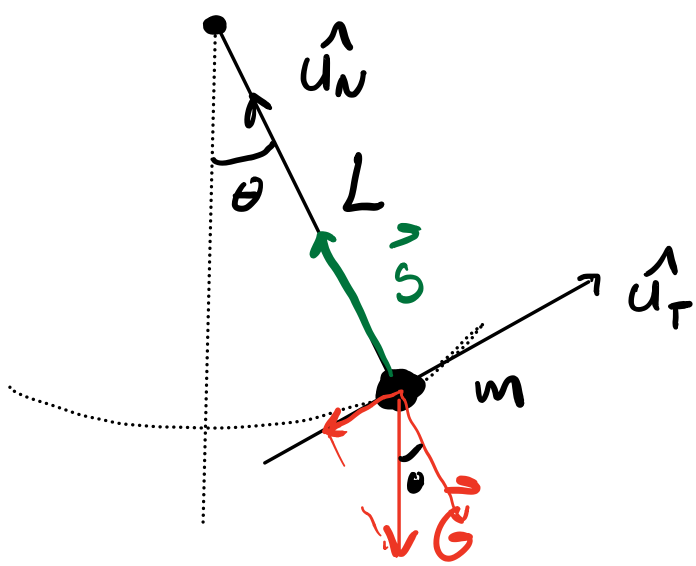
Rotasjon Kap 14

θ - vinkel
ω - vinkel hastighet
α - vinkel akselerasjon.

Pendel



$$\vec{G} = \vec{G_T} + \vec{G_N}$$

$$\vec{G_T} = -G \cdot \sin\theta \, \hat{u}_T$$

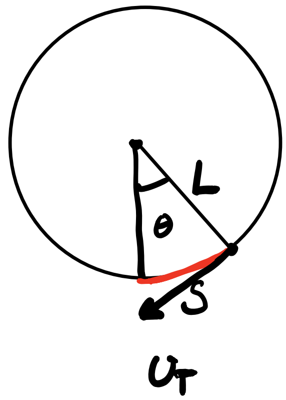
$$\boxed{\vec{G_T} = -mg\sin\theta \, \hat{u}_T}$$

N. 2. lov

$\hat{u}_T$-retning:

$$\vec{G_T} = m\vec{a_T}$$

$$\boxed{-mg\sin\theta = m a_T}$$



$$s = \theta \cdot L$$

$$v_T = \frac{d}{dt} s = \frac{d}{dt}(\theta L)$$

$$v_T = \omega L \qquad \left(\omega = \frac{d\theta}{dt}\right)$$

$$a_T = \frac{d}{dt}(v_T) = \frac{d}{dt}(\omega L)$$

$$a_T = \alpha \cdot L \qquad \left(\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt}\right)$$

$$-\cancel{m} g \sin\theta = \cancel{m} \cdot \alpha L$$

$$\boxed{\alpha = -\frac{g}{L}\sin\theta} \qquad (\sin\theta \approx \theta)$$

når θ er liten

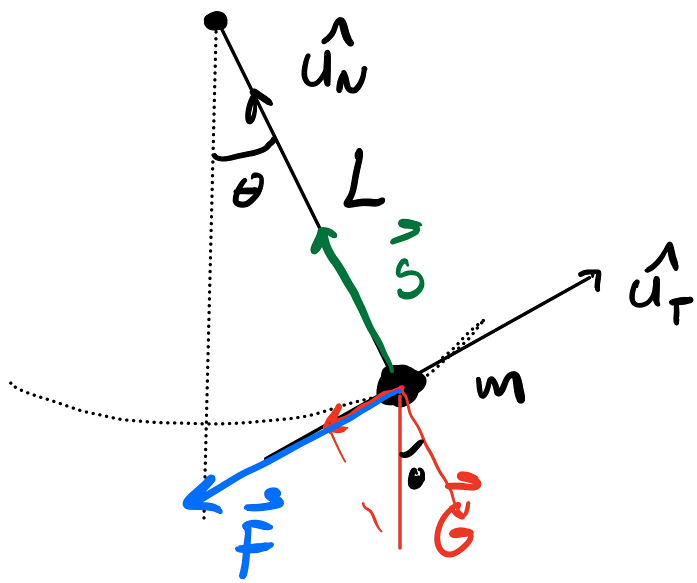
Løser ved Euler-Cromer

$$\omega(t+\Delta t) = \omega(t) + \alpha \cdot \Delta t$$

$$\theta(t+\Delta t) = \theta(t) + \omega(t+\Delta t) \cdot \Delta t$$

Pendel

$\vec{F}$ - ekstern kraft.



N. 2. lov

$\hat{u}_T$ -retning:

$$\vec{F} + \vec{G_T} = m\vec{a_T}$$

$$m\vec{a_T} = \vec{G_T} + \vec{F}$$

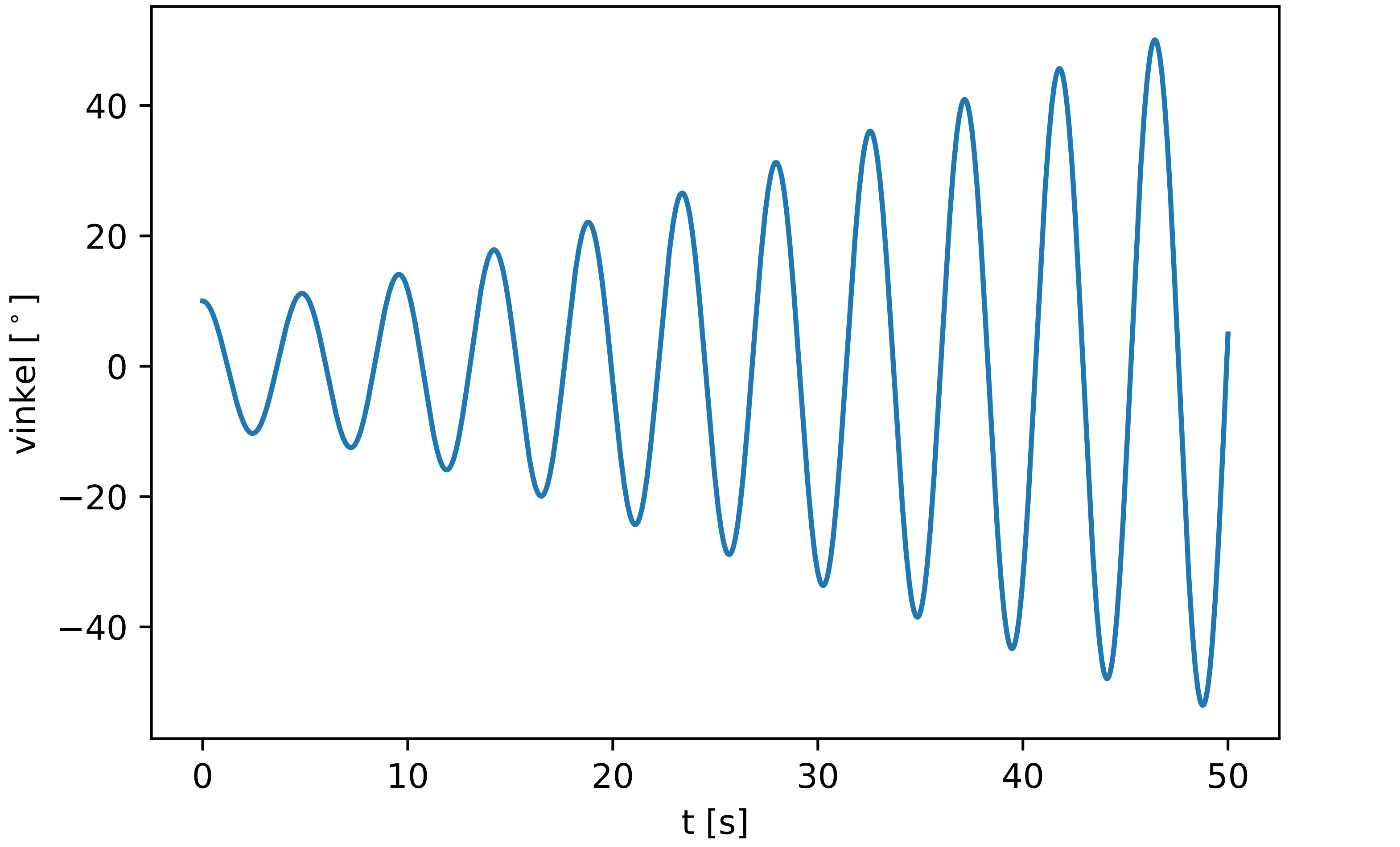
$$m\alpha \cdot L = -mg\sin\theta + F \quad \Big| \cdot \frac{1}{mL}$$

$$\alpha = -\frac{g}{L}\sin\theta + \frac{F}{mL}$$

$$F = F_0 \cdot \cos(\omega_F \cdot t)$$

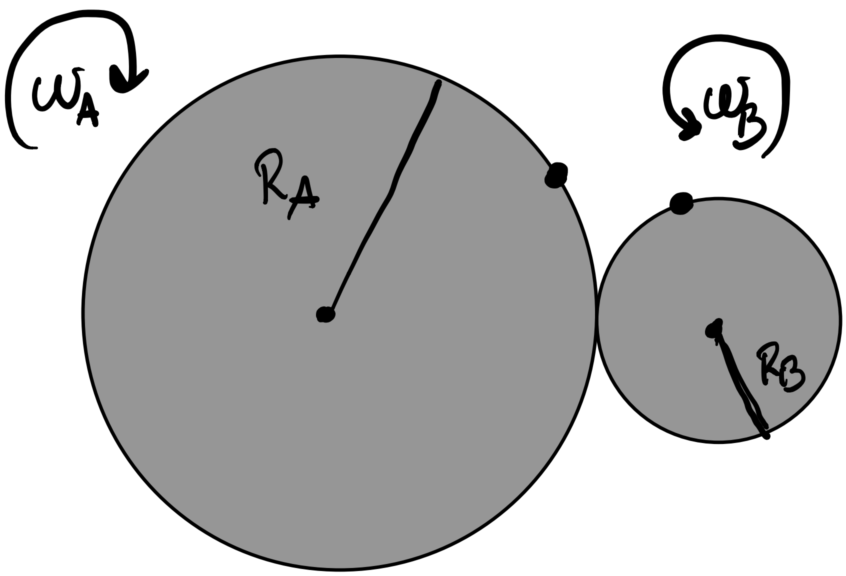
$$\alpha = -\frac{g}{L}\sin\theta + \frac{F_0}{mL}\cdot\cos(\omega_F \cdot t)$$

$$\omega_F = \sqrt{\frac{g}{L}} \quad \text{- Egenfrekvens}$$



40
20
0
−20
−40
vinkel [∘]
0
10
20
30
40
50
t [s]

Tannhjul



Rullebetingelse

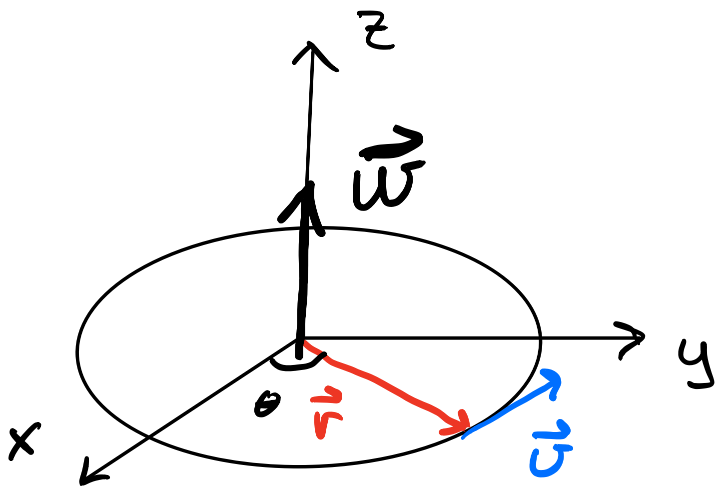
$$v_A = R_A \cdot \omega_A \qquad v_B = R_B \cdot \omega_B$$

I kontaktpunktet:

$$v_A = -v_B$$

$$R_A \cdot \omega_A = -R_B \omega_B \Rightarrow \boxed{\omega_B = -\frac{R_A}{R_B}\omega_A}$$

Rotasjon i 3D



$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$|\vec{A} \times \vec{B}| = |\vec{A}| \cdot |\vec{B}| \cdot \sin \gamma$$

I sirkelbane er vinkel mellom $\vec{r}$ og $\vec{\omega}$ lik $90°$.

$$\vec{v} = \vec{\omega} \times \vec{r} = |\vec{v}| = |\vec{\omega}| \cdot |\vec{r}| \cdot \overbrace{\sin 90°}^{=1}$$

$$\boxed{v = \omega \cdot r}$$

$$\vec{a} = \frac{d}{dt}\vec{v} = \frac{d}{dt}(\vec{\omega} \times \vec{r})$$

$$= \underbrace{\frac{d\vec{\omega}}{dt}}_{=\alpha} \times \vec{r} + \vec{\omega} \times \underbrace{\frac{d\vec{r}}{dt}}_{=\vec{v}}$$

$$= \vec{\alpha} \times \vec{r} + \vec{\omega} \times \underline{\vec{v}}$$

La alle vinkler være 90°

$$\vec{a} = \alpha \cdot r \hat{u}_T + \omega \cdot \omega \cdot r \, \hat{u}_N$$

$$\omega^2 \cdot r \qquad \omega = \frac{v}{r}$$

$$\left(\frac{v}{r}\right)^2 \cdot r = \boxed{\frac{v^2}{r}}$$