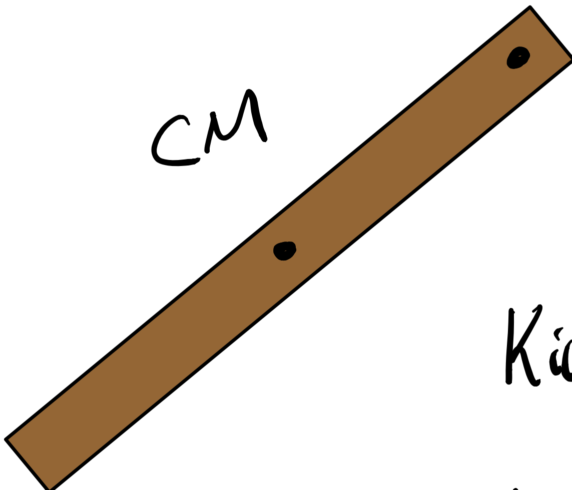
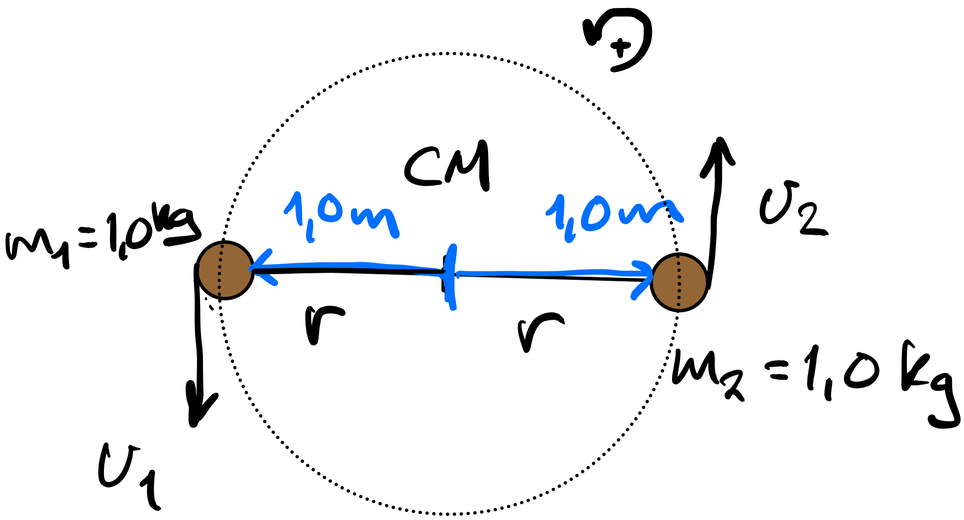
Kap 15 rotasjon av stive legemer



Kinetisk energi

$$K = \underbrace{\tfrac{1}{2} M v^2}_{\text{Energi pga tranlatorisk bevegelse av massesenter}} + \underbrace{\sum \tfrac{1}{2} m_i v_i^2}_{\text{Energi pga bevegelse i forhold til massesenter}}$$



roterer med vinkelhastighet $\omega = 3{,}14\ rad/s$

Massesenter er i ro $V = 0$

$$K = \underbrace{\tfrac{1}{2} M V^2}_{=0} + \underbrace{\sum \tfrac{1}{2} m_i v_i^2}_{\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2}$$

$$v_1 = r \cdot \omega$$

$$v_2 = r \cdot \omega$$

$$K = \tfrac{1}{2} \underbrace{(m_1 + m_2) \cdot r^2}_{I} \omega^2$$

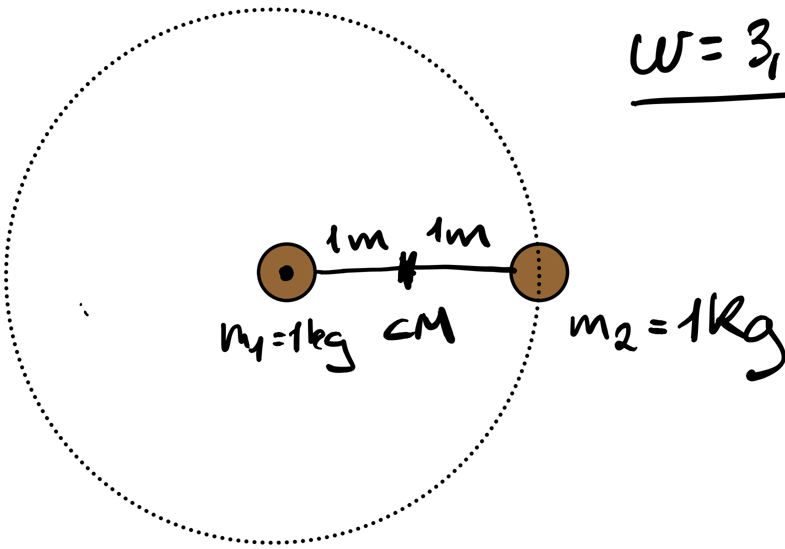
I - treghetsmoment.

$$K = \frac{1}{2} I \omega^2$$

$$I = (m_1 + m_2) \cdot r^2 = (1{,}0\,kg + 1{,}0\,kg) \cdot (1{,}0\,m)^2$$

Treghetsmoment om massesenter $\rightarrow$ $\underline{\underline{I = 2\,kg\,m^2}}$

$\underline{\omega = 3{,}14\ rad/s}$



Finn treghetsmoment I når vi roterer om m_1.

$$\boxed{I = \sum m_i r_i^2}$$

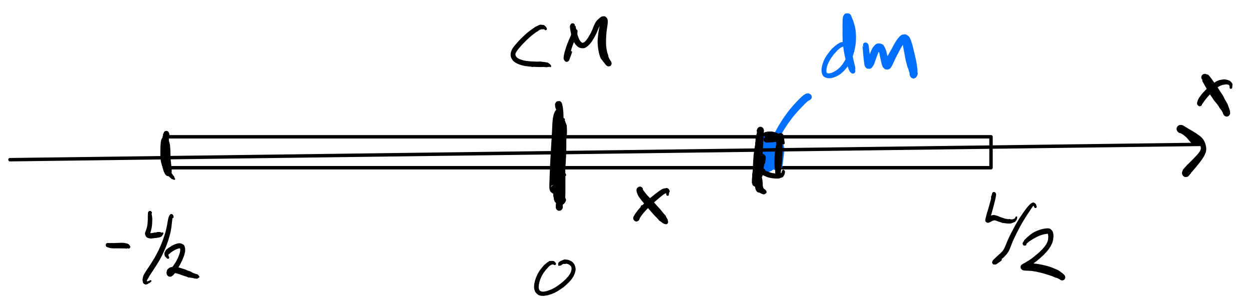
$$I = m_1 \cdot \underbrace{r_1^2}_{=0} + m_2 \cdot \underbrace{r_2^2}_{2{,}0m}$$

$$I = 1{,}0\,\text{kg} \cdot 0^2 + 1{,}0\,\text{kg} \cdot (2{,}0\,\text{m})^2$$

$$\underline{I = 4{,}0\,\text{kgm}^2}$$

Treghetsmoment til kontinuerlig fordelt masse.

$$I = \int \underline{r}^2\, dm$$



Homogen stav: $\boxed{\lambda = \frac{M}{L}}$ - masse per lengde

$$dI = x^2\, dm \qquad \Big|\ dm = \lambda\, dx$$

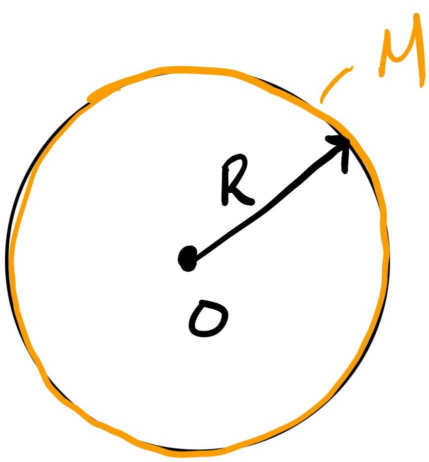
$$\int_0^I dI = \int_{-L/2}^{L/2} x^2 \lambda\, dx$$

$$I = \lambda \int_{-L/2}^{L/2} x^2 \, dx = \lambda \cdot \frac{1}{3} x^3 \Big|_{-L/2}^{L/2}$$

$$I = \lambda \cdot \frac{1}{3} \left(\left(\frac{L}{2}\right)^3 - \left(-\frac{L}{2}\right)^3 \right) = \frac{\lambda \cdot L^3}{12}$$

$$I = \frac{\frac{M}{L} \cdot L^3}{12} = \underline{\underline{\frac{ML^2}{12}}}$$

Treghetsmoment til sylinderskall.



$$I_O = \int r^2 \, dm$$

$$I_O = R^2 \underbrace{\int dm}_{M}$$

$$I_O = R^2 M$$

Eksempler på treghetsmoment

Lærebok s. 464.

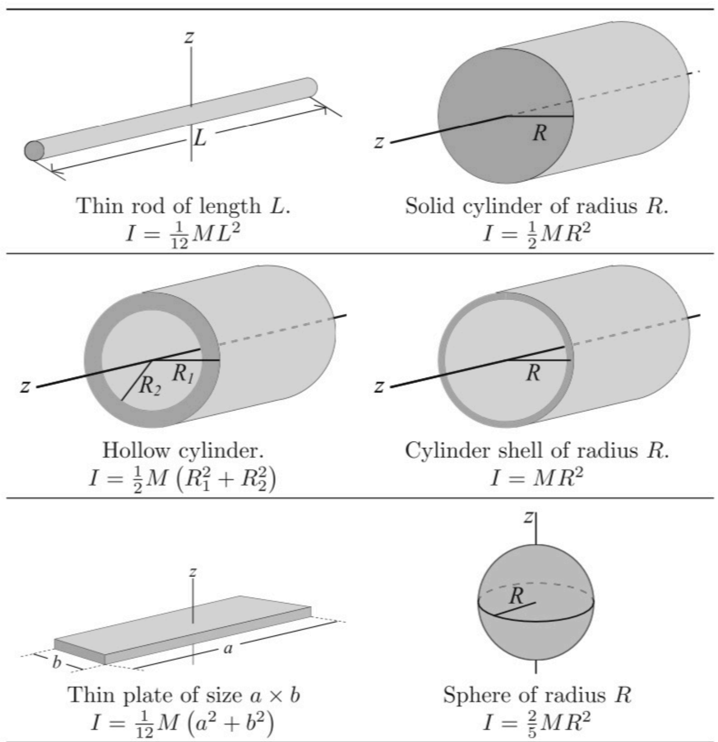
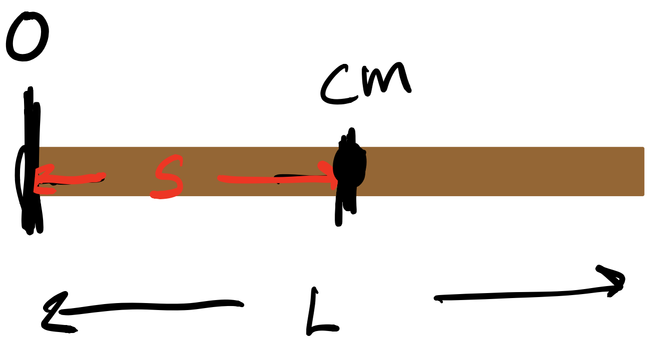


Fig. 15.5 Moments of inertia for various solid bodies

Parallellakse - teoremet

$$I_o = I_{CM} + Ms^2$$



$$I_{cm} = \frac{ML^2}{12} \qquad \left(s = \frac{L}{2}\right)$$

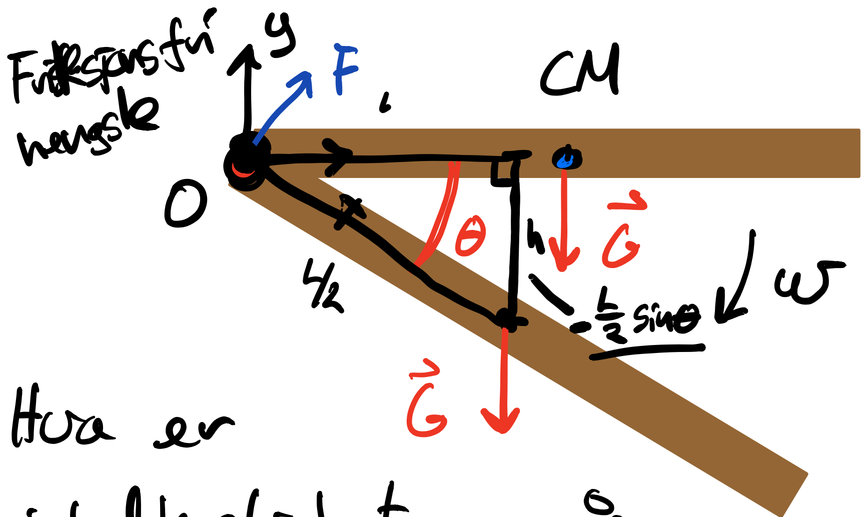
$$I_o = I_{cm} + M \cdot s^2$$

$$I_o = \frac{ML^2}{12} + M \cdot \left(\frac{L}{2}\right)^2 = \underline{\frac{1}{3} ML^2}$$

Bevaring av Energi

$$E = K + U$$

$$= \frac{1}{2} M v^2 + \frac{1}{2} I_{cm} \omega^2 + U_{cM}$$



Hva er vinkelhastigheten når denne staven roterer om hengslen?

<u>Bruker Energibevaring.</u>

$$K_0 + U_0 = K_1 + U_1$$

$$\left.\begin{aligned} K_0 &= \tfrac{1}{2} m \underbrace{v_0^2}_{=0} + \tfrac{1}{2} I_0 \underbrace{\omega^2}_{=0} \\ U_0 &= mg\underbrace{y}_{=0} \end{aligned}\right\} K_0 + U_0 = 0$$

$$K_1 = \tfrac{1}{2} m \underbrace{v_1^2}_{=0} + \tfrac{1}{2} I_0 \underbrace{\omega_1^2}_{=?} = \boxed{\tfrac{1}{2} I_0 \omega_1^2}$$

Hastigheten til hengslen

$$U_1 = mgy_1 \qquad y_1 = -\frac{L}{2}\sin\theta$$

$$\boxed{U_1 = -mg\cdot\frac{L}{2}\sin\theta}$$

$$K_1 + U_1 = 0$$

$$\frac{1}{\cancel{2}} I_o \omega^2 + -mg\frac{L}{\cancel{2}}\sin\theta = 0 \quad \Big| \cdot 2$$

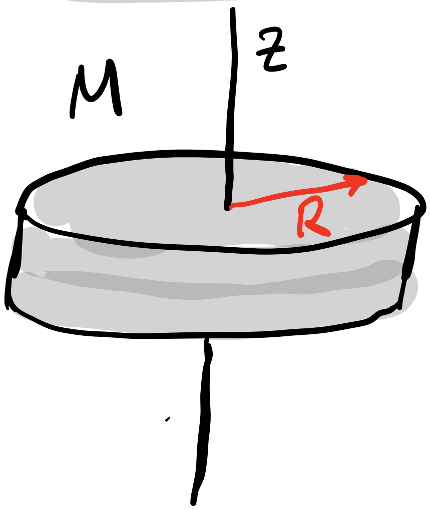
$$\omega^2 = \frac{mgL\sin\theta}{I_o}$$

$$\omega = \sqrt{\frac{mgL\sin\theta}{I_o}} \quad \Bigg| \quad I_o = \frac{mL^2}{3}$$

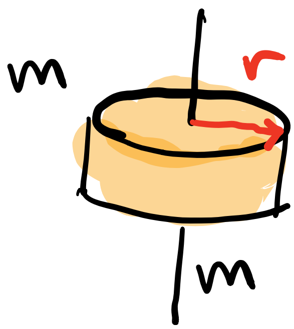
$$\omega = \sqrt{\frac{\cancel{m}g\cancel{L}\sin\theta}{\cancel{m}L^{\cancel{2}}\cdot\frac{1}{3}}}$$

$$\omega = \sqrt{\frac{3g}{L}\sin\theta}$$

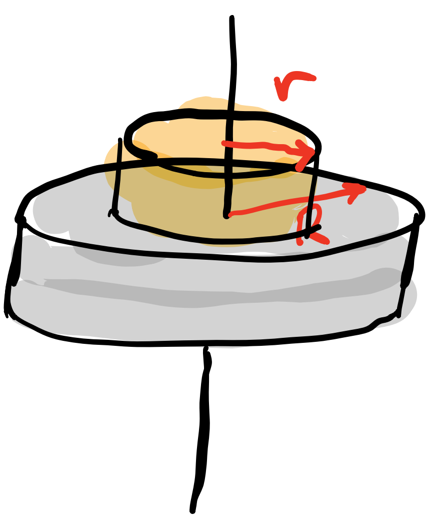
Superposisjonsprinsippet



$$I_z = \frac{1}{2}MR^2$$



$$I_z = \frac{1}{2}mr^2$$



$$I = \frac{1}{2}mr^2 + \frac{1}{2}MR^2$$

Summer treghetsmoment.