UMASS AMHERST

ECE 603
Probability and Random Processes

Lessons 18-20
Chapter 10
Introduction to Random Processes

Objectives

- Examine the concept of random processes.
- Apply methods of analysis to multiple random processes.
- Explore mean and correlation functions.
- Examine stationary processes.

Rationale

In the real world, you may be interested in multiple observations of random values over a period of time. One example might be watching a company's stock price fluctuate over time.

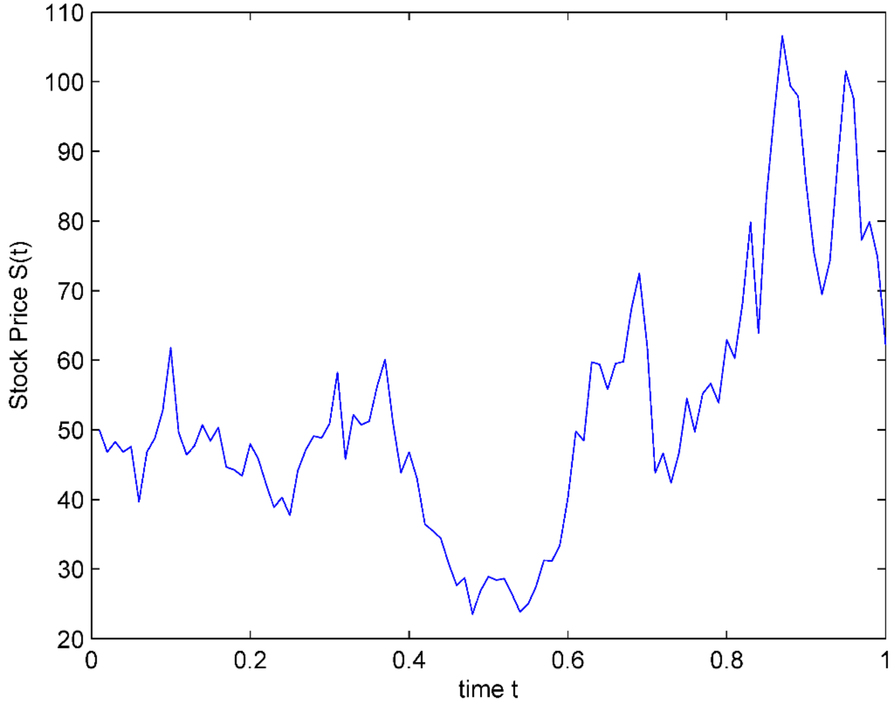
Knowing that random processes are collections of random variables, you possess the knowledge needed to analyze these random processes.

Prior Learning

- Basic Concepts
- Counting Methods
- Random Variables
- Access to the online textbook: https://www.probabilitycourse.com/

Random Process

Read book.

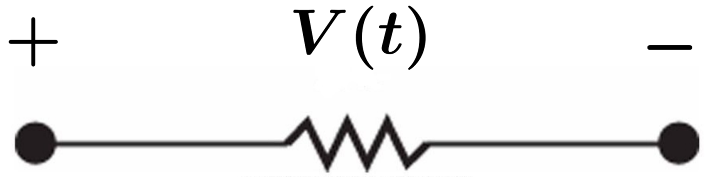


Random Process

- Random process (stochastic process)

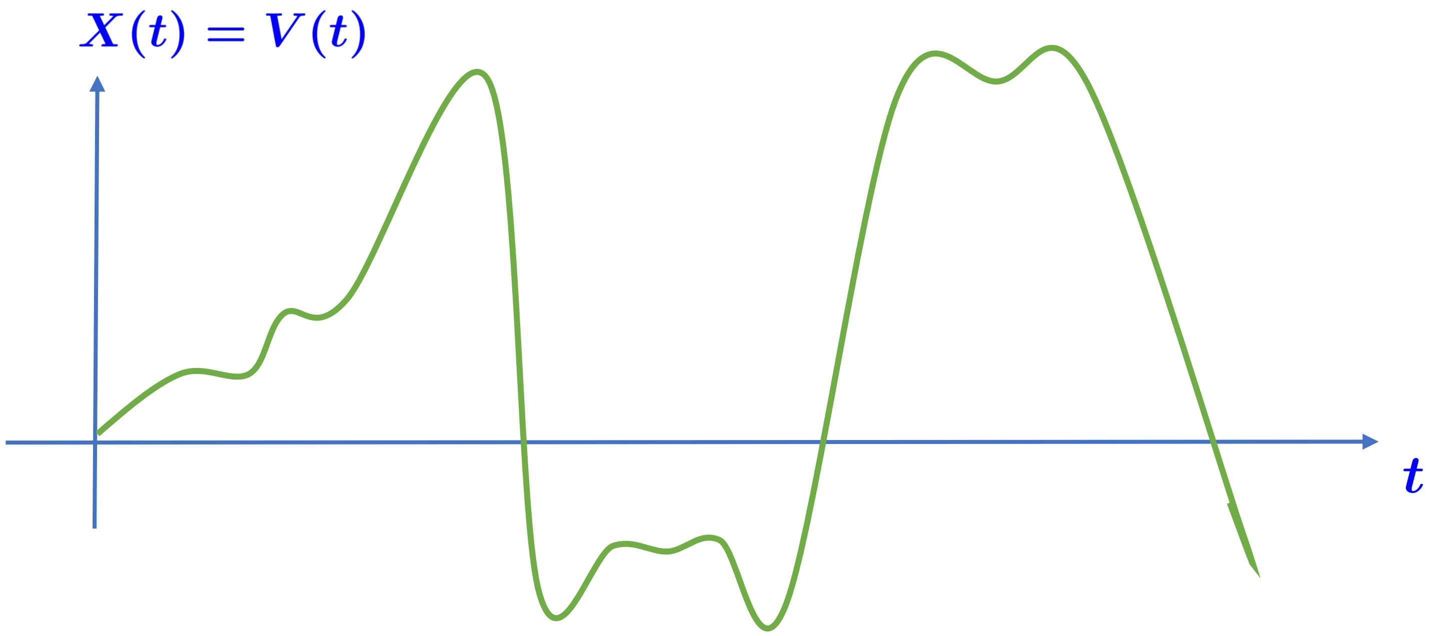
  A family of random variable (usually infinite)

- Example. Many random phenomena that occur in nature are function of time.



Random Process

The thermal noise voltage generated in a resistor

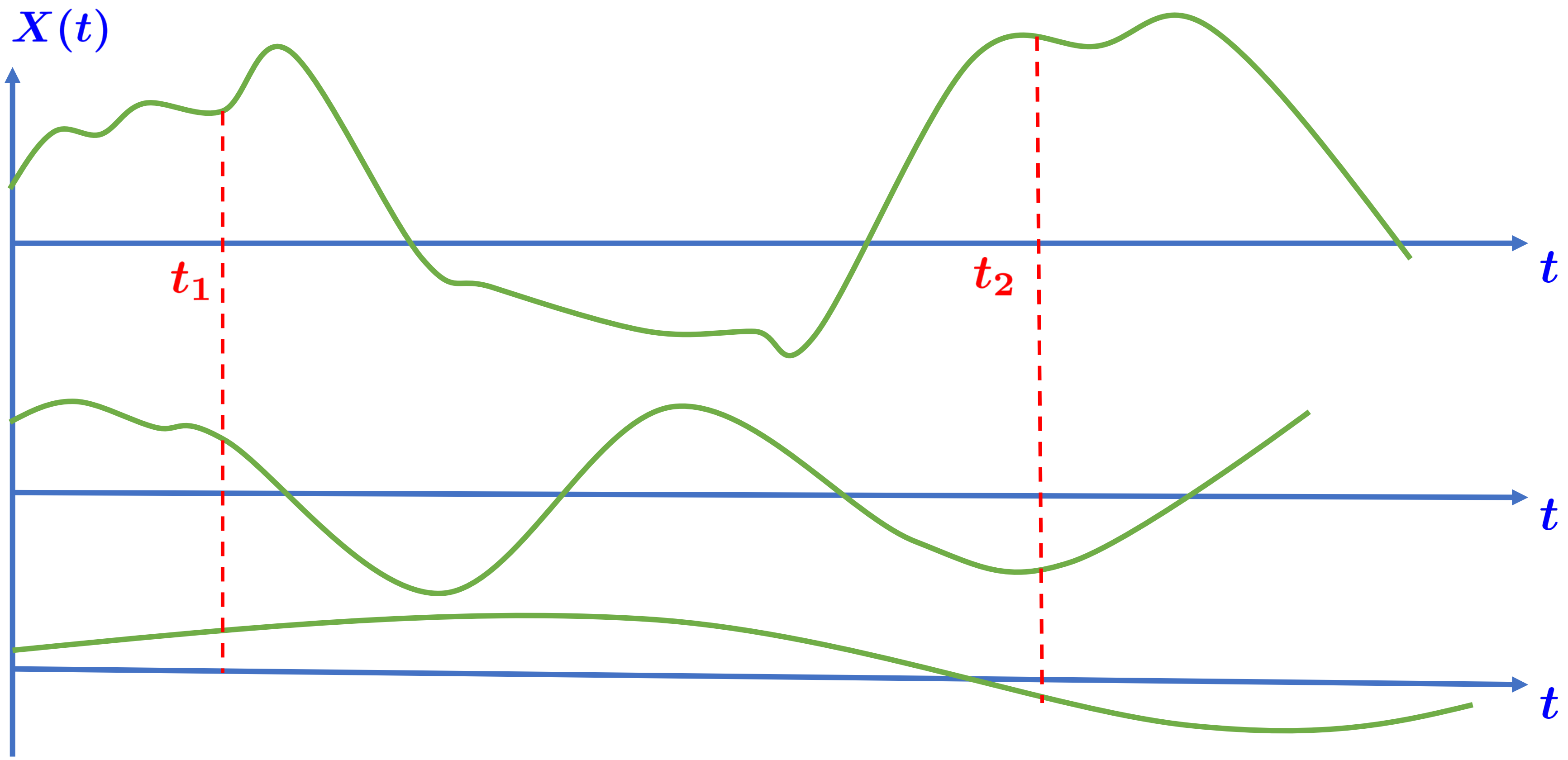


Random Process

For any fixed t, the voltage $V(t)$ is a random variable $V(t)$. This an example of random process. Thus, a random process (r.p.) is just a random function.

Example. $X(t)$ is a temperature in Amherst at time t

Random Process



Random Process

At any time $t = t_1$, $X(t_1)$ is a random variable

- Each of the possible outcomes (functions) is called a sample function.

These are examples of continuous-time random processes:

$$\{X(t), t \in [0, 1]\}$$

Random Process

Discrete-time random process:

$$\{X[n], n \in \mathbb{Z}\} \quad \text{or} \quad \{X_n, n \in \mathbb{Z}\}.$$

$$\{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$$

Random Process

A random process is a collection of random variables usually indexed by time.

$$\{X(t), t \in J\}.$$

Random Process

A continuous-time random process is a random process $\{X(t), t \in J\}$, where J is an interval on the real line such as $[-1, 1]$, $[0, \infty)$, $(-\infty, \infty)$, etc.

A discrete-time random process (or a random sequence) is a random process $\{X(n) = X_n, n \in J\}$, where J is a countable set such as $\mathbb{N}$ or $\mathbb{Z}$.

Random Process

A random process is a random function of time.

Example. You have 1000 dollars to put in an account with interest rate R, compounded annually. That is, if X_n is the value of the account at year n, then

$$X_n = 1000(1+R)^n, \qquad \text{for } n = 0, 1, 2, \cdots .$$

The value of R is a random variable that is determined when you put the money in the bank, but it does not change after that. In particular, assume that

$$R \sim Uniform(0.04, 0.05).$$

Orchestrated Conversation: Random Process

a) Find all possible sample functions for the random process

$$\{X_n, n = 0, 1, 2, \ldots\}.$$

b) Find the expected value of your account at year three. That is, find $E[X_3]$.

Random Process

Example. Let $\{X(t), t \in [0, \infty)\}$ be defined as

$$X(t) = A + Bt, \qquad \text{for all } t \in [0, \infty),$$

where A and B are independent normal $N(1,1)$ random variables.

a) Find all possible sample functions for this random process.

b) Define the random variable $Y = X(1)$. Find the PDF of Y.

c) Let also $Z = X(2)$. Find $E[YZ]$.

Mean and Correlation Functions

Mean Function of a Random Process

For a random process $\{X(t), t \in J\}$ the **mean function** $\mu_X(t) : J \to \mathbb{R}$, is defined as

$$\mu_X(t) = E[X(t)].$$

Mean and Correlation Functions

For a random process $\{X(t), t \in J\}$, the **autocorrelation function** or, simply, the **correlation function**, $R_X(t_1, t_2)$, is defined by

$$R_X(t_1, t_2) = E[X(t_1)X(t_2)], \quad \text{for } t_1, t_2 \in J.$$

Mean and Correlation Functions

For a random process $\{X(t), t \in J\}$, the autocovariance function or, simply, the covariance function, $C_X(t_1, t_2)$, is defined by

$$\begin{aligned} C_X(t_1, t_2) &= \mathrm{Cov}\big(X(t_1), X(t_2)\big) \\ &= R_X(t_1, t_2) - \mu_X(t_1)\mu_X(t_2), \quad \text{for } t_1, t_2 \in J. \end{aligned}$$

Multiple Random Processes

For two random processes $\{X(t), t \in J\}$ and $\{Y(t), t \in J\}$:

- the **cross-correlation** function $R_{XY}(t_1, t_2)$, is defined by

$$R_{XY}(t_1, t_2) = E[X(t_1)Y(t_2)], \quad \text{for } t_1, t_2 \in J;$$

- the **cross-covariance** function $C_{XY}(t_1, t_2)$, is defined by

$$\begin{aligned} C_{XY}(t_1, t_2) &= \text{Cov}\big(X(t_1), Y(t_2)\big) \\ &= R_{XY}(t_1, t_2) - \mu_X(t_1)\mu_Y(t_2), \quad \text{for } t_1, t_2 \in J. \end{aligned}$$

Multiple Random Processes

Two random processes $\{X(t), t \in J\}$ and $\{Y(t), t \in J'\}$ are said to be independent if, for all

$$t_1, t_2, \ldots, t_m \in J$$
$$\text{and}$$
$$t'_1, t'_2, \ldots, t'_n \in J',$$

the set of random variables

$$X(t_1), X(t_2), \cdots, X(t_m)$$

are independent of the set of random variables

$$Y(t'_1), Y(t'_2), \cdots, Y(t'_n).$$

Stationary Processes

A continuous-time random process $\{X(t), t \in \mathbb{R}\}$ is **strict-sense stationary** or simply **stationary** if, for all $t_1, t_2, \cdots, t_r \in \mathbb{R}$ and all $\Delta \in \mathbb{R}$, the joint CDF of

$$X(t_1), X(t_2), \cdots, X(t_r)$$

is the same as the joint CDF of

$$X(t_1 + \Delta), X(t_2 + \Delta), \cdots, X(t_r + \Delta).$$

Stationary Processes

That is, for all real numbers $x_1, x_2, \cdots, x_r$, we have

$$F_{X(t_1)X(t_2)\cdots X(t_r)}(x_1, x_2, \cdots, x_r)$$
$$= F_{X(t_1+\Delta)X(t_2+\Delta)\cdots X(t_r+\Delta)}(x_1, x_2, \cdots, x_r).$$

Stationary Processes

A discrete-time random process $\{X(n), n \in \mathbb{Z}\}$ is **strict-sense stationary** or simply **stationary**, if for all $n_1, n_2, \cdots, n_r \in \mathbb{Z}$ and all $D \in \mathbb{Z}$, the joint CDF of

$$X(n_1), X(n_2), \cdots, X(n_r)$$

is the same as the joint CDF of

$$X(n_1 + D), X(n_2 + D), \cdots, X(n_r + D).$$

Stationary Processes

That is, for all real numbers $x_1, x_2, \cdots, x_r$, we have

$$F_{X(n_1)X(n_2)\cdots X(n_r)}(x_1, x_2, \cdots, x_n) \\ = F_{X(n_1+D)X(n_2+D)\cdots X(n_r+D)}(x_1, x_2, \cdots, x_r).$$

Stationary Processes

Weak-Sense Stationary Processes:

A continuous-time random process $\{X(t), t \in \mathbb{R}\}$ is **weak-sense stationary** or **wide-stationary (WSS)** if

1) $\mu_X(t) = \mu_X, \text{ for all } t \in \mathbb{R}.$

2) $R_X(t_1, t_2) = R_X(t_1 - t_2), \text{ for all } t_1, t_2 \in \mathbb{R}.$

Stationary Processes

Weak-Sense Stationary Processes:

A discrete-time random process $\{X(n), n \in \mathbb{Z}\}$ is weak-sense stationary or wide-stationary (WSS) if

1) $\mu_X(n) = \mu_X, \text{ for all } n \in \mathbb{Z}.$

2) $R_X(n_1, n_2) = R_X(n_1 - n_2), \text{ for all } n_1, n_2 \in \mathbb{Z}.$

Stationary Processes

Example. **Consider the random process** $\{X(t), t \in \mathbb{R}\}$ **defined as**

$$X(t) = \cos(t + U),$$

where $U \sim Uniform(0, 2\pi)$. **Show that** $X(t)$ **is a WSS process.**

Stationary Processes

Properties of $R_X(\tau)$:

$\tau = t_1 - t_2$

If $X(t)$ WSS, $R_X(t_1 - t_2) = R_X(\tau)$

1) $t_1 = t_2 \ \rightarrow \ E[X(t_1)^2] \geq 0 \quad \Rightarrow \ R_X(0) = E[X(t)^2] \geq 0,$

2) $R_X(-\tau) = E[X(t-\tau)X(t)] = E[X(t)X(t-\tau)] = R_X(\tau)$

$$\Rightarrow \ R_X(\tau) = R_X(-\tau), \quad \text{for all } \tau \in \mathbb{R}.$$

Stationary Processes

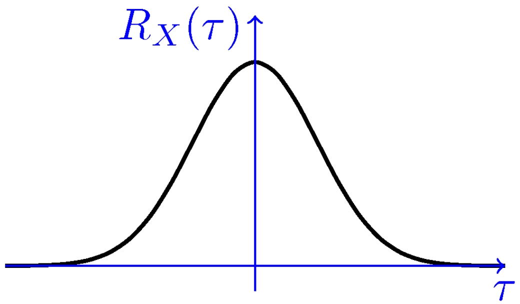
3) $|R_X(\tau)| \leq R_X(0), \quad \text{for all } \tau \in \mathbb{R}.$

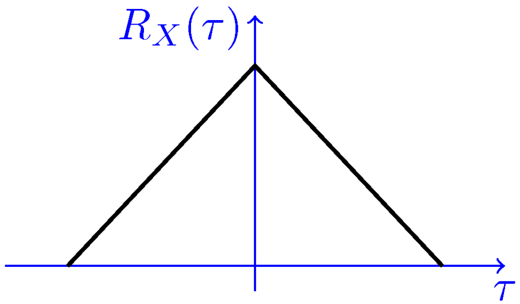
Proof:

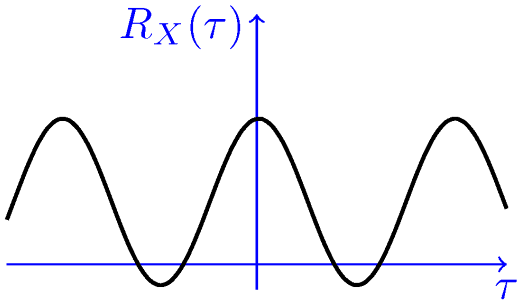
$$X = X(t)$$

$$Y = X(t-\tau)$$

$$\underbrace{|EXY|}_{R_X(\tau)} \leq \sqrt{\underbrace{E[X^2]}_{R_X(0)}\,\underbrace{E[Y^2]}_{R_X(0)}}$$







Stationary Processes

Jointly Wide-Sense Stationary Processes:

Two random processes $\{X(t), t \in \mathbb{R}\}$ and $\{Y(t), t \in \mathbb{R}\}$ are said to be jointly wide-sense stationary if

1) $X(t)$ and $Y(t)$ are each wide-sense stationary.

2) $R_{XY}(t_1, t_2) = R_{XY}(t_1 - t_2)$.

- For WSS $X(t)$ & $Y(t)$, $\rightarrow$ $R_{XY}(\tau) = R_{XY}(-\tau)$.

Stationary Processes

Cross Covariance:

$$C_{XY}(t_1, t_2) = R_{XY}(t_1, t_2) - \mu_X(t_1) \cdot \mu_Y(t_2).$$

Orchestrated Conversation: Stationary Processes

Example. Let $X(t)$ and $Y(t)$ be two jointly WSS random processes. Consider the random process $Z(t)$ defined as

$$Z(t) = X(t) + Y(t).$$

a) Find $M_Z(t)$ and $R_Z(t_1, t_2)$.

b) Is $Z(t)$ a WSS random processes?

Gaussian Random Processes

A random process $\{X(t), t \in J\}$ is said to be a **Gaussian (normal) random process** if, for all

$$t_1, t_2, \ldots, t_n \in J,$$

the random variables $X(t_1), X(t_2), \cdots, X(t_n)$ are jointly normal.

Summary of Random Process

$$X(t), \quad t \in (0, \infty) \ or \ (-\infty, \infty) \qquad \text{Continuous-time}$$

$$X(n), \quad n \in \mathbb{Z} \qquad \text{Discrete-time}$$

$$\mu_X(t) = E[X(t)] = \mu_X,$$

$$R_X(t_1, t_2) = E[X(t_1)X(t_2)] = R_X(t_1 - t_2) \quad \text{for WSS}$$

Summary of Random Process

$$R_X(0) = E[X(t)^2] \geq 0$$

$$R_X(\tau) = R_X(-\tau), \quad \text{for all } \tau \in \mathbb{R}.$$

$$|R_X(\tau)| \leq R_X(0), \quad \text{for all } \tau \in \mathbb{R}.$$

Stationary Processes

Cyclostationary Processes:

Some practical random processes have a periodic structure. That is, the statistical properties are repeated every T units of time (e.g., every T seconds).

Example. Temperature in a city

Stationary Processes

A continuous-time random process $\{X(t), t \in \mathbb{R}\}$ is cyclostationary if there exists a positive real number T such that, for all $t_1, t_2, \cdots, t_r \in \mathbb{R}$, the joint CDF

$$X(t_1), X(t_2), \cdots, X(t_r)$$

is the same as the joint CDF of

$$X(t_1 + T), X(t_2 + T), \cdots, X(t_r + T).$$

Stationary Processes

A continuous-time random process $\{X(t), t \in \mathbb{R}\}$ is **weak-sense cyclostationary** or **wide-sense cyclostationary** if there exists a positive real number T such that

1) $\mu_X(t+T) = \mu_X(t), \text{ for all } t \in \mathbb{R}.$

2) $R_X(t_1+T, t_2+T) = R_X(t_1, t_2), \text{ for all } t_1, t_2 \in \mathbb{R}.$

Stationary Processes

Derivatives and Integrals of Random Processes:

Example.

$$X(t) = At + Bt^2, \quad \text{where } A,\ B \text{ are RV.}$$

Then

$$\frac{d}{dt}X(t) = A + 2Bt.$$

We need to first talk about limits and continuity.

Stationary Processes

Mean-square continuity:

$$\lim_{\delta \to 0} E\left[\left|X(t+\delta) - X(t)\right|^2\right] = 0.$$

Stationary Processes

Example. The Poisson process is discussed in detail in Chapter 11. If $X(t)$ is a Poisson process with intensity λ , then for all $t > s \geq 0$, we have

$$X(t) - X(s) \sim Poisson\big(\lambda(t-s)\big).$$

Show that $X(t)$ is mean-square continuous at any time $t \geq 0$.

Stationary Processes

More specifically, we can write

$$\boldsymbol{E}\left[\int_0^t \boldsymbol{X}(u)du\right] = \int_0^t \boldsymbol{E}[\boldsymbol{X}(u)]du.$$

$$\boldsymbol{E}\left[\frac{d}{dt}\boldsymbol{X}(t)\right] = \frac{d}{dt}\boldsymbol{E}[\boldsymbol{X}(t)].$$

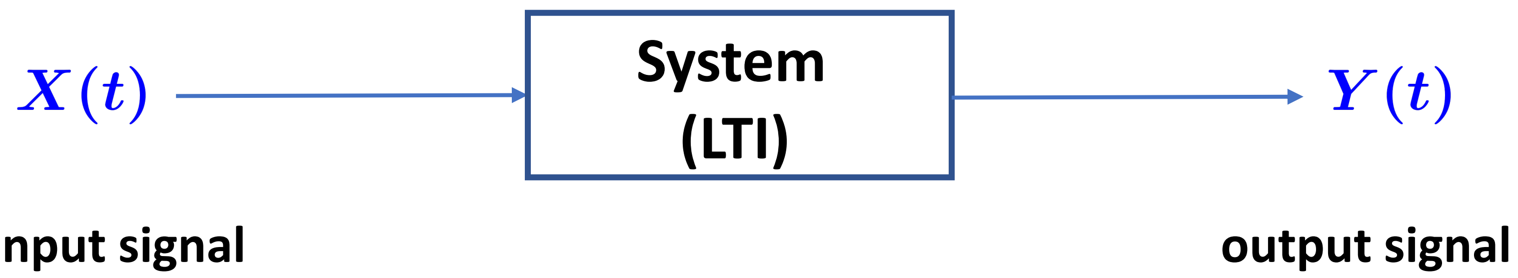
Gaussian Random Processes

A random process $\{X(t), t \in J\}$ is said to be a **Gaussian (normal) random process** if, for all

$$t_1, t_2, \ldots, t_n \in J,$$

the random variables $X(t_1), X(t_2), \cdots, X(t_n)$ are jointly normal.

Processing of Random Signals



If $X(t)$ is a random process, then $Y(t)$ is a random process.

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Power Spectral Density

Power Spectral Density (PSD)

$$S_X(f) = \mathcal{F}\{R_X(\tau)\} = \int_{-\infty}^{\infty} R_X(\tau) e^{-2j\pi f\tau} \, d\tau, \quad \text{where } j = \sqrt{-1}.$$

$$\downarrow$$

$$R_X(\tau) = \mathcal{F}^{-1}\{S_X(f)\} = \int_{-\infty}^{\infty} S_X(f) e^{2j\pi f\tau} \, df.$$

Power Spectral Density

Properties:

1) $S_X(-f) = S_X(f), \text{ for all } f.$

2) $S_X(f) \geq 0, \text{ for all } f.$

Power Spectral Density

Expected power: $\boldsymbol{E\left[X(t)^2\right]}$

$$R_X(\tau) = E\left[X(t)X(t-\tau)\right]$$

$$\Rightarrow \; R_X(0) = E\left[X(t)^2\right]$$

$$R_X(\tau) = \mathcal{F}^{-1}\{S_X(f)\} = \int_{-\infty}^{\infty} S_X(f)e^{2j\pi f\cdot\tau}\, df$$

$$\Rightarrow \; E\left[X(t)^2\right] = R_X(0) = \int_{-\infty}^{\infty} S_X(f)\, df. \qquad \text{(Power Spectral Density)}$$

Power Spectral Density

The expected power in $X(t)$ can be obtained as

$$E\left[X(t)^2\right] = R_X(0) = \int_{-\infty}^{\infty} S_X(f)\, df.$$

Power Spectral Density

Example. Consider a WSS random process $X(t)$ with

$$R_X(\tau) = e^{-a|\tau|},$$

where a is a positive real number. Find the PSD of $X(t)$.

Power Spectral Density

Cross Spectral Density:

$$S_{XY}(f) = \mathcal{F}\{R_{XY}(\tau)\} = \int_{-\infty}^{\infty} R_{XY}(\tau)e^{-2j\pi f\tau}\, d\tau.$$

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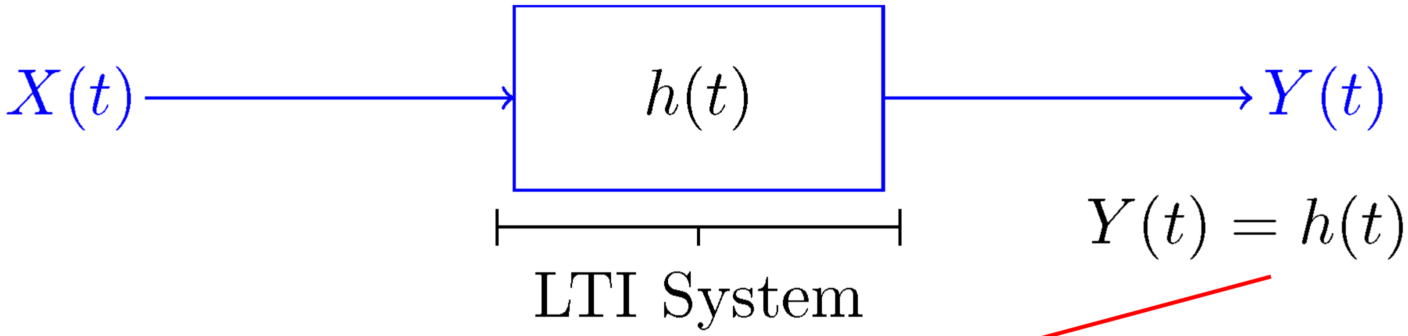
Linear Time-Invariant (LTI) Systems

Linear Time-Invariant (LTI) Systems:

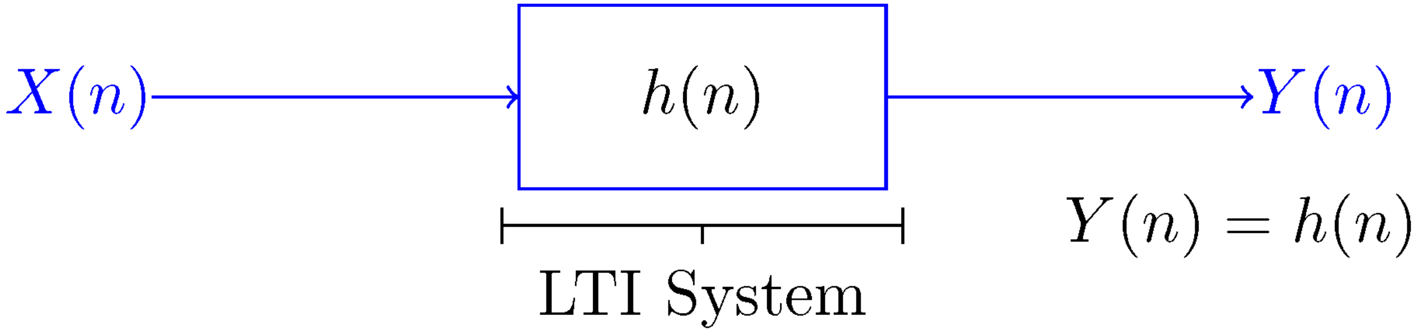


$$Y(t) = h(t) * X(t)$$

$$\underbrace{\boldsymbol{Y(t)}}_{\text{Random}} = \int_{-\infty}^{\infty} \boldsymbol{h(\alpha)}\, \underbrace{\boldsymbol{X(t-\alpha)}}_{\text{Random}} \, \boldsymbol{d\alpha} = \int_{-\infty}^{\infty} \boldsymbol{X(\alpha) h(t-\alpha)} \, \boldsymbol{d\alpha}.$$

Linear Time-Invariant (LTI) Systems

Linear Time-Invariant (LTI) Systems:



$$Y(n) = h(n) * X(n)$$

$$\begin{aligned} Y(n) = h(n) * X(n) &= X(n) * h(n) \\ &= \sum_{k=-\infty}^{\infty} h(k)X(n-k) = \sum_{k=-\infty}^{\infty} X(k)h(n-k). \end{aligned}$$

Linear Time-Invariant (LTI) Systems

LTI Systems with Random Inputs:

Let $X(t)$ be a WSS random process.

$$Y(t) = h(t) * X(t) = \int_{-\infty}^{\infty} h(\alpha) X(t-\alpha)\, d\alpha.$$

Linear Time-Invariant (LTI) Systems

The mean function of $Y(t)$:

$$\mu_Y(t) = E[Y(t)] = E\left[\int_{-\infty}^{\infty} h(\alpha)X(t-\alpha)\, d\alpha\right]$$

$$= \int_{-\infty}^{\infty} h(\alpha)\underbrace{E[X(t-\alpha)]}_{\mu_X}\, d\alpha$$

$$\Rightarrow \quad \mu_Y(t) = \mu_Y = \mu_X \int_{-\infty}^{\infty} h(\alpha)\, d\alpha.$$

Linear Time-Invariant (LTI) Systems

The cross-correlation function:

$$
\begin{aligned}
R_{XY}(t_1, t_2) = E[X(t_1)Y(t_2)] &= E\left[X(t_1)\int_{-\infty}^{\infty} h(\alpha)X(t_2-\alpha)\,d\alpha\right] \\
&= E\left[\int_{-\infty}^{\infty} h(\alpha)X(t_1)X(t_2-\alpha)\,d\alpha\right] \\
&= \int_{-\infty}^{\infty} h(\alpha)\,\underbrace{E[X(t_1)X(t_2-\alpha)]}_{R_X(t_1-t_2+\alpha)}\,d\alpha
\end{aligned}
$$

Linear Time-Invariant (LTI) Systems

The cross-correlation function:

$$\Rightarrow \; R_{XY}(\underbrace{t_1, t_2}_{\tau}) = \int_{-\infty}^{\infty} h(\alpha) R_X(\underbrace{t_1 - t_2}_{\tau} + \alpha)\, d\alpha$$

$$\begin{aligned} R_{XY}(\tau) &= \int_{-\infty}^{\infty} h(\alpha) R_X(\tau + \alpha)\, d\alpha \\ &= h(\tau) * R_X(-\tau) = h(-\tau) * R_X(\tau). \end{aligned}$$

Linear Time-Invariant (LTI) Systems

Similarly

$$\begin{aligned}
R_Y(t_1, t_2) &= E[Y(t_1)Y(t_2)] \\
&= E\left[\int_{-\infty}^{\infty} h(\alpha)X(t_1 - \alpha)\, d\alpha \int_{-\infty}^{\infty} h(\beta)X(t_2 - \beta)\, d\beta\right] \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha)h(\beta)E[X(t_1 - \alpha)X(t_2 - \beta)]\, d\alpha d\beta \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha)h(\beta)R_X(t_1 - t_2 - \alpha + \beta)\, d\alpha d\beta \\
&= h(\tau) * h(-\tau) * R_X(\tau).
\end{aligned}$$

Linear Time-Invariant (LTI) Systems

Theorem. Let $X(t)$ be a WSS random process and $Y(t)$ be given by

$$Y(t) = h(t) * X(t),$$

Where $h(t)$ is the impulse response of the system. Then $X(t)$ and $Y(t)$ are jointly WSS. Moreover,

Linear Time-Invariant (LTI) Systems

1. $\mu_Y(t) = \mu_Y = \mu_X \int_{-\infty}^{\infty} h(\alpha)\, d\alpha$
2. $R_{XY}(\tau) = h(-\tau) * R_X(\tau) = \int_{-\infty}^{\infty} h(-\alpha) R_X(t-\alpha)\, d\alpha$
3. $R_Y(\tau) = h(\tau) * h(-\tau) * R_X(\tau)$

Linear Time-Invariant (LTI) Systems

Frequency Domain Analysis: let $H(f)$ be the Fourier transform of $h(t)$.

$$H(f) = \mathcal{F}\{h(t)\} = \int_{-\infty}^{\infty} h(t)e^{-2j\pi ft}\,dt.$$

$$H(0) = \int_{-\infty}^{\infty} h(t)\,dt.$$

$$\mu_Y = \mu_X \int_{-\infty}^{\infty} h(\alpha)\,d\alpha, \quad \Rightarrow \quad \mu_Y = \mu_X H(0).$$

Linear Time-Invariant (LTI) Systems

$$R_{XY}(\tau) = R_X(\tau) * h(-\tau),$$

by taking the Fourier transform from both sides

$$\mathcal{F}\{R_{XY}(\tau)\} = \mathcal{F}\{R_X(\tau)\} \cdot \mathcal{F}\{h(-\tau)\},$$

$$\Rightarrow \; S_{XY}(f) = S_X(f)H(-f) = S_X(f)H^*(f).$$

Linear Time-Invariant (LTI) Systems

$$R_Y(\tau) = h(\tau) * h(-\tau) * R_X(\tau),$$

by taking the Fourier transform from both sides

$$S_Y(f) = S_X(f)H^*(f)H(f) = S_X(f)|H(f)|^2.,$$

$$\Rightarrow \; S_Y(f) = S_X(f)|H(f)|^2.$$

Linear Time-Invariant (LTI) Systems

Example. Let $X(t)$ be a zero-mean WSS process with $R_X(\tau) = e^{-|\tau|}$. $X(t)$ is input to an LTI system with

$$|H(f)| = \begin{cases} \sqrt{1 + 4\pi^2 f^2} & |f| < 2 \\ 0 & \text{otherwise} \end{cases}$$

Let $Y(t)$ be the output.

a) Find $\mu_Y(t) = E[Y(t)]$.

b) Find $R_Y(\tau)$.

c) Find $E[Y(t)^2]$.

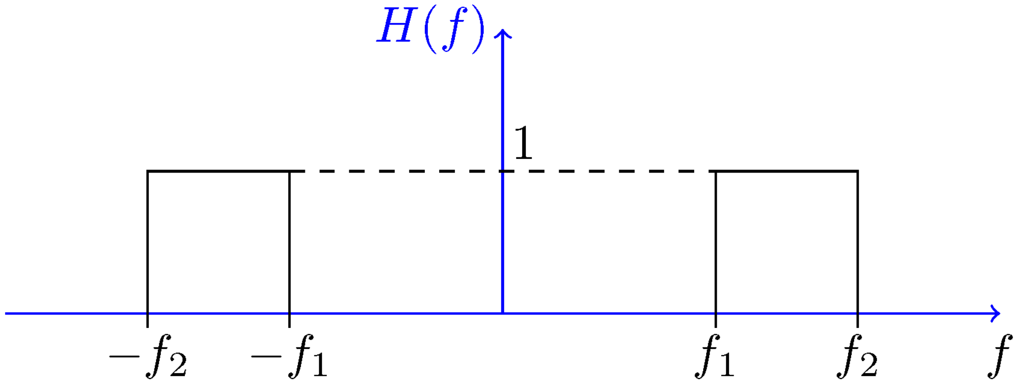
Power in a Frequency Band

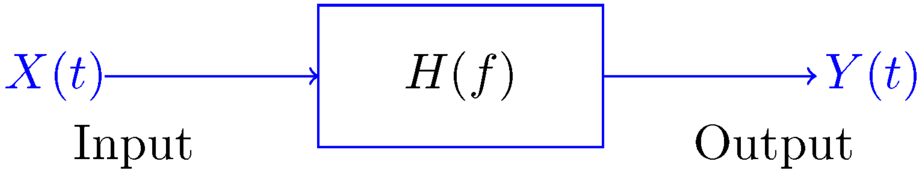
Total Power:

$$E[X(t)^2] = \int_{-\infty}^{\infty} S_X(f)\, df$$

Power in a Frequency Band

Power in frequency range f_1 to f_2 : $f_1 < |f| < f_2$





Power in a Frequency Band

$$S_Y(f) = S_X(f)|H(f)|^2 = \begin{cases} S_X(f) & f_1 < |f| < f_2 \\ 0 & \text{otherwise} \end{cases}$$

Thus, the power in $Y(t)$ is

$$E[Y(t)^2] = \int_{-\infty}^{\infty} S_Y(f)\, df = \int_{-f_2}^{-f_1} S_X(f)\, df + \int_{f_1}^{f_2} S_X(f)\, df$$

$$= 2\int_{f_1}^{f_2} S_X(f)\, df \quad \big(\text{since } S_X(-f) = S_X(f)\big)$$

Power in a Frequency Band

Gaussian Processes through LTI Systems:

$$X(t) \longrightarrow \boxed{h(t)} \longrightarrow Y(t)$$

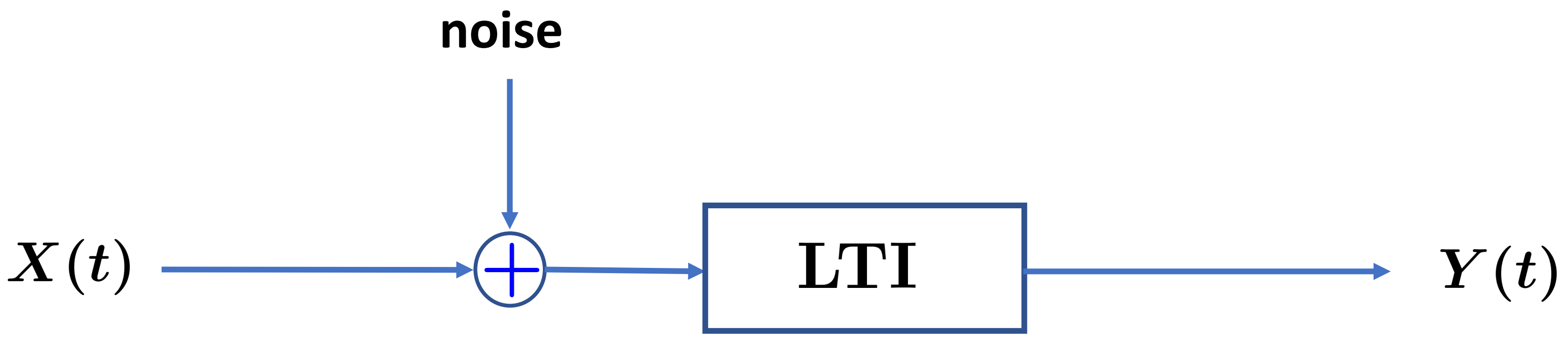
Gaussian WSS **WSS**

$$\begin{aligned} Y(t) &= h(t) * X(t) \\ &= \int_{-\infty}^{\infty} h(\alpha)X(t-\alpha)\, d\alpha. \end{aligned}$$

Linear combination of $X(t - \alpha_i)$

Power in a Frequency Band

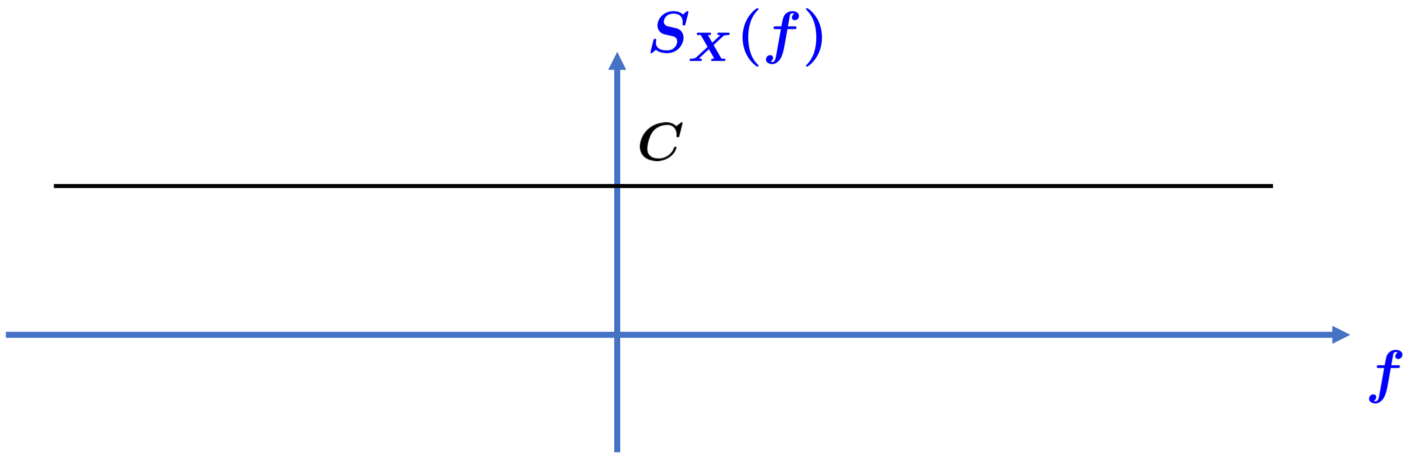
Example.



White Processes

A process $X(t)$ is called a **white process** if

$$S_X(f) = C \quad \text{for some } C \in \mathbb{R}.$$



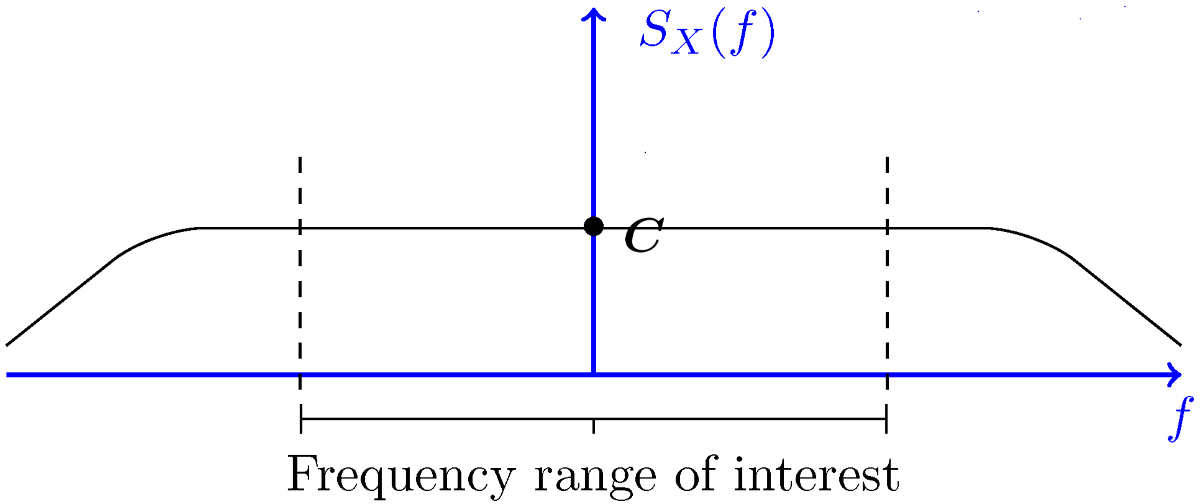
White Processes

$$P_X = E\left[X(t)^2\right] = \int_{-\infty}^{\infty} S_X(f)\, df = \infty.$$

It is Not a meaningful physical process.

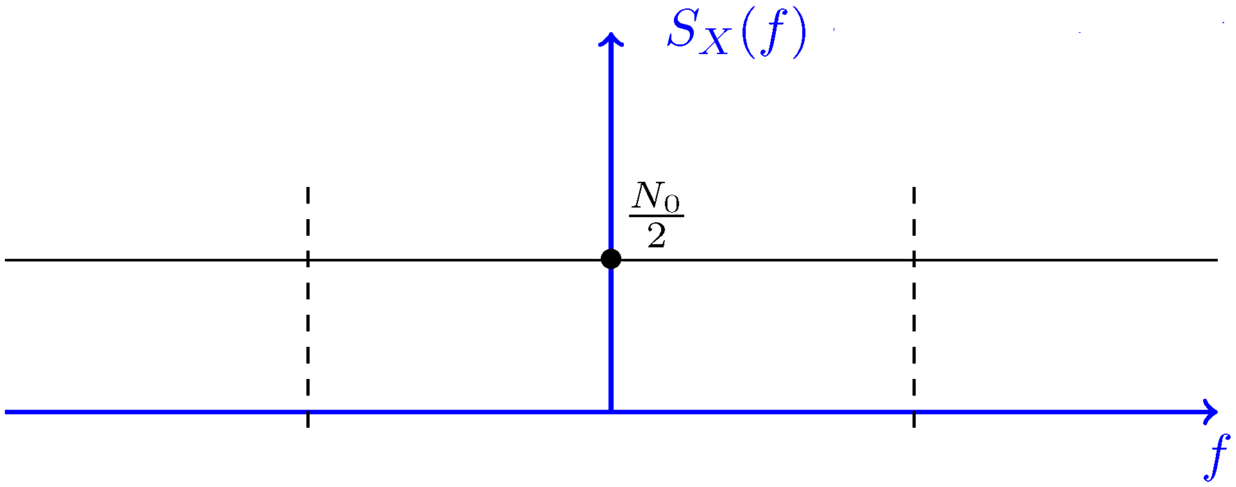
White Processes

In reality:



➢ **The constant C is usually denoted as $\frac{N_0}{2}$ for white noise.**

White Processes



$$R_X(\tau) = \mathcal{F}^{-1}\left\{\frac{N_0}{2}\right\} = \frac{N_0}{2}\delta(\tau),$$

White Processes

Where $\delta(\tau)$ is the dirac delta function

$$\delta(x) = \begin{cases} \infty & x = 0 \\ 0 & \text{otherwise} \end{cases}$$

Thus $R_X(\tau) = 0$ for any $\tau \neq 0$.

- The thermal noise ($n(t)$) that affects electronic systems has a very wide range BW and can be modeled by white noise.

White Processes

Properties of thermal noise, $n(t)$:

1) $n(t)$ is WSS.
2) $\mu_n = E[n(t)] = 0.$
3) $n(t)$ is a Gaussian Process.
4) $n(t)$ is white $S_n(f) = \dfrac{kT}{2}.$

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Filtered Noise Process

Usually the white noise generated in one stage of the system is filtered by the next stage.

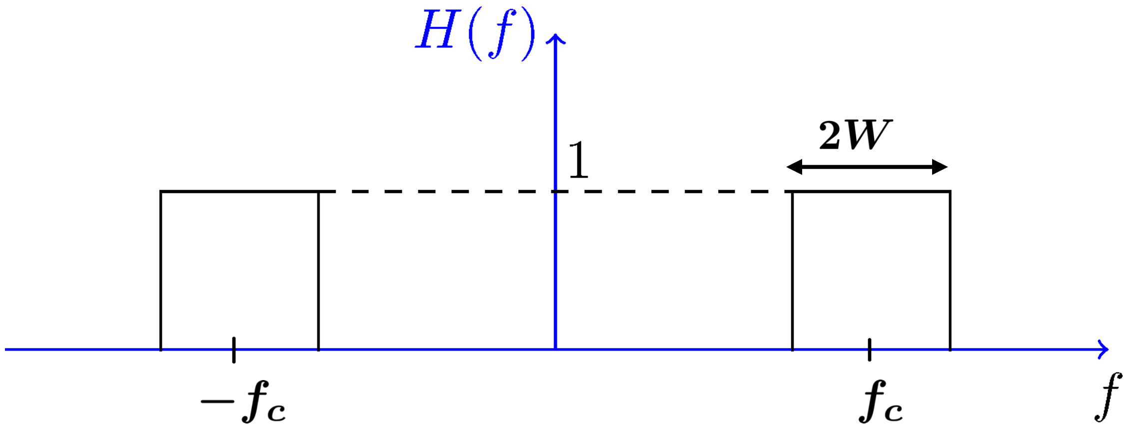
Band Pass Process:

$$n(t) \longrightarrow \boxed{H(f)} \longrightarrow X(t)$$

$$S_X(f) = S_n(f)|H(f)|^2 = \frac{N_0}{2}|H(f)|^2.$$

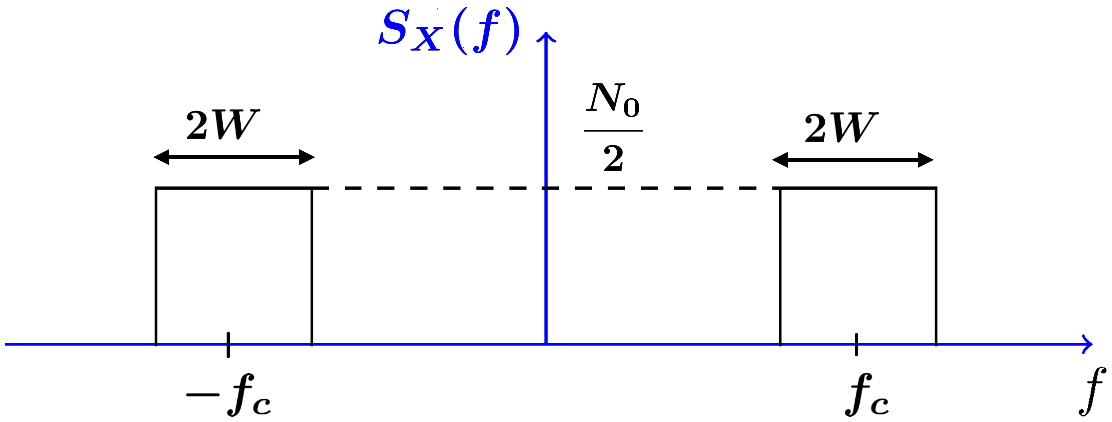
Filtered Noise Process

Example. if



Filtered Noise Process

then



Filtered noise (a Band pass signal)

Post-work for Lesson

- **Complete homework assignment for Lessons 18-20:**

HW#10

Go to the online classroom for details.

To Prepare for the Next Lesson

- Read Chapter 11 in your online textbook:

  https://www.probabilitycourse.com/chapter11/11_0_0_intro.php

- Complete the Pre-work for Lessons 21-24.

Visit the online classroom for details.