Arbeid (kap 10) Energi (kap 11)

Newtons 2. lov $\vec{F} = m\vec{a}$

$$\vec{F}\cdot\vec{v} = m\vec{a}\cdot\vec{v} \qquad \big|\cdot\vec{v}$$

$$\underbrace{\int_{t_0}^{t_1} \vec{F}\cdot\underbrace{\vec{v}\,dt}_{d\vec{r}}}_{\int_{r_1}^{r_2}\vec{F}\cdot d\vec{r}} = \int_{t_0}^{t_1} m\vec{a}\cdot\vec{v}\,dt \qquad \Big|\int dt$$

$$\boxed{\begin{aligned}
&m(\vec{v}\cdot\vec{v})\Big|_{t_0}^{t_1} - \int_{t_0}^{t_1} m\vec{v}\cdot\vec{a}\,dt \\
&2\int_{t_0}^{t_1} m\vec{a}\cdot\vec{v}\,dt = m\vec{v}\cdot\vec{v}\Big|_{t_0}^{t_0} \\
&\int_{t_0}^{t_1} m\vec{a}\cdot\vec{v}\,dt = \frac{1}{2}mv^2\Big|_{t_0}^{t_1}
\end{aligned}}$$

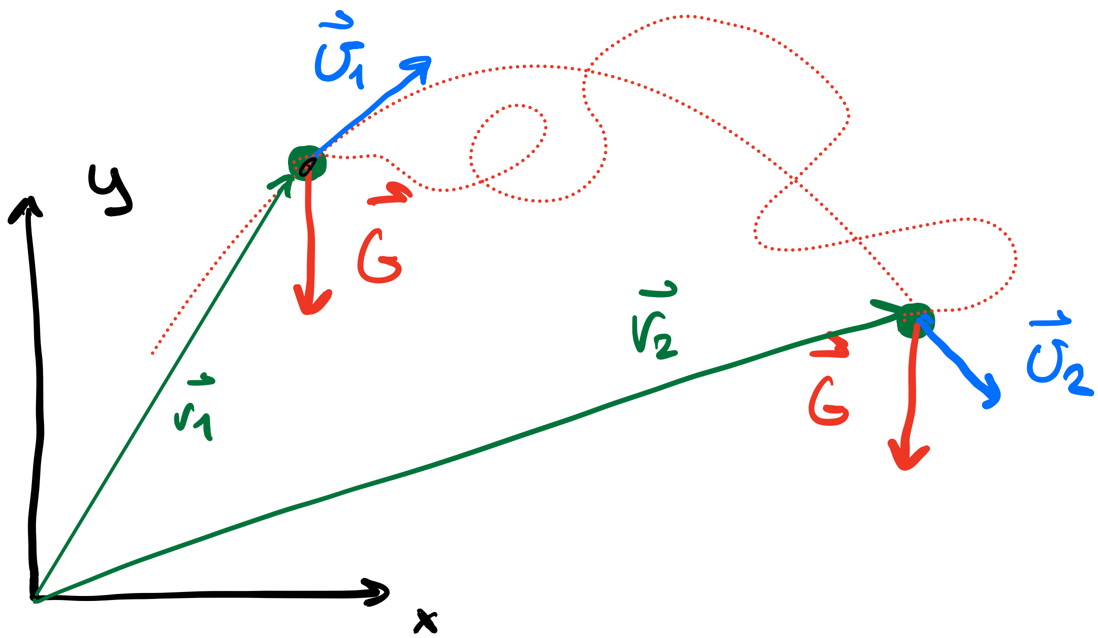
$$\int \vec{F}\cdot d\vec{r} = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2$$

Arbeid-Energi-Teorem

$$\int \vec{F} \cdot d\vec{r} = \Delta E_K = \frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2$$

$$W = \Delta E_K \quad \text{enhet } [Nm] = [J]$$

Arbeid i et tyngdefelt



Arbeid gjort av tyngdekraft:

$$W = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$$

$$\vec{F} = -mg\vec{j} = \vec{G}$$

$$d\vec{r} = dx\,\vec{i} + dy\,\vec{j}$$

$$\int_{(x_1,y_1)}^{(x_2,y_2)} -mg\vec{j} \cdot (dx\,\vec{i} + dy\,\vec{j})$$

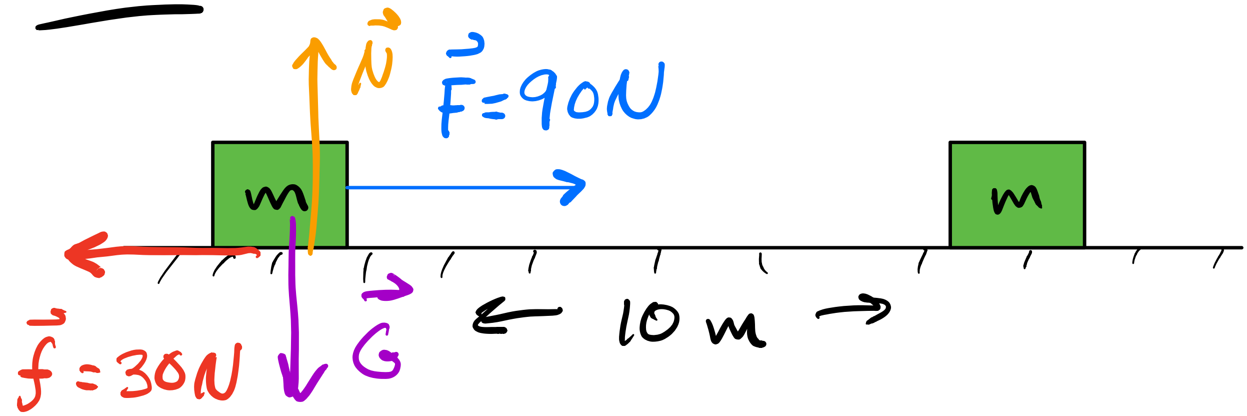
$$\vec{j} \cdot \vec{i} = 0 \,, \quad \vec{j} \cdot \vec{j} = 1$$

$$\int_{(x_1,y_1)}^{(x_2,y_2)} -mg\,dy = -mgy\Big|_{y_1}^{y_2}$$

$$W = -mg\underbrace{(y_2 - y_1)}_{h} = -mgh$$

10,7

$m = 10\,kg$



$\vec{G}$ og $\vec{N}$ gjør ikke arbeid på klossen når den flytter seg 10 m horisontalt. Fordi: $\vec{G} \perp d\vec{r}$ og
$\vec{N} \perp d\vec{r}$.

a) Finn arbeidet fra $\vec{F}$

$$W_F = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} = \vec{F} \cdot \int_{r_1}^{r_2} d\vec{r} = \vec{F} \cdot \underbrace{\int_{x_1}^{x_2} dx}_{10m}$$

$$W_F = 90N \cdot 10m = 900\,Nm$$

$$\underline{\underline{W_F = 0{,}90\,kJ}}$$

b) Finn arbeidet fra $\vec{f}$.

$$W_f = \int \vec{f} \cdot d\vec{r} = \vec{f} \cdot \vec{r}$$

$$W_f = -30N \cdot 10m = -300Nm$$

$$\underline{\underline{W_f = -0{,}30\ kJ}}$$

c) Finn farten til klossen

$$\sum W = \Delta E_K$$

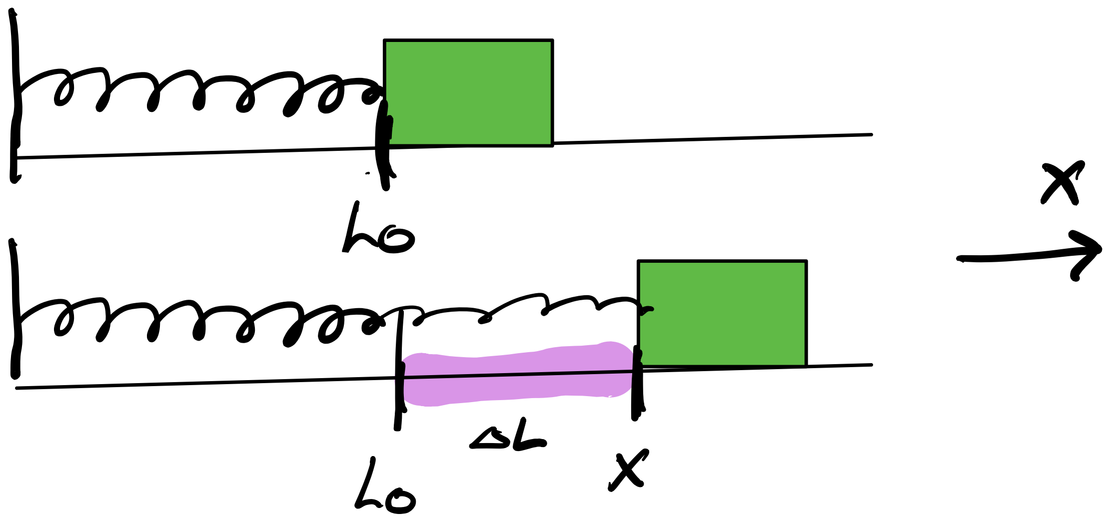
$$W_F + W_f = \frac{1}{2} m v_2^2 - \frac{1}{2} m \underbrace{v_0^2}_{=0}$$

$$2 \cdot m (W_F + W_f) = v_2^2$$

$$v_2 = \sqrt{\frac{2}{m} \cdot (W_F + W_f)}$$

$$v_2 = \sqrt{\frac{2}{10kg}(0{,}60\ kJ)} = \underline{\underline{11\ m/s}}$$

Arbeid av Fjær



$$\vec{F} = -k \cdot \Delta L \, \vec{i} \qquad (\Delta L = x - L_0)$$

$$\vec{F} = -k(x - L_0)\,\vec{i}$$

$$W = \int_{x_1}^{x_2} \vec{F} \cdot d\vec{r} = \int_{x_1}^{x_2} -k(x - L_0)\,dx$$

$u = x - L_0$, $du = dx$ Substitusjon

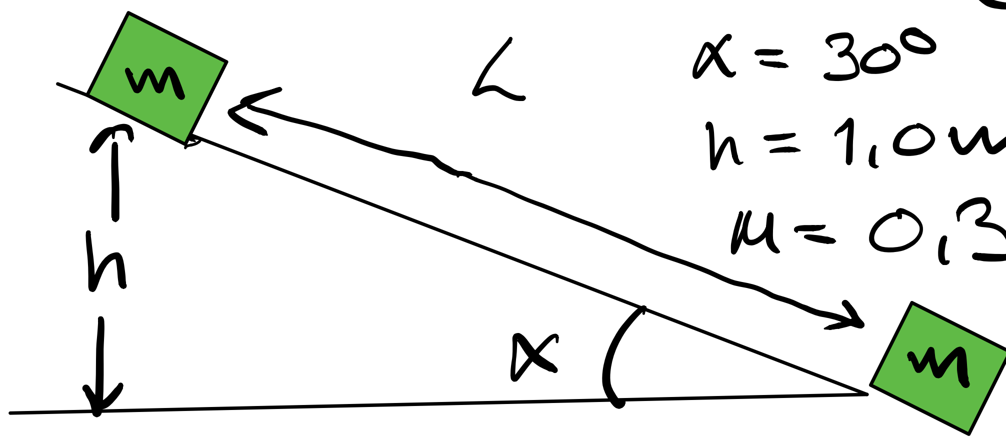
$$\int -ku\,du = -\frac{1}{2}ku^2$$

$$\int_{x_1}^{x_2} -k(x-L_0)\,dx = -\frac{1}{2}k(x-L_0)^2 \Big|_{x_1}^{x_2}$$

$$W = -\frac{1}{2}k(x_2-L_0)^2 - -\frac{1}{2}k(x_1-L_0)^2$$

$$W = \frac{1}{2}k(\underbrace{x_1-L_0}_{\Delta L_1})^2 - \frac{1}{2}k(\underbrace{x_2-L_0}_{\Delta L_2})^2$$

10,4



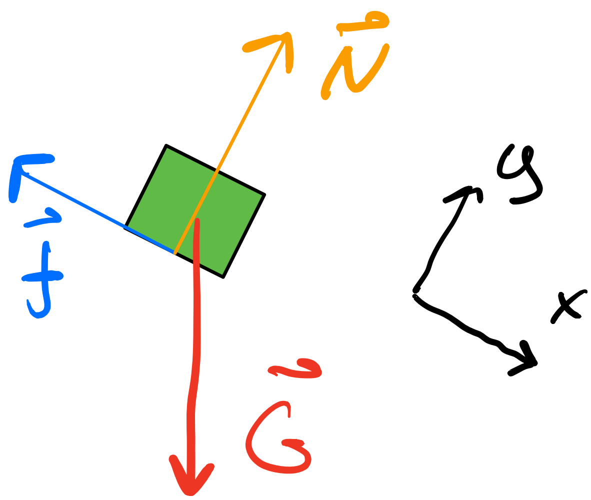
$m = 1{,}0\,kg$

$\alpha = 30^\circ$

$h = 1{,}0\,m$

$\mu = 0{,}3$

Hva er farten til klossen?



$G_x = mg\sin\alpha$

$G_y = -mg\cos\alpha$

$f = -\mu N$

N. 1. lov

$N = mg\cos\alpha$

$G_y + N = 0$

$f = -\mu mg\cos\alpha$

$$W_G = \int \vec{G} \cdot d\vec{r} = mgh$$

$$W_f = \int \vec{f} \cdot d\vec{r} = -\mu mg\cos\alpha \cdot L$$

$$W_N = \int \vec{N} \cdot d\vec{r} = 0 \quad (\vec{N} \perp d\vec{r})$$

$$W_G + W_f + W_N = \Delta E_K$$

$$mgh - \mu mg\cos\alpha \cdot L = \frac{1}{2}mv_2^2 - \underbrace{\cancel{\frac{1}{2}mv_1^2}}_{v_1=0}$$

$$\boxed{\frac{h}{L} = \sin\alpha \Rightarrow L = \frac{h}{\sin\alpha}}$$

$$\cancel{m}gh - \mu \cancel{m}g\cos\alpha \cdot \frac{h}{\sin\alpha} = \frac{1}{2}\cancel{m}v_2^2$$

$$v_2 = \sqrt{2 \cdot gh - \mu g\cos\alpha \cdot \frac{h}{\sin\alpha}}$$