

$$\vec{l} = \vec{R} \times \vec{p} = \text{konstant.}$$

$$\vec{R} \times m\vec{v} = \vec{R} \times mR\vec{\omega} = R^2 m\omega \vec{k}$$

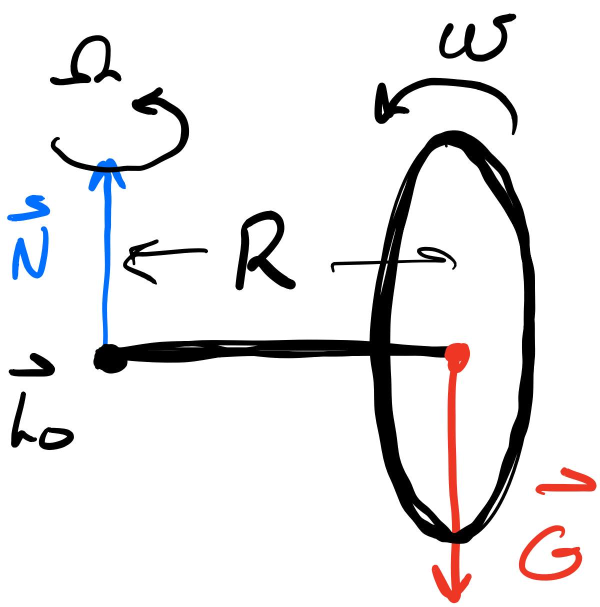
$$\vec{l} = R^2 m\omega \vec{k}$$

$$R^2 m\omega = \text{konstant}$$

$$R^2 \cancel{m} \omega_0 = r^2 \cancel{m} \omega_1$$

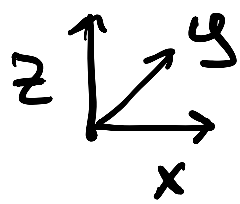
$$\boxed{\omega_1 = \frac{R^2}{r^2}\omega_0}$$

Gyroskop.



Spinn

$$\vec{L}_{cm} = I_{cm}\,\vec{\omega}$$



$$\vec{L}_o = \vec{L}_{cm} + \underbrace{\vec{L}_\Omega}$$

Anta $\vec{L}_\Omega \ll \vec{L}_{cm}$

$$\vec{L}_o \simeq \vec{L}_{cm}$$

Kraftmoment på gyroskopet

$$\vec{\tau} = \vec{R} \times \vec{G} \quad \Bigg| \quad \begin{aligned} \vec{R} &= R\,\hat{e} \\ \vec{G} &= -mg\,\vec{k} \end{aligned}$$

$$\vec{R} \times \vec{G} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ R & 0 & 0 \\ 0 & 0 & -mg \end{vmatrix} = \underline{Rmg\,\vec{j}}$$

$$\underline{\underline{\vec{\tau} = Rmg\,\vec{j}}}$$

Del 2

Starter

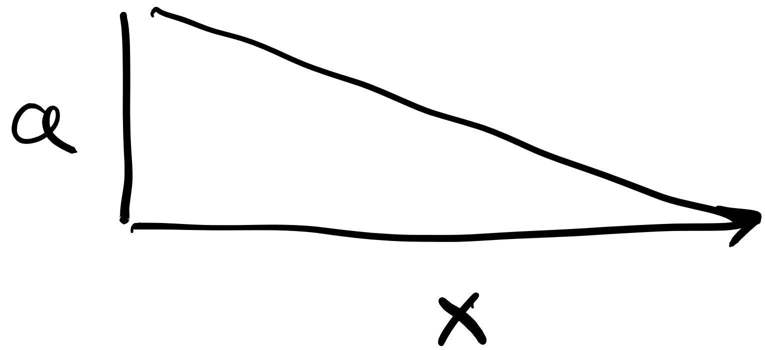
$F = -kx$

N.2.lov $\quad \Sigma F = ma$

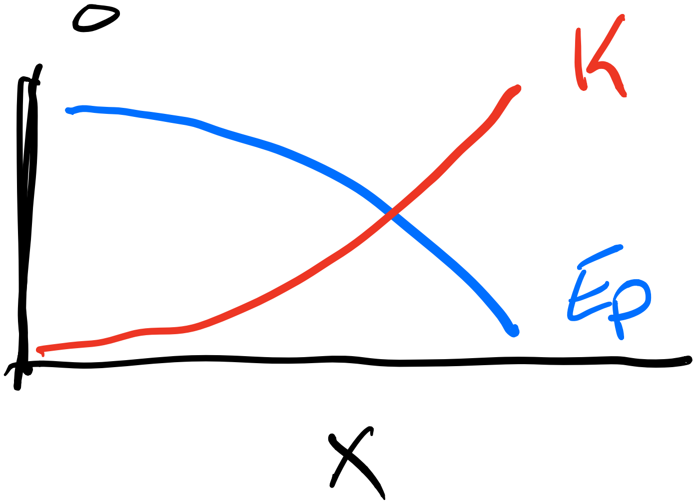
$$F = ma$$

$$-kx = ma$$

$$a = -\frac{k}{m} x$$



$$W = \int_0^x F\,dx = \int -kx\,dx = \underline{\tfrac{1}{2}kx^2}$$

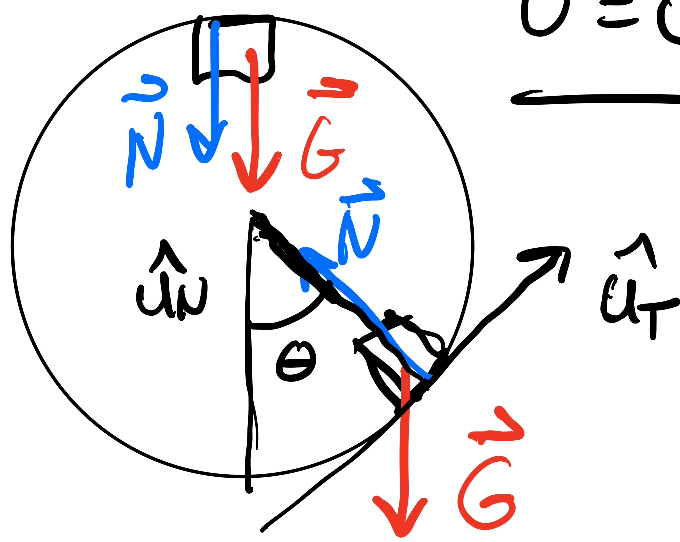


<u>Energibevaring:</u>

$$K_0 + U_0 = K_1 + U_1$$

Loop

$v = \sqrt{Rg}$



$$\Sigma F = ma$$

$$a = \frac{v^2}{R}$$

$$\vec{N} + \vec{G} = m\vec{a}$$

Velge koordinatsystem $\hat{u}_T$, $\hat{u}_N$

$$\vec{G} = -mg\sin\theta\,\hat{u}_N + mg\cos\theta\,\hat{u}_N$$

N.2.lov $\hat{u}_T$: $-mg\sin\theta = a_T$

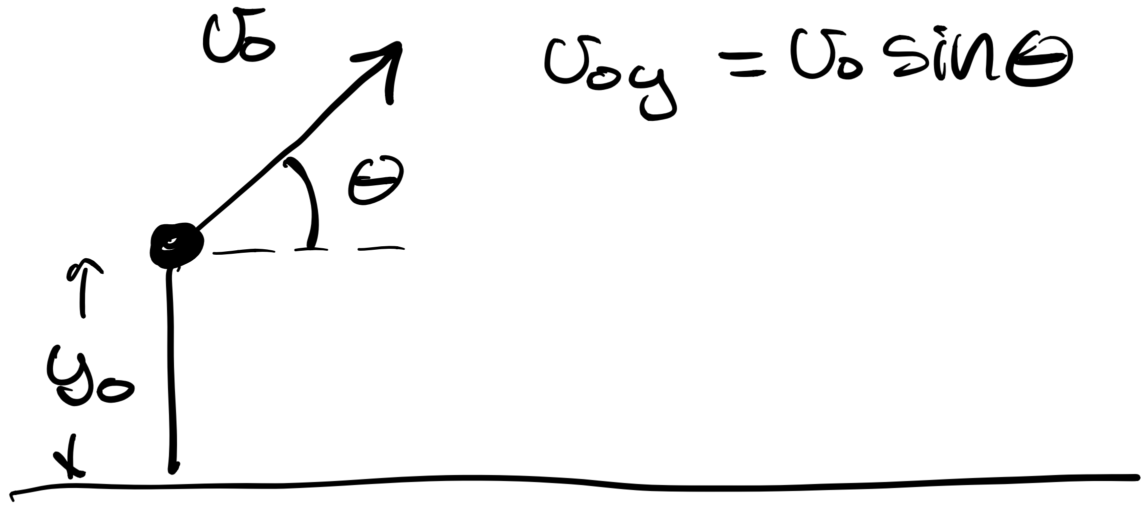
$\hat{u}_N$: $-mg\cos\theta + N = a_N$

$$a_N = \frac{v^2}{R}$$

Hopp

$$v_{0x} = v_0 \cos\theta$$

$$v_{0y} = v_0 \sin\theta$$



U-2-b

x

$$a = 0$$

$$x = v_0 \cdot \cos\theta \cdot t$$

y

$$a = -g$$

$$y = -\frac{1}{2} g t^2 + v_0 \sin\theta t + y_0$$

Treffer bakken når $y=0$

$$t = \frac{-v_0 \sin\theta \pm \sqrt{v_0^2 \sin^2\theta - 4 \cdot \frac{1}{2} g \cdot y_0}}{2 \cdot \left(-\frac{1}{2} g\right)}$$