ECE 603
Probability and Random Processes

Lessons 1-3
Chapter 1
Basic Concepts

Objectives

- Review set theory
- Explore random experiments and probabilities
- Examine conditional probability
- Examine independence
- Review law of total probability
- Review Bayes’ rule
- Explore conditional independence

Rationale

- An exploration of the basic concepts of probability will provide a foundation for discussion.
- Mathematical concepts are necessary for understanding probability theory.
- This lesson focuses on random experiments and the axioms of probability.
- You will explore discrete and continuous probability models, before discussing conditional probability.

Prior Learning

- A level of mathematical maturity consistent with the graduate engineering level.
- Access to the online textbook: https://www.probabilitycourse.com/

Review of Set Theory

A set is an unordered collection of things (elements).

- $A = \{\clubsuit, \diamondsuit\}$ $\qquad \diamondsuit \in A; \quad \heartsuit \notin A$
- $B = \{1, 2, 3\};$
- $C = \{x^2 : x = 1, 2, 3\} = \{1, 4, 9\};$
- $D = \{H, T\};$

Review of Set Theory

- **The set of natural numbers,** $\mathbb{N} = \{1, 2, 3, \cdots\}$.
- **The set of integers,** $\mathbb{Z} = \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$.
- **The set of real number** $\mathbb{R}$.

Review of Set Theory

Set A is a subset of set B if every element of A is also an element of B.

We write $A \subset B$, where " $\subset$ " indicates "subset ".

$$A \subset B \;\equiv\; (x \in A) \;\Rightarrow\; (x \in B)$$

Example:

- $E = \{1,4\}; \quad C = \{1,4,9\} \;\Rightarrow\; E \subset C.$
- $\mathbb{N} \subset \mathbb{Z}.$

Review of Set Theory

$A = B$ **if and only if** $A \subset B$ **and** $B \subset A$.

Example:

- $\{1,2,3\} = \{3,2,1\}$
- $\{a,a,b\} = \{a,b\}$

Universal set: The set of all things that we could possibly consider in a given context.

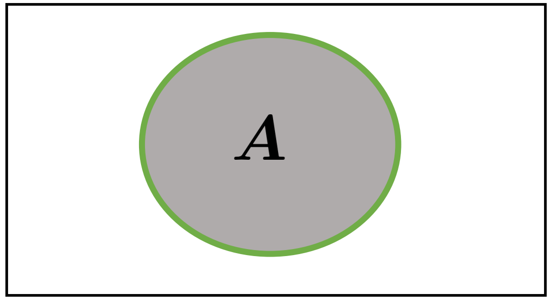
$S =$ **Universal set** $= \Omega$;

$\emptyset =$ **Null set ;** $\emptyset = \{\}$;

For any set A; $\emptyset \subset A$.

Venn Diagrams

In a Venn diagram any set is depicted by a closed surface.



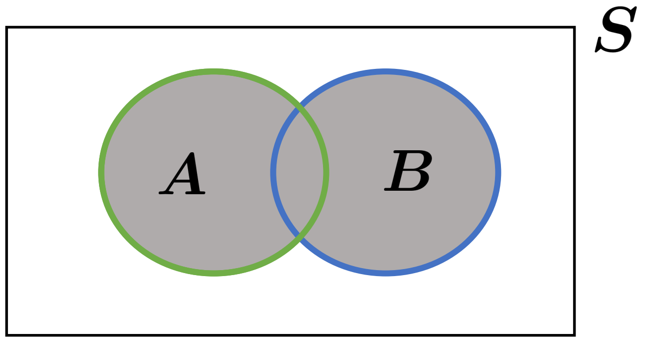
$\Omega = S$

Set Operations

Union: The union of two sets A and B is denoted by $A \cup B$ and consist of all objects in A or B.

$x \in (A \cup B)$ if and only if $(x \in A)$ or $(x \in B)$.

$\{1, 2\} \cup \{3\} = \{1, 2, 3\}$

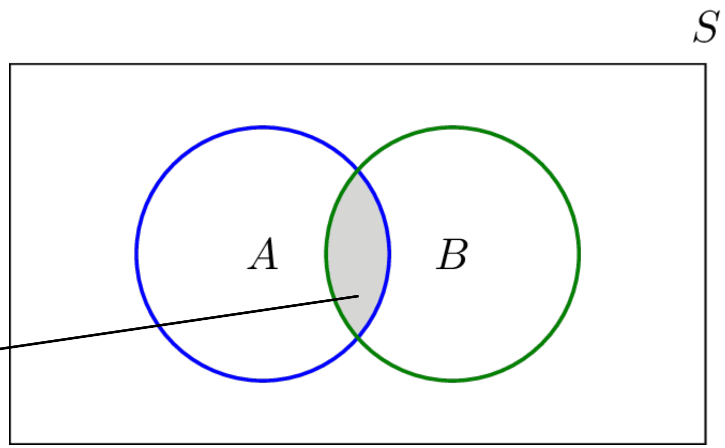


Set Operations

Intersection:

The intersection of two sets A and B is denoted by $A \cap B$ and consist of all objects in both A and B.

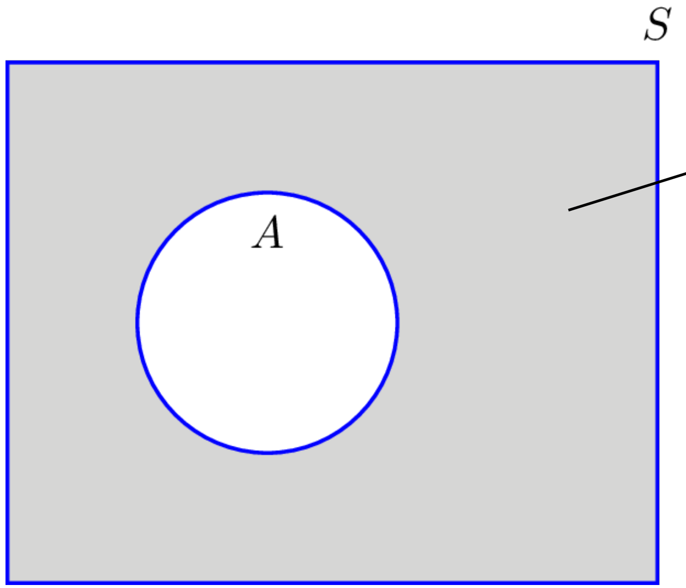
$A \cap B$



Set Operations

Complement:

The complement of a set A, denoted by A^c, is the set of all elements in S (Ω) that are Not in A.



$\boldsymbol{A^c}$

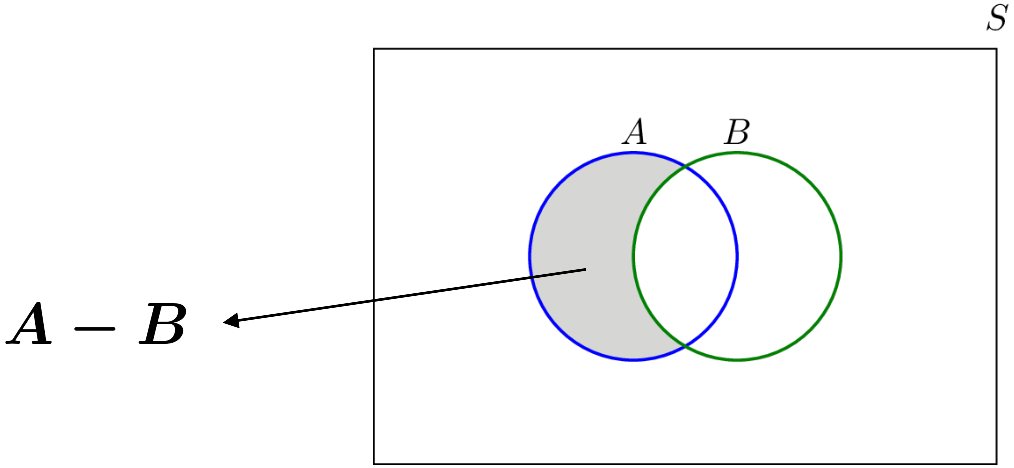
$$\bar{A} = A^c$$

Set Operations

Difference (subtraction):

The subtraction of set B from A $(A - B)$ is all elements in A that are not in B.

$$A - B = A \cap B^c; \ (x \in A) \text{ and } (x \notin B).$$

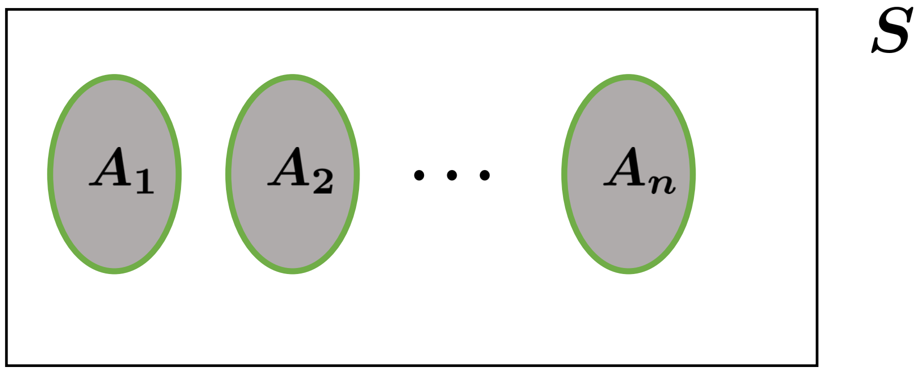


Set Operations

Mutually exclusive set (disjoint):

Two sets A and B are mutually exclusive (or disjoint) if $A \cap B = \emptyset$.

❖ $\boldsymbol{A_1, A_2, \cdots, A_n}$ are m.e. if $\boldsymbol{A_i \cap A_j = \emptyset,\ i \neq j}$.



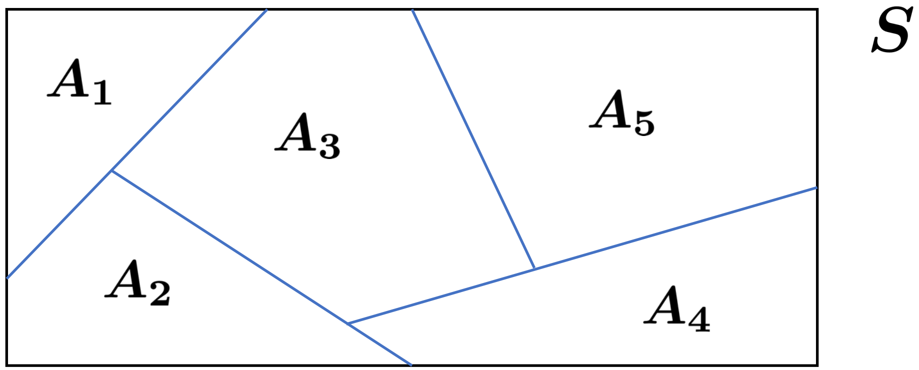
Set Operations

Partition:

A collection of sets $A_1, A_2, \cdots, A_n$ is a Partition of S if

a) They are disjoint .

b) $A_1 \cup A_2 \cup A_3 \cup \cdots A_n = S$



Set Operations

Theorem : De Morgan's law

$$(A \cup B)^c = \overline{A \cup B} = A^c \cap B^c$$

Set Operations

Example:

Let $S = \{1, 2, 3, 4, 5, 6\}$, **and** $A = \{1, 2\}$, $B = \{2, 4, 6\}$.

a) $A \cup B$ **b)** $A \cap B$ **c)** A^c

d) B^c **f)** $(A \cup B)^c$ **g)** $A^c \cap B^c$

- **The sets** $\{1, 2\}$, $\{3, 4, 5\}$, $\{6\}$ **form a partition of** S.

Set Operations

Theorem : Distributive law

- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

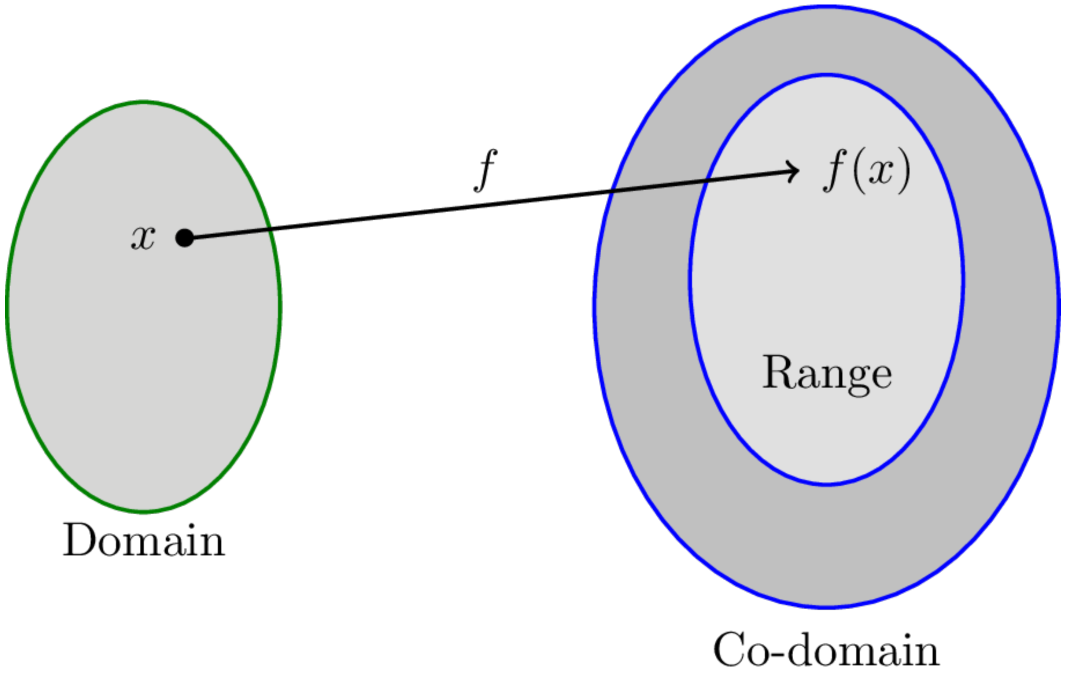
Functions

$f : X \to Y.$

X : Domain

Y : Co-domain

$\forall\, x \in X,\ f(x) \in Y$



Range: the set of all the possible values of $f(x)$. (Range $\subset Y$)

Functions

Example:

Consider the function $f : \mathbb{R} \to \mathbb{R}$, defined as $f(x) = x^2$.

$$X = Y = \mathbb{R};$$

$$\text{Range}(f) = \mathbb{R}^+ = \{x \in \mathbb{R} | x \geq 0\}.$$

- **one-to-one (invertible):** $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

Countable and Uncountable Sets

Cardinality of a set A is the number of elements in A; $|A|$.

- A set is finite if $|A| < \infty$.

- A set if countable if it is finite Or the elements of A can be enumerated or listed in a sequence $a_1, a_2, a_3, \cdots$, that is,

$$A = \bigcup_{k=1}^{\infty} \{a_k\}, \qquad A = \{a_1, a_2, a_3, \cdots\}$$

Ex: $\mathbb{N} = \{1, 2, 3, \cdots\}$ is countable.

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Countable and Uncountable Sets

Uncountable: Not countable.

e.g., $\mathbb{R}$; $[\,0, 1]$

Equivalently: A set is countably infinite if it is in one-to-one correspondence with

$$\boldsymbol{N = \{1, 2, 3, \cdots\} = \bigcup_{k=1}^{\infty} \{k\}}$$

Countable and Uncountable Sets

Example:

$\mathbb{Z}$ (set of integers) is countable (countably infinite).

Because $\mathbb{Z} = \{0, 1, -1, 2, -2, 3, -3, \cdots\}$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ a_1 & a_2 & a_3 \end{array}$$

Countable and Uncountable Sets

Example:

Show that a set of the form $B = \bigcup_{i,j=1}^{\infty} \{b_{ij}\} = \bigcup_{i}^{\infty} \bigcup_{j}^{\infty} \{b_{ij}\}$ **is countable.**

Example:

Show that the positive rational number form a countable set: $\mathbb{Q}^{+} = \bigcup_{i,j=1}^{\infty} \{\frac{i}{j}\}$.

Countable and Uncountable Sets

Example:

Show that the positive rational number form a countable set:

$$\mathbb{Q}^{+} = \bigcup_{i,j=1}^{\infty} \{\frac{i}{j}\}.$$

Countable and Uncountable Sets

But $\mathbb{R}$ is not countable.

In fact, any interval $[a, b]$ where $b > a$ is not countable.

$$[a, b] = \{x \in \mathbb{R},\ a \leq x \leq b\}$$

$$[a, b) = \{x \in \mathbb{R},\ a \leq x < b\}$$

Orchestrated Conversation: Review and Discussion

Review of Video and Exercises from Lesson 1 Video 1

Random experiment

Random experiment: A phenomenon whose outcome cannot be predicted with certainty, such as

Random experiment:

- Roll a die
- Roll a die three times
- Flip a coin

Random experiment

Outcome:

An outcome is the result of a random experiment.

- **Roll a die** $\longrightarrow$ **3**
- **Roll a die 3 times** $\longrightarrow$ **(2, 3, 6)**

Random experiment

Events:

An event is collection of possible outcomes.

- **Roll a die (Event=E)**

$$E_1 = \{1, 3, 5\}, \quad E_2 = \{2, 4\}, \quad E_3 = \{6\}$$

Random experiment

Sample Space:

The sample space is the set of all possible outcomes.

- **Roll a die:** ← random experiment

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

- **Roll a die three times**

$$\Omega = \{(1,1,1), (1,1,2), \cdots, (1,1,6), (2,1,1), \cdots, (6,6,6)\}$$

an outcome

Random experiment

- Event $\longleftrightarrow$ Set
- Sample space $\longleftrightarrow$ Universal Set
- Outcome $\longleftrightarrow$ Element

We say that an event A occurs if the outcome of the experiment is an element of A.

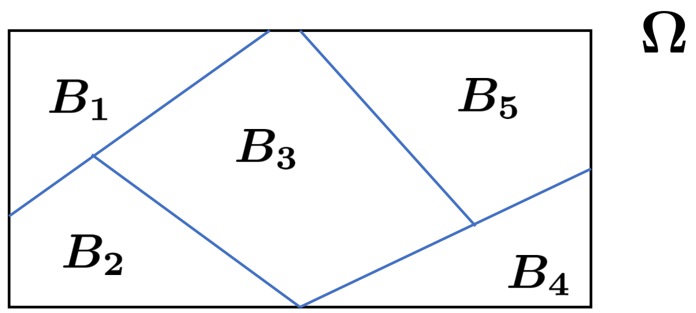
Random experiment

Partition:

A partition is a collectively exhaustive, and mutually exclusive set of events, i.e.,

$B_1, B_2, \cdots, B_n$ **is a Partition if**

- $B_1 \cup B_2 \cup B_3 \cup \cdots B_n = \Omega$
- $B_i \cap B_j = \emptyset, \ i \neq j.$



Summary of Random experiment

a) Review of set theory

b) Random experiments: Roll a die, etc.

- **Outcome:** An outcome is a result of random experiment.
  - Roll a die $\longrightarrow$ 3
  - Roll a die three times $\longrightarrow$ (3,6,2)

Summary of Random experiment

- **Sample Space:** The set of all possible outcomes (S).
  - Roll a die $\longrightarrow$ $S = \{1, 2, 3, 4, 5, 6\}$
- **Event:** An event is a collection of possible outcomes.

  $\Rightarrow$ An event is subset of S.
  - Roll a die : $E_1 = \{1, 2\}, \quad E_2 = \{4\}$

Summary of Random experiment

- We also say that an event A has occurred if the outcome of the experiment is an element of A.
  - Roll a die $\longrightarrow$ 2 , $\quad E_1$: has occurred

    E_2 : has not occurred

  - Roll a die 3 times

$$S = \{(1,1,1),(1,1,2),\cdots,(1,1,6),(2,1,1),\cdots,(6,6,6)\}$$

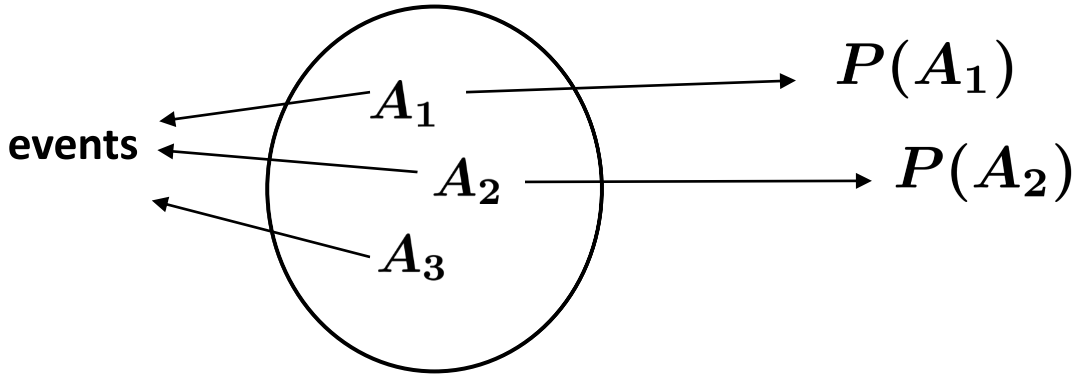
$\Rightarrow\ 6^3$ elements.

Probability

Event $A \longrightarrow P(A)$: Probability of A.

We assign a probability $P(A)$ to every event A.

$P(A)$: The portion of times event A is observed in a large number of runs of the experiment.



Probability

Axioms of Probability

Definition. A probability measure $P(.)$ is a function that maps events in the sample space S to real numbers. Such that:

1) For any event A, $P(A) \geq 0$.

2) Probability of the sample space S is $P(S) = 1$.

3) For any countable collection $A_1, A_2, A_3, \cdots$ of disjoint events

$$P(A_1 \cup A_2 \cup A_3 \cdots) = P(A_1) + P(A_2) + P(A_3) + \cdots$$

Orchestrated Conversation: Review and Discussion

Review of Video and Exercises from Lesson 1 Video 2

Probability

Roll a fair die (fair: outcomes are equally likely).

$$P(\{1\}) = P(\{2\}) = \cdots = P(\{6\})$$

disjoint

3rd axiom:

$$P(\{1\} \cup \{2\} \cup \{3\} \cup \cdots \{6\}) = P(\{1\}) + P(\{2\}) + \cdots + P(\{6\}) = 6P(\{1\})$$

$$1 = P(S) = P(\{1, 2, \cdots, 6\})$$

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Probability

$$\Rightarrow \; P(1) = P(2) = \cdots = P(6) = \frac{1}{6}$$

$$P(\{1,2\}) = P(\{1\} \cup \{2\}) = P(1) + P(2) = \frac{2}{6} = \frac{1}{3}.$$

disjoint

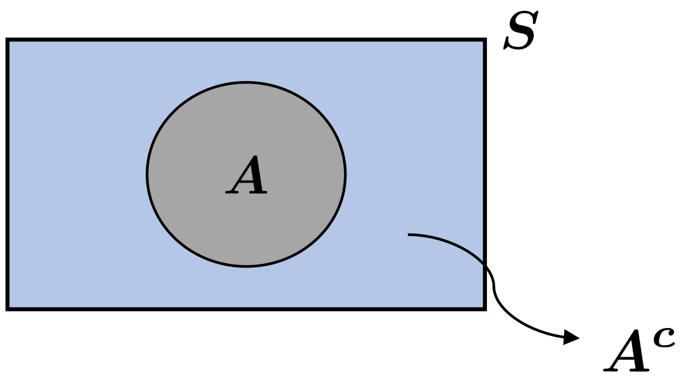
- **Equally likely outcomes:**

$$P(A) = \frac{|A|}{|S|}$$

Probability

Using the axioms:

1) $P(A)$, what is $P(A^c)$.



$$A \cup A^c = S$$

$$\Rightarrow \; P(A \cup A^c) = P(s) = 1$$

disjoint

$$\Rightarrow \; P(A) + P(A^c) = 1 \;\Rightarrow\; P(A^c) = 1 - P(A)$$

Probability

2) $P(\emptyset) = 0,$

$\emptyset$: empty

$$P(\emptyset) = P(S^c) = 1 - P(S) = 1 - 1 = 0.$$

3) $P(A) \leq 1,$

$$P(A) = 1 - \underbrace{P(A^c)}_{\geq 0} \ \Rightarrow \ P(A) \leq 1.$$

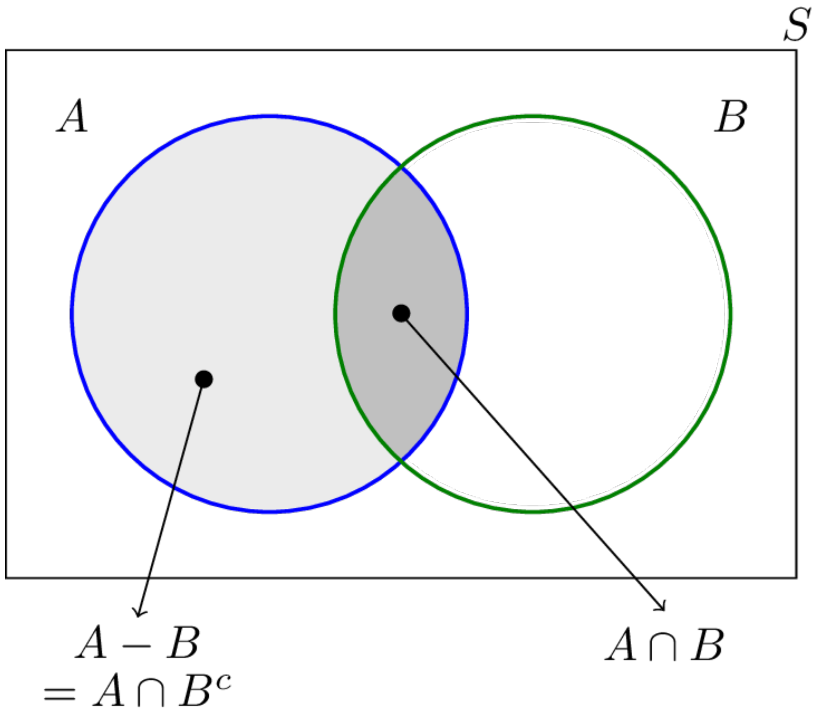
Probability

4) $P(A-B) = P(A) - P(A \cap B)$,

$$A = (A \cap B) \cup (A - B)$$

$(A \cap B)$ **and** $(A - B)$ **are disjoint.**

$$\begin{aligned} P(A) &= P\big((A \cap B) \cup (A - B)\big) \\ &= P(A \cap B) + P(A - B). \end{aligned}$$



Probability

5) $P(A \cup B) = P(A) + P(B) - P(A \cap B),$

$\cup$: or , $\cap$: and

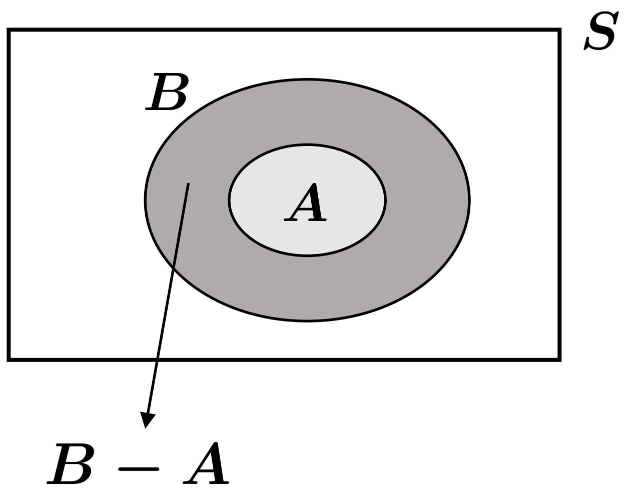
Use Venn diagram.

6) $A \subset B \;\Rightarrow\; P(A) \leq P(B),$

$P(B) = P(A) + P(B - A)$

$P(B-A) \geq 0,\;\; B$ and $(A-B)$ are disjoint.

$\Rightarrow\;\; P(B) \geq P(A)$



Probability

Example:

Roll a die twice and observed X_1 and X_2.

(a) Find S.

(b) $A : X_1 + X_2 = 4,$ Find the elements in A, and $P(A)$.

(c) $B : X_1 + X_2 = 6$ or 7, Find $P(B)$.

Orchestrated Conversation: Review and Discussion

Review of Video and Exercises from Lesson 1 Video 3

Whiteboard

Summary

Probability:

$A \subset S,\ P(A):$

$(1)\ P(A) \geq 0$

$(2)\ P(S) = 1$

$(3)\ A_1, A_2, \cdots \text{ disjoint } P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$

➢ **Finite Sample Space with equally likely outcomes:** $P(A) = \dfrac{|A|}{|S|}$

Sample Space

Sample Space:

a) Countable: $S = \{s_1, s_2, s_3, \cdots\}.$

S : Discrete Probability Space

$$A = \{a_1, a_2, a_3, \cdots\},$$

$$\Rightarrow \; P(A) = P(a_1) + P(a_2) + P(a_3) + \cdots$$

Sample Space

b) Uncountable

S : **Continuous Probability Space**

$$S = \mathbb{R}^+ = \{x \in \mathbb{R},\ x \geq 0\}$$

Continuous Probability Space

Example: I choose a point completely at random in $[0, 1]$.

a) $P([0, 0.5]) = 0.5$

b) $P([0, 0.25]) = 0.25$

c) $P([a, b]) = b - a, \quad 0 \leq a \leq b \leq 1$

d) $P(\{0.5\}) = 0 = P([0.5, 0.5]) = 0.5 - 0.5 = 0$

Continuous Probability Space

Key point: **Axioms of Probability applies to continuous probability spaces.**

Example: **Suppose we know that the probability that a certain machine lasts more than or equal to x years is :**

$$P(T \geq x) = \frac{1}{2^x}, \quad T: \text{ Lifetime}$$

Find the following sets:

a) $P(T \geq 1)$

b) $P(T \geq 2)$

c) $P(1 \leq T \leq 2)$

Conditional Probability

Suppose that in a certain city, 0.3 percent of the days are rainy.

$$P(\text{Rain}) = 0.3$$

The probability that it rains given that it is cloudy might be:

$$P(\text{Rain}|\text{Cloudy}) = 0.9$$

Conditional probability: The probability A given B

$$P(A|B)$$

Conditional Probability

Example. **Roll a fair die, what is the probability that the outcome is an even number given it was less than or equal to 3, i.e.,**

$$A = \{2, 4, 6\}$$

$$B = \{1, 2, 3\} \quad \longrightarrow \quad \text{has occurred}$$

Note:

$$\frac{|A \cap B|}{|B|} = \frac{\frac{|A \cap B|}{|S|}}{\frac{|B|}{|S|}} = \frac{P(A \cap B)}{P(B)}.$$

Conditional Probability

Definition: The **Conditional Probability** $P(A|B)$, the probability that A occurred given that B has occurred is given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \text{ when } P(B) > 0.$$

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Summary of Probability

Axioms of Probability:

a) For any event A, $P(A) \geq 0.$

b) Probability of the sample space S is $P(S) = 1.$

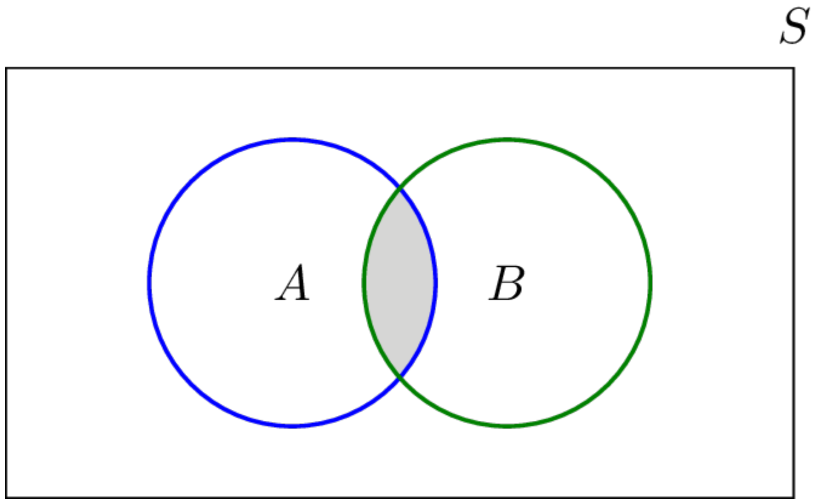
c) If $A_1, A_2, A_3, \cdots$ are disjoint events, then

$$P(A_1 \cup A_2 \cup A_3 \cdots) = P(A_1) + P(A_2) + P(A_3) + \cdots$$

Conditional Probability

If A and B are two events in a sample space S, then

$$P(A \text{ given } B) = P(A|B) = \frac{P(A \cap B)}{P(B)}, \text{ when } P(B) > 0.$$



Conditional Probability

Conditional probability satisfies the probability axioms :

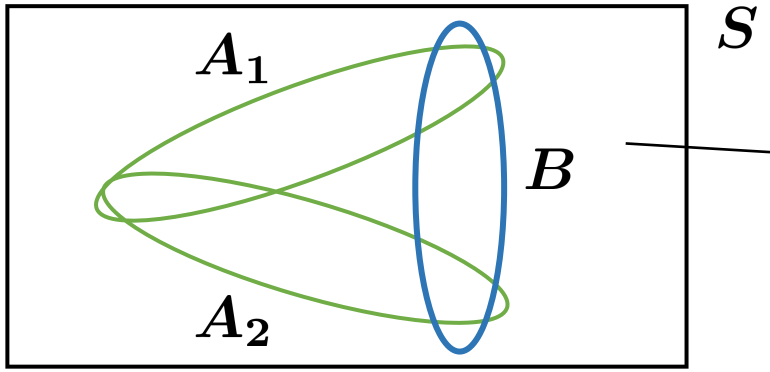
a) For any event A, $P(A|B) \geq 0.$

b) Conditional probability of B given B is $P(B|B) = 1.$

Conditional Probability

c) If $A_1, A_2, A_3, \cdots$ are disjoint events, then

$$P(A_1 \cup A_2 \cup A_3 \cdots |B) = P(A_1|B) + P(A_2|B) + P(A_3|B) + \cdots .$$



$$P(A_1 \cup A_2|B) = P(A_1|B) + P(A_2|B).$$

Conditional Probability

Example. Roll two dice $X_1, \ X_2$.

A : 3 dots are shown at least on one die

$$X_1 = 3 \ \text{ or } \ X_2 = 3,$$

B : $X_1 + X_2 = 6,$

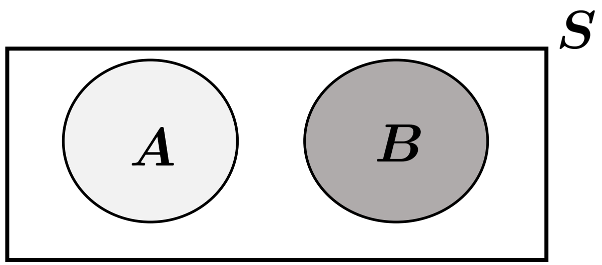
Find $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$.

Conditional Probability

Special cases:

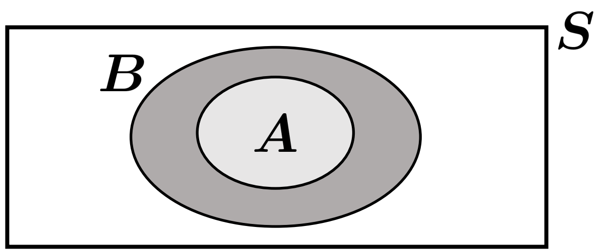
1) A and B are disjoint:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{P(B)} = 0.$$



2) $A \subset B$

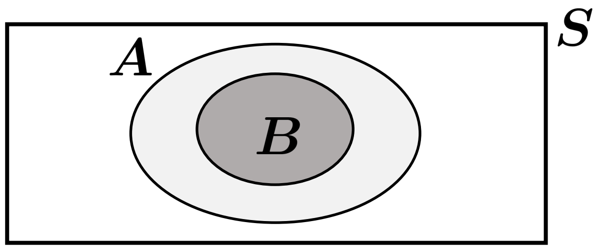
$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}.$$



Conditional Probability

3) $B \subset A$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1.$$



Conditional Probability

Example. **Roll a die, what is the probability that it is larger than or equal to 5, given that it is an even number ?**

$A: \;\; \geq 5,$ $\qquad\qquad$ $B:$ **even number**

Conditional Probability

Definition: Two events A and B are independent if and only if

$P(A|B) = P(A),$ equivalently $P(A \cap B) = P(A)P(B).$

$$\Rightarrow P(A|B)P(B) = P(A)P(B) \;\Rightarrow\; P(A \cap B) = P(A)P(B).$$

Conditional Probability

Warning!

Disjoint (mutually exclusive) $\neq$ **Independent**

Disjoint: $A \cap B = \emptyset, \quad P(A \cup B) = P(A) + P(B)$

Independent: $P(A \cap B) = P(A)P(B), \quad P(A|B) = P(A), P(B|A) = P(B)$

Conditional Probability

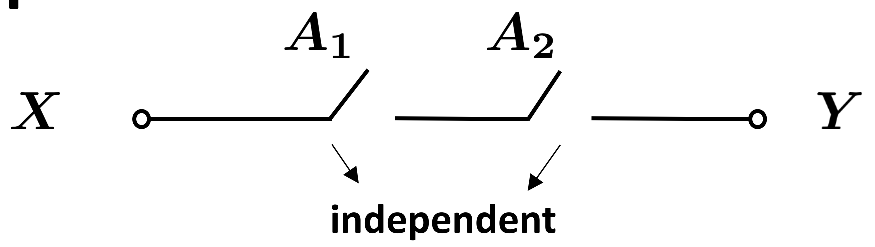
Suppose A and B are disjoint:

If $P(B) \neq 0, \quad P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0}{P(B)} = 0.$

If $P(A) \neq 0, P(B) \neq 0$ $\&$ **disjoint** $\Rightarrow$ **Not independent.**

Conditional Probability

Example:



$$P(A_1 \text{ is closed }) = P(A_2 \text{ is closed }) = p, \quad 0 < p < 1$$

B **: Event that there is a connection from node X to node Y.**

Conditional Probability

Remark:

1) $P(A|B) = P(A) \;\Rightarrow\; P(B|A) = P(B), \quad (P(A), P(B) \neq 0)$

2) If $A \;\&\; B$ are independent, then

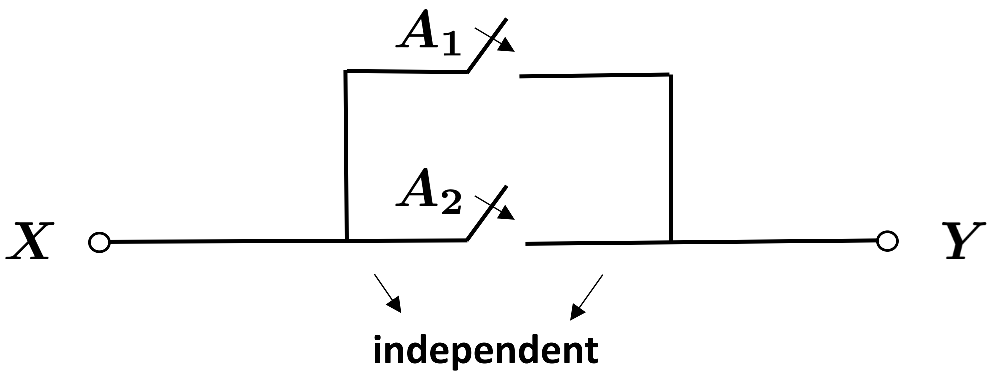
   a) $A^c \;\&\; B$ are independent.

   b) $A \;\&\; B^c$ are independent.

   c) $A^c \;\&\; B^c$ are independent.

Conditional Probability

Example:



$E_1 : P(A_1 \text{ closed }) = p_1$

$E_2 : P(A_2 \text{ closed }) = p_2$

Find $P(E_1 \cup E_2)$.

Note that E_1^c & E_2^c **are independent.**

Conditional Probability

If A and B are two events in a sample space S, then the **conditional probability** of A **given** B is defined as

$$P(A \text{ given } B) = P(A|B) = \frac{P(A \cap B)}{P(B)}, \text{ when } P(B) > 0.$$

Independence

Two events A and B are independent if and only if

$$P(A \cap B) = P(A)P(B)$$

Independence

Three events A, B and C are independent if all of the following conditions hold

$$P(A \cap B) = P(A)P(B),$$

$$P(A \cap C) = P(A)P(C),$$

$$P(B \cap C) = P(B)P(C),$$

$$P(A \cap B \cap C) = P(A)P(B)P(C).$$

Independence

Example.

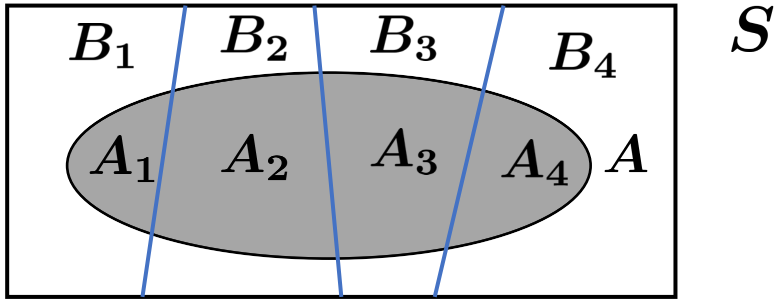
Two darts players throw alternately at a board and the first to score a bull wins. On each of their throws player A has probability p_A and player B p_B of success; the result of different throws are independent. If A starts, calculate the probability that he/she wins.

Law of Total Probability

Let $B_1, B_2, B_3, \cdots$ be a partition of the sample space S with $P(B_i) > 0$.

For any event A we have

$$P(A) = \sum_{i=1}^{m} P(A \cap B_i) = \sum_{i=1}^{m} P(A|B_i)P(B_i).$$



Law of Total Probability

$$A_i = A \cap B_i$$

$$P(A) = P(A_1) + P(A_2) + P(A_3) + \cdots$$

$$P(A) = \sum_{i=1}^{m} P(A_i) = \sum_{i=1}^{m} P(A \cap B_i).$$

$$P(A|B_i) = \frac{P(A \cap B_i)}{P(B_i)} \Rightarrow P(A \cap B_i) = P(A|B_i)P(B_i).$$

Orchestrated Conversation: Law of Total Probability

Example:

Three coins are in a bag:

a) Coin 1: probability of heads is 0.9. $P(H|C_1) = 0.9$

b) Coin 2: probability of heads is 0.6. $P(H|C_2) = 0.6$

c) Coin 3: probability of heads is 0.3. $P(H|C_3) = 0.3$

I draw a coin at random and toss it. What is the probability of heads?

Bayes' Rule

For any two events A and B, where $P(A) \neq 0$, we have

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}.$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(A|B)P(B).$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

Bayes' Rule

If $B_1, B_2, B_3, \cdots$ is a partition of the sample space S, and A is any event with $P(A) > 0$, we have

$$P(B_j|A) = \frac{P(A|B_j)P(B_j)}{P(A)} = \frac{P(A|B_j)P(B_j)}{\sum_i P(A|B_i)P(B_i)}.$$

Orchestrated Conversation: Bayes' Rule

Example.

In the previous problem, suppose that we know the result is heads; what is the probability that Coin 1 was chosen?

Bayes' Rule

Example. In a communication system a zero or a one is transmitted with the probability $\mathrm{Prob}\{X = 0\} = p_0$ or $\mathrm{Prob}\{X = 1\} = 1 - p_0 = p_1$ respectively. Due to the noise in the channel, a zero can be received as a one, with probability β and a one can be received as a zero also with probability β. A one was observed, what is the probability that a one was transmitted?

Conditional Independence

Two events A and B are independent if and only if

$$P(A \cap B) = P(A)P(B), \quad \text{or equivalently, } P(A|B) = P(A).$$

Two events A and B are conditionally independent given an event C if and only if

$$P(A \cap B|C) = P(A|C)P(B|C).$$

Chain Rule for Conditional Probability

$$P(B|A) = \frac{P(B \cap A)}{P(A)} \;\Rightarrow\; P(B \cap A) = P(A)P(B|A).$$

We can extend this to 3 or more events:

$$P(B \cap A \cap C) = P(A)P(B|A)P(C|A \cap B).$$

Summary of this Lesson

You explored the basic concepts of probability that will provide a foundation for discussion of probability throughout this term. You also reviewed mathematical concepts needed to understand probability theory. You had the opportunity to examine random experiments and the axioms of probability. Additionally, you explored discrete and continuous probability models and discussed conditional probability.

Post-work for Lessons 1-3

- Complete homework assignment for Lessons 1-3: HW#1

Go to the online classroom for details.

To Prepare for the Next Lesson

- Read Chapter 2 in your online textbook:

  https://www.probabilitycourse.com/chapter2/2_1_0_counting.php

- Complete the Pre-work for Lesson 4.

Visit the online classroom for details.