Artificial Intelligence
A Modern Approach
Third Edition

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1 INTRODUCTION

In which we try to explain why we consider artificial intelligence to be a subject most worthy of study, and in which we try to decide what exactly it is, this being a good thing to decide before embarking.

INTELLIGENCE

We call ourselves *Homo sapiens*—man the wise—because our **intelligence** is so important to us. For thousands of years, we have tried to understand *how we think*; that is, how a mere handful of matter can perceive, understand, predict, and manipulate a world far larger and more complicated than itself. The field of **artificial intelligence**, or AI, goes further still: it attempts not just to understand but also to *build* intelligent entities.

ARTIFICIAL INTELLIGENCE

AI is one of the newest fields in science and engineering. Work started in earnest soon after World War II, and the name itself was coined in 1956. Along with molecular biology, AI is regularly cited as the "field I would most like to be in" by scientists in other disciplines. A student in physics might reasonably feel that all the good ideas have already been taken by Galileo, Newton, Einstein, and the rest. AI, on the other hand, still has openings for several full-time Einsteins and Edisons.

AI currently encompasses a huge variety of subfields, ranging from the general (learning and perception) to the specific, such as playing chess, proving mathematical theorems, writing poetry, driving a car on a crowded street, and diagnosing diseases. AI is relevant to any intellectual task; it is truly a universal field.

1.1 WHAT IS AI?

We have claimed that AI is exciting, but we have not said what it *is*. In Figure 1.1 we see eight definitions of AI, laid out along two dimensions. The definitions on top are concerned with *thought processes* and *reasoning*, whereas the ones on the bottom address *behavior*. The definitions on the left measure success in terms of fidelity to *human* performance, whereas the ones on the right measure against an *ideal* performance measure, called **rationality**. A system is rational if it does the "right thing," given what it knows.

RATIONALITY

Historically, all four approaches to AI have been followed, each by different people with different methods. A human-centered approach must be in part an empirical science, in-

Thinking Humanly	**Thinking Rationally**
"The exciting new effort to make computers think … *machines with minds*, in the full and literal sense." (Haugeland, 1985) "[The automation of] activities that we associate with human thinking, activities such as decision-making, problem solving, learning …" (Bellman, 1978)	"The study of mental faculties through the use of computational models." (Charniak and McDermott, 1985) "The study of the computations that make it possible to perceive, reason, and act." (Winston, 1992)
Acting Humanly	**Acting Rationally**
"The art of creating machines that perform functions that require intelligence when performed by people." (Kurzweil, 1990) "The study of how to make computers do things at which, at the moment, people are better." (Rich and Knight, 1991)	"Computational Intelligence is the study of the design of intelligent agents." (Poole *et al.*, 1998) "AI …is concerned with intelligent behavior in artifacts." (Nilsson, 1998)

Figure 1.1   Some definitions of artificial intelligence, organized into four categories.

volving observations and hypotheses about human behavior. A rationalist[1] approach involves a combination of mathematics and engineering. The various group have both disparaged and helped each other. Let us look at the four approaches in more detail.

1.1.1   Acting humanly: The Turing Test approach

TURING TEST

The **Turing Test**, proposed by Alan Turing (1950), was designed to provide a satisfactory operational definition of intelligence. A computer passes the test if a human interrogator, after posing some written questions, cannot tell whether the written responses come from a person or from a computer. Chapter 26 discusses the details of the test and whether a computer would really be intelligent if it passed. For now, we note that programming a computer to pass a rigorously applied test provides plenty to work on. The computer would need to possess the following capabilities:

NATURAL LANGUAGE
PROCESSING
- **natural language processing** to enable it to communicate successfully in English;

KNOWLEDGE
REPRESENTATION
- **knowledge representation** to store what it knows or hears;

AUTOMATED
REASONING
- **automated reasoning** to use the stored information to answer questions and to draw new conclusions;

MACHINE LEARNING
- **machine learning** to adapt to new circumstances and to detect and extrapolate patterns.

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[1]   By distinguishing between *human* and *rational* behavior, we are not suggesting that humans are necessarily "irrational" in the sense of "emotionally unstable" or "insane." One merely need note that we are not perfect: not all chess players are grandmasters; and, unfortunately, not everyone gets an A on the exam. Some systematic errors in human reasoning are cataloged by Kahneman *et al.* (1982).

Turing's test deliberately avoided direct physical interaction between the interrogator and the computer, because *physical* simulation of a person is unnecessary for intelligence. However, the so-called **total Turing Test** includes a video signal so that the interrogator can test the subject's perceptual abilities, as well as the opportunity for the interrogator to pass physical objects "through the hatch." To pass the total Turing Test, the computer will need

- **computer vision** to perceive objects, and

- **robotics** to manipulate objects and move about.

These six disciplines compose most of AI, and Turing deserves credit for designing a test that remains relevant 60 years later. Yet AI researchers have devoted little effort to passing the Turing Test, believing that it is more important to study the underlying principles of intelligence than to duplicate an exemplar. The quest for "artificial flight" succeeded when the Wright brothers and others stopped imitating birds and started using wind tunnels and learning about aerodynamics. Aeronautical engineering texts do not define the goal of their field as making "machines that fly so exactly like pigeons that they can fool even other pigeons."

1.1.2   Thinking humanly: The cognitive modeling approach

If we are going to say that a given program thinks like a human, we must have some way of determining how humans think. We need to get *inside* the actual workings of human minds. There are three ways to do this: through introspection—trying to catch our own thoughts as they go by; through psychological experiments—observing a person in action; and through brain imaging—observing the brain in action. Once we have a sufficiently precise theory of the mind, it becomes possible to express the theory as a computer program. If the program's input–output behavior matches corresponding human behavior, that is evidence that some of the program's mechanisms could also be operating in humans. For example, Allen Newell and Herbert Simon, who developed GPS, the "General Problem Solver" (Newell and Simon, 1961), were not content merely to have their program solve problems correctly. They were more concerned with comparing the trace of its reasoning steps to traces of human subjects

solving the same problems. The interdisciplinary field of **cognitive science** brings together computer models from AI and experimental techniques from psychology to construct precise and testable theories of the human mind.

Cognitive science is a fascinating field in itself, worthy of several textbooks and at least one encyclopedia (Wilson and Keil, 1999). We will occasionally comment on similarities or differences between AI techniques and human cognition. Real cognitive science, however, is necessarily based on experimental investigation of actual humans or animals. We will leave that for other books, as we assume the reader has only a computer for experimentation.

In the early days of AI there was often confusion between the approaches: an author would argue that an algorithm performs well on a task and that it is *therefore* a good model of human performance, or vice versa. Modern authors separate the two kinds of claims; this distinction has allowed both AI and cognitive science to develop more rapidly. The two fields continue to fertilize each other, most notably in computer vision, which incorporates neurophysiological evidence into computational models.

1.1.3   Thinking rationally: The "laws of thought" approach

SYLLOGISM

LOGIC

The Greek philosopher Aristotle was one of the first to attempt to codify "right thinking," that is, irrefutable reasoning processes. His **syllogisms** provided patterns for argument structures that always yielded correct conclusions when given correct premises—for example, "Socrates is a man; all men are mortal; therefore, Socrates is mortal." These laws of thought were supposed to govern the operation of the mind; their study initiated the field called **logic**.

LOGICIST

Logicians in the 19th century developed a precise notation for statements about all kinds of objects in the world and the relations among them. (Contrast this with ordinary arithmetic notation, which provides only for statements about *numbers*.) By 1965, programs existed that could, in principle, solve *any* solvable problem described in logical notation. (Although if no solution exists, the program might loop forever.) The so-called **logicist** tradition within artificial intelligence hopes to build on such programs to create intelligent systems.

There are two main obstacles to this approach. First, it is not easy to take informal knowledge and state it in the formal terms required by logical notation, particularly when the knowledge is less than 100% certain. Second, there is a big difference between solving a problem "in principle" and solving it in practice. Even problems with just a few hundred facts can exhaust the computational resources of any computer unless it has some guidance as to which reasoning steps to try first. Although both of these obstacles apply to *any* attempt to build computational reasoning systems, they appeared first in the logicist tradition.

1.1.4   Acting rationally: The rational agent approach

AGENT

RATIONAL AGENT

An **agent** is just something that acts (*agent* comes from the Latin *agere*, to do). Of course, all computer programs do something, but computer agents are expected to do more: operate autonomously, perceive their environment, persist over a prolonged time period, adapt to change, and create and pursue goals. A **rational agent** is one that acts so as to achieve the best outcome or, when there is uncertainty, the best expected outcome.

In the "laws of thought" approach to AI, the emphasis was on correct inferences. Making correct inferences is sometimes *part* of being a rational agent, because one way to act rationally is to reason logically to the conclusion that a given action will achieve one's goals and then to act on that conclusion. On the other hand, correct inference is not *all* of rationality; in some situations, there is no provably correct thing to do, but something must still be done. There are also ways of acting rationally that cannot be said to involve inference. For example, recoiling from a hot stove is a reflex action that is usually more successful than a slower action taken after careful deliberation.

All the skills needed for the Turing Test also allow an agent to act rationally. Knowledge representation and reasoning enable agents to reach good decisions. We need to be able to generate comprehensible sentences in natural language to get by in a complex society. We need learning not only for erudition, but also because it improves our ability to generate effective behavior.

The rational-agent approach has two advantages over the other approaches. First, it is more general than the "laws of thought" approach because correct inference is just one of several possible mechanisms for achieving rationality. Second, it is more amenable to

scientific development than are approaches based on human behavior or human thought. The standard of rationality is mathematically well defined and completely general, and can be "unpacked" to generate agent designs that provably achieve it. Human behavior, on the other hand, is well adapted for one specific environment and is defined by, well, the sum total of all the things that humans do. *This book therefore concentrates on general principles of rational agents and on components for constructing them.* We will see that despite the apparent simplicity with which the problem can be stated, an enormous variety of issues come up when we try to solve it. Chapter 2 outlines some of these issues in more detail.

One important point to keep in mind: We will see before too long that achieving perfect rationality—always doing the right thing—is not feasible in complicated environments. The computational demands are just too high. For most of the book, however, we will adopt the working hypothesis that perfect rationality is a good starting point for analysis. It simplifies the problem and provides the appropriate setting for most of the foundational material in the field. Chapters 5 and 17 deal explicitly with the issue of **limited rationality**—acting appropriately when there is not enough time to do all the computations one might like.

LIMITED
RATIONALITY

1.2   THE FOUNDATIONS OF ARTIFICIAL INTELLIGENCE

In this section, we provide a brief history of the disciplines that contributed ideas, viewpoints, and techniques to AI. Like any history, this one is forced to concentrate on a small number of people, events, and ideas and to ignore others that also were important. We organize the history around a series of questions. We certainly would not wish to give the impression that these questions are the only ones the disciplines address or that the disciplines have all been working toward AI as their ultimate fruition.

1.2.1   Philosophy

- Can formal rules be used to draw valid conclusions?
- How does the mind arise from a physical brain?
- Where does knowledge come from?
- How does knowledge lead to action?

Aristotle (384–322 B.C.), whose bust appears on the front cover of this book, was the first to formulate a precise set of laws governing the rational part of the mind. He developed an informal system of syllogisms for proper reasoning, which in principle allowed one to generate conclusions mechanically, given initial premises. Much later, Ramon Lull (d. 1315) had the idea that useful reasoning could actually be carried out by a mechanical artifact. Thomas Hobbes (1588–1679) proposed that reasoning was like numerical computation, that "we add and subtract in our silent thoughts." The automation of computation itself was already well under way. Around 1500, Leonardo da Vinci (1452–1519) designed but did not build a mechanical calculator; recent reconstructions have shown the design to be functional. The first known calculating machine was constructed around 1623 by the German scientist Wilhelm Schickard (1592–1635), although the Pascaline, built in 1642 by Blaise Pascal (1623–1662),

is more famous. Pascal wrote that "the arithmetical machine produces effects which appear nearer to thought than all the actions of animals." Gottfried Wilhelm Leibniz (1646–1716) built a mechanical device intended to carry out operations on concepts rather than numbers, but its scope was rather limited. Leibniz did surpass Pascal by building a calculator that could add, subtract, multiply, and take roots, whereas the Pascaline could only add and subtract. Some speculated that machines might not just do calculations but actually be able to think and act on their own. In his 1651 book *Leviathan*, Thomas Hobbes suggested the idea of an "artificial animal," arguing "For what is the heart but a spring; and the nerves, but so many strings; and the joints, but so many wheels."

It's one thing to say that the mind operates, at least in part, according to logical rules, and to build physical systems that emulate some of those rules; it's another to say that the mind itself *is* such a physical system. René Descartes (1596–1650) gave the first clear discussion of the distinction between mind and matter and of the problems that arise. One problem with a purely physical conception of the mind is that it seems to leave little room for free will: if the mind is governed entirely by physical laws, then it has no more free will than a rock "deciding" to fall toward the center of the earth. Descartes was a strong advocate of the power of reasoning in understanding the world, a philosophy now called **rationalism**, and one that counts Aristotle and Leibnitz as members. But Descartes was also a proponent of **dualism**. He held that there is a part of the human mind (or soul or spirit) that is outside of nature, exempt from physical laws. Animals, on the other hand, did not possess this dual quality; they could be treated as machines. An alternative to dualism is **materialism**, which holds that the brain's operation according to the laws of physics *constitutes* the mind. Free will is simply the way that the perception of available choices appears to the choosing entity.

Given a physical mind that manipulates knowledge, the next problem is to establish the source of knowledge. The **empiricism** movement, starting with Francis Bacon's (1561–1626) *Novum Organum*,[2] is characterized by a dictum of John Locke (1632–1704): "Nothing is in the understanding, which was not first in the senses." David Hume's (1711–1776) *A Treatise of Human Nature* (Hume, 1739) proposed what is now known as the principle of **induction**: that general rules are acquired by exposure to repeated associations between their elements. Building on the work of Ludwig Wittgenstein (1889–1951) and Bertrand Russell (1872–1970), the famous Vienna Circle, led by Rudolf Carnap (1891–1970), developed the doctrine of **logical positivism**. This doctrine holds that all knowledge can be characterized by logical theories connected, ultimately, to **observation sentences** that correspond to sensory inputs; thus logical positivism combines rationalism and empiricism.[3] The **confirmation theory** of Carnap and Carl Hempel (1905–1997) attempted to analyze the acquisition of knowledge from experience. Carnap's book *The Logical Structure of the World* (1928) defined an explicit computational procedure for extracting knowledge from elementary experiences. It was probably the first theory of mind as a computational process.

RATIONALISM

DUALISM

MATERIALISM

EMPIRICISM

INDUCTION

LOGICAL POSITIVISM

OBSERVATION SENTENCES

CONFIRMATION THEORY

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[2] The *Novum Organum* is an update of Aristotle's *Organon*, or instrument of thought. Thus Aristotle can be seen as both an empiricist and a rationalist.

[3] In this picture, all meaningful statements can be verified or falsified either by experimentation or by analysis of the meaning of the words. Because this rules out most of metaphysics, as was the intention, logical positivism was unpopular in some circles.

The final element in the philosophical picture of the mind is the connection between knowledge and action. This question is vital to AI because intelligence requires action as well as reasoning. Moreover, only by understanding how actions are justified can we understand how to build an agent whose actions are justifiable (or rational). Aristotle argued (in *De Motu Animalium*) that actions are justified by a logical connection between goals and knowledge of the action's outcome (the last part of this extract also appears on the front cover of this book, in the original Greek):

> But how does it happen that thinking is sometimes accompanied by action and sometimes not, sometimes by motion, and sometimes not? It looks as if almost the same thing happens as in the case of reasoning and making inferences about unchanging objects. But in that case the end is a speculative proposition . . . whereas here the conclusion which results from the two premises is an action. . . . I need covering; a cloak is a covering. I need a cloak. What I need, I have to make; I need a cloak. I have to make a cloak. And the conclusion, the "I have to make a cloak," is an action.

In the *Nicomachean Ethics* (Book III. 3, 1112b), Aristotle further elaborates on this topic, suggesting an algorithm:

> We deliberate not about ends, but about means. For a doctor does not deliberate whether he shall heal, nor an orator whether he shall persuade, . . . They assume the end and consider how and by what means it is attained, and if it seems easily and best produced thereby; while if it is achieved by one means only they consider *how* it will be achieved by this and by what means *this* will be achieved, till they come to the first cause, . . . and what is last in the order of analysis seems to be first in the order of becoming. And if we come on an impossibility, we give up the search, e.g., if we need money and this cannot be got; but if a thing appears possible we try to do it.

Aristotle's algorithm was implemented 2300 years later by Newell and Simon in their GPS program. We would now call it a regression planning system (see Chapter 10).

Goal-based analysis is useful, but does not say what to do when several actions will achieve the goal or when no action will achieve it completely. Antoine Arnauld (1612–1694) correctly described a quantitative formula for deciding what action to take in cases like this (see Chapter 16). John Stuart Mill's (1806–1873) book *Utilitarianism* (Mill, 1863) promoted the idea of rational decision criteria in all spheres of human activity. The more formal theory of decisions is discussed in the following section.

1.2.2   Mathematics

- What are the formal rules to draw valid conclusions?
- What can be computed?
- How do we reason with uncertain information?

Philosophers staked out some of the fundamental ideas of AI, but the leap to a formal science required a level of mathematical formalization in three fundamental areas: logic, computation, and probability.

The idea of formal logic can be traced back to the philosophers of ancient Greece, but its mathematical development really began with the work of George Boole (1815–1864), who

worked out the details of propositional, or Boolean, logic (Boole, 1847). In 1879, Gottlob Frege (1848–1925) extended Boole's logic to include objects and relations, creating the first-order logic that is used today.[4] Alfred Tarski (1902–1983) introduced a theory of reference that shows how to relate the objects in a logic to objects in the real world.

ALGORITHM

The next step was to determine the limits of what could be done with logic and computation. The first nontrivial **algorithm** is thought to be Euclid's algorithm for computing greatest common divisors. The word *algorithm* (and the idea of studying them) comes from al-Khowarazmi, a Persian mathematician of the 9th century, whose writings also introduced Arabic numerals and algebra to Europe. Boole and others discussed algorithms for logical deduction, and, by the late 19th century, efforts were under way to formalize general mathematical reasoning as logical deduction. In 1930, Kurt Gödel (1906–1978) showed that there exists an effective procedure to prove any true statement in the first-order logic of Frege and Russell, but that first-order logic could not capture the principle of mathematical induction needed to characterize the natural numbers. In 1931, Gödel showed that limits on deduction do exist. His **incompleteness theorem** showed that in any formal theory as strong as Peano arithmetic (the elementary theory of natural numbers), there are true statements that are undecidable in the sense that they have no proof within the theory.

INCOMPLETENESS
THEOREM

This fundamental result can also be interpreted as showing that some functions on the integers cannot be represented by an algorithm—that is, they cannot be computed. This motivated Alan Turing (1912–1954) to try to characterize exactly which functions *are* **computable**—capable of being computed. This notion is actually slightly problematic because the notion of a computation or effective procedure really cannot be given a formal definition. However, the Church–Turing thesis, which states that the Turing machine (Turing, 1936) is capable of computing any computable function, is generally accepted as providing a sufficient definition. Turing also showed that there were some functions that no Turing machine can compute. For example, no machine can tell *in general* whether a given program will return an answer on a given input or run forever.

COMPUTABLE

Although decidability and computability are important to an understanding of computation, the notion of **tractability** has had an even greater impact. Roughly speaking, a problem is called intractable if the time required to solve instances of the problem grows exponentially with the size of the instances. The distinction between polynomial and exponential growth in complexity was first emphasized in the mid-1960s (Cobham, 1964; Edmonds, 1965). It is important because exponential growth means that even moderately large instances cannot be solved in any reasonable time. Therefore, one should strive to divide the overall problem of generating intelligent behavior into tractable subproblems rather than intractable ones.

TRACTABILITY

How can one recognize an intractable problem? The theory of **NP-completeness**, pioneered by Steven Cook (1971) and Richard Karp (1972), provides a method. Cook and Karp showed the existence of large classes of canonical combinatorial search and reasoning problems that are NP-complete. Any problem class to which the class of NP-complete problems can be reduced is likely to be intractable. (Although it has not been proved that NP-complete

NP-COMPLETENESS

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[4] Frege's proposed notation for first-order logic—an arcane combination of textual and geometric features—never became popular.

problems are necessarily intractable, most theoreticians believe it.) These results contrast with the optimism with which the popular press greeted the first computers—"Electronic Super-Brains" that were "Faster than Einstein!" Despite the increasing speed of computers, careful use of resources will characterize intelligent systems. Put crudely, the world is an *extremely* large problem instance! Work in AI has helped explain why some instances of NP-complete problems are hard, yet others are easy (Cheeseman *et al.*, 1991).

PROBABILITY

Besides logic and computation, the third great contribution of mathematics to AI is the theory of **probability**. The Italian Gerolamo Cardano (1501–1576) first framed the idea of probability, describing it in terms of the possible outcomes of gambling events. In 1654, Blaise Pascal (1623–1662), in a letter to Pierre Fermat (1601–1665), showed how to predict the future of an unfinished gambling game and assign average payoffs to the gamblers. Probability quickly became an invaluable part of all the quantitative sciences, helping to deal with uncertain measurements and incomplete theories. James Bernoulli (1654–1705), Pierre Laplace (1749–1827), and others advanced the theory and introduced new statistical methods. Thomas Bayes (1702–1761), who appears on the front cover of this book, proposed a rule for updating probabilities in the light of new evidence. Bayes' rule underlies most modern approaches to uncertain reasoning in AI systems.

1.2.3   Economics

- How should we make decisions so as to maximize payoff?
- How should we do this when others may not go along?
- How should we do this when the payoff may be far in the future?

The science of economics got its start in 1776, when Scottish philosopher Adam Smith (1723–1790) published *An Inquiry into the Nature and Causes of the Wealth of Nations.* While the ancient Greeks and others had made contributions to economic thought, Smith was the first to treat it as a science, using the idea that economies can be thought of as consisting of individual agents maximizing their own economic well-being. Most people think of economics as being about money, but economists will say that they are really studying how people make choices that lead to preferred outcomes. When McDonald's offers a hamburger for a dollar, they are asserting that they would prefer the dollar and hoping that customers will

UTILITY

prefer the hamburger. The mathematical treatment of "preferred outcomes" or **utility** was first formalized by Léon Walras (pronounced "Valrasse") (1834-1910) and was improved by Frank Ramsey (1931) and later by John von Neumann and Oskar Morgenstern in their book *The Theory of Games and Economic Behavior* (1944).

DECISION THEORY

Decision theory, which combines probability theory with utility theory, provides a formal and complete framework for decisions (economic or otherwise) made under uncertainty—that is, in cases where probabilistic descriptions appropriately capture the decision maker's environment. This is suitable for "large" economies where each agent need pay no attention to the actions of other agents as individuals. For "small" economies, the situation is much more like a **game**: the actions of one player can significantly affect the utility of another (either positively or negatively). Von Neumann and Morgenstern's development of **game**

GAME THEORY

theory (see also Luce and Raiffa, 1957) included the surprising result that, for some games,

a rational agent should adopt policies that are (or least appear to be) randomized. Unlike decision theory, game theory does not offer an unambiguous prescription for selecting actions.

OPERATIONS
RESEARCH

For the most part, economists did not address the third question listed above, namely, how to make rational decisions when payoffs from actions are not immediate but instead result from several actions taken *in sequence*. This topic was pursued in the field of **operations research**, which emerged in World War II from efforts in Britain to optimize radar installations, and later found civilian applications in complex management decisions. The work of Richard Bellman (1957) formalized a class of sequential decision problems called **Markov decision processes**, which we study in Chapters 17 and 21.

SATISFICING

Work in economics and operations research has contributed much to our notion of rational agents, yet for many years AI research developed along entirely separate paths. One reason was the apparent complexity of making rational decisions. The pioneering AI researcher Herbert Simon (1916–2001) won the Nobel Prize in economics in 1978 for his early work showing that models based on **satisficing**—making decisions that are "good enough," rather than laboriously calculating an optimal decision—gave a better description of actual human behavior (Simon, 1947). Since the 1990s, there has been a resurgence of interest in decision-theoretic techniques for agent systems (Wellman, 1995).

1.2.4   Neuroscience

- How do brains process information?

NEUROSCIENCE

Neuroscience is the study of the nervous system, particularly the brain. Although the exact way in which the brain enables thought is one of the great mysteries of science, the fact that it *does* enable thought has been appreciated for thousands of years because of the evidence that strong blows to the head can lead to mental incapacitation. It has also long been known that human brains are somehow different; in about 335 B.C. Aristotle wrote, "Of all the animals, man has the largest brain in proportion to his size."[5] Still, it was not until the middle of the 18th century that the brain was widely recognized as the seat of consciousness. Before then, candidate locations included the heart and the spleen.

NEURON

Paul Broca's (1824–1880) study of aphasia (speech deficit) in brain-damaged patients in 1861 demonstrated the existence of localized areas of the brain responsible for specific cognitive functions. In particular, he showed that speech production was localized to the portion of the left hemisphere now called Broca's area.[6] By that time, it was known that the brain consisted of nerve cells, or **neurons**, but it was not until 1873 that Camillo Golgi (1843–1926) developed a staining technique allowing the observation of individual neurons in the brain (see Figure 1.2). This technique was used by Santiago Ramon y Cajal (1852–1934) in his pioneering studies of the brain's neuronal structures.[7] Nicolas Rashevsky (1936, 1938) was the first to apply mathematical models to the study of the nervous sytem.

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[5]   Since then, it has been discovered that the tree shrew (*Scandentia*) has a higher ratio of brain to body mass.

[6]   Many cite Alexander Hood (1824) as a possible prior source.

[7]   Golgi persisted in his belief that the brain's functions were carried out primarily in a continuous medium in which neurons were embedded, whereas Cajal propounded the "neuronal doctrine." The two shared the Nobel prize in 1906 but gave mutually antagonistic acceptance speeches.
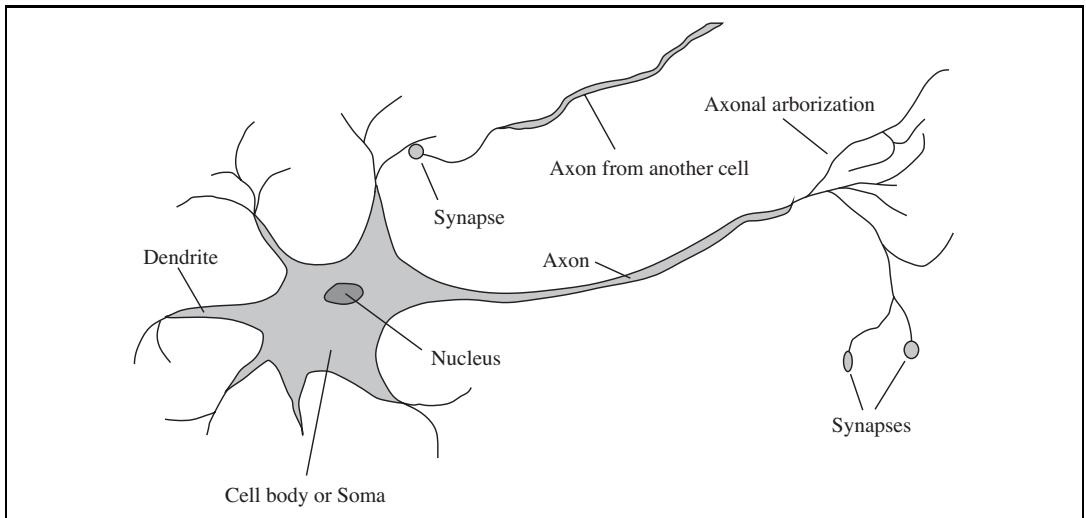
Figure 1.2   The parts of a nerve cell or neuron. Each neuron consists of a cell body, or soma, that contains a cell nucleus. Branching out from the cell body are a number of fibers called dendrites and a single long fiber called the axon. The axon stretches out for a long distance, much longer than the scale in this diagram indicates. Typically, an axon is 1 cm long (100 times the diameter of the cell body), but can reach up to 1 meter. A neuron makes connections with 10 to 100,000 other neurons at junctions called synapses. Signals are propagated from neuron to neuron by a complicated electrochemical reaction. The signals control brain activity in the short term and also enable long-term changes in the connectivity of neurons. These mechanisms are thought to form the basis for learning in the brain. Most information processing goes on in the cerebral cortex, the outer layer of the brain. The basic organizational unit appears to be a column of tissue about 0.5 mm in diameter, containing about 20,000 neurons and extending the full depth of the cortex about 4 mm in humans).

We now have some data on the mapping between areas of the brain and the parts of the body that they control or from which they receive sensory input. Such mappings are able to change radically over the course of a few weeks, and some animals seem to have multiple maps. Moreover, we do not fully understand how other areas can take over functions when one area is damaged. There is almost no theory on how an individual memory is stored.

The measurement of intact brain activity began in 1929 with the invention by Hans Berger of the electroencephalograph (EEG). The recent development of functional magnetic resonance imaging (fMRI) (Ogawa *et al.*, 1990; Cabeza and Nyberg, 2001) is giving neuroscientists unprecedentedly detailed images of brain activity, enabling measurements that correspond in interesting ways to ongoing cognitive processes. These are augmented by advances in single-cell recording of neuron activity. Individual neurons can be stimulated electrically, chemically, or even optically (Han and Boyden, 2007), allowing neuronal input–output relationships to be mapped. Despite these advances, we are still a long way from understanding how cognitive processes actually work.

The truly amazing conclusion is that *a collection of simple cells can lead to thought, action, and consciousness* or, in the pithy words of John Searle (1992), *brains cause minds*.

	Supercomputer	Personal Computer	Human Brain
Computational units	10^4 CPUs, 10^{12} transistors	4 CPUs, 10^9 transistors	10^{11} neurons
Storage units	10^{14} bits RAM	10^{11} bits RAM	10^{11} neurons
	10^{15} bits disk	10^{13} bits disk	10^{14} synapses
Cycle time	10^{-9} sec	10^{-9} sec	10^{-3} sec
Operations/sec	10^{15}	10^{10}	10^{17}
Memory updates/sec	10^{14}	10^{10}	10^{14}

Figure 1.3   A crude comparison of the raw computational resources available to the IBM BLUE GENE supercomputer, a typical personal computer of 2008, and the human brain. The brain's numbers are essentially fixed, whereas the supercomputer's numbers have been increasing by a factor of 10 every 5 years or so, allowing it to achieve rough parity with the brain. The personal computer lags behind on all metrics except cycle time.

The only real alternative theory is mysticism: that minds operate in some mystical realm that is beyond physical science.

Brains and digital computers have somewhat different properties. Figure 1.3 shows that computers have a cycle time that is a million times faster than a brain. The brain makes up for that with far more storage and interconnection than even a high-end personal computer, although the largest supercomputers have a capacity that is similar to the brain's. (It should be noted, however, that the brain does not seem to use all of its neurons simultaneously.) SINGULARITY Futurists make much of these numbers, pointing to an approaching **singularity** at which computers reach a superhuman level of performance (Vinge, 1993; Kurzweil, 2005), but the raw comparisons are not especially informative. Even with a computer of virtually unlimited capacity, we still would not know how to achieve the brain's level of intelligence.

1.2.5   Psychology

- How do humans and animals think and act?

The origins of scientific psychology are usually traced to the work of the German physicist Hermann von Helmholtz (1821–1894) and his student Wilhelm Wundt (1832–1920). Helmholtz applied the scientific method to the study of human vision, and his *Handbook of Physiological Optics* is even now described as "the single most important treatise on the physics and physiology of human vision" (Nalwa, 1993, p.15). In 1879, Wundt opened the first laboratory of experimental psychology, at the University of Leipzig. Wundt insisted on carefully controlled experiments in which his workers would perform a perceptual or associative task while introspecting on their thought processes. The careful controls went a long way toward making psychology a science, but the subjective nature of the data made it unlikely that an experimenter would ever disconfirm his or her own theories. Biologists studying animal behavior, on the other hand, lacked introspective data and developed an objective methodology, as described by H. S. Jennings (1906) in his influential work *Behavior of* BEHAVIORISM *the Lower Organisms*. Applying this viewpoint to humans, the **behaviorism** movement, led by John Watson (1878–1958), rejected *any* theory involving mental processes on the grounds

that introspection could not provide reliable evidence. Behaviorists insisted on studying only objective measures of the percepts (or *stimulus*) given to an animal and its resulting actions (or *response*). Behaviorism discovered a lot about rats and pigeons but had less success at understanding humans.

COGNITIVE
PSYCHOLOGY

Cognitive psychology, which views the brain as an information-processing device, can be traced back at least to the works of William James (1842–1910). Helmholtz also insisted that perception involved a form of unconscious logical inference. The cognitive viewpoint was largely eclipsed by behaviorism in the United States, but at Cambridge's Applied Psychology Unit, directed by Frederic Bartlett (1886–1969), cognitive modeling was able to flourish. *The Nature of Explanation*, by Bartlett's student and successor Kenneth Craik (1943), forcefully reestablished the legitimacy of such "mental" terms as beliefs and goals, arguing that they are just as scientific as, say, using pressure and temperature to talk about gases, despite their being made of molecules that have neither. Craik specified the three key steps of a knowledge-based agent: (1) the stimulus must be translated into an internal representation, (2) the representation is manipulated by cognitive processes to derive new internal representations, and (3) these are in turn retranslated back into action. He clearly explained why this was a good design for an agent:

> If the organism carries a "small-scale model" of external reality and of its own possible actions within its head, it is able to try out various alternatives, conclude which is the best of them, react to future situations before they arise, utilize the knowledge of past events in dealing with the present and future, and in every way to react in a much fuller, safer, and more competent manner to the emergencies which face it. (Craik, 1943)

After Craik's death in a bicycle accident in 1945, his work was continued by Donald Broadbent, whose book *Perception and Communication* (1958) was one of the first works to model psychological phenomena as information processing. Meanwhile, in the United States, the development of computer modeling led to the creation of the field of **cognitive science**. The field can be said to have started at a workshop in September 1956 at MIT. (We shall see that this is just two months after the conference at which AI itself was "born.") At the workshop, George Miller presented *The Magic Number Seven*, Noam Chomsky presented *Three Models of Language*, and Allen Newell and Herbert Simon presented *The Logic Theory Machine*. These three influential papers showed how computer models could be used to address the psychology of memory, language, and logical thinking, respectively. It is now a common (although far from universal) view among psychologists that "a cognitive theory should be like a computer program" (Anderson, 1980); that is, it should describe a detailed information-processing mechanism whereby some cognitive function might be implemented.

1.2.6   Computer engineering

- How can we build an efficient computer?

For artificial intelligence to succeed, we need two things: intelligence and an artifact. The computer has been the artifact of choice. The modern digital electronic computer was invented independently and almost simultaneously by scientists in three countries embattled in

World War II. The first *operational* computer was the electromechanical Heath Robinson,[8] built in 1940 by Alan Turing's team for a single purpose: deciphering German messages. In 1943, the same group developed the Colossus, a powerful general-purpose machine based on vacuum tubes.[9] The first operational *programmable* computer was the Z-3, the invention of Konrad Zuse in Germany in 1941. Zuse also invented floating-point numbers and the first high-level programming language, Plankalkül. The first *electronic* computer, the ABC, was assembled by John Atanasoff and his student Clifford Berry between 1940 and 1942 at Iowa State University. Atanasoff's research received little support or recognition; it was the ENIAC, developed as part of a secret military project at the University of Pennsylvania by a team including John Mauchly and John Eckert, that proved to be the most influential forerunner of modern computers.

Since that time, each generation of computer hardware has brought an increase in speed and capacity and a decrease in price. Performance doubled every 18 months or so until around 2005, when power dissipation problems led manufacturers to start multiplying the number of CPU cores rather than the clock speed. Current expectations are that future increases in power will come from massive parallelism—a curious convergence with the properties of the brain.

Of course, there were calculating devices before the electronic computer. The earliest automated machines, dating from the 17th century, were discussed on page 6. The first *programmable* machine was a loom, devised in 1805 by Joseph Marie Jacquard (1752–1834), that used punched cards to store instructions for the pattern to be woven. In the mid-19th century, Charles Babbage (1792–1871) designed two machines, neither of which he completed. The Difference Engine was intended to compute mathematical tables for engineering and scientific projects. It was finally built and shown to work in 1991 at the Science Museum in London (Swade, 2000). Babbage's Analytical Engine was far more ambitious: it included addressable memory, stored programs, and conditional jumps and was the first artifact capable of universal computation. Babbage's colleague Ada Lovelace, daughter of the poet Lord Byron, was perhaps the world's first programmer. (The programming language Ada is named after her.) She wrote programs for the unfinished Analytical Engine and even speculated that the machine could play chess or compose music.

AI also owes a debt to the software side of computer science, which has supplied the operating systems, programming languages, and tools needed to write modern programs (and papers about them). But this is one area where the debt has been repaid: work in AI has pioneered many ideas that have made their way back to mainstream computer science, including time sharing, interactive interpreters, personal computers with windows and mice, rapid development environments, the linked list data type, automatic storage management, and key concepts of symbolic, functional, declarative, and object-oriented programming.

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[8]   Heath Robinson was a cartoonist famous for his depictions of whimsical and absurdly complicated contraptions for everyday tasks such as buttering toast.

[9]   In the postwar period, Turing wanted to use these computers for AI research—for example, one of the first chess programs (Turing *et al.*, 1953). His efforts were blocked by the British government.

1.2.7   Control theory and cybernetics

- How can artifacts operate under their own control?

Ktesibios of Alexandria (c. 250 B.C.) built the first self-controlling machine: a water clock with a regulator that maintained a constant flow rate. This invention changed the definition of what an artifact could do. Previously, only living things could modify their behavior in response to changes in the environment. Other examples of self-regulating feedback control systems include the steam engine governor, created by James Watt (1736–1819), and the thermostat, invented by Cornelis Drebbel (1572–1633), who also invented the submarine. The mathematical theory of stable feedback systems was developed in the 19th century.

CONTROL THEORY

The central figure in the creation of what is now called **control theory** was Norbert Wiener (1894–1964). Wiener was a brilliant mathematician who worked with Bertrand Russell, among others, before developing an interest in biological and mechanical control systems and their connection to cognition. Like Craik (who also used control systems as psychological models), Wiener and his colleagues Arturo Rosenblueth and Julian Bigelow challenged the behaviorist orthodoxy (Rosenblueth *et al.*, 1943). They viewed purposive behavior as arising from a regulatory mechanism trying to minimize "error"—the difference between current state and goal state. In the late 1940s, Wiener, along with Warren McCulloch, Walter Pitts, and John von Neumann, organized a series of influential conferences that explored the new

CYBERNETICS

mathematical and computational models of cognition. Wiener's book *Cybernetics* (1948) became a bestseller and awoke the public to the possibility of artificially intelligent machines. Meanwhile, in Britain, W. Ross Ashby (Ashby, 1940) pioneered similar ideas. Ashby, Alan Turing, Grey Walter, and others formed the Ratio Club for "those who had Wiener's ideas before Wiener's book appeared." Ashby's *Design for a Brain* (1948, 1952) elaborated on his

HOMEOSTATIC

idea that intelligence could be created by the use of **homeostatic** devices containing appropriate feedback loops to achieve stable adaptive behavior.

OBJECTIVE FUNCTION

Modern control theory, especially the branch known as stochastic optimal control, has as its goal the design of systems that maximize an **objective function** over time. This roughly matches our view of AI: designing systems that behave optimally. Why, then, are AI and control theory two different fields, despite the close connections among their founders? The answer lies in the close coupling between the mathematical techniques that were familiar to the participants and the corresponding sets of problems that were encompassed in each world view. Calculus and matrix algebra, the tools of control theory, lend themselves to systems that are describable by fixed sets of continuous variables, whereas AI was founded in part as a way to escape from the these perceived limitations. The tools of logical inference and computation allowed AI researchers to consider problems such as language, vision, and planning that fell completely outside the control theorist's purview.

1.2.8   Linguistics

- How does language relate to thought?

In 1957, B. F. Skinner published *Verbal Behavior*. This was a comprehensive, detailed account of the behaviorist approach to language learning, written by the foremost expert in

the field. But curiously, a review of the book became as well known as the book itself, and served to almost kill off interest in behaviorism. The author of the review was the linguist Noam Chomsky, who had just published a book on his own theory, *Syntactic Structures*. Chomsky pointed out that the behaviorist theory did not address the notion of creativity in language—it did not explain how a child could understand and make up sentences that he or she had never heard before. Chomsky's theory—based on syntactic models going back to the Indian linguist Panini (c. 350 B.C.)—could explain this, and unlike previous theories, it was formal enough that it could in principle be programmed.

COMPUTATIONAL LINGUISTICS

Modern linguistics and AI, then, were "born" at about the same time, and grew up together, intersecting in a hybrid field called **computational linguistics** or **natural language processing**. The problem of understanding language soon turned out to be considerably more complex than it seemed in 1957. Understanding language requires an understanding of the subject matter and context, not just an understanding of the structure of sentences. This might seem obvious, but it was not widely appreciated until the 1960s. Much of the early work in **knowledge representation** (the study of how to put knowledge into a form that a computer can reason with) was tied to language and informed by research in linguistics, which was connected in turn to decades of work on the philosophical analysis of language.

1.3 THE HISTORY OF ARTIFICIAL INTELLIGENCE

With the background material behind us, we are ready to cover the development of AI itself.

1.3.1 The gestation of artificial intelligence (1943–1955)

The first work that is now generally recognized as AI was done by Warren McCulloch and Walter Pitts (1943). They drew on three sources: knowledge of the basic physiology and function of neurons in the brain; a formal analysis of propositional logic due to Russell and Whitehead; and Turing's theory of computation. They proposed a model of artificial neurons in which each neuron is characterized as being "on" or "off," with a switch to "on" occurring in response to stimulation by a sufficient number of neighboring neurons. The state of a neuron was conceived of as "factually equivalent to a proposition which proposed its adequate stimulus." They showed, for example, that any computable function could be computed by some network of connected neurons, and that all the logical connectives (and, or, not, etc.) could be implemented by simple net structures. McCulloch and Pitts also suggested that suitably defined networks could learn. Donald Hebb (1949) demonstrated a simple updating rule for modifying the connection strengths between neurons. His rule, now called **Hebbian learning**, remains an influential model to this day.

HEBBIAN LEARNING

Two undergraduate students at Harvard, Marvin Minsky and Dean Edmonds, built the first neural network computer in 1950. The SNARC, as it was called, used 3000 vacuum tubes and a surplus automatic pilot mechanism from a B-24 bomber to simulate a network of 40 neurons. Later, at Princeton, Minsky studied universal computation in neural networks. His Ph.D. committee was skeptical about whether this kind of work should be considered

mathematics, but von Neumann reportedly said, "If it isn't now, it will be someday." Minsky was later to prove influential theorems showing the limitations of neural network research.

There were a number of early examples of work that can be characterized as AI, but Alan Turing's vision was perhaps the most influential. He gave lectures on the topic as early as 1947 at the London Mathematical Society and articulated a persuasive agenda in his 1950 article "Computing Machinery and Intelligence." Therein, he introduced the Turing Test, machine learning, genetic algorithms, and reinforcement learning. He proposed the *Child Programme* idea, explaining "Instead of trying to produce a programme to simulate the adult mind, why not rather try to produce one which simulated the child's?"

1.3.2   The birth of artificial intelligence (1956)

Princeton was home to another influential figure in AI, John McCarthy. After receiving his PhD there in 1951 and working for two years as an instructor, McCarthy moved to Stanford and then to Dartmouth College, which was to become the official birthplace of the field. McCarthy convinced Minsky, Claude Shannon, and Nathaniel Rochester to help him bring together U.S. researchers interested in automata theory, neural nets, and the study of intelligence. They organized a two-month workshop at Dartmouth in the summer of 1956. The proposal states:[10]

> We propose that a 2 month, 10 man study of artificial intelligence be carried out during the summer of 1956 at Dartmouth College in Hanover, New Hampshire. The study is to proceed on the basis of the conjecture that every aspect of learning or any other feature of intelligence can in principle be so precisely described that a machine can be made to simulate it. An attempt will be made to find how to make machines use language, form abstractions and concepts, solve kinds of problems now reserved for humans, and improve themselves. We think that a significant advance can be made in one or more of these problems if a carefully selected group of scientists work on it together for a summer.

There were 10 attendees in all, including Trenchard More from Princeton, Arthur Samuel from IBM, and Ray Solomonoff and Oliver Selfridge from MIT.

Two researchers from Carnegie Tech,[11] Allen Newell and Herbert Simon, rather stole the show. Although the others had ideas and in some cases programs for particular applications such as checkers, Newell and Simon already had a reasoning program, the Logic Theorist (LT), about which Simon claimed, "We have invented a computer program capable of thinking non-numerically, and thereby solved the venerable mind–body problem."[12] Soon after the workshop, the program was able to prove most of the theorems in Chapter 2 of Rus-

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[10] This was the first official usage of McCarthy's term *artificial intelligence*. Perhaps "computational rationality" would have been more precise and less threatening, but "AI" has stuck. At the 50th anniversary of the Dartmouth conference, McCarthy stated that he resisted the terms "computer" or "computational" in deference to Norbert Weiner, who was promoting analog cybernetic devices rather than digital computers.

[11] Now Carnegie Mellon University (CMU).

[12] Newell and Simon also invented a list-processing language, IPL, to write LT. They had no compiler and translated it into machine code by hand. To avoid errors, they worked in parallel, calling out binary numbers to each other as they wrote each instruction to make sure they agreed.

sell and Whitehead's *Principia Mathematica.* Russell was reportedly delighted when Simon showed him that the program had come up with a proof for one theorem that was shorter than the one in *Principia.* The editors of the *Journal of Symbolic Logic* were less impressed; they rejected a paper coauthored by Newell, Simon, and Logic Theorist.

The Dartmouth workshop did not lead to any new breakthroughs, but it did introduce all the major figures to each other. For the next 20 years, the field would be dominated by these people and their students and colleagues at MIT, CMU, Stanford, and IBM.

Looking at the proposal for the Dartmouth workshop (McCarthy *et al.*, 1955), we can see why it was necessary for AI to become a separate field. Why couldn't all the work done in AI have taken place under the name of control theory or operations research or decision theory, which, after all, have objectives similar to those of AI? Or why isn't AI a branch of mathematics? The first answer is that AI from the start embraced the idea of duplicating human faculties such as creativity, self-improvement, and language use. None of the other fields were addressing these issues. The second answer is methodology. AI is the only one of these fields that is clearly a branch of computer science (although operations research does share an emphasis on computer simulations), and AI is the only field to attempt to build machines that will function autonomously in complex, changing environments.

1.3.3   Early enthusiasm, great expectations (1952–1969)

The early years of AI were full of successes—in a limited way. Given the primitive computers and programming tools of the time and the fact that only a few years earlier computers were seen as things that could do arithmetic and no more, it was astonishing whenever a computer did anything remotely clever. The intellectual establishment, by and large, preferred to believe that "a machine can never do X." (See Chapter 26 for a long list of X's gathered by Turing.) AI researchers naturally responded by demonstrating one X after another. John McCarthy referred to this period as the "Look, Ma, no hands!" era.

Newell and Simon's early success was followed up with the General Problem Solver, or GPS. Unlike Logic Theorist, this program was designed from the start to imitate human problem-solving protocols. Within the limited class of puzzles it could handle, it turned out that the order in which the program considered subgoals and possible actions was similar to that in which humans approached the same problems. Thus, GPS was probably the first program to embody the "thinking humanly" approach. The success of GPS and subsequent programs as models of cognition led Newell and Simon (1976) to formulate the famous **physical symbol system** hypothesis, which states that "a physical symbol system has the necessary and sufficient means for general intelligent action." What they meant is that any system (human or machine) exhibiting intelligence must operate by manipulating data structures composed of symbols. We will see later that this hypothesis has been challenged from many directions.

PHYSICAL SYMBOL
SYSTEM

At IBM, Nathaniel Rochester and his colleagues produced some of the first AI programs. Herbert Gelernter (1959) constructed the Geometry Theorem Prover, which was able to prove theorems that many students of mathematics would find quite tricky. Starting in 1952, Arthur Samuel wrote a series of programs for checkers (draughts) that eventually learned to play at a strong amateur level. Along the way, he disproved the idea that comput-

ers can do only what they are told to: his program quickly learned to play a better game than its creator. The program was demonstrated on television in February 1956, creating a strong impression. Like Turing, Samuel had trouble finding computer time. Working at night, he used machines that were still on the testing floor at IBM's manufacturing plant. Chapter 5 covers game playing, and Chapter 21 explains the learning techniques used by Samuel.

John McCarthy moved from Dartmouth to MIT and there made three crucial contributions in one historic year: 1958. In MIT AI Lab Memo No. 1, McCarthy defined the high-level language **Lisp**, which was to become the dominant AI programming language for the next 30 years. With Lisp, McCarthy had the tool he needed, but access to scarce and expensive computing resources was also a serious problem. In response, he and others at MIT invented time sharing. Also in 1958, McCarthy published a paper entitled *Programs with Common Sense*, in which he described the Advice Taker, a hypothetical program that can be seen as the first complete AI system. Like the Logic Theorist and Geometry Theorem Prover, McCarthy's program was designed to use knowledge to search for solutions to problems. But unlike the others, it was to embody general knowledge of the world. For example, he showed how some simple axioms would enable the program to generate a plan to drive to the airport. The program was also designed to accept new axioms in the normal course of operation, thereby allowing it to achieve competence in new areas *without being reprogrammed*. The Advice Taker thus embodied the central principles of knowledge representation and reasoning: that it is useful to have a formal, explicit representation of the world and its workings and to be able to manipulate that representation with deductive processes. It is remarkable how much of the 1958 paper remains relevant today.

LISP

1958 also marked the year that Marvin Minsky moved to MIT. His initial collaboration with McCarthy did not last, however. McCarthy stressed representation and reasoning in formal logic, whereas Minsky was more interested in getting programs to work and eventually developed an anti-logic outlook. In 1963, McCarthy started the AI lab at Stanford. His plan to use logic to build the ultimate Advice Taker was advanced by J. A. Robinson's discovery in 1965 of the resolution method (a complete theorem-proving algorithm for first-order logic; see Chapter 9). Work at Stanford emphasized general-purpose methods for logical reasoning. Applications of logic included Cordell Green's question-answering and planning systems (Green, 1969b) and the Shakey robotics project at the Stanford Research Institute (SRI). The latter project, discussed further in Chapter 25, was the first to demonstrate the complete integration of logical reasoning and physical activity.

Minsky supervised a series of students who chose limited problems that appeared to require intelligence to solve. These limited domains became known as **microworlds**. James Slagle's SAINT program (1963) was able to solve closed-form calculus integration problems typical of first-year college courses. Tom Evans's ANALOGY program (1968) solved geometric analogy problems that appear in IQ tests. Daniel Bobrow's STUDENT program (1967) solved algebra story problems, such as the following:

MICROWORLD

> If the number of customers Tom gets is twice the square of 20 percent of the number of advertisements he runs, and the number of advertisements he runs is 45, what is the number of customers Tom gets?

Figure 1.4   A scene from the blocks world. SHRDLU (Winograd, 1972) has just completed the command "Find a block which is taller than the one you are holding and put it in the box."
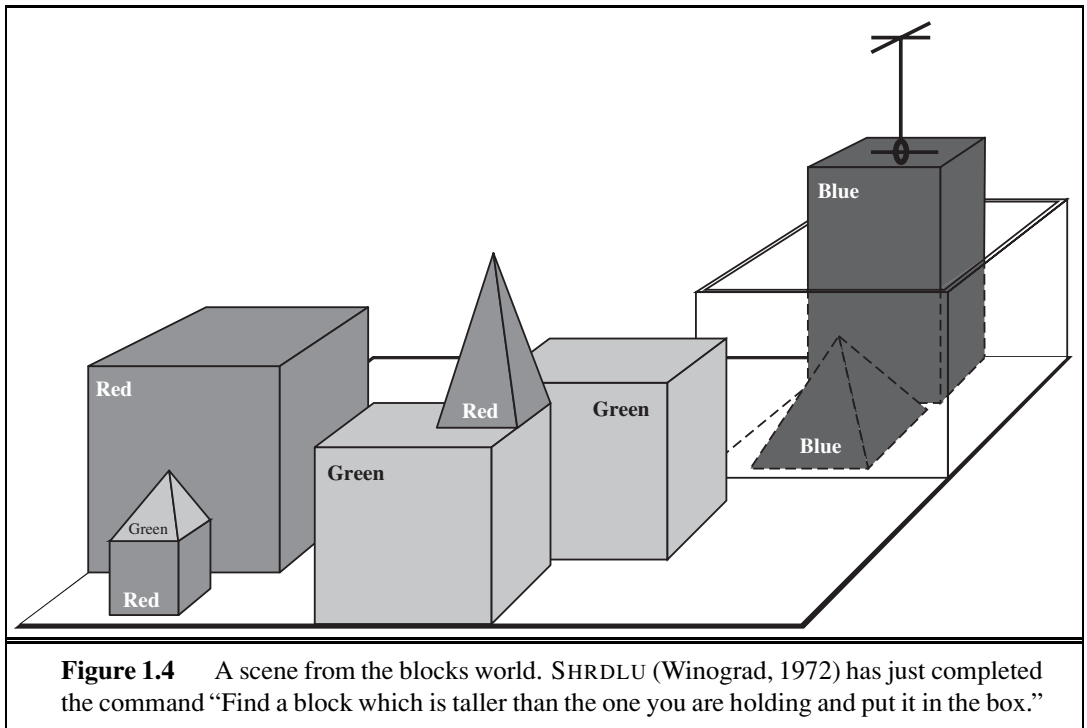
The most famous microworld was the blocks world, which consists of a set of solid blocks placed on a tabletop (or more often, a simulation of a tabletop), as shown in Figure 1.4. A typical task in this world is to rearrange the blocks in a certain way, using a robot hand that can pick up one block at a time. The blocks world was home to the vision project of David Huffman (1971), the vision and constraint-propagation work of David Waltz (1975), the learning theory of Patrick Winston (1970), the natural-language-understanding program of Terry Winograd (1972), and the planner of Scott Fahlman (1974).

Early work building on the neural networks of McCulloch and Pitts also flourished. The work of Winograd and Cowan (1963) showed how a large number of elements could collectively represent an individual concept, with a corresponding increase in robustness and parallelism. Hebb's learning methods were enhanced by Bernie Widrow (Widrow and Hoff, 1960; Widrow, 1962), who called his networks **adalines**, and by Frank Rosenblatt (1962) with his **perceptrons**. The **perceptron convergence theorem** (Block *et al.*, 1962) says that the learning algorithm can adjust the connection strengths of a perceptron to match any input data, provided such a match exists. These topics are covered in Chapter 20.

1.3.4   A dose of reality (1966–1973)

From the beginning, AI researchers were not shy about making predictions of their coming successes. The following statement by Herbert Simon in 1957 is often quoted:

It is not my aim to surprise or shock you—but the simplest way I can summarize is to say that there are now in the world machines that think, that learn and that create. Moreover,

their ability to do these things is going to increase rapidly until—in a visible future—the range of problems they can handle will be coextensive with the range to which the human mind has been applied.

Terms such as "visible future" can be interpreted in various ways, but Simon also made more concrete predictions: that within 10 years a computer would be chess champion, and a significant mathematical theorem would be proved by machine. These predictions came true (or approximately true) within 40 years rather than 10. Simon's overconfidence was due to the promising performance of early AI systems on simple examples. In almost all cases, however, these early systems turned out to fail miserably when tried out on wider selections of problems and on more difficult problems.

The first kind of difficulty arose because most early programs knew nothing of their subject matter; they succeeded by means of simple syntactic manipulations. A typical story occurred in early machine translation efforts, which were generously funded by the U.S. National Research Council in an attempt to speed up the translation of Russian scientific papers in the wake of the Sputnik launch in 1957. It was thought initially that simple syntactic transformations based on the grammars of Russian and English, and word replacement from an electronic dictionary, would suffice to preserve the exact meanings of sentences. The fact is that accurate translation requires background knowledge in order to resolve ambiguity and establish the content of the sentence. The famous retranslation of "the spirit is willing but the flesh is weak" as "the vodka is good but the meat is rotten" illustrates the difficulties encountered. In 1966, a report by an advisory committee found that "there has been no machine translation of general scientific text, and none is in immediate prospect." All U.S. government funding for academic translation projects was canceled. Today, machine translation is an imperfect but widely used tool for technical, commercial, government, and Internet documents.

The second kind of difficulty was the intractability of many of the problems that AI was attempting to solve. Most of the early AI programs solved problems by trying out different combinations of steps until the solution was found. This strategy worked initially because microworlds contained very few objects and hence very few possible actions and very short solution sequences. Before the theory of computational complexity was developed, it was widely thought that "scaling up" to larger problems was simply a matter of faster hardware and larger memories. The optimism that accompanied the development of resolution theorem proving, for example, was soon dampened when researchers failed to prove theorems involving more than a few dozen facts. *The fact that a program can find a solution in principle does not mean that the program contains any of the mechanisms needed to find it in practice.*

MACHINE EVOLUTION
GENETIC
ALGORITHM

The illusion of unlimited computational power was not confined to problem-solving programs. Early experiments in **machine evolution** (now called **genetic algorithms**) (Friedberg, 1958; Friedberg *et al.*, 1959) were based on the undoubtedly correct belief that by making an appropriate series of small mutations to a machine-code program, one can generate a program with good performance for any particular task. The idea, then, was to try random mutations with a selection process to preserve mutations that seemed useful. Despite thousands of hours of CPU time, almost no progress was demonstrated. Modern genetic algorithms use better representations and have shown more success.

Failure to come to grips with the "combinatorial explosion" was one of the main criticisms of AI contained in the Lighthill report (Lighthill, 1973), which formed the basis for the decision by the British government to end support for AI research in all but two universities. (Oral tradition paints a somewhat different and more colorful picture, with political ambitions and personal animosities whose description is beside the point.)

A third difficulty arose because of some fundamental limitations on the basic structures being used to generate intelligent behavior. For example, Minsky and Papert's book *Perceptrons* (1969) proved that, although perceptrons (a simple form of neural network) could be shown to learn anything they were capable of representing, they could represent very little. In particular, a two-input perceptron (restricted to be simpler than the form Rosenblatt originally studied) could not be trained to recognize when its two inputs were different. Although their results did not apply to more complex, multilayer networks, research funding for neural-net research soon dwindled to almost nothing. Ironically, the new back-propagation learning algorithms for multilayer networks that were to cause an enormous resurgence in neural-net research in the late 1980s were actually discovered first in 1969 (Bryson and Ho, 1969).

1.3.5   Knowledge-based systems: The key to power? (1969–1979)

The picture of problem solving that had arisen during the first decade of AI research was of a general-purpose search mechanism trying to string together elementary reasoning steps to find complete solutions. Such approaches have been called **weak methods** because, although general, they do not scale up to large or difficult problem instances. The alternative to weak methods is to use more powerful, domain-specific knowledge that allows larger reasoning steps and can more easily handle typically occurring cases in narrow areas of expertise. One might say that to solve a hard problem, you have to almost know the answer already.

WEAK METHOD

The DENDRAL program (Buchanan *et al.*, 1969) was an early example of this approach. It was developed at Stanford, where Ed Feigenbaum (a former student of Herbert Simon), Bruce Buchanan (a philosopher turned computer scientist), and Joshua Lederberg (a Nobel laureate geneticist) teamed up to solve the problem of inferring molecular structure from the information provided by a mass spectrometer. The input to the program consists of the elementary formula of the molecule (e.g., $C_6H_{13}NO_2$) and the mass spectrum giving the masses of the various fragments of the molecule generated when it is bombarded by an electron beam. For example, the mass spectrum might contain a peak at $m = 15$, corresponding to the mass of a methyl (CH_3) fragment.

The naive version of the program generated all possible structures consistent with the formula, and then predicted what mass spectrum would be observed for each, comparing this with the actual spectrum. As one might expect, this is intractable for even moderate-sized molecules. The DENDRAL researchers consulted analytical chemists and found that they worked by looking for well-known patterns of peaks in the spectrum that suggested common substructures in the molecule. For example, the following rule is used to recognize a ketone (C=O) subgroup (which weighs 28):

if there are two peaks at x_1 and x_2 such that
(a) $x_1 + x_2 = M + 28$ (M is the mass of the whole molecule);

(b) $x_1 - 28$ is a high peak;
(c) $x_2 - 28$ is a high peak;
(d) At least one of x_1 and x_2 is high.
then there is a ketone subgroup

Recognizing that the molecule contains a particular substructure reduces the number of possible candidates enormously. DENDRAL was powerful because

> All the relevant theoretical knowledge to solve these problems has been mapped over from its general form in the [spectrum prediction component] ("first principles") to efficient special forms ("cookbook recipes"). (Feigenbaum *et al.*, 1971)

The significance of DENDRAL was that it was the first successful *knowledge-intensive* system: its expertise derived from large numbers of special-purpose rules. Later systems also incorporated the main theme of McCarthy's Advice Taker approach—the clean separation of the knowledge (in the form of rules) from the reasoning component.

EXPERT SYSTEMS

With this lesson in mind, Feigenbaum and others at Stanford began the Heuristic Programming Project (HPP) to investigate the extent to which the new methodology of **expert systems** could be applied to other areas of human expertise. The next major effort was in the area of medical diagnosis. Feigenbaum, Buchanan, and Dr. Edward Shortliffe developed MYCIN to diagnose blood infections. With about 450 rules, MYCIN was able to perform as well as some experts, and considerably better than junior doctors. It also contained two major differences from DENDRAL. First, unlike the DENDRAL rules, no general theoretical model existed from which the MYCIN rules could be deduced. They had to be acquired from extensive interviewing of experts, who in turn acquired them from textbooks, other experts, and direct experience of cases. Second, the rules had to reflect the uncertainty associated with

CERTAINTY FACTOR

medical knowledge. MYCIN incorporated a calculus of uncertainty called **certainty factors** (see Chapter 14), which seemed (at the time) to fit well with how doctors assessed the impact of evidence on the diagnosis.

The importance of domain knowledge was also apparent in the area of understanding natural language. Although Winograd's SHRDLU system for understanding natural language had engendered a good deal of excitement, its dependence on syntactic analysis caused some of the same problems as occurred in the early machine translation work. It was able to overcome ambiguity and understand pronoun references, but this was mainly because it was designed specifically for one area—the blocks world. Several researchers, including Eugene Charniak, a fellow graduate student of Winograd's at MIT, suggested that robust language understanding would require general knowledge about the world and a general method for using that knowledge.

At Yale, linguist-turned-AI-researcher Roger Schank emphasized this point, claiming, "There is no such thing as syntax," which upset a lot of linguists but did serve to start a useful discussion. Schank and his students built a series of programs (Schank and Abelson, 1977; Wilensky, 1978; Schank and Riesbeck, 1981; Dyer, 1983) that all had the task of understanding natural language. The emphasis, however, was less on language *per se* and more on the problems of representing and reasoning with the knowledge required for language understanding. The problems included representing stereotypical situations (Cullingford, 1981),

describing human memory organization (Rieger, 1976; Kolodner, 1983), and understanding plans and goals (Wilensky, 1983).

The widespread growth of applications to real-world problems caused a concurrent increase in the demands for workable knowledge representation schemes. A large number of different representation and reasoning languages were developed. Some were based on logic—for example, the Prolog language became popular in Europe, and the PLANNER family in the United States. Others, following Minsky's idea of **frames** (1975), adopted a more structured approach, assembling facts about particular object and event types and arranging the types into a large taxonomic hierarchy analogous to a biological taxonomy.

FRAMES

1.3.6   AI becomes an industry (1980–present)

The first successful commercial expert system, R1, began operation at the Digital Equipment Corporation (McDermott, 1982). The program helped configure orders for new computer systems; by 1986, it was saving the company an estimated $40 million a year. By 1988, DEC's AI group had 40 expert systems deployed, with more on the way. DuPont had 100 in use and 500 in development, saving an estimated $10 million a year. Nearly every major U.S. corporation had its own AI group and was either using or investigating expert systems.

In 1981, the Japanese announced the "Fifth Generation" project, a 10-year plan to build intelligent computers running Prolog. In response, the United States formed the Microelectronics and Computer Technology Corporation (MCC) as a research consortium designed to assure national competitiveness. In both cases, AI was part of a broad effort, including chip design and human-interface research. In Britain, the Alvey report reinstated the funding that was cut by the Lighthill report.[13] In all three countries, however, the projects never met their ambitious goals.

Overall, the AI industry boomed from a few million dollars in 1980 to billions of dollars in 1988, including hundreds of companies building expert systems, vision systems, robots, and software and hardware specialized for these purposes. Soon after that came a period called the "AI Winter," in which many companies fell by the wayside as they failed to deliver on extravagant promises.

1.3.7   The return of neural networks (1986–present)

BACK-PROPAGATION

In the mid-1980s at least four different groups reinvented the **back-propagation** learning algorithm first found in 1969 by Bryson and Ho. The algorithm was applied to many learning problems in computer science and psychology, and the widespread dissemination of the results in the collection *Parallel Distributed Processing* (Rumelhart and McClelland, 1986) caused great excitement.

CONNECTIONIST

These so-called **connectionist** models of intelligent systems were seen by some as direct competitors both to the symbolic models promoted by Newell and Simon and to the logicist approach of McCarthy and others (Smolensky, 1988). It might seem obvious that at some level humans manipulate symbols—in fact, Terrence Deacon's book *The Symbolic*

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[13] To save embarrassment, a new field called IKBS (Intelligent Knowledge-Based Systems) was invented because Artificial Intelligence had been officially canceled.

Species (1997) suggests that this is the *defining characteristic* of humans—but the most ardent connectionists questioned whether symbol manipulation had any real explanatory role in detailed models of cognition. This question remains unanswered, but the current view is that connectionist and symbolic approaches are complementary, not competing. As occurred with the separation of AI and cognitive science, modern neural network research has bifurcated into two fields, one concerned with creating effective network architectures and algorithms and understanding their mathematical properties, the other concerned with careful modeling of the empirical properties of actual neurons and ensembles of neurons.

1.3.8   AI adopts the scientific method (1987–present)

Recent years have seen a revolution in both the content and the methodology of work in artificial intelligence.[14] It is now more common to build on existing theories than to propose brand-new ones, to base claims on rigorous theorems or hard experimental evidence rather than on intuition, and to show relevance to real-world applications rather than toy examples.

AI was founded in part as a rebellion against the limitations of existing fields like control theory and statistics, but now it is embracing those fields. As David McAllester (1998) put it:

> In the early period of AI it seemed plausible that new forms of symbolic computation, e.g., frames and semantic networks, made much of classical theory obsolete. This led to a form of isolationism in which AI became largely separated from the rest of computer science. This isolationism is currently being abandoned. There is a recognition that machine learning should not be isolated from information theory, that uncertain reasoning should not be isolated from stochastic modeling, that search should not be isolated from classical optimization and control, and that automated reasoning should not be isolated from formal methods and static analysis.

In terms of methodology, AI has finally come firmly under the scientific method. To be accepted, hypotheses must be subjected to rigorous empirical experiments, and the results must be analyzed statistically for their importance (Cohen, 1995). It is now possible to replicate experiments by using shared repositories of test data and code.

The field of speech recognition illustrates the pattern. In the 1970s, a wide variety of different architectures and approaches were tried. Many of these were rather *ad hoc* and fragile, and were demonstrated on only a few specially selected examples. In recent years, approaches based on **hidden Markov models** (HMMs) have come to dominate the area. Two aspects of HMMs are relevant. First, they are based on a rigorous mathematical theory. This has allowed speech researchers to build on several decades of mathematical results developed in other fields. Second, they are generated by a process of training on a large corpus of real speech data. This ensures that the performance is robust, and in rigorous blind tests the HMMs have been improving their scores steadily. Speech technology and the related field of handwritten character recognition are already making the transition to widespread industrial

HIDDEN MARKOV
MODELS

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[14] Some have characterized this change as a victory of the **neats**—those who think that AI theories should be grounded in mathematical rigor—over the **scruffies**—those who would rather try out lots of ideas, write some programs, and then assess what seems to be working. Both approaches are important. A shift toward neatness implies that the field has reached a level of stability and maturity. Whether that stability will be disrupted by a new scruffy idea is another question.

and consumer applications. Note that there is no scientific claim that humans use HMMs to recognize speech; rather, HMMs provide a mathematical framework for understanding the problem and support the engineering claim that they work well in practice.

Machine translation follows the same course as speech recognition. In the 1950s there was initial enthusiasm for an approach based on sequences of words, with models learned according to the principles of information theory. That approach fell out of favor in the 1960s, but returned in the late 1990s and now dominates the field.

Neural networks also fit this trend. Much of the work on neural nets in the 1980s was done in an attempt to scope out what could be done and to learn how neural nets differ from "traditional" techniques. Using improved methodology and theoretical frameworks, the field arrived at an understanding in which neural nets can now be compared with corresponding techniques from statistics, pattern recognition, and machine learning, and the most promising technique can be applied to each application. As a result of these developments, so-called **data mining** technology has spawned a vigorous new industry.

DATA MINING

Judea Pearl's (1988) *Probabilistic Reasoning in Intelligent Systems* led to a new acceptance of probability and decision theory in AI, following a resurgence of interest epitomized by Peter Cheeseman's (1985) article "In Defense of Probability." The **Bayesian network** formalism was invented to allow efficient representation of, and rigorous reasoning with, uncertain knowledge. This approach largely overcomes many problems of the probabilistic reasoning systems of the 1960s and 1970s; it now dominates AI research on uncertain reasoning and expert systems. The approach allows for learning from experience, and it combines the best of classical AI and neural nets. Work by Judea Pearl (1982a) and by Eric Horvitz and David Heckerman (Horvitz and Heckerman, 1986; Horvitz *et al.*, 1986) promoted the idea of *normative* expert systems: ones that act rationally according to the laws of decision theory and do not try to imitate the thought steps of human experts. The Windows$^{\text{TM}}$ operating system includes several normative diagnostic expert systems for correcting problems. Chapters 13 to 16 cover this area.

BAYESIAN NETWORK

Similar gentle revolutions have occurred in robotics, computer vision, and knowledge representation. A better understanding of the problems and their complexity properties, combined with increased mathematical sophistication, has led to workable research agendas and robust methods. Although increased formalization and specialization led fields such as vision and robotics to become somewhat isolated from "mainstream" AI in the 1990s, this trend has reversed in recent years as tools from machine learning in particular have proved effective for many problems. The process of reintegration is already yielding significant benefits

1.3.9   The emergence of intelligent agents (1995–present)

Perhaps encouraged by the progress in solving the subproblems of AI, researchers have also started to look at the "whole agent" problem again. The work of Allen Newell, John Laird, and Paul Rosenbloom on SOAR (Newell, 1990; Laird *et al.*, 1987) is the best-known example of a complete agent architecture. One of the most important environments for intelligent agents is the Internet. AI systems have become so common in Web-based applications that the "-bot" suffix has entered everyday language. Moreover, AI technologies underlie many

Internet tools, such as search engines, recommender systems, and Web site aggregators.

One consequence of trying to build complete agents is the realization that the previously isolated subfields of AI might need to be reorganized somewhat when their results are to be tied together. In particular, it is now widely appreciated that sensory systems (vision, sonar, speech recognition, etc.) cannot deliver perfectly reliable information about the environment. Hence, reasoning and planning systems must be able to handle uncertainty. A second major consequence of the agent perspective is that AI has been drawn into much closer contact with other fields, such as control theory and economics, that also deal with agents. Recent progress in the control of robotic cars has derived from a mixture of approaches ranging from better sensors, control-theoretic integration of sensing, localization and mapping, as well as a degree of high-level planning.

Despite these successes, some influential founders of AI, including John McCarthy (2007), Marvin Minsky (2007), Nils Nilsson (1995, 2005) and Patrick Winston (Beal and Winston, 2009), have expressed discontent with the progress of AI. They think that AI should put less emphasis on creating ever-improved versions of applications that are good at a specific task, such as driving a car, playing chess, or recognizing speech. Instead, they believe AI should return to its roots of striving for, in Simon's words, "machines that think, that learn and that create." They call the effort **human-level AI** or HLAI; their first symposium was in 2004 (Minsky *et al.*, 2004). The effort will require very large knowledge bases; Hendler *et al.* (1995) discuss where these knowledge bases might come from.

HUMAN-LEVEL AI

A related idea is the subfield of **Artificial General Intelligence** or AGI (Goertzel and Pennachin, 2007), which held its first conference and organized the *Journal of Artificial General Intelligence* in 2008. AGI looks for a universal algorithm for learning and acting in any environment, and has its roots in the work of Ray Solomonoff (1964), one of the attendees of the original 1956 Dartmouth conference. Guaranteeing that what we create is really **Friendly AI** is also a concern (Yudkowsky, 2008; Omohundro, 2008), one we will return to in Chapter 26.

ARTIFICIAL GENERAL
INTELLIGENCE

FRIENDLY AI

1.3.10   The availability of very large data sets (2001–present)

Throughout the 60-year history of computer science, the emphasis has been on the *algorithm* as the main subject of study. But some recent work in AI suggests that for many problems, it makes more sense to worry about the *data* and be less picky about what algorithm to apply. This is true because of the increasing availability of very large data sources: for example, trillions of words of English and billions of images from the Web (Kilgarriff and Grefenstette, 2006); or billions of base pairs of genomic sequences (Collins *et al.*, 2003).

One influential paper in this line was Yarowsky's (1995) work on word-sense disambiguation: given the use of the word "plant" in a sentence, does that refer to flora or factory? Previous approaches to the problem had relied on human-labeled examples combined with machine learning algorithms. Yarowsky showed that the task can be done, with accuracy above 96%, with no labeled examples at all. Instead, given a very large corpus of unannotated text and just the dictionary definitions of the two senses—"works, industrial plant" and "flora, plant life"—one can label examples in the corpus, and from there **bootstrap** to learn

new patterns that help label new examples. Banko and Brill (2001) show that techniques like this perform even better as the amount of available text goes from a million words to a billion and that the increase in performance from using more data exceeds any difference in algorithm choice; a mediocre algorithm with 100 million words of unlabeled training data outperforms the best known algorithm with 1 million words.

As another example, Hays and Efros (2007) discuss the problem of filling in holes in a photograph. Suppose you use Photoshop to mask out an ex-friend from a group photo, but now you need to fill in the masked area with something that matches the background. Hays and Efros defined an algorithm that searches through a collection of photos to find something that will match. They found the performance of their algorithm was poor when they used a collection of only ten thousand photos, but crossed a threshold into excellent performance when they grew the collection to two million photos.

Work like this suggests that the "knowledge bottleneck" in AI—the problem of how to express all the knowledge that a system needs—may be solved in many applications by learning methods rather than hand-coded knowledge engineering, provided the learning algorithms have enough data to go on (Halevy *et al.*, 2009). Reporters have noticed the surge of new applications and have written that "AI Winter" may be yielding to a new Spring (Havenstein, 2005). As Kurzweil (2005) writes, "today, many thousands of AI applications are deeply embedded in the infrastructure of every industry."

1.4   THE STATE OF THE ART

What can AI do today? A concise answer is difficult because there are so many activities in so many subfields. Here we sample a few applications; others appear throughout the book.

Robotic vehicles: A driverless robotic car named STANLEY sped through the rough terrain of the Mojave dessert at 22 mph, finishing the 132-mile course first to win the 2005 DARPA Grand Challenge. STANLEY is a Volkswagen Touareg outfitted with cameras, radar, and laser rangefinders to sense the environment and onboard software to command the steering, braking, and acceleration (Thrun, 2006). The following year CMU's BOSS won the Urban Challenge, safely driving in traffic through the streets of a closed Air Force base, obeying traffic rules and avoiding pedestrians and other vehicles.

Speech recognition: A traveler calling United Airlines to book a flight can have the entire conversation guided by an automated speech recognition and dialog management system.

Autonomous planning and scheduling: A hundred million miles from Earth, NASA's Remote Agent program became the first on-board autonomous planning program to control the scheduling of operations for a spacecraft (Jonsson *et al.*, 2000). REMOTE AGENT generated plans from high-level goals specified from the ground and monitored the execution of those plans—detecting, diagnosing, and recovering from problems as they occurred. Successor program MAPGEN (Al-Chang *et al.*, 2004) plans the daily operations for NASA's Mars Exploration Rovers, and MEXAR2 (Cesta *et al.*, 2007) did mission planning—both logistics and science planning—for the European Space Agency's Mars Express mission in 2008.

Game playing: IBM's DEEP BLUE became the first computer program to defeat the world champion in a chess match when it bested Garry Kasparov by a score of 3.5 to 2.5 in an exhibition match (Goodman and Keene, 1997). Kasparov said that he felt a "new kind of intelligence" across the board from him. *Newsweek* magazine described the match as "The brain's last stand." The value of IBM's stock increased by $18 billion. Human champions studied Kasparov's loss and were able to draw a few matches in subsequent years, but the most recent human-computer matches have been won convincingly by the computer.

Spam fighting: Each day, learning algorithms classify over a billion messages as spam, saving the recipient from having to waste time deleting what, for many users, could comprise 80% or 90% of all messages, if not classified away by algorithms. Because the spammers are continually updating their tactics, it is difficult for a static programmed approach to keep up, and learning algorithms work best (Sahami *et al.*, 1998; Goodman and Heckerman, 2004).

Logistics planning: During the Persian Gulf crisis of 1991, U.S. forces deployed a Dynamic Analysis and Replanning Tool, DART (Cross and Walker, 1994), to do automated logistics planning and scheduling for transportation. This involved up to 50,000 vehicles, cargo, and people at a time, and had to account for starting points, destinations, routes, and conflict resolution among all parameters. The AI planning techniques generated in hours a plan that would have taken weeks with older methods. The Defense Advanced Research Project Agency (DARPA) stated that this single application more than paid back DARPA's 30-year investment in AI.

Robotics: The iRobot Corporation has sold over two million Roomba robotic vacuum cleaners for home use. The company also deploys the more rugged PackBot to Iraq and Afghanistan, where it is used to handle hazardous materials, clear explosives, and identify the location of snipers.

Machine Translation: A computer program automatically translates from Arabic to English, allowing an English speaker to see the headline "Ardogan Confirms That Turkey Would Not Accept Any Pressure, Urging Them to Recognize Cyprus." The program uses a statistical model built from examples of Arabic-to-English translations and from examples of English text totaling two trillion words (Brants *et al.*, 2007). None of the computer scientists on the team speak Arabic, but they do understand statistics and machine learning algorithms.

These are just a few examples of artificial intelligence systems that exist today. Not magic or science fiction—but rather science, engineering, and mathematics, to which this book provides an introduction.

1.5   SUMMARY

This chapter defines AI and establishes the cultural background against which it has developed. Some of the important points are as follows:

- Different people approach AI with different goals in mind. Two important questions to ask are: Are you concerned with thinking or behavior? Do you want to model humans or work from an ideal standard?

- In this book, we adopt the view that intelligence is concerned mainly with **rational action**. Ideally, an **intelligent agent** takes the best possible action in a situation. We study the problem of building agents that are intelligent in this sense.

- Philosophers (going back to 400 B.C.) made AI conceivable by considering the ideas that the mind is in some ways like a machine, that it operates on knowledge encoded in some internal language, and that thought can be used to choose what actions to take.

- Mathematicians provided the tools to manipulate statements of logical certainty as well as uncertain, probabilistic statements. They also set the groundwork for understanding computation and reasoning about algorithms.

- Economists formalized the problem of making decisions that maximize the expected outcome to the decision maker.

- Neuroscientists discovered some facts about how the brain works and the ways in which it is similar to and different from computers.

- Psychologists adopted the idea that humans and animals can be considered information-processing machines. Linguists showed that language use fits into this model.

- Computer engineers provided the ever-more-powerful machines that make AI applications possible.

- Control theory deals with designing devices that act optimally on the basis of feedback from the environment. Initially, the mathematical tools of control theory were quite different from AI, but the fields are coming closer together.

- The history of AI has had cycles of success, misplaced optimism, and resulting cutbacks in enthusiasm and funding. There have also been cycles of introducing new creative approaches and systematically refining the best ones.

- AI has advanced more rapidly in the past decade because of greater use of the scientific method in experimenting with and comparing approaches.

- Recent progress in understanding the theoretical basis for intelligence has gone hand in hand with improvements in the capabilities of real systems. The subfields of AI have become more integrated, and AI has found common ground with other disciplines.

BIBLIOGRAPHICAL AND HISTORICAL NOTES

The methodological status of artificial intelligence is investigated in *The Sciences of the Artificial*, by Herb Simon (1981), which discusses research areas concerned with complex artifacts. It explains how AI can be viewed as both science and mathematics. Cohen (1995) gives an overview of experimental methodology within AI.

The Turing Test (Turing, 1950) is discussed by Shieber (1994), who severely criticizes the usefulness of its instantiation in the Loebner Prize competition, and by Ford and Hayes (1995), who argue that the test itself is not helpful for AI. Bringsjord (2008) gives advice for a Turing Test judge. Shieber (2004) and Epstein *et al.* (2008) collect a number of essays on the Turing Test. *Artificial Intelligence: The Very Idea*, by John Haugeland (1985), gives a

readable account of the philosophical and practical problems of AI. Significant early papers in AI are anthologized in the collections by Webber and Nilsson (1981) and by Luger (1995). The *Encyclopedia of AI* (Shapiro, 1992) contains survey articles on almost every topic in AI, as does Wikipedia. These articles usually provide a good entry point into the research literature on each topic. An insightful and comprehensive history of AI is given by Nils Nillson (2009), one of the early pioneers of the field.

The most recent work appears in the proceedings of the major AI conferences: the biennial International Joint Conference on AI (IJCAI), the annual European Conference on AI (ECAI), and the National Conference on AI, more often known as AAAI, after its sponsoring organization. The major journals for general AI are *Artificial Intelligence*, *Computational Intelligence*, the *IEEE Transactions on Pattern Analysis and Machine Intelligence*, *IEEE Intelligent Systems*, and the electronic *Journal of Artificial Intelligence Research*. There are also many conferences and journals devoted to specific areas, which we cover in the appropriate chapters. The main professional societies for AI are the American Association for Artificial Intelligence (AAAI), the ACM Special Interest Group in Artificial Intelligence (SIGART), and the Society for Artificial Intelligence and Simulation of Behaviour (AISB). AAAI's *AI Magazine* contains many topical and tutorial articles, and its Web site, `aaai.org`, contains news, tutorials, and background information.

EXERCISES

These exercises are intended to stimulate discussion, and some might be set as term projects. Alternatively, preliminary attempts can be made now, and these attempts can be reviewed after the completion of the book.

1.1 Define in your own words: (a) intelligence, (b) artificial intelligence, (c) agent, (d) rationality, (e) logical reasoning.
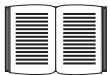
1.2 Read Turing's original paper on AI (Turing, 1950). In the paper, he discusses several objections to his proposed enterprise and his test for intelligence. Which objections still carry weight? Are his refutations valid? Can you think of new objections arising from developments since he wrote the paper? In the paper, he predicts that, by the year 2000, a computer will have a 30% chance of passing a five-minute Turing Test with an unskilled interrogator. What chance do you think a computer would have today? In another 50 years?

1.3 Are reflex actions (such as flinching from a hot stove) rational? Are they intelligent?

1.4 Suppose we extend Evans's ANALOGY program so that it can score 200 on a standard IQ test. Would we then have a program more intelligent than a human? Explain.

1.5 The neural structure of the sea slug *Aplysia* has been widely studied (first by Nobel Laureate Eric Kandel) because it has only about 20,000 neurons, most of them large and easily manipulated. Assuming that the cycle time for an *Aplysia* neuron is roughly the same as for a human neuron, how does the computational power, in terms of memory updates per second, compare with the high-end computer described in Figure 1.3?

1.6   How could introspection—reporting on one's inner thoughts—be inaccurate? Could I be wrong about what I'm thinking? Discuss.

1.7   To what extent are the following computer systems instances of artificial intelligence:

- Supermarket bar code scanners.
- Web search engines.
- Voice-activated telephone menus.
- Internet routing algorithms that respond dynamically to the state of the network.

1.8   Many of the computational models of cognitive activities that have been proposed involve quite complex mathematical operations, such as convolving an image with a Gaussian or finding a minimum of the entropy function. Most humans (and certainly all animals) never learn this kind of mathematics at all, almost no one learns it before college, and almost no one can compute the convolution of a function with a Gaussian in their head. What sense does it make to say that the "vision system" is doing this kind of mathematics, whereas the actual person has no idea how to do it?

1.9   Why would evolution tend to result in systems that act rationally? What goals are such systems designed to achieve?

1.10   Is AI a science, or is it engineering? Or neither or both? Explain.

1.11   "Surely computers cannot be intelligent—they can do only what their programmers tell them." Is the latter statement true, and does it imply the former?

1.12   "Surely animals cannot be intelligent—they can do only what their genes tell them." Is the latter statement true, and does it imply the former?

1.13   "Surely animals, humans, and computers cannot be intelligent—they can do only what their constituent atoms are told to do by the laws of physics." Is the latter statement true, and does it imply the former?
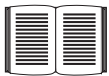
1.14   Examine the AI literature to discover whether the following tasks can currently be solved by computers:

  a. Playing a decent game of table tennis (Ping-Pong).
  b. Driving in the center of Cairo, Egypt.
  c. Driving in Victorville, California.
  d. Buying a week's worth of groceries at the market.
  e. Buying a week's worth of groceries on the Web.
  f. Playing a decent game of bridge at a competitive level.
  g. Discovering and proving new mathematical theorems.
  h. Writing an intentionally funny story.
  i. Giving competent legal advice in a specialized area of law.
  j. Translating spoken English into spoken Swedish in real time.
  k. Performing a complex surgical operation.

For the currently infeasible tasks, try to find out what the difficulties are and predict when, if ever, they will be overcome.

1.15   Various subfields of AI have held contests by defining a standard task and inviting researchers to do their best. Examples include the DARPA Grand Challenge for robotic cars, The International Planning Competition, the Robocup robotic soccer league, the TREC information retrieval event, and contests in machine translation, speech recognition. Investigate five of these contests, and describe the progress made over the years. To what degree have the contests advanced toe state of the art in AI? Do what degree do they hurt the field by drawing energy away from new ideas?

2 INTELLIGENT AGENTS

In which we discuss the nature of agents, perfect or otherwise, the diversity of environments, and the resulting menagerie of agent types.

Chapter 1 identified the concept of **rational agents** as central to our approach to artificial intelligence. In this chapter, we make this notion more concrete. We will see that the concept of rationality can be applied to a wide variety of agents operating in any imaginable environment. Our plan in this book is to use this concept to develop a small set of design principles for building successful agents—systems that can reasonably be called **intelligent**.

We begin by examining agents, environments, and the coupling between them. The observation that some agents behave better than others leads naturally to the idea of a rational agent—one that behaves as well as possible. How well an agent can behave depends on the nature of the environment; some environments are more difficult than others. We give a crude categorization of environments and show how properties of an environment influence the design of suitable agents for that environment. We describe a number of basic "skeleton" agent designs, which we flesh out in the rest of the book.

2.1 AGENTS AND ENVIRONMENTS

ENVIRONMENT

SENSOR

ACTUATOR

An **agent** is anything that can be viewed as perceiving its **environment** through **sensors** and acting upon that environment through **actuators**. This simple idea is illustrated in Figure 2.1. A human agent has eyes, ears, and other organs for sensors and hands, legs, vocal tract, and so on for actuators. A robotic agent might have cameras and infrared range finders for sensors and various motors for actuators. A software agent receives keystrokes, file contents, and network packets as sensory inputs and acts on the environment by displaying on the screen, writing files, and sending network packets.

PERCEPT

PERCEPT SEQUENCE

We use the term **percept** to refer to the agent's perceptual inputs at any given instant. An agent's **percept sequence** is the complete history of everything the agent has ever perceived. In general, *an agent's choice of action at any given instant can depend on the entire percept sequence observed to date, but not on anything it hasn't perceived.* By specifying the agent's choice of action for every possible percept sequence, we have said more or less everything

Figure 2.1   Agents interact with environments through sensors and actuators.
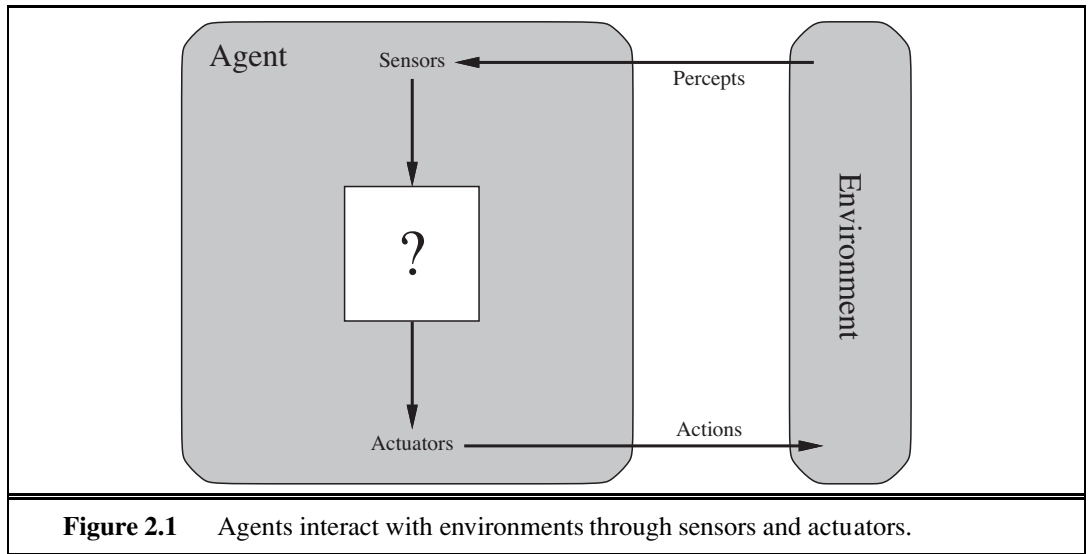
AGENT FUNCTION

there is to say about the agent. Mathematically speaking, we say that an agent's behavior is described by the **agent function** that maps any given percept sequence to an action.

We can imagine *tabulating* the agent function that describes any given agent; for most agents, this would be a very large table—infinite, in fact, unless we place a bound on the length of percept sequences we want to consider. Given an agent to experiment with, we can, in principle, construct this table by trying out all possible percept sequences and recording which actions the agent does in response.[1] The table is, of course, an *external* characterization of the agent. *Internally*, the agent function for an artificial agent will be implemented by an

AGENT PROGRAM

agent program. It is important to keep these two ideas distinct. The agent function is an abstract mathematical description; the agent program is a concrete implementation, running within some physical system.

To illustrate these ideas, we use a very simple example—the vacuum-cleaner world shown in Figure 2.2. This world is so simple that we can describe everything that happens; it's also a made-up world, so we can invent many variations. This particular world has just two locations: squares A and B. The vacuum agent perceives which square it is in and whether there is dirt in the square. It can choose to move left, move right, suck up the dirt, or do nothing. One very simple agent function is the following: if the current square is dirty, then suck; otherwise, move to the other square. A partial tabulation of this agent function is shown in Figure 2.3 and an agent program that implements it appears in Figure 2.8 on page 48.

Looking at Figure 2.3, we see that various vacuum-world agents can be defined simply by filling in the right-hand column in various ways. The obvious question, then, is this: *What is the right way to fill out the table?* In other words, what makes an agent good or bad, intelligent or stupid? We answer these questions in the next section.

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[1] If the agent uses some randomization to choose its actions, then we would have to try each sequence many times to identify the probability of each action. One might imagine that acting randomly is rather silly, but we show later in this chapter that it can be very intelligent.
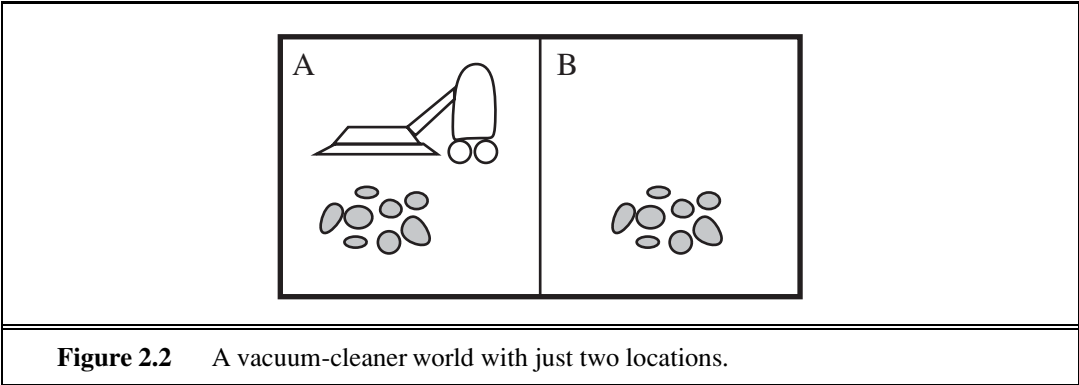
Figure 2.2   A vacuum-cleaner world with just two locations.

Percept sequence	Action
$[A, Clean]$	*Right*
$[A, Dirty]$	*Suck*
$[B, Clean]$	*Left*
$[B, Dirty]$	*Suck*
$[A, Clean], [A, Clean]$	*Right*
$[A, Clean], [A, Dirty]$	*Suck*
⋮	⋮
$[A, Clean], [A, Clean], [A, Clean]$	*Right*
$[A, Clean], [A, Clean], [A, Dirty]$	*Suck*
⋮	⋮

Figure 2.3   Partial tabulation of a simple agent function for the vacuum-cleaner world shown in Figure 2.2.

Before closing this section, we should emphasize that the notion of an agent is meant to be a tool for analyzing systems, not an absolute characterization that divides the world into agents and non-agents. One could view a hand-held calculator as an agent that chooses the action of displaying "4" when given the percept sequence "2 + 2 =," but such an analysis would hardly aid our understanding of the calculator. In a sense, all areas of engineering can be seen as designing artifacts that interact with the world; AI operates at (what the authors consider to be) the most interesting end of the spectrum, where the artifacts have significant computational resources and the task environment requires nontrivial decision making.

2.2   GOOD BEHAVIOR: THE CONCEPT OF RATIONALITY

RATIONAL AGENT   A **rational agent** is one that does the right thing—conceptually speaking, every entry in the table for the agent function is filled out correctly. Obviously, doing the right thing is better than doing the wrong thing, but what does it mean to do the right thing?

We answer this age-old question in an age-old way: by considering the *consequences* of the agent's behavior. When an agent is plunked down in an environment, it generates a sequence of actions according to the percepts it receives. This sequence of actions causes the environment to go through a sequence of states. If the sequence is desirable, then the agent has performed well. This notion of desirability is captured by a **performance measure** that evaluates any given sequence of environment states.

Notice that we said *environment* states, not *agent* states. If we define success in terms of agent's opinion of its own performance, an agent could achieve perfect rationality simply by deluding itself that its performance was perfect. Human agents in particular are notorious for "sour grapes"—believing they did not really want something (e.g., a Nobel Prize) after not getting it.

Obviously, there is not one fixed performance measure for all tasks and agents; typically, a designer will devise one appropriate to the circumstances. This is not as easy as it sounds. Consider, for example, the vacuum-cleaner agent from the preceding section. We might propose to measure performance by the amount of dirt cleaned up in a single eight-hour shift. With a rational agent, of course, what you ask for is what you get. A rational agent can maximize this performance measure by cleaning up the dirt, then dumping it all on the floor, then cleaning it up again, and so on. A more suitable performance measure would reward the agent for having a clean floor. For example, one point could be awarded for each clean square at each time step (perhaps with a penalty for electricity consumed and noise generated). *As a general rule, it is better to design performance measures according to what one actually wants in the environment, rather than according to how one thinks the agent should behave.*

Even when the obvious pitfalls are avoided, there remain some knotty issues to untangle. For example, the notion of "clean floor" in the preceding paragraph is based on average cleanliness over time. Yet the same average cleanliness can be achieved by two different agents, one of which does a mediocre job all the time while the other cleans energetically but takes long breaks. Which is preferable might seem to be a fine point of janitorial science, but in fact it is a deep philosophical question with far-reaching implications. Which is better—a reckless life of highs and lows, or a safe but humdrum existence? Which is better—an economy where everyone lives in moderate poverty, or one in which some live in plenty while others are very poor? We leave these questions as an exercise for the diligent reader.

2.2.1   Rationality

What is rational at any given time depends on four things:

- The performance measure that defines the criterion of success.
- The agent's prior knowledge of the environment.
- The actions that the agent can perform.
- The agent's percept sequence to date.

This leads to a **definition of a rational agent**:

> *For each possible percept sequence, a rational agent should select an action that is expected to maximize its performance measure, given the evidence provided by the percept sequence and whatever built-in knowledge the agent has.*

Consider the simple vacuum-cleaner agent that cleans a square if it is dirty and moves to the other square if not; this is the agent function tabulated in Figure 2.3. Is this a rational agent? That depends! First, we need to say what the performance measure is, what is known about the environment, and what sensors and actuators the agent has. Let us assume the following:

- The performance measure awards one point for each clean square at each time step, over a "lifetime" of 1000 time steps.
- The "geography" of the environment is known *a priori* (Figure 2.2) but the dirt distribution and the initial location of the agent are not. Clean squares stay clean and sucking cleans the current square. The *Left* and *Right* actions move the agent left and right except when this would take the agent outside the environment, in which case the agent remains where it is.
- The only available actions are *Left*, *Right*, and *Suck*.
- The agent correctly perceives its location and whether that location contains dirt.

We claim that *under these circumstances* the agent is indeed rational; its expected performance is at least as high as any other agent's. Exercise 2.2 asks you to prove this.

One can see easily that the same agent would be irrational under different circumstances. For example, once all the dirt is cleaned up, the agent will oscillate needlessly back and forth; if the performance measure includes a penalty of one point for each movement left or right, the agent will fare poorly. A better agent for this case would do nothing once it is sure that all the squares are clean. If clean squares can become dirty again, the agent should occasionally check and re-clean them if needed. If the geography of the environment is unknown, the agent will need to explore it rather than stick to squares A and B. Exercise 2.2 asks you to design agents for these cases.

2.2.2 Omniscience, learning, and autonomy

OMNISCIENCE

We need to be careful to distinguish between rationality and **omniscience**. An omniscient agent knows the *actual* outcome of its actions and can act accordingly; but omniscience is impossible in reality. Consider the following example: I am walking along the Champs Elysées one day and I see an old friend across the street. There is no traffic nearby and I'm not otherwise engaged, so, being rational, I start to cross the street. Meanwhile, at 33,000 feet, a cargo door falls off a passing airliner,[2] and before I make it to the other side of the street I am flattened. Was I irrational to cross the street? It is unlikely that my obituary would read "Idiot attempts to cross street."

This example shows that rationality is not the same as perfection. Rationality maximizes *expected* performance, while perfection maximizes *actual* performance. Retreating from a requirement of perfection is not just a question of being fair to agents. The point is that if we expect an agent to do what turns out to be the best action after the fact, it will be impossible to design an agent to fulfill this specification—unless we improve the performance of crystal balls or time machines.

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[2] See N. Henderson, "New door latches urged for Boeing 747 jumbo jets," *Washington Post*, August 24, 1989.

Our definition of rationality does not require omniscience, then, because the rational choice depends only on the percept sequence *to date*. We must also ensure that we haven't inadvertently allowed the agent to engage in decidedly underintelligent activities. For example, if an agent does not look both ways before crossing a busy road, then its percept sequence will not tell it that there is a large truck approaching at high speed. Does our definition of rationality say that it's now OK to cross the road? Far from it! First, it would not be rational to cross the road given this uninformative percept sequence: the risk of accident from crossing without looking is too great. Second, a rational agent should choose the "looking" action before stepping into the street, because looking helps maximize the expected performance. Doing actions *in order to modify future percepts*—sometimes called **information gathering**—is an important part of rationality and is covered in depth in Chapter 16. A second example of information gathering is provided by the **exploration** that must be undertaken by a vacuum-cleaning agent in an initially unknown environment.

INFORMATION GATHERING

EXPLORATION

LEARNING

Our definition requires a rational agent not only to gather information but also to **learn** as much as possible from what it perceives. The agent's initial configuration could reflect some prior knowledge of the environment, but as the agent gains experience this may be modified and augmented. There are extreme cases in which the environment is completely known *a priori*. In such cases, the agent need not perceive or learn; it simply acts correctly. Of course, such agents are fragile. Consider the lowly dung beetle. After digging its nest and laying its eggs, it fetches a ball of dung from a nearby heap to plug the entrance. If the ball of dung is removed from its grasp *en route*, the beetle continues its task and pantomimes plugging the nest with the nonexistent dung ball, never noticing that it is missing. Evolution has built an assumption into the beetle's behavior, and when it is violated, unsuccessful behavior results. Slightly more intelligent is the sphex wasp. The female sphex will dig a burrow, go out and sting a caterpillar and drag it to the burrow, enter the burrow again to check all is well, drag the caterpillar inside, and lay its eggs. The caterpillar serves as a food source when the eggs hatch. So far so good, but if an entomologist moves the caterpillar a few inches away while the sphex is doing the check, it will revert to the "drag" step of its plan and will continue the plan without modification, even after dozens of caterpillar-moving interventions. The sphex is unable to learn that its innate plan is failing, and thus will not change it.

To the extent that an agent relies on the prior knowledge of its designer rather than on its own percepts, we say that the agent lacks **autonomy**. A rational agent should be autonomous—it should learn what it can to compensate for partial or incorrect prior knowledge. For example, a vacuum-cleaning agent that learns to foresee where and when additional dirt will appear will do better than one that does not. As a practical matter, one seldom requires complete autonomy from the start: when the agent has had little or no experience, it would have to act randomly unless the designer gave some assistance. So, just as evolution provides animals with enough built-in reflexes to survive long enough to learn for themselves, it would be reasonable to provide an artificial intelligent agent with some initial knowledge as well as an ability to learn. After sufficient experience of its environment, the behavior of a rational agent can become effectively *independent* of its prior knowledge. Hence, the incorporation of learning allows one to design a single rational agent that will succeed in a vast variety of environments.

AUTONOMY

2.3   THE NATURE OF ENVIRONMENTS

Now that we have a definition of rationality, we are almost ready to think about building rational agents. First, however, we must think about **task environments**, which are essentially the "problems" to which rational agents are the "solutions." We begin by showing how to specify a task environment, illustrating the process with a number of examples. We then show that task environments come in a variety of flavors. The flavor of the task environment directly affects the appropriate design for the agent program.

2.3.1   Specifying the task environment

In our discussion of the rationality of the simple vacuum-cleaner agent, we had to specify the performance measure, the environment, and the agent's actuators and sensors. We group all these under the heading of the **task environment**. For the acronymically minded, we call
this the **PEAS** (**P**erformance, **E**nvironment, **A**ctuators, **S**ensors) description. In designing an agent, the first step must always be to specify the task environment as fully as possible.

The vacuum world was a simple example; let us consider a more complex problem: an automated taxi driver. We should point out, before the reader becomes alarmed, that a fully automated taxi is currently somewhat beyond the capabilities of existing technology. (page 28 describes an existing driving robot.) The full driving task is extremely *open-ended*. There is no limit to the novel combinations of circumstances that can arise—another reason we chose it as a focus for discussion. Figure 2.4 summarizes the PEAS description for the taxi's task environment. We discuss each element in more detail in the following paragraphs.

Agent Type	Performance Measure	Environment	Actuators	Sensors
Taxi driver	Safe, fast, legal, comfortable trip, maximize profits	Roads, other traffic, pedestrians, customers	Steering, accelerator, brake, signal, horn, display	Cameras, sonar, speedometer, GPS, odometer, accelerometer, engine sensors, keyboard

Figure 2.4   PEAS description of the task environment for an automated taxi.

First, what is the **performance measure** to which we would like our automated driver to aspire? Desirable qualities include getting to the correct destination; minimizing fuel consumption and wear and tear; minimizing the trip time or cost; minimizing violations of traffic laws and disturbances to other drivers; maximizing safety and passenger comfort; maximizing profits. Obviously, some of these goals conflict, so tradeoffs will be required.

Next, what is the driving **environment** that the taxi will face? Any taxi driver must deal with a variety of roads, ranging from rural lanes and urban alleys to 12-lane freeways. The roads contain other traffic, pedestrians, stray animals, road works, police cars, puddles,

and potholes. The taxi must also interact with potential and actual passengers. There are also some optional choices. The taxi might need to operate in Southern California, where snow is seldom a problem, or in Alaska, where it seldom is not. It could always be driving on the right, or we might want it to be flexible enough to drive on the left when in Britain or Japan. Obviously, the more restricted the environment, the easier the design problem.

The **actuators** for an automated taxi include those available to a human driver: control over the engine through the accelerator and control over steering and braking. In addition, it will need output to a display screen or voice synthesizer to talk back to the passengers, and perhaps some way to communicate with other vehicles, politely or otherwise.

The basic **sensors** for the taxi will include one or more controllable video cameras so that it can see the road; it might augment these with infrared or sonar sensors to detect distances to other cars and obstacles. To avoid speeding tickets, the taxi should have a speedometer, and to control the vehicle properly, especially on curves, it should have an accelerometer. To determine the mechanical state of the vehicle, it will need the usual array of engine, fuel, and electrical system sensors. Like many human drivers, it might want a global positioning system (GPS) so that it doesn't get lost. Finally, it will need a keyboard or microphone for the passenger to request a destination.

In Figure 2.5, we have sketched the basic PEAS elements for a number of additional agent types. Further examples appear in Exercise 2.4. It may come as a surprise to some readers that our list of agent types includes some programs that operate in the entirely artificial environment defined by keyboard input and character output on a screen. "Surely," one might say, "this is not a real environment, is it?" In fact, what matters is not the distinction between "real" and "artificial" environments, but the complexity of the relationship among the behavior of the agent, the percept sequence generated by the environment, and the performance measure. Some "real" environments are actually quite simple. For example, a robot designed to inspect parts as they come by on a conveyor belt can make use of a number of simplifying assumptions: that the lighting is always just so, that the only thing on the conveyor belt will be parts of a kind that it knows about, and that only two actions (accept or reject) are possible.

SOFTWARE AGENT

SOFTBOT

In contrast, some **software agents** (or software robots or **softbots**) exist in rich, unlimited domains. Imagine a softbot Web site operator designed to scan Internet news sources and show the interesting items to its users, while selling advertising space to generate revenue. To do well, that operator will need some natural language processing abilities, it will need to learn what each user and advertiser is interested in, and it will need to change its plans dynamically—for example, when the connection for one news source goes down or when a new one comes online. The Internet is an environment whose complexity rivals that of the physical world and whose inhabitants include many artificial and human agents.

2.3.2   Properties of task environments

The range of task environments that might arise in AI is obviously vast. We can, however, identify a fairly small number of dimensions along which task environments can be categorized. These dimensions determine, to a large extent, the appropriate agent design and the applicability of each of the principal families of techniques for agent implementation. First,

Agent Type	Performance Measure	Environment	Actuators	Sensors
Medical diagnosis system	Healthy patient, reduced costs	Patient, hospital, staff	Display of questions, tests, diagnoses, treatments, referrals	Keyboard entry of symptoms, findings, patient's answers
Satellite image analysis system	Correct image categorization	Downlink from orbiting satellite	Display of scene categorization	Color pixel arrays
Part-picking robot	Percentage of parts in correct bins	Conveyor belt with parts; bins	Jointed arm and hand	Camera, joint angle sensors
Refinery controller	Purity, yield, safety	Refinery, operators	Valves, pumps, heaters, displays	Temperature, pressure, chemical sensors
Interactive English tutor	Student's score on test	Set of students, testing agency	Display of exercises, suggestions, corrections	Keyboard entry

Figure 2.5   Examples of agent types and their PEAS descriptions.

we list the dimensions, then we analyze several task environments to illustrate the ideas. The definitions here are informal; later chapters provide more precise statements and examples of each kind of environment.

Fully observable vs. **partially observable**: If an agent's sensors give it access to the complete state of the environment at each point in time, then we say that the task environment is fully observable. A task environment is effectively fully observable if the sensors detect all aspects that are *relevant* to the choice of action; relevance, in turn, depends on the performance measure. Fully observable environments are convenient because the agent need not maintain any internal state to keep track of the world. An environment might be partially observable because of noisy and inaccurate sensors or because parts of the state are simply missing from the sensor data—for example, a vacuum agent with only a local dirt sensor cannot tell whether there is dirt in other squares, and an automated taxi cannot see what other drivers are thinking. If the agent has no sensors at all then the environment is **unobservable**. One might think that in such cases the agent's plight is hopeless, but, as we discuss in Chapter 4, the agent's goals may still be achievable, sometimes with certainty.

Single agent vs. **multiagent**: The distinction between single-agent and multiagent en-

FULLY OBSERVABLE
PARTIALLY OBSERVABLE

UNOBSERVABLE

SINGLE AGENT
MULTIAGENT

vironments may seem simple enough. For example, an agent solving a crossword puzzle by itself is clearly in a single-agent environment, whereas an agent playing chess is in a two-agent environment. There are, however, some subtle issues. First, we have described how an entity *may* be viewed as an agent, but we have not explained which entities *must* be viewed as agents. Does an agent A (the taxi driver for example) have to treat an object B (another vehicle) as an agent, or can it be treated merely as an object behaving according to the laws of physics, analogous to waves at the beach or leaves blowing in the wind? The key distinction is whether B's behavior is best described as maximizing a performance measure whose value depends on agent A's behavior. For example, in chess, the opponent entity B is trying to maximize its performance measure, which, by the rules of chess, minimizes agent A's per-

COMPETITIVE
formance measure. Thus, chess is a **competitive** multiagent environment. In the taxi-driving environment, on the other hand, avoiding collisions maximizes the performance measure of

COOPERATIVE
all agents, so it is a partially **cooperative** multiagent environment. It is also partially competitive because, for example, only one car can occupy a parking space. The agent-design problems in multiagent environments are often quite different from those in single-agent environments; for example, **communication** often emerges as a rational behavior in multiagent environments; in some competitive environments, **randomized behavior** is rational because it avoids the pitfalls of predictability.

DETERMINISTIC
STOCHASTIC
    Deterministic vs. **stochastic**. If the next state of the environment is completely determined by the current state and the action executed by the agent, then we say the environment is deterministic; otherwise, it is stochastic. In principle, an agent need not worry about uncertainty in a fully observable, deterministic environment. (In our definition, we ignore uncertainty that arises purely from the actions of other agents in a multiagent environment; thus, a game can be deterministic even though each agent may be unable to predict the actions of the others.) If the environment is partially observable, however, then it could *appear* to be stochastic. Most real situations are so complex that it is impossible to keep track of all the unobserved aspects; for practical purposes, they must be treated as stochastic. Taxi driving is clearly stochastic in this sense, because one can never predict the behavior of traffic exactly; moreover, one's tires blow out and one's engine seizes up without warning. The vacuum world as we described it is deterministic, but variations can include stochastic elements such as randomly appearing dirt and an unreliable suction mechanism (Exercise 2.13). We say an

UNCERTAIN
environment is **uncertain** if it is not fully observable or not deterministic. One final note: our use of the word "stochastic" generally implies that uncertainty about outcomes is quan-

NONDETERMINISTIC
tified in terms of probabilities; a **nondeterministic** environment is one in which actions are characterized by their *possible* outcomes, but no probabilities are attached to them. Nondeterministic environment descriptions are usually associated with performance measures that require the agent to succeed for *all possible* outcomes of its actions.

EPISODIC
SEQUENTIAL
    Episodic vs. **sequential**: In an episodic task environment, the agent's experience is divided into atomic episodes. In each episode the agent receives a percept and then performs a single action. Crucially, the next episode does not depend on the actions taken in previous episodes. Many classification tasks are episodic. For example, an agent that has to spot defective parts on an assembly line bases each decision on the current part, regardless of previous decisions; moreover, the current decision doesn't affect whether the next part is

defective. In sequential environments, on the other hand, the current decision could affect all future decisions.[3] Chess and taxi driving are sequential: in both cases, short-term actions can have long-term consequences. Episodic environments are much simpler than sequential environments because the agent does not need to think ahead.

STATIC
DYNAMIC

SEMIDYNAMIC

Static vs. **dynamic**: If the environment can change while an agent is deliberating, then we say the environment is dynamic for that agent; otherwise, it is static. Static environments are easy to deal with because the agent need not keep looking at the world while it is deciding on an action, nor need it worry about the passage of time. Dynamic environments, on the other hand, are continuously asking the agent what it wants to do; if it hasn't decided yet, that counts as deciding to do nothing. If the environment itself does not change with the passage of time but the agent's performance score does, then we say the environment is **semidynamic**. Taxi driving is clearly dynamic: the other cars and the taxi itself keep moving while the driving algorithm dithers about what to do next. Chess, when played with a clock, is semidynamic. Crossword puzzles are static.

DISCRETE
CONTINUOUS

Discrete vs. **continuous**: The discrete/continuous distinction applies to the *state* of the environment, to the way *time* is handled, and to the *percepts* and *actions* of the agent. For example, the chess environment has a finite number of distinct states (excluding the clock). Chess also has a discrete set of percepts and actions. Taxi driving is a continuous-state and continuous-time problem: the speed and location of the taxi and of the other vehicles sweep through a range of continuous values and do so smoothly over time. Taxi-driving actions are also continuous (steering angles, etc.). Input from digital cameras is discrete, strictly speaking, but is typically treated as representing continuously varying intensities and locations.

KNOWN
UNKNOWN

Known vs. **unknown**: Strictly speaking, this distinction refers not to the environment itself but to the agent's (or designer's) state of knowledge about the "laws of physics" of the environment. In a known environment, the outcomes (or outcome probabilities if the environment is stochastic) for all actions are given. Obviously, if the environment is unknown, the agent will have to learn how it works in order to make good decisions. Note that the distinction between known and unknown environments is not the same as the one between fully and partially observable environments. It is quite possible for a *known* environment to be *partially* observable—for example, in solitaire card games, I know the rules but am still unable to see the cards that have not yet been turned over. Conversely, an *unknown* environment can be *fully* observable—in a new video game, the screen may show the entire game state but I still don't know what the buttons do until I try them.

As one might expect, the hardest case is *partially observable*, *multiagent*, *stochastic*, *sequential*, *dynamic*, *continuous*, and *unknown*. Taxi driving is hard in all these senses, except that for the most part the driver's environment is known. Driving a rented car in a new country with unfamiliar geography and traffic laws is a lot more exciting.

Figure 2.6 lists the properties of a number of familiar environments. Note that the answers are not always cut and dried. For example, we describe the part-picking robot as episodic, because it normally considers each part in isolation. But if one day there is a large

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[3] The word "sequential" is also used in computer science as the antonym of "parallel." The two meanings are largely unrelated.

Task Environment	Observable	Agents	Deterministic	Episodic	Static	Discrete
Crossword puzzle	Fully	Single	Deterministic	Sequential	Static	Discrete
Chess with a clock	Fully	Multi	Deterministic	Sequential	Semi	Discrete
Poker	Partially	Multi	Stochastic	Sequential	Static	Discrete
Backgammon	Fully	Multi	Stochastic	Sequential	Static	Discrete
Taxi driving	Partially	Multi	Stochastic	Sequential	Dynamic	Continuous
Medical diagnosis	Partially	Single	Stochastic	Sequential	Dynamic	Continuous
Image analysis	Fully	Single	Deterministic	Episodic	Semi	Continuous
Part-picking robot	Partially	Single	Stochastic	Episodic	Dynamic	Continuous
Refinery controller	Partially	Single	Stochastic	Sequential	Dynamic	Continuous
Interactive English tutor	Partially	Multi	Stochastic	Sequential	Dynamic	Discrete

Figure 2.6   Examples of task environments and their characteristics.

batch of defective parts, the robot should learn from several observations that the distribution of defects has changed, and should modify its behavior for subsequent parts. We have not included a "known/unknown" column because, as explained earlier, this is not strictly a property of the environment. For some environments, such as chess and poker, it is quite easy to supply the agent with full knowledge of the rules, but it is nonetheless interesting to consider how an agent might learn to play these games without such knowledge.

Several of the answers in the table depend on how the task environment is defined. We have listed the medical-diagnosis task as single-agent because the disease process in a patient is not profitably modeled as an agent; but a medical-diagnosis system might also have to deal with recalcitrant patients and skeptical staff, so the environment could have a multiagent aspect. Furthermore, medical diagnosis is episodic if one conceives of the task as selecting a diagnosis given a list of symptoms; the problem is sequential if the task can include proposing a series of tests, evaluating progress over the course of treatment, and so on. Also, many environments are episodic at higher levels than the agent's individual actions. For example, a chess tournament consists of a sequence of games; each game is an episode because (by and large) the contribution of the moves in one game to the agent's overall performance is not affected by the moves in its previous game. On the other hand, decision making within a single game is certainly sequential.

The code repository associated with this book (aima.cs.berkeley.edu) includes implementations of a number of environments, together with a general-purpose environment simulator that places one or more agents in a simulated environment, observes their behavior over time, and evaluates them according to a given performance measure. Such experiments are often carried out not for a single environment but for many environments drawn from an **environment class**. For example, to evaluate a taxi driver in simulated traffic, we would want to run many simulations with different traffic, lighting, and weather conditions. If we designed the agent for a single scenario, we might be able to take advantage of specific properties of the particular case but might not identify a good design for driving in general. For this

ENVIRONMENT
CLASS

ENVIRONMENT
GENERATOR

reason, the code repository also includes an **environment generator** for each environment class that selects particular environments (with certain likelihoods) in which to run the agent. For example, the vacuum environment generator initializes the dirt pattern and agent location randomly. We are then interested in the agent's average performance over the environment class. A rational agent for a given environment class maximizes this average performance. Exercises 2.8 to 2.13 take you through the process of developing an environment class and evaluating various agents therein.

2.4   THE STRUCTURE OF AGENTS

So far we have talked about agents by describing *behavior*—the action that is performed after any given sequence of percepts. Now we must bite the bullet and talk about how the insides

AGENT PROGRAM

work. The job of AI is to design an **agent program** that implements the agent function— the mapping from percepts to actions. We assume this program will run on some sort of

ARCHITECTURE

computing device with physical sensors and actuators—we call this the **architecture**:

$$agent = architecture + program .$$

Obviously, the program we choose has to be one that is appropriate for the architecture. If the program is going to recommend actions like *Walk*, the architecture had better have legs. The architecture might be just an ordinary PC, or it might be a robotic car with several onboard computers, cameras, and other sensors. In general, the architecture makes the percepts from the sensors available to the program, runs the program, and feeds the program's action choices to the actuators as they are generated. Most of this book is about designing agent programs, although Chapters 24 and 25 deal directly with the sensors and actuators.

2.4.1   Agent programs

The agent programs that we design in this book all have the same skeleton: they take the current percept as input from the sensors and return an action to the actuators.[4] Notice the difference between the agent program, which takes the current percept as input, and the agent function, which takes the entire percept history. The agent program takes just the current percept as input because nothing more is available from the environment; if the agent's actions need to depend on the entire percept sequence, the agent will have to remember the percepts.

We describe the agent programs in the simple pseudocode language that is defined in Appendix B. (The online code repository contains implementations in real programming languages.) For example, Figure 2.7 shows a rather trivial agent program that keeps track of the percept sequence and then uses it to index into a table of actions to decide what to do. The table—an example of which is given for the vacuum world in Figure 2.3—represents explicitly the agent function that the agent program embodies. To build a rational agent in

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[4] There are other choices for the agent program skeleton; for example, we could have the agent programs be **coroutines** that run asynchronously with the environment. Each such coroutine has an input and output port and consists of a loop that reads the input port for percepts and writes actions to the output port.

---

function TABLE-DRIVEN-AGENT(*percept*) **returns** an action
   persistent: *percepts*, a sequence, initially empty
            table, a table of actions, indexed by percept sequences, initially fully specified

   append *percept* to the end of *percepts*
   action ← LOOKUP(*percepts*, *table*)
   return *action*

Figure 2.7   The TABLE-DRIVEN-AGENT program is invoked for each new percept and
returns an action each time. It retains the complete percept sequence in memory.

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this way, we as designers must construct a table that contains the appropriate action for every
possible percept sequence.

It is instructive to consider why the table-driven approach to agent construction is
doomed to failure. Let $\mathcal{P}$ be the set of possible percepts and let T be the lifetime of the
agent (the total number of percepts it will receive). The lookup table will contain $\sum_{t=1}^{T} |\mathcal{P}|^t$
entries. Consider the automated taxi: the visual input from a single camera comes in at the
rate of roughly 27 megabytes per second (30 frames per second, 640×480 pixels with 24
bits of color information). This gives a lookup table with over $10^{250,000,000,000}$ entries for an
hour's driving. Even the lookup table for chess—a tiny, well-behaved fragment of the real
world—would have at least 10^{150} entries. The daunting size of these tables (the number of
atoms in the observable universe is less than 10^{80}) means that (a) no physical agent in this
universe will have the space to store the table, (b) the designer would not have time to create
the table, (c) no agent could ever learn all the right table entries from its experience, and (d)
even if the environment is simple enough to yield a feasible table size, the designer still has
no guidance about how to fill in the table entries.

Despite all this, TABLE-DRIVEN-AGENT *does* do what we want: it implements the
desired agent function. The key challenge for AI is to find out how to write programs that,
to the extent possible, produce rational behavior from a smallish program rather than from
a vast table. We have many examples showing that this can be done successfully in other
areas: for example, the huge tables of square roots used by engineers and schoolchildren prior
to the 1970s have now been replaced by a five-line program for Newton's method running
on electronic calculators. The question is, can AI do for general intelligent behavior what
Newton did for square roots? We believe the answer is yes.

In the remainder of this section, we outline four basic kinds of agent programs that
embody the principles underlying almost all intelligent systems:

- Simple reflex agents;
- Model-based reflex agents;
- Goal-based agents; and
- Utility-based agents.

Each kind of agent program combines particular components in particular ways to generate
actions. Section 2.4.6 explains in general terms how to convert all these agents into *learning*

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function REFLEX-VACUUM-AGENT([$location, status$]) **returns** an action

   if $status = Dirty$ **then return** $Suck$
   else if $location = A$ **then return** $Right$
   else if $location = B$ **then return** $Left$

Figure 2.8   The agent program for a simple reflex agent in the two-state vacuum environment. This program implements the agent function tabulated in Figure 2.3.

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agents that can improve the performance of their components so as to generate better actions. Finally, Section 2.4.7 describes the variety of ways in which the components themselves can be represented within the agent. This variety provides a major organizing principle for the field and for the book itself.

2.4.2   Simple reflex agents

SIMPLE REFLEX
AGENT

The simplest kind of agent is the **simple reflex agent**. These agents select actions on the basis of the *current* percept, ignoring the rest of the percept history. For example, the vacuum agent whose agent function is tabulated in Figure 2.3 is a simple reflex agent, because its decision is based only on the current location and on whether that location contains dirt. An agent program for this agent is shown in Figure 2.8.

     Notice that the vacuum agent program is very small indeed compared to the corresponding table. The most obvious reduction comes from ignoring the percept history, which cuts down the number of possibilities from 4^T to just 4. A further, small reduction comes from the fact that when the current square is dirty, the action does not depend on the location.

     Simple reflex behaviors occur even in more complex environments. Imagine yourself as the driver of the automated taxi. If the car in front brakes and its brake lights come on, then you should notice this and initiate braking. In other words, some processing is done on the visual input to establish the condition we call "The car in front is braking." Then, this triggers some established connection in the agent program to the action "initiate braking." We call

CONDITION–ACTION
RULE

such a connection a **condition–action rule**,[5] written as

     if *car-in-front-is-braking* **then** *initiate-braking*.

Humans also have many such connections, some of which are learned responses (as for driving) and some of which are innate reflexes (such as blinking when something approaches the eye). In the course of the book, we show several different ways in which such connections can be learned and implemented.

     The program in Figure 2.8 is specific to one particular vacuum environment. A more general and flexible approach is first to build a general-purpose interpreter for condition–action rules and then to create rule sets for specific task environments. Figure 2.9 gives the structure of this general program in schematic form, showing how the condition–action rules allow the agent to make the connection from percept to action. (Do not worry if this seems

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[5]   Also called **situation–action rules**, **productions**, or **if–then rules**.

Figure 2.9   Schematic diagram of a simple reflex agent.

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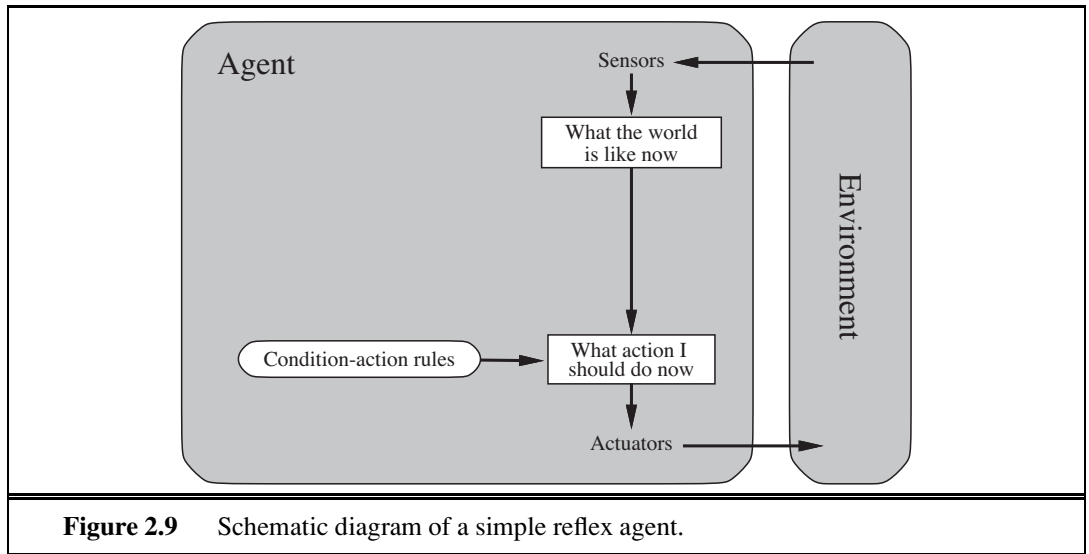
function SIMPLE-REFLEX-AGENT(*percept*) **returns** an action
    persistent: *rules*, a set of condition–action rules

    state ← INTERPRET-INPUT(*percept*)
    rule ← RULE-MATCH(*state*, *rules*)
    action ← *rule*.ACTION
    return *action*

Figure 2.10   A simple reflex agent. It acts according to a rule whose condition matches
the current state, as defined by the percept.

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trivial; it gets more interesting shortly.) We use rectangles to denote the current internal state
of the agent's decision process, and ovals to represent the background information used in
the process. The agent program, which is also very simple, is shown in Figure 2.10. The
INTERPRET-INPUT function generates an abstracted description of the current state from the
percept, and the RULE-MATCH function returns the first rule in the set of rules that matches
the given state description. Note that the description in terms of "rules" and "matching" is
purely conceptual; actual implementations can be as simple as a collection of logic gates
implementing a Boolean circuit.

Simple reflex agents have the admirable property of being simple, but they turn out to be
of limited intelligence. The agent in Figure 2.10 will work *only if the correct decision can be
made on the basis of only the current percept—that is, only if the environment is fully observ-
able.* Even a little bit of unobservability can cause serious trouble. For example, the braking
rule given earlier assumes that the condition *car-in-front-is-braking* can be determined from
the current percept—a single frame of video. This works if the car in front has a centrally
mounted brake light. Unfortunately, older models have different configurations of taillights,

brake lights, and turn-signal lights, and it is not always possible to tell from a single image whether the car is braking. A simple reflex agent driving behind such a car would either brake continuously and unnecessarily, or, worse, never brake at all.

We can see a similar problem arising in the vacuum world. Suppose that a simple reflex vacuum agent is deprived of its location sensor and has only a dirt sensor. Such an agent has just two possible percepts: $[Dirty]$ and $[Clean]$. It can $Suck$ in response to $[Dirty]$; what should it do in response to $[Clean]$? Moving $Left$ fails (forever) if it happens to start in square A, and moving $Right$ fails (forever) if it happens to start in square B. Infinite loops are often unavoidable for simple reflex agents operating in partially observable environments.

RANDOMIZATION   Escape from infinite loops is possible if the agent can **randomize** its actions. For example, if the vacuum agent perceives $[Clean]$, it might flip a coin to choose between $Left$ and $Right$. It is easy to show that the agent will reach the other square in an average of two steps. Then, if that square is dirty, the agent will clean it and the task will be complete. Hence, a randomized simple reflex agent might outperform a deterministic simple reflex agent.

We mentioned in Section 2.3 that randomized behavior of the right kind can be rational in some multiagent environments. In single-agent environments, randomization is usually *not* rational. It is a useful trick that helps a simple reflex agent in some situations, but in most cases we can do much better with more sophisticated deterministic agents.

2.4.3   Model-based reflex agents

The most effective way to handle partial observability is for the agent to *keep track of the part of the world it can't see now*. That is, the agent should maintain some sort of **internal**
INTERNAL STATE   **state** that depends on the percept history and thereby reflects at least some of the unobserved aspects of the current state. For the braking problem, the internal state is not too extensive— just the previous frame from the camera, allowing the agent to detect when two red lights at the edge of the vehicle go on or off simultaneously. For other driving tasks such as changing lanes, the agent needs to keep track of where the other cars are if it can't see them all at once. And for any driving to be possible at all, the agent needs to keep track of where its keys are.

Updating this internal state information as time goes by requires two kinds of knowledge to be encoded in the agent program. First, we need some information about how the world evolves independently of the agent—for example, that an overtaking car generally will be closer behind than it was a moment ago. Second, we need some information about how the agent's own actions affect the world—for example, that when the agent turns the steering wheel clockwise, the car turns to the right, or that after driving for five minutes northbound on the freeway, one is usually about five miles north of where one was five minutes ago. This knowledge about "how the world works"—whether implemented in simple Boolean circuits or in complete scientific theories—is called a **model** of the world. An agent that uses such a
MODEL-BASED
AGENT   model is called a **model-based agent**.

Figure 2.11 gives the structure of the model-based reflex agent with internal state, showing how the current percept is combined with the old internal state to generate the updated description of the current state, based on the agent's model of how the world works. The agent program is shown in Figure 2.12. The interesting part is the function UPDATE-STATE, which

Figure 2.11   A model-based reflex agent.

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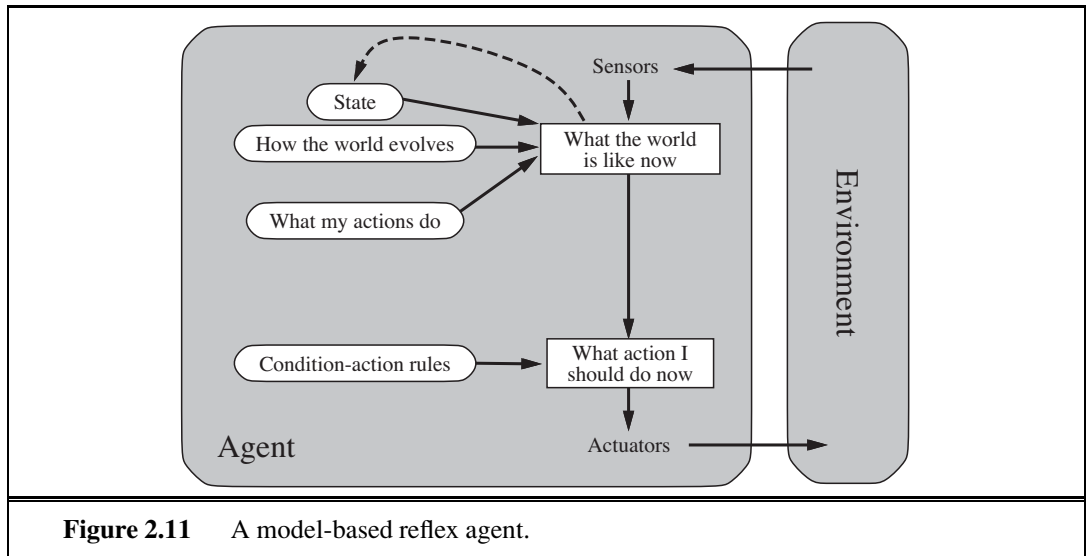
function MODEL-BASED-REFLEX-AGENT(*percept*) **returns** an action
   persistent: *state*, the agent's current conception of the world state
                model, a description of how the next state depends on current state and action
                rules, a set of condition–action rules
                action, the most recent action, initially none

   $state \leftarrow$ UPDATE-STATE($state, action, percept, model$)
   $rule \leftarrow$ RULE-MATCH($state, rules$)
   $action \leftarrow rule$.ACTION
   return $action$

Figure 2.12   A model-based reflex agent. It keeps track of the current state of the world,
using an internal model. It then chooses an action in the same way as the reflex agent.

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is responsible for creating the new internal state description. The details of how models and
states are represented vary widely depending on the type of environment and the particular
technology used in the agent design. Detailed examples of models and updating algorithms
appear in Chapters 4, 12, 11, 15, 17, and 25.

Regardless of the kind of representation used, it is seldom possible for the agent to
determine the current state of a partially observable environment *exactly*. Instead, the box
labeled "what the world is like now" (Figure 2.11) represents the agent's "best guess" (or
sometimes best guesses). For example, an automated taxi may not be able to see around the
large truck that has stopped in front of it and can only guess about what may be causing the
hold-up. Thus, uncertainty about the current state may be unavoidable, but the agent still has
to make a decision.

A perhaps less obvious point about the internal "state" maintained by a model-based
agent is that it does not have to describe "what the world is like now" in a literal sense. For

Figure 2.13   A model-based, goal-based agent. It keeps track of the world state as well as a set of goals it is trying to achieve, and chooses an action that will (eventually) lead to the achievement of its goals.
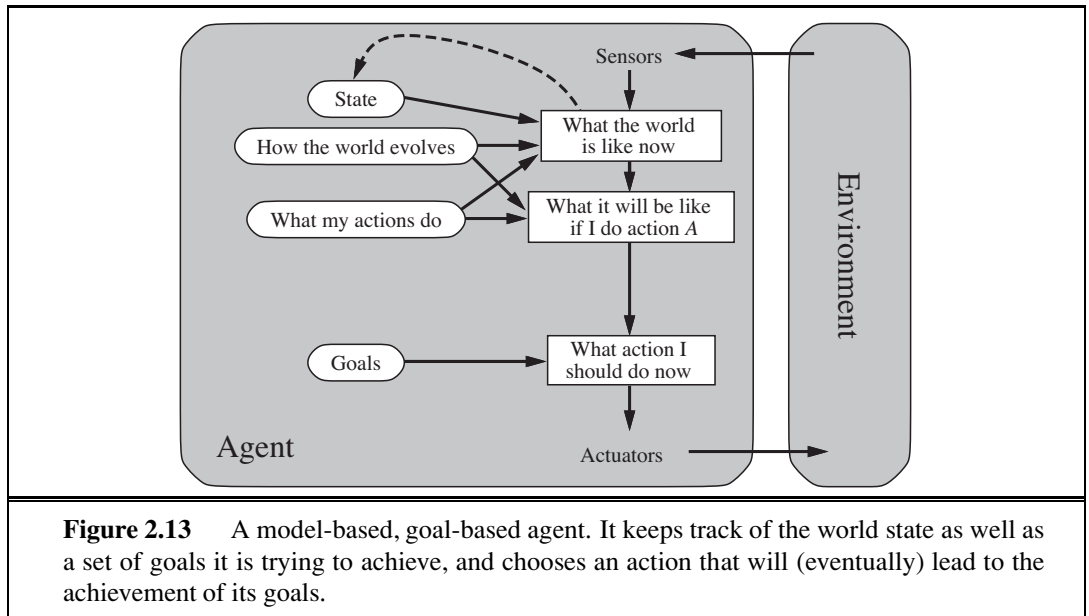
example, the taxi may be driving back home, and it may have a rule telling it to fill up with gas on the way home unless it has at least half a tank. Although "driving back home" may *seem* to an aspect of the world state, the fact of the taxi's *destination* is actually an aspect of the agent's internal state. If you find this puzzling, consider that the taxi could be in exactly the same place at the same time, but intending to reach a different destination.

2.4.4   Goal-based agents

Knowing something about the current state of the environment is not always enough to decide what to do. For example, at a road junction, the taxi can turn left, turn right, or go straight on. The correct decision depends on where the taxi is trying to get to. In other words, as well GOAL as a current state description, the agent needs some sort of **goal** information that describes situations that are desirable—for example, being at the passenger's destination. The agent program can combine this with the model (the same information as was used in the model-based reflex agent) to choose actions that achieve the goal. Figure 2.13 shows the goal-based agent's structure.

Sometimes goal-based action selection is straightforward—for example, when goal satisfaction results immediately from a single action. Sometimes it will be more tricky—for example, when the agent has to consider long sequences of twists and turns in order to find a way to achieve the goal. **Search** (Chapters 3 to 5) and **planning** (Chapters 10 and 11) are the subfields of AI devoted to finding action sequences that achieve the agent's goals.

Notice that decision making of this kind is fundamentally different from the condition–action rules described earlier, in that it involves consideration of the future—both "What will happen if I do such-and-such?" and "Will that make me happy?" In the reflex agent designs, this information is not explicitly represented, because the built-in rules map directly from

percepts to actions. The reflex agent brakes when it sees brake lights. A goal-based agent, in principle, could reason that if the car in front has its brake lights on, it will slow down. Given the way the world usually evolves, the only action that will achieve the goal of not hitting other cars is to brake.

Although the goal-based agent appears less efficient, it is more flexible because the knowledge that supports its decisions is represented explicitly and can be modified. If it starts to rain, the agent can update its knowledge of how effectively its brakes will operate; this will automatically cause all of the relevant behaviors to be altered to suit the new conditions. For the reflex agent, on the other hand, we would have to rewrite many condition–action rules. The goal-based agent's behavior can easily be changed to go to a different destination, simply by specifying that destination as the goal. The reflex agent's rules for when to turn and when to go straight will work only for a single destination; they must all be replaced to go somewhere new.

2.4.5   Utility-based agents

Goals alone are not enough to generate high-quality behavior in most environments. For example, many action sequences will get the taxi to its destination (thereby achieving the goal) but some are quicker, safer, more reliable, or cheaper than others. Goals just provide a crude binary distinction between "happy" and "unhappy" states. A more general performance measure should allow a comparison of different world states according to exactly how happy they would make the agent. Because "happy" does not sound very scientific, economists and
UTILITY   computer scientists use the term **utility** instead.[6]

We have already seen that a performance measure assigns a score to any given sequence of environment states, so it can easily distinguish between more and less desirable ways of
UTILITY FUNCTION   getting to the taxi's destination. An agent's **utility function** is essentially an internalization of the performance measure. If the internal utility function and the external performance measure are in agreement, then an agent that chooses actions to maximize its utility will be rational according to the external performance measure.

Let us emphasize again that this is not the *only* way to be rational—we have already seen a rational agent program for the vacuum world (Figure 2.8) that has no idea what its utility function is—but, like goal-based agents, a utility-based agent has many advantages in terms of flexibility and learning. Furthermore, in two kinds of cases, goals are inadequate but a utility-based agent can still make rational decisions. First, when there are conflicting goals, only some of which can be achieved (for example, speed and safety), the utility function specifies the appropriate tradeoff. Second, when there are several goals that the agent can aim for, none of which can be achieved with certainty, utility provides a way in which the likelihood of success can be weighed against the importance of the goals.

Partial observability and stochasticity are ubiquitous in the real world, and so, therefore, is decision making under uncertainty. Technically speaking, a rational utility-based agent
EXPECTED UTILITY   chooses the action that maximizes the **expected utility** of the action outcomes—that is, the utility the agent expects to derive, on average, given the probabilities and utilities of each

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[6]   The word "utility" here refers to "the quality of being useful," not to the electric company or waterworks.

Figure 2.14   A model-based, utility-based agent. It uses a model of the world, along with a utility function that measures its preferences among states of the world. Then it chooses the action that leads to the best expected utility, where expected utility is computed by averaging over all possible outcome states, weighted by the probability of the outcome.
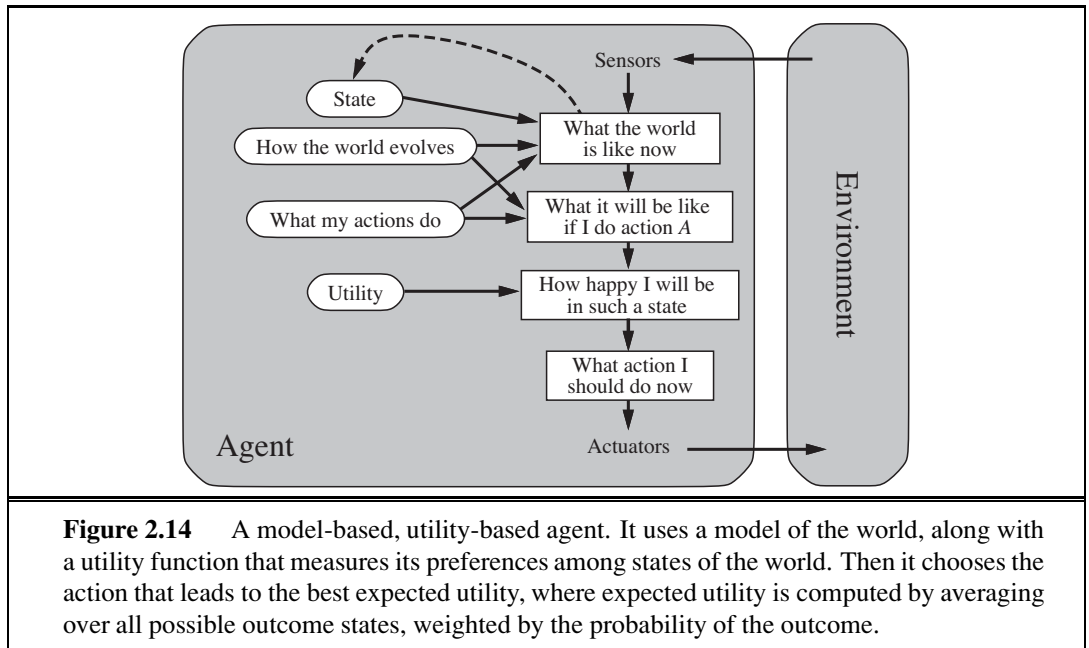
outcome. (Appendix A defines expectation more precisely.) In Chapter 16, we show that any rational agent must behave *as if* it possesses a utility function whose expected value it tries to maximize. An agent that possesses an *explicit* utility function can make rational decisions with a general-purpose algorithm that does not depend on the specific utility function being maximized. In this way, the "global" definition of rationality—designating as rational those agent functions that have the highest performance—is turned into a "local" constraint on rational-agent designs that can be expressed in a simple program.

The utility-based agent structure appears in Figure 2.14. Utility-based agent programs appear in Part IV, where we design decision-making agents that must handle the uncertainty inherent in stochastic or partially observable environments.

At this point, the reader may be wondering, "Is it that simple? We just build agents that maximize expected utility, and we're done?" It's true that such agents would be intelligent, but it's not simple. A utility-based agent has to model and keep track of its environment, tasks that have involved a great deal of research on perception, representation, reasoning, and learning. The results of this research fill many of the chapters of this book. Choosing the utility-maximizing course of action is also a difficult task, requiring ingenious algorithms that fill several more chapters. Even with these algorithms, perfect rationality is usually unachievable in practice because of computational complexity, as we noted in Chapter 1.

2.4.6   Learning agents

We have described agent programs with various methods for selecting actions. We have not, so far, explained how the agent programs *come into being*. In his famous early paper, Turing (1950) considers the idea of actually programming his intelligent machines by hand.
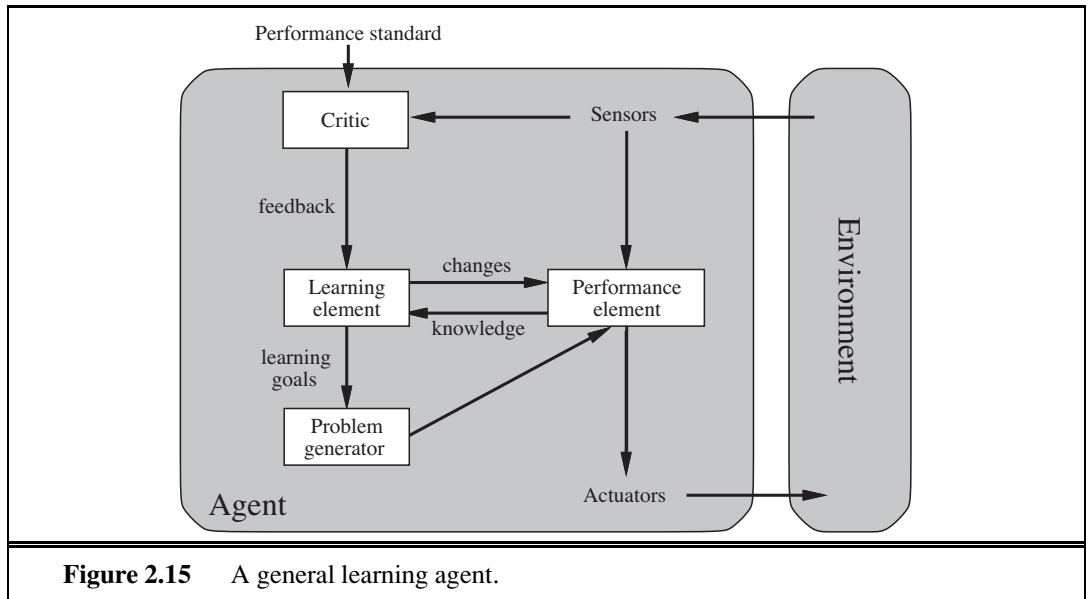
Figure 2.15   A general learning agent.

He estimates how much work this might take and concludes "Some more expeditious method seems desirable." The method he proposes is to build learning machines and then to teach them. In many areas of AI, this is now the preferred method for creating state-of-the-art systems. Learning has another advantage, as we noted earlier: it allows the agent to operate in initially unknown environments and to become more competent than its initial knowledge alone might allow. In this section, we briefly introduce the main ideas of learning agents. Throughout the book, we comment on opportunities and methods for learning in particular kinds of agents. Part V goes into much more depth on the learning algorithms themselves.

A learning agent can be divided into four conceptual components, as shown in Figure 2.15. The most important distinction is between the **learning element**, which is responsible for making improvements, and the **performance element**, which is responsible for selecting external actions. The performance element is what we have previously considered to be the entire agent: it takes in percepts and decides on actions. The learning element uses feedback from the **critic** on how the agent is doing and determines how the performance element should be modified to do better in the future.

The design of the learning element depends very much on the design of the performance element. When trying to design an agent that learns a certain capability, the first question is not "How am I going to get it to learn this?" but "What kind of performance element will my agent need to do this once it has learned how?" Given an agent design, learning mechanisms can be constructed to improve every part of the agent.

The critic tells the learning element how well the agent is doing with respect to a fixed performance standard. The critic is necessary because the percepts themselves provide no indication of the agent's success. For example, a chess program could receive a percept indicating that it has checkmated its opponent, but it needs a performance standard to know that this is a good thing; the percept itself does not say so. It is important that the performance

LEARNING ELEMENT

PERFORMANCE
ELEMENT

CRITIC

standard be fixed. Conceptually, one should think of it as being outside the agent altogether because the agent must not modify it to fit its own behavior.

The last component of the learning agent is the **problem generator**. It is responsible for suggesting actions that will lead to new and informative experiences. The point is that if the performance element had its way, it would keep doing the actions that are best, given what it knows. But if the agent is willing to explore a little and do some perhaps suboptimal actions in the short run, it might discover much better actions for the long run. The problem generator's job is to suggest these exploratory actions. This is what scientists do when they carry out experiments. Galileo did not think that dropping rocks from the top of a tower in Pisa was valuable in itself. He was not trying to break the rocks or to modify the brains of unfortunate passers-by. His aim was to modify his own brain by identifying a better theory of the motion of objects.

To make the overall design more concrete, let us return to the automated taxi example. The performance element consists of whatever collection of knowledge and procedures the taxi has for selecting its driving actions. The taxi goes out on the road and drives, using this performance element. The critic observes the world and passes information along to the learning element. For example, after the taxi makes a quick left turn across three lanes of traffic, the critic observes the shocking language used by other drivers. From this experience, the learning element is able to formulate a rule saying this was a bad action, and the performance element is modified by installation of the new rule. The problem generator might identify certain areas of behavior in need of improvement and suggest experiments, such as trying out the brakes on different road surfaces under different conditions.

The learning element can make changes to any of the "knowledge" components shown in the agent diagrams (Figures 2.9, 2.11, 2.13, and 2.14). The simplest cases involve learning directly from the percept sequence. Observation of pairs of successive states of the environment can allow the agent to learn "How the world evolves," and observation of the results of its actions can allow the agent to learn "What my actions do." For example, if the taxi exerts a certain braking pressure when driving on a wet road, then it will soon find out how much deceleration is actually achieved. Clearly, these two learning tasks are more difficult if the environment is only partially observable.

The forms of learning in the preceding paragraph do not need to access the external performance standard—in a sense, the standard is the universal one of making predictions that agree with experiment. The situation is slightly more complex for a utility-based agent that wishes to learn utility information. For example, suppose the taxi-driving agent receives no tips from passengers who have been thoroughly shaken up during the trip. The external performance standard must inform the agent that the loss of tips is a negative contribution to its overall performance; then the agent might be able to learn that violent maneuvers do not contribute to its own utility. In a sense, the performance standard distinguishes part of the incoming percept as a **reward** (or **penalty**) that provides direct feedback on the quality of the agent's behavior. Hard-wired performance standards such as pain and hunger in animals can be understood in this way. This issue is discussed further in Chapter 21.

In summary, agents have a variety of components, and those components can be represented in many ways within the agent program, so there appears to be great variety among

learning methods. There is, however, a single unifying theme. Learning in intelligent agents can be summarized as a process of modification of each component of the agent to bring the components into closer agreement with the available feedback information, thereby improving the overall performance of the agent.

2.4.7   How the components of agent programs work

We have described agent programs (in very high-level terms) as consisting of various components, whose function it is to answer questions such as: "What is the world like now?" "What action should I do now?" "What do my actions do?" The next question for a student of AI is, "How on earth do these components work?" It takes about a thousand pages to begin to answer that question properly, but here we want to draw the reader's attention to some basic distinctions among the various ways that the components can represent the environment that the agent inhabits.

Roughly speaking, we can place the representations along an axis of increasing complexity and expressive power—**atomic**, **factored**, and **structured**. To illustrate these ideas, it helps to consider a particular agent component, such as the one that deals with "What my actions do." This component describes the changes that might occur in the environment as the result of taking an action, and Figure 2.16 provides schematic depictions of how those transitions might be represented.

(a) Atomic   (b) Factored   (b) Structured

Figure 2.16   Three ways to represent states and the transitions between them. (a) Atomic representation: a state (such as B or C) is a black box with no internal structure; (b) Factored representation: a state consists of a vector of attribute values; values can be Boolean, real-valued, or one of a fixed set of symbols. (c) Structured representation: a state includes objects, each of which may have attributes of its own as well as relationships to other objects.

In an **atomic representation** each state of the world is indivisible—it has no internal structure. Consider the problem of finding a driving route from one end of a country to the other via some sequence of cities (we address this problem in Figure 3.2 on page 68). For the purposes of solving this problem, it may suffice to reduce the state of world to just the name of the city we are in—a single atom of knowledge; a "black box" whose only discernible property is that of being identical to or different from another black box. The algorithms

underlying **search** and **game-playing** (Chapters 3–5), **Hidden Markov models** (Chapter 15), and **Markov decision processes** (Chapter 17) all work with atomic representations—or, at least, they treat representations *as if* they were atomic.

Now consider a higher-fidelity description for the same problem, where we need to be concerned with more than just atomic location in one city or another; we might need to pay attention to how much gas is in the tank, our current GPS coordinates, whether or not the oil warning light is working, how much spare change we have for toll crossings, what station is on the radio, and so on. A **factored representation** splits up each state into a fixed set of **variables** or **attributes**, each of which can have a **value**. While two different atomic states have nothing in common—they are just different black boxes—two different factored states can share some attributes (such as being at some particular GPS location) and not others (such as having lots of gas or having no gas); this makes it much easier to work out how to turn one state into another. With factored representations, we can also represent *uncertainty*—for example, ignorance about the amount of gas in the tank can be represented by leaving that attribute blank. Many important areas of AI are based on factored representations, including **constraint satisfaction** algorithms (Chapter 6), **propositional logic** (Chapter 7), **planning** (Chapters 10 and 11), **Bayesian networks** (Chapters 13–16), and the **machine learning** algorithms in Chapters 18, 20, and 21.

For many purposes, we need to understand the world as having *things* in it that are *related* to each other, not just variables with values. For example, we might notice that a large truck ahead of us is reversing into the driveway of a dairy farm but a cow has got loose and is blocking the truck's path. A factored representation is unlikely to be pre-equipped with the attribute *TruckAheadBackingIntoDairyFarmDrivewayBlockedByLooseCow* with value *true* or *false*. Instead, we would need a **structured representation**, in which objects such as cows and trucks and their various and varying relationships can be described explicitly. (See Figure 2.16(c).) Structured representations underlie **relational databases** and **first-order logic** (Chapters 8, 9, and 12), **first-order probability models** (Chapter 14), **knowledge-based learning** (Chapter 19) and much of **natural language understanding** (Chapters 22 and 23). In fact, almost everything that humans express in natural language concerns objects and their relationships.

As we mentioned earlier, the axis along which atomic, factored, and structured representations lie is the axis of increasing **expressiveness**. Roughly speaking, a more expressive representation can capture, at least as concisely, everything a less expressive one can capture, plus some more. Often, the more expressive language is *much* more concise; for example, the rules of chess can be written in a page or two of a structured-representation language such as first-order logic but require thousands of pages when written in a factored-representation language such as propositional logic. On the other hand, reasoning and learning become more complex as the expressive power of the representation increases. To gain the benefits of expressive representations while avoiding their drawbacks, intelligent systems for the real world may need to operate at all points along the axis simultaneously.

FACTORED
REPRESENTATION

VARIABLE

ATTRIBUTE

VALUE

STRUCTURED
REPRESENTATION

EXPRESSIVENESS

2.5   SUMMARY

This chapter has been something of a whirlwind tour of AI, which we have conceived of as the science of agent design. The major points to recall are as follows:

- An **agent** is something that perceives and acts in an environment. The **agent function** for an agent specifies the action taken by the agent in response to any percept sequence.
- The **performance measure** evaluates the behavior of the agent in an environment. A **rational agent** acts so as to maximize the expected value of the performance measure, given the percept sequence it has seen so far.
- A **task environment** specification includes the performance measure, the external environment, the actuators, and the sensors. In designing an agent, the first step must always be to specify the task environment as fully as possible.
- Task environments vary along several significant dimensions. They can be fully or partially observable, single-agent or multiagent, deterministic or stochastic, episodic or sequential, static or dynamic, discrete or continuous, and known or unknown.
- The **agent program** implements the agent function. There exists a variety of basic agent-program designs reflecting the kind of information made explicit and used in the decision process. The designs vary in efficiency, compactness, and flexibility. The appropriate design of the agent program depends on the nature of the environment.
- **Simple reflex agents** respond directly to percepts, whereas **model-based reflex agents** maintain internal state to track aspects of the world that are not evident in the current percept. **Goal-based agents** act to achieve their goals, and **utility-based agents** try to maximize their own expected "happiness."
- All agents can improve their performance through **learning**.

BIBLIOGRAPHICAL AND HISTORICAL NOTES

The central role of action in intelligence—the notion of practical reasoning—goes back at least as far as Aristotle's *Nicomachean Ethics*. Practical reasoning was also the subject of McCarthy's (1958) influential paper "Programs with Common Sense." The fields of robotics and control theory are, by their very nature, concerned principally with physical agents. The concept of a **controller** in control theory is identical to that of an agent in AI. Perhaps surprisingly, AI has concentrated for most of its history on isolated components of agents—question-answering systems, theorem-provers, vision systems, and so on—rather than on whole agents. The discussion of agents in the text by Genesereth and Nilsson (1987) was an influential exception. The whole-agent view is now widely accepted and is a central theme in recent texts (Poole *et al.*, 1998; Nilsson, 1998; Padgham and Winikoff, 2004; Jones, 2007).

CONTROLLER

Chapter 1 traced the roots of the concept of rationality in philosophy and economics. In AI, the concept was of peripheral interest until the mid-1980s, when it began to suffuse many

discussions about the proper technical foundations of the field. A paper by Jon Doyle (1983) predicted that rational agent design would come to be seen as the core mission of AI, while other popular topics would spin off to form new disciplines.

Careful attention to the properties of the environment and their consequences for rational agent design is most apparent in the control theory tradition—for example, classical control systems (Dorf and Bishop, 2004; Kirk, 2004) handle fully observable, deterministic environments; stochastic optimal control (Kumar and Varaiya, 1986; Bertsekas and Shreve, 2007) handles partially observable, stochastic environments; and hybrid control (Henzinger and Sastry, 1998; Cassandras and Lygeros, 2006) deals with environments containing both discrete and continuous elements. The distinction between fully and partially observable environments is also central in the **dynamic programming** literature developed in the field of operations research (Puterman, 1994), which we discuss in Chapter 17.

Reflex agents were the primary model for psychological behaviorists such as Skinner (1953), who attempted to reduce the psychology of organisms strictly to input/output or stimulus/response mappings. The advance from behaviorism to functionalism in psychology, which was at least partly driven by the application of the computer metaphor to agents (Putnam, 1960; Lewis, 1966), introduced the internal state of the agent into the picture. Most work in AI views the idea of pure reflex agents with state as too simple to provide much leverage, but work by Rosenschein (1985) and Brooks (1986) questioned this assumption (see Chapter 25). In recent years, a great deal of work has gone into finding efficient algorithms for keeping track of complex environments (Hamscher *et al.*, 1992; Simon, 2006). The Remote Agent program (described on page 28) that controlled the Deep Space One spacecraft is a particularly impressive example (Muscettola *et al.*, 1998; Jonsson *et al.*, 2000).

Goal-based agents are presupposed in everything from Aristotle's view of practical reasoning to McCarthy's early papers on logical AI. Shakey the Robot (Fikes and Nilsson, 1971; Nilsson, 1984) was the first robotic embodiment of a logical, goal-based agent. A full logical analysis of goal-based agents appeared in Genesereth and Nilsson (1987), and a goal-based programming methodology called agent-oriented programming was developed by Shoham (1993). The agent-based approach is now extremely popular in software engineering (Ciancarini and Wooldridge, 2001). It has also infiltrated the area of operating systems, where **autonomic computing** refers to computer systems and networks that monitor and control themselves with a perceive–act loop and machine learning methods (Kephart and Chess, 2003). Noting that a collection of agent programs designed to work well together in a true multiagent environment necessarily exhibits modularity—the programs share no internal state and communicate with each other only through the environment—it is common within the field of **multiagent systems** to design the agent program of a single agent as a collection of autonomous sub-agents. In some cases, one can even prove that the resulting system gives the same optimal solutions as a monolithic design.

AUTONOMIC
COMPUTING

MULTIAGENT
SYSTEMS

The goal-based view of agents also dominates the cognitive psychology tradition in the area of problem solving, beginning with the enormously influential *Human Problem Solving* (Newell and Simon, 1972) and running through all of Newell's later work (Newell, 1990). Goals, further analyzed as *desires* (general) and *intentions* (currently pursued), are central to the theory of agents developed by Bratman (1987). This theory has been influential both in

natural language understanding and multiagent systems.

Horvitz *et al.* (1988) specifically suggest the use of rationality conceived as the maximization of expected utility as a basis for AI. The text by Pearl (1988) was the first in AI to cover probability and utility theory in depth; its exposition of practical methods for reasoning and decision making under uncertainty was probably the single biggest factor in the rapid shift towards utility-based agents in the 1990s (see Part IV).

The general design for learning agents portrayed in Figure 2.15 is classic in the machine learning literature (Buchanan *et al.*, 1978; Mitchell, 1997). Examples of the design, as embodied in programs, go back at least as far as Arthur Samuel's (1959, 1967) learning program for playing checkers. Learning agents are discussed in depth in Part V.

Interest in agents and in agent design has risen rapidly in recent years, partly because of the growth of the Internet and the perceived need for automated and mobile **softbot** (Etzioni and Weld, 1994). Relevant papers are collected in *Readings in Agents* (Huhns and Singh, 1998) and *Foundations of Rational Agency* (Wooldridge and Rao, 1999). Texts on multiagent systems usually provide a good introduction to many aspects of agent design (Weiss, 2000a; Wooldridge, 2002). Several conference series devoted to agents began in the 1990s, including the International Workshop on Agent Theories, Architectures, and Languages (ATAL), the International Conference on Autonomous Agents (AGENTS), and the International Conference on Multi-Agent Systems (ICMAS). In 2002, these three merged to form the International Joint Conference on Autonomous Agents and Multi-Agent Systems (AAMAS). The journal *Autonomous Agents and Multi-Agent Systems* was founded in 1998. Finally, *Dung Beetle Ecology* (Hanski and Cambefort, 1991) provides a wealth of interesting information on the behavior of dung beetles. YouTube features inspiring video recordings of their activities.

EXERCISES

2.1   Suppose that the performance measure is concerned with just the first T time steps of the environment and ignores everything thereafter. Show that a rational agent's action may depend not just on the state of the environment but also on the time step it has reached.

2.2   Let us examine the rationality of various vacuum-cleaner agent functions.

  a. Show that the simple vacuum-cleaner agent function described in Figure 2.3 is indeed rational under the assumptions listed on page 38.

  b. Describe a rational agent function for the case in which each movement costs one point. Does the corresponding agent program require internal state?

  c. Discuss possible agent designs for the cases in which clean squares can become dirty and the geography of the environment is unknown. Does it make sense for the agent to learn from its experience in these cases? If so, what should it learn? If not, why not?

2.3   For each of the following assertions, say whether it is true or false and support your answer with examples or counterexamples where appropriate.

  a. An agent that senses only partial information about the state cannot be perfectly rational.

    b. There exist task environments in which no pure reflex agent can behave rationally.

    c. There exists a task environment in which every agent is rational.

    d. The input to an agent program is the same as the input to the agent function.

    e. Every agent function is implementable by some program/machine combination.

    f. Suppose an agent selects its action uniformly at random from the set of possible actions. There exists a deterministic task environment in which this agent is rational.

    g. It is possible for a given agent to be perfectly rational in two distinct task environments.

    h. Every agent is rational in an unobservable environment.

    i. A perfectly rational poker-playing agent never loses.

2.4   For each of the following activities, give a PEAS description of the task environment and characterize it in terms of the properties listed in Section 2.3.2.

- Playing soccer.
- Exploring the subsurface oceans of Titan.
- Shopping for used AI books on the Internet.
- Playing a tennis match.
- Practicing tennis against a wall.
- Performing a high jump.
- Knitting a sweater.
- Bidding on an item at an auction.

2.5   Define in your own words the following terms: agent, agent function, agent program, rationality, autonomy, reflex agent, model-based agent, goal-based agent, utility-based agent, learning agent.

2.6   This exercise explores the differences between agent functions and agent programs.

    a. Can there be more than one agent program that implements a given agent function? Give an example, or show why one is not possible.

    b. Are there agent functions that cannot be implemented by any agent program?

    c. Given a fixed machine architecture, does each agent program implement exactly one agent function?

    d. Given an architecture with n bits of storage, how many different possible agent programs are there?

    e. Suppose we keep the agent program fixed but speed up the machine by a factor of two. Does that change the agent function?

2.7   Write pseudocode agent programs for the goal-based and utility-based agents.

The following exercises all concern the implementation of environments and agents for the vacuum-cleaner world.

2.8   Implement a performance-measuring environment simulator for the vacuum-cleaner world depicted in Figure 2.2 and specified on page 38. Your implementation should be modular so that the sensors, actuators, and environment characteristics (size, shape, dirt placement, etc.) can be changed easily. (*Note:* for some choices of programming language and operating system there are already implementations in the online code repository.)

2.9   Implement a simple reflex agent for the vacuum environment in Exercise 2.8. Run the environment with this agent for all possible initial dirt configurations and agent locations. Record the performance score for each configuration and the overall average score.

2.10   Consider a modified version of the vacuum environment in Exercise 2.8, in which the agent is penalized one point for each movement.

   a. Can a simple reflex agent be perfectly rational for this environment? Explain.
   b. What about a reflex agent with state? Design such an agent.
   c. How do your answers to **a** and **b** change if the agent's percepts give it the clean/dirty status of every square in the environment?

2.11   Consider a modified version of the vacuum environment in Exercise 2.8, in which the geography of the environment—its extent, boundaries, and obstacles—is unknown, as is the initial dirt configuration. (The agent can go *Up* and *Down* as well as *Left* and *Right*.)

   a. Can a simple reflex agent be perfectly rational for this environment? Explain.
   b. Can a simple reflex agent with a *randomized* agent function outperform a simple reflex agent? Design such an agent and measure its performance on several environments.
   c. Can you design an environment in which your randomized agent will perform poorly? Show your results.
   d. Can a reflex agent with state outperform a simple reflex agent? Design such an agent and measure its performance on several environments. Can you design a rational agent of this type?

2.12   Repeat Exercise 2.11 for the case in which the location sensor is replaced with a "bump" sensor that detects the agent's attempts to move into an obstacle or to cross the boundaries of the environment. Suppose the bump sensor stops working; how should the agent behave?

2.13   The vacuum environments in the preceding exercises have all been deterministic. Discuss possible agent programs for each of the following stochastic versions:

   a. Murphy's law: twenty-five percent of the time, the *Suck* action fails to clean the floor if it is dirty and deposits dirt onto the floor if the floor is clean. How is your agent program affected if the dirt sensor gives the wrong answer 10% of the time?
   b. Small children: At each time step, each clean square has a 10% chance of becoming dirty. Can you come up with a rational agent design for this case?

3 SOLVING PROBLEMS BY SEARCHING

In which we see how an agent can find a sequence of actions that achieves its goals when no single action will do.

The simplest agents discussed in Chapter 2 were the reflex agents, which base their actions on a direct mapping from states to actions. Such agents cannot operate well in environments for which this mapping would be too large to store and would take too long to learn. Goal-based agents, on the other hand, consider future actions and the desirability of their outcomes.

PROBLEM-SOLVING
AGENT

This chapter describes one kind of goal-based agent called a **problem-solving agent**. Problem-solving agents use **atomic** representations, as described in Section 2.4.7—that is, states of the world are considered as wholes, with no internal structure visible to the problem-solving algorithms. Goal-based agents that use more advanced **factored** or **structured** representations are usually called **planning agents** and are discussed in Chapters 7 and 10.

Our discussion of problem solving begins with precise definitions of **problems** and their **solutions** and give several examples to illustrate these definitions. We then describe several general-purpose search algorithms that can be used to solve these problems. We will see several **uninformed** search algorithms—algorithms that are given no information about the problem other than its definition. Although some of these algorithms can solve any solvable problem, none of them can do so efficiently. **Informed** search algorithms, on the other hand, can do quite well given some guidance on where to look for solutions.

In this chapter, we limit ourselves to the simplest kind of task environment, for which the solution to a problem is always a *fixed sequence* of actions. The more general case—where the agent's future actions may vary depending on future percepts—is handled in Chapter 4.

This chapter uses the concepts of asymptotic complexity (that is, $O()$ notation) and NP-completeness. Readers unfamiliar with these concepts should consult Appendix A.

3.1 PROBLEM-SOLVING AGENTS

Intelligent agents are supposed to maximize their performance measure. As we mentioned in Chapter 2, achieving this is sometimes simplified if the agent can adopt a **goal** and aim at satisfying it. Let us first look at why and how an agent might do this.

Imagine an agent in the city of Arad, Romania, enjoying a touring holiday. The agent's performance measure contains many factors: it wants to improve its suntan, improve its Romanian, take in the sights, enjoy the nightlife (such as it is), avoid hangovers, and so on. The decision problem is a complex one involving many tradeoffs and careful reading of guidebooks. Now, suppose the agent has a nonrefundable ticket to fly out of Bucharest the following day. In that case, it makes sense for the agent to adopt the **goal** of getting to Bucharest. Courses of action that don't reach Bucharest on time can be rejected without further consideration and the agent's decision problem is greatly simplified. Goals help organize behavior by limiting the objectives that the agent is trying to achieve and hence the actions it needs to consider. **Goal formulation**, based on the current situation and the agent's performance measure, is the first step in problem solving.

GOAL FORMULATION

We will consider a goal to be a set of world states—exactly those states in which the goal is satisfied. The agent's task is to find out how to act, now and in the future, so that it reaches a goal state. Before it can do this, it needs to decide (or we need to decide on its behalf) what sorts of actions and states it should consider. If it were to consider actions at the level of "move the left foot forward an inch" or "turn the steering wheel one degree left," the agent would probably never find its way out of the parking lot, let alone to Bucharest, because at that level of detail there is too much uncertainty in the world and there would be too many steps in a solution. **Problem formulation** is the process of deciding what actions and states to consider, given a goal. We discuss this process in more detail later. For now, let us assume that the agent will consider actions at the level of driving from one major town to another. Each state therefore corresponds to being in a particular town.

PROBLEM
FORMULATION

Our agent has now adopted the goal of driving to Bucharest and is considering where to go from Arad. Three roads lead out of Arad, one toward Sibiu, one to Timisoara, and one to Zerind. None of these achieves the goal, so unless the agent is familiar with the geography of Romania, it will not know which road to follow.[1] In other words, the agent will not know which of its possible actions is best, because it does not yet know enough about the state that results from taking each action. If the agent has no additional information—i.e., if the environment is **unknown** in the sense defined in Section 2.3—then it is has no choice but to try one of the actions at random. This sad situation is discussed in Chapter 4.

But suppose the agent has a map of Romania. The point of a map is to provide the agent with information about the states it might get itself into and the actions it can take. The agent can use this information to consider *subsequent* stages of a hypothetical journey via each of the three towns, trying to find a journey that eventually gets to Bucharest. Once it has found a path on the map from Arad to Bucharest, it can achieve its goal by carrying out the driving actions that correspond to the legs of the journey. In general, *an agent with several immediate options of unknown value can decide what to do by first examining* future *actions that eventually lead to states of known value.*

To be more specific about what we mean by "examining future actions," we have to be more specific about properties of the environment, as defined in Section 2.3. For now,

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[1]   We are assuming that most readers are in the same position and can easily imagine themselves to be as clueless as our agent. We apologize to Romanian readers who are unable to take advantage of this pedagogical device.

we assume that the environment is **observable**, so the agent always knows the current state. For the agent driving in Romania, it's reasonable to suppose that each city on the map has a sign indicating its presence to arriving drivers. We also assume the environment is **discrete**, so at any given state there are only finitely many actions to choose from. This is true for navigating in Romania because each city is connected to a small number of other cities. We will assume the environment is **known**, so the agent knows which states are reached by each action. (Having an accurate map suffices to meet this condition for navigation problems.) Finally, we assume that the environment is **deterministic**, so each action has exactly one outcome. Under ideal conditions, this is true for the agent in Romania—it means that if it chooses to drive from Arad to Sibiu, it does end up in Sibiu. Of course, conditions are not always ideal, as we show in Chapter 4.

Under these assumptions, the solution to any problem is a fixed sequence of actions. "Of course!" one might say, "What else could it be?" Well, in general it could be a branching strategy that recommends different actions in the future depending on what percepts arrive. For example, under less than ideal conditions, the agent might plan to drive from Arad to Sibiu and then to Rimnicu Vilcea but may also need to have a contingency plan in case it arrives by accident in Zerind instead of Sibiu. Fortunately, if the agent knows the initial state and the environment is known and deterministic, it knows exactly where it will be after the first action and what it will perceive. Since only one percept is possible after the first action, the solution can specify only one possible second action, and so on.

SEARCH

SOLUTION

EXECUTION

The process of looking for a sequence of actions that reaches the goal is called **search**. A search algorithm takes a problem as input and returns a **solution** in the form of an action sequence. Once a solution is found, the actions it recommends can be carried out. This is called the **execution** phase. Thus, we have a simple "formulate, search, execute" design for the agent, as shown in Figure 3.1. After formulating a goal and a problem to solve, the agent calls a search procedure to solve it. It then uses the solution to guide its actions, doing whatever the solution recommends as the next thing to do—typically, the first action of the sequence—and then removing that step from the sequence. Once the solution has been executed, the agent will formulate a new goal.

OPEN-LOOP

Notice that while the agent is executing the solution sequence it *ignores its percepts* when choosing an action because it knows in advance what they will be. An agent that carries out its plans with its eyes closed, so to speak, must be quite certain of what is going on. Control theorists call this an **open-loop** system, because ignoring the percepts breaks the loop between agent and environment.

We first describe the process of problem formulation, and then devote the bulk of the chapter to various algorithms for the SEARCH function. We do not discuss the workings of the UPDATE-STATE and FORMULATE-GOAL functions further in this chapter.

3.1.1   Well-defined problems and solutions

PROBLEM

A **problem** can be defined formally by five components:

INITIAL STATE

- The **initial state** that the agent starts in. For example, the initial state for our agent in Romania might be described as $In(Arad)$.

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function SIMPLE-PROBLEM-SOLVING-AGENT(*percept*) **returns** an action
  persistent: *seq*, an action sequence, initially empty
            state, some description of the current world state
            goal, a goal, initially null
            problem, a problem formulation

  state ← UPDATE-STATE(*state*, *percept*)
  if *seq* is empty **then**
    goal ← FORMULATE-GOAL(*state*)
    problem ← FORMULATE-PROBLEM(*state*, *goal*)
    seq ← SEARCH(*problem*)
    if *seq* = *failure* **then return** a null action
  action ← FIRST(*seq*)
  seq ← REST(*seq*)
  return *action*

Figure 3.1   A simple problem-solving agent. It first formulates a goal and a problem, searches for a sequence of actions that would solve the problem, and then executes the actions one at a time. When this is complete, it formulates another goal and starts over.

ACTIONS

APPLICABLE

- A description of the possible **actions** available to the agent. Given a particular state s, ACTIONS(s) returns the set of actions that can be executed in s. We say that each of these actions is **applicable** in s. For example, from the state $In(Arad)$, the applicable actions are $\{Go(Sibiu), Go(Timisoara), Go(Zerind)\}$.

TRANSITION MODEL

SUCCESSOR

- A description of what each action does; the formal name for this is the **transition model**, specified by a function RESULT(s, a) that returns the state that results from doing action a in state s. We also use the term **successor** to refer to any state reachable from a given state by a single action.[2] For example, we have

$$\text{RESULT}(In(Arad), Go(Zerind)) = In(Zerind) .$$

STATE SPACE

GRAPH

PATH

Together, the initial state, actions, and transition model implicitly define the **state space** of the problem—the set of all states reachable from the initial state by any sequence of actions. The state space forms a directed network or **graph** in which the nodes are states and the links between nodes are actions. (The map of Romania shown in Figure 3.2 can be interpreted as a state-space graph if we view each road as standing for two driving actions, one in each direction.) A **path** in the state space is a sequence of states connected by a sequence of actions.

GOAL TEST

- The **goal test**, which determines whether a given state is a goal state. Sometimes there is an explicit set of possible goal states, and the test simply checks whether the given state is one of them. The agent's goal in Romania is the singleton set $\{In(Bucharest)\}$.

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[2] Many treatments of problem solving, including previous editions of this book, use a **successor function**, which returns the set of all successors, instead of separate ACTIONS and RESULT functions. The successor function makes it difficult to describe an agent that knows what actions it can try but not what they achieve. Also, note some author use RESULT(a, s) instead of RESULT(s, a), and some use DO instead of RESULT.
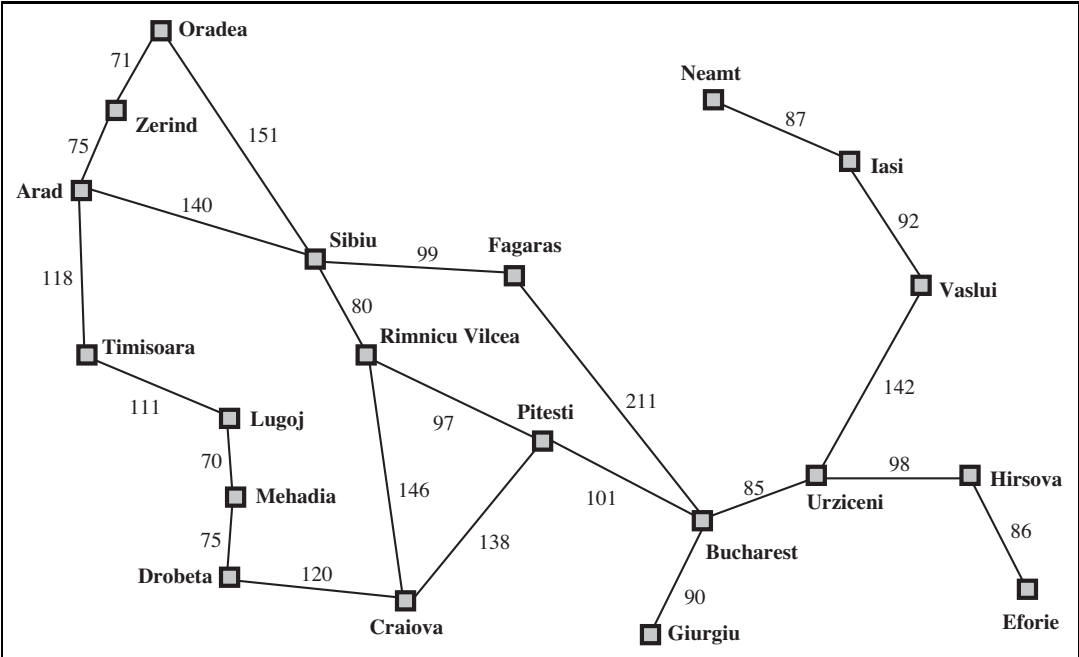
Figure 3.2   A simplified road map of part of Romania.

Sometimes the goal is specified by an abstract property rather than an explicitly enumerated set of states. For example, in chess, the goal is to reach a state called "checkmate," where the opponent's king is under attack and can't escape.

PATH COST
- A **path cost** function that assigns a numeric cost to each path. The problem-solving agent chooses a cost function that reflects its own performance measure. For the agent trying to get to Bucharest, time is of the essence, so the cost of a path might be its length in kilometers. In this chapter, we assume that the cost of a path can be described as the *sum* of the costs of the individual actions along the path.[3] The **step cost** of taking action
STEP COST
  a in state s to reach state s' is denoted by $c(s, a, s')$. The step costs for Romania are shown in Figure 3.2 as route distances. We assume that step costs are nonnegative.[4]

The preceding elements define a problem and can be gathered into a single data structure that is given as input to a problem-solving algorithm. A **solution** to a problem is an action sequence that leads from the initial state to a goal state. Solution quality is measured by the path cost function, and an **optimal solution** has the lowest path cost among all solutions.

OPTIMAL SOLUTION

3.1.2   Formulating problems

In the preceding section we proposed a formulation of the problem of getting to Bucharest in terms of the initial state, actions, transition model, goal test, and path cost. This formulation seems reasonable, but it is still a *model*—an abstract mathematical description—and not the

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[3]   This assumption is algorithmically convenient but also theoretically justifiable—see page 649 in Chapter 17.
[4]   The implications of negative costs are explored in Exercise 3.8.

real thing. Compare the simple state description we have chosen, *In(Arad)*, to an actual cross-country trip, where the state of the world includes so many things: the traveling companions, the current radio program, the scenery out of the window, the proximity of law enforcement officers, the distance to the next rest stop, the condition of the road, the weather, and so on. All these considerations are left out of our state descriptions because they are irrelevant to the problem of finding a route to Bucharest. The process of removing detail from a representation

ABSTRACTION

is called **abstraction**.

In addition to abstracting the state description, we must abstract the actions themselves. A driving action has many effects. Besides changing the location of the vehicle and its occupants, it takes up time, consumes fuel, generates pollution, and changes the agent (as they say, travel is broadening). Our formulation takes into account only the change in location. Also, there are many actions that we omit altogether: turning on the radio, looking out of the window, slowing down for law enforcement officers, and so on. And of course, we don't specify actions at the level of "turn steering wheel to the left by one degree."

Can we be more precise about defining the appropriate level of abstraction? Think of the abstract states and actions we have chosen as corresponding to large sets of detailed world states and detailed action sequences. Now consider a solution to the abstract problem: for example, the path from Arad to Sibiu to Rimnicu Vilcea to Pitesti to Bucharest. This abstract solution corresponds to a large number of more detailed paths. For example, we could drive with the radio on between Sibiu and Rimnicu Vilcea, and then switch it off for the rest of the trip. The abstraction is *valid* if we can expand any abstract solution into a solution in the more detailed world; a sufficient condition is that for every detailed state that is "in Arad," there is a detailed path to some state that is "in Sibiu," and so on.[5] The abstraction is *useful* if carrying out each of the actions in the solution is easier than the original problem; in this case they are easy enough that they can be carried out without further search or planning by an average driving agent. The choice of a good abstraction thus involves removing as much detail as possible while retaining validity and ensuring that the abstract actions are easy to carry out. Were it not for the ability to construct useful abstractions, intelligent agents would be completely swamped by the real world.

3.2   EXAMPLE PROBLEMS

The problem-solving approach has been applied to a vast array of task environments. We list some of the best known here, distinguishing between *toy* and *real-world* problems. A

TOY PROBLEM

toy problem is intended to illustrate or exercise various problem-solving methods. It can be given a concise, exact description and hence is usable by different researchers to compare the

REAL-WORLD
PROBLEM

performance of algorithms. A **real-world problem** is one whose solutions people actually care about. Such problems tend not to have a single agreed-upon description, but we can give the general flavor of their formulations.

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[5]   See Section 11.2 for a more complete set of definitions and algorithms.

Figure 3.3 The state space for the vacuum world. Links denote actions: L = *Left*, R = *Right*, S = *Suck*.
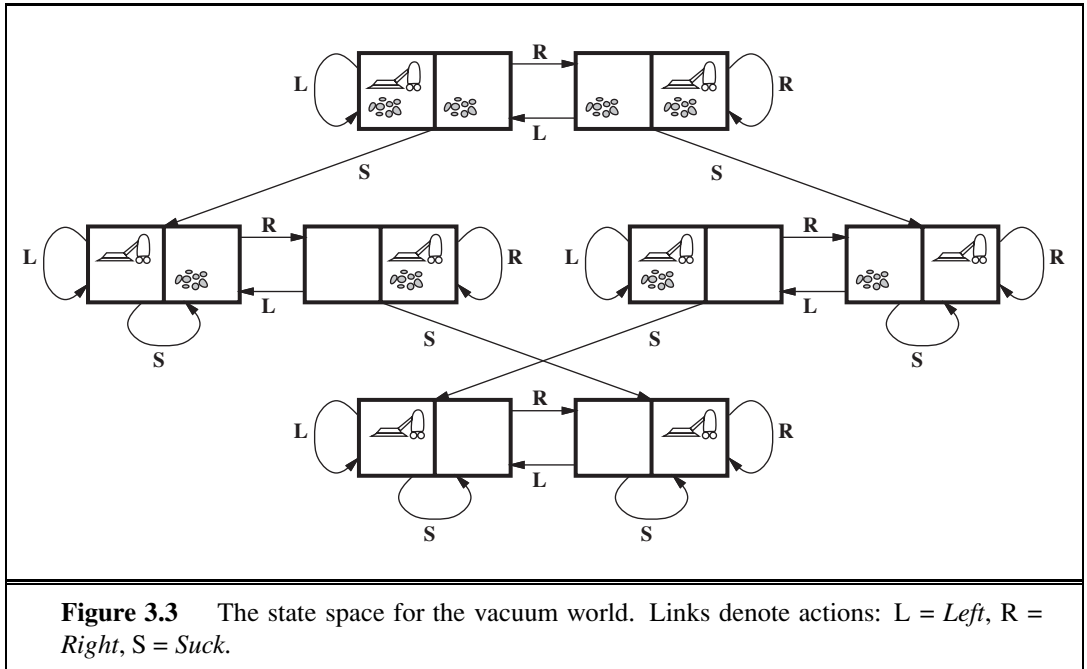
3.2.1 Toy problems

The first example we examine is the **vacuum world** first introduced in Chapter 2. (See Figure 2.2.) This can be formulated as a problem as follows:

- **States**: The state is determined by both the agent location and the dirt locations. The agent is in one of two locations, each of which might or might not contain dirt. Thus, there are $2 \times 2^2 = 8$ possible world states. A larger environment with n locations has $n \cdot 2^n$ states.
- **Initial state**: Any state can be designated as the initial state.
- **Actions**: In this simple environment, each state has just three actions: *Left*, *Right*, and *Suck*. Larger environments might also include *Up* and *Down*.
- **Transition model**: The actions have their expected effects, except that moving *Left* in the leftmost square, moving *Right* in the rightmost square, and *Suck*ing in a clean square have no effect. The complete state space is shown in Figure 3.3.
- **Goal test**: This checks whether all the squares are clean.
- **Path cost**: Each step costs 1, so the path cost is the number of steps in the path.

Compared with the real world, this toy problem has discrete locations, discrete dirt, reliable cleaning, and it never gets any dirtier. Chapter 4 relaxes some of these assumptions.

8-PUZZLE   The **8-puzzle**, an instance of which is shown in Figure 3.4, consists of a 3×3 board with eight numbered tiles and a blank space. A tile adjacent to the blank space can slide into the space. The object is to reach a specified goal state, such as the one shown on the right of the figure. The standard formulation is as follows:
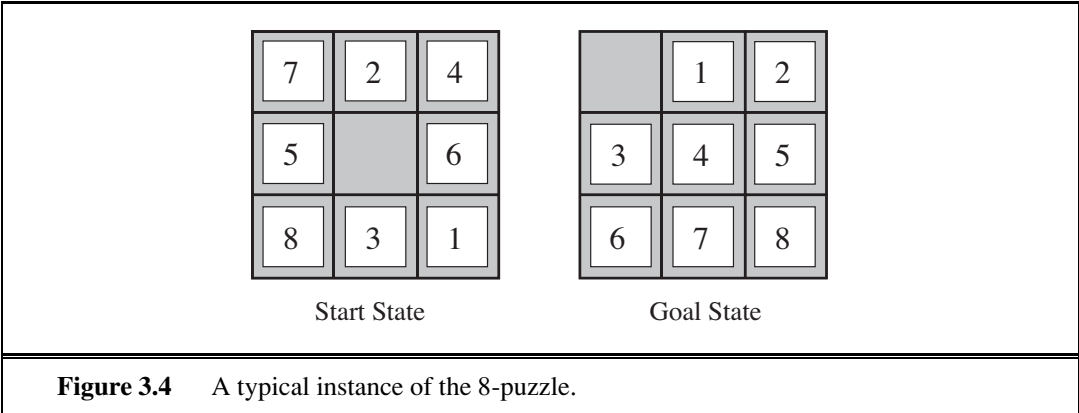
Figure 3.4   A typical instance of the 8-puzzle.

- **States**: A state description specifies the location of each of the eight tiles and the blank in one of the nine squares.
- **Initial state**: Any state can be designated as the initial state. Note that any given goal can be reached from exactly half of the possible initial states (Exercise 3.4).
- **Actions**: The simplest formulation defines the actions as movements of the blank space *Left*, *Right*, *Up*, or *Down*. Different subsets of these are possible depending on where the blank is.
- **Transition model**: Given a state and action, this returns the resulting state; for example, if we apply *Left* to the start state in Figure 3.4, the resulting state has the 5 and the blank switched.
- **Goal test**: This checks whether the state matches the goal configuration shown in Figure 3.4. (Other goal configurations are possible.)
- **Path cost**: Each step costs 1, so the path cost is the number of steps in the path.

What abstractions have we included here? The actions are abstracted to their beginning and final states, ignoring the intermediate locations where the block is sliding. We have abstracted away actions such as shaking the board when pieces get stuck and ruled out extracting the pieces with a knife and putting them back again. We are left with a description of the rules of the puzzle, avoiding all the details of physical manipulations.

SLIDING-BLOCK
PUZZLES
     The 8-puzzle belongs to the family of **sliding-block puzzles**, which are often used as test problems for new search algorithms in AI. This family is known to be NP-complete, so one does not expect to find methods significantly better in the worst case than the search algorithms described in this chapter and the next. The 8-puzzle has $9!/2 = 181,440$ reachable states and is easily solved. The 15-puzzle (on a 4×4 board) has around 1.3 trillion states, and random instances can be solved optimally in a few milliseconds by the best search algorithms. The 24-puzzle (on a 5×5 board) has around 10^{25} states, and random instances take several hours to solve optimally.

8-QUEENS PROBLEM
     The goal of the **8-queens problem** is to place eight queens on a chessboard such that no queen attacks any other. (A queen attacks any piece in the same row, column or diagonal.) Figure 3.5 shows an attempted solution that fails: the queen in the rightmost column is attacked by the queen at the top left.
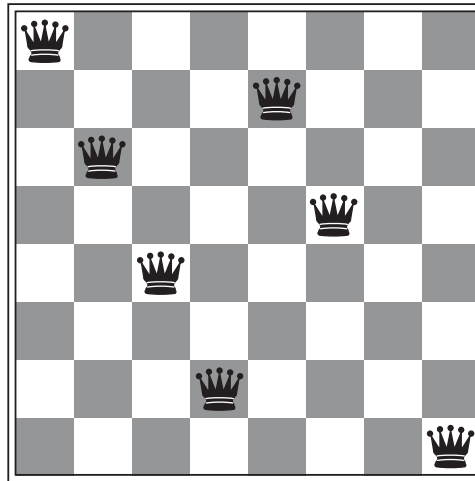
Figure 3.5   Almost a solution to the 8-queens problem. (Solution is left as an exercise.)

Although efficient special-purpose algorithms exist for this problem and for the whole n-queens family, it remains a useful test problem for search algorithms. There are two main kinds of formulation. An **incremental formulation** involves operators that *augment* the state description, starting with an empty state; for the 8-queens problem, this means that each action adds a queen to the state. A **complete-state formulation** starts with all 8 queens on the board and moves them around. In either case, the path cost is of no interest because only the final state counts. The first incremental formulation one might try is the following:

INCREMENTAL
FORMULATION

COMPLETE-STATE
FORMULATION

- **States**: Any arrangement of 0 to 8 queens on the board is a state.
- **Initial state**: No queens on the board.
- **Actions**: Add a queen to any empty square.
- **Transition model**: Returns the board with a queen added to the specified square.
- **Goal test**: 8 queens are on the board, none attacked.

In this formulation, we have $64 \cdot 63 \cdots 57 \approx 1.8 \times 10^{14}$ possible sequences to investigate. A better formulation would prohibit placing a queen in any square that is already attacked:

- **States**: All possible arrangements of n queens ($0 \leq n \leq 8$), one per column in the leftmost n columns, with no queen attacking another.
- **Actions**: Add a queen to any square in the leftmost empty column such that it is not attacked by any other queen.

This formulation reduces the 8-queens state space from 1.8×10^{14} to just 2,057, and solutions are easy to find. On the other hand, for 100 queens the reduction is from roughly 10^{400} states to about 10^{52} states (Exercise 3.5)—a big improvement, but not enough to make the problem tractable. Section 4.1 describes the complete-state formulation, and Chapter 6 gives a simple algorithm that solves even the million-queens problem with ease.

Our final toy problem was devised by Donald Knuth (1964) and illustrates how infinite state spaces can arise. Knuth conjectured that, starting with the number 4, a sequence of factorial, square root, and floor operations will reach any desired positive integer. For example, we can reach 5 from 4 as follows:

$$\left\lfloor \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{(4!)!}}}}} \right\rfloor = 5 \ .$$

The problem definition is very simple:

- **States**: Positive numbers.
- **Initial state**: 4.
- **Actions**: Apply factorial, square root, or floor operation (factorial for integers only).
- **Transition model**: As given by the mathematical definitions of the operations.
- **Goal test**: State is the desired positive integer.

To our knowledge there is no bound on how large a number might be constructed in the process of reaching a given target—for example, the number 620,448,401,733,239,439,360,000 is generated in the expression for 5—so the state space for this problem is infinite. Such state spaces arise frequently in tasks involving the generation of mathematical expressions, circuits, proofs, programs, and other recursively defined objects.

3.2.2   Real-world problems

ROUTE-FINDING
PROBLEM

We have already seen how the **route-finding problem** is defined in terms of specified locations and transitions along links between them. Route-finding algorithms are used in a variety of applications. Some, such as Web sites and in-car systems that provide driving directions, are relatively straightforward extensions of the Romania example. Others, such as routing video streams in computer networks, military operations planning, and airline travel-planning systems, involve much more complex specifications. Consider the airline travel problems that must be solved by a travel-planning Web site:

- **States**: Each state obviously includes a location (e.g., an airport) and the current time. Furthermore, because the cost of an action (a flight segment) may depend on previous segments, their fare bases, and their status as domestic or international, the state must record extra information about these "historical" aspects.
- **Initial state**: This is specified by the user's query.
- **Actions**: Take any flight from the current location, in any seat class, leaving after the current time, leaving enough time for within-airport transfer if needed.
- **Transition model**: The state resulting from taking a flight will have the flight's destination as the current location and the flight's arrival time as the current time.
- **Goal test**: Are we at the final destination specified by the user?
- **Path cost**: This depends on monetary cost, waiting time, flight time, customs and immigration procedures, seat quality, time of day, type of airplane, frequent-flyer mileage awards, and so on.

Commercial travel advice systems use a problem formulation of this kind, with many additional complications to handle the byzantine fare structures that airlines impose. Any seasoned traveler knows, however, that not all air travel goes according to plan. A really good system should include contingency plans—such as backup reservations on alternate flights—to the extent that these are justified by the cost and likelihood of failure of the original plan.

TOURING PROBLEM

Touring problems are closely related to route-finding problems, but with an important difference. Consider, for example, the problem "Visit every city in Figure 3.2 at least once, starting and ending in Bucharest." As with route finding, the actions correspond to trips between adjacent cities. The state space, however, is quite different. Each state must include not just the current location but also the *set of cities the agent has visited*. So the initial state would be $In(Bucharest)$, $Visited(\{Bucharest\})$, a typical intermediate state would be $In(Vaslui)$, $Visited(\{Bucharest, Urziceni, Vaslui\})$, and the goal test would check whether the agent is in Bucharest and all 20 cities have been visited.

TRAVELING
SALESPERSON
PROBLEM

The **traveling salesperson problem** (TSP) is a touring problem in which each city must be visited exactly once. The aim is to find the *shortest* tour. The problem is known to be NP-hard, but an enormous amount of effort has been expended to improve the capabilities of TSP algorithms. In addition to planning trips for traveling salespersons, these algorithms have been used for tasks such as planning movements of automatic circuit-board drills and of stocking machines on shop floors.

VLSI LAYOUT

A **VLSI layout** problem requires positioning millions of components and connections on a chip to minimize area, minimize circuit delays, minimize stray capacitances, and maximize manufacturing yield. The layout problem comes after the logical design phase and is usually split into two parts: **cell layout** and **channel routing**. In cell layout, the primitive components of the circuit are grouped into cells, each of which performs some recognized function. Each cell has a fixed footprint (size and shape) and requires a certain number of connections to each of the other cells. The aim is to place the cells on the chip so that they do not overlap and so that there is room for the connecting wires to be placed between the cells. Channel routing finds a specific route for each wire through the gaps between the cells. These search problems are extremely complex, but definitely worth solving. Later in this chapter, we present some algorithms capable of solving them.

ROBOT NAVIGATION

Robot navigation is a generalization of the route-finding problem described earlier. Rather than following a discrete set of routes, a robot can move in a continuous space with (in principle) an infinite set of possible actions and states. For a circular robot moving on a flat surface, the space is essentially two-dimensional. When the robot has arms and legs or wheels that must also be controlled, the search space becomes many-dimensional. Advanced techniques are required just to make the search space finite. We examine some of these methods in Chapter 25. In addition to the complexity of the problem, real robots must also deal with errors in their sensor readings and motor controls.

AUTOMATIC
ASSEMBLY
SEQUENCING

Automatic assembly sequencing of complex objects by a robot was first demonstrated by FREDDY (Michie, 1972). Progress since then has been slow but sure, to the point where the assembly of intricate objects such as electric motors is economically feasible. In assembly problems, the aim is to find an order in which to assemble the parts of some object. If the wrong order is chosen, there will be no way to add some part later in the sequence without

undoing some of the work already done. Checking a step in the sequence for feasibility is a difficult geometrical search problem closely related to robot navigation. Thus, the generation of legal actions is the expensive part of assembly sequencing. Any practical algorithm must avoid exploring all but a tiny fraction of the state space. Another important assembly problem is **protein design**, in which the goal is to find a sequence of amino acids that will fold into a three-dimensional protein with the right properties to cure some disease.

PROTEIN DESIGN

3.3   SEARCHING FOR SOLUTIONS

Having formulated some problems, we now need to solve them. A solution is an action sequence, so search algorithms work by considering various possible action sequences. The possible action sequences starting at the initial state form a **search tree** with the initial state at the root; the branches are actions and the **nodes** correspond to states in the state space of the problem. Figure 3.6 shows the first few steps in growing the search tree for finding a route from Arad to Bucharest. The root node of the tree corresponds to the initial state, *In(Arad)*. The first step is to test whether this is a goal state. (Clearly it is not, but it is important to check so that we can solve trick problems like "starting in Arad, get to Arad.") Then we need to consider taking various actions. We do this by **expanding** the current state; that is, applying each legal action to the current state, thereby **generating** a new set of states. In this case, we add three branches from the **parent node** *In(Arad)* leading to three new **child nodes**: *In(Sibiu), In(Timisoara),* and *In(Zerind)*. Now we must choose which of these three possibilities to consider further.

SEARCH TREE

NODE

EXPANDING

GENERATING

PARENT NODE

CHILD NODE

   This is the essence of search—following up one option now and putting the others aside for later, in case the first choice does not lead to a solution. Suppose we choose Sibiu first. We check to see whether it is a goal state (it is not) and then expand it to get *In(Arad)*, *In(Fagaras)*, *In(Oradea)*, and *In(RimnicuVilcea)*. We can then choose any of these four or go back and choose Timisoara or Zerind. Each of these six nodes is a **leaf node**, that is, a node with no children in the tree. The set of all leaf nodes available for expansion at any given point is called the **frontier**. (Many authors call it the **open list**, which is both geographically less evocative and less accurate, because other data structures are better suited than a list.) In Figure 3.6, the frontier of each tree consists of those nodes with bold outlines.

LEAF NODE

FRONTIER

OPEN LIST

   The process of expanding nodes on the frontier continues until either a solution is found or there are no more states to expand. The general TREE-SEARCH algorithm is shown informally in Figure 3.7. Search algorithms all share this basic structure; they vary primarily according to how they choose which state to expand next—the so-called **search strategy**.

SEARCH STRATEGY

   The eagle-eyed reader will notice one peculiar thing about the search tree shown in Figure 3.6: it includes the path from Arad to Sibiu and back to Arad again! We say that *In(Arad)* is a **repeated state** in the search tree, generated in this case by a **loopy path**. Considering such loopy paths means that the complete search tree for Romania is *infinite* because there is no limit to how often one can traverse a loop. On the other hand, the state space—the map shown in Figure 3.2—has only 20 states. As we discuss in Section 3.4, loops can cause

REPEATED STATE

LOOPY PATH

certain algorithms to fail, making otherwise solvable problems unsolvable. Fortunately, there
is no need to consider loopy paths. We can rely on more than intuition for this: because path
costs are additive and step costs are nonnegative, a loopy path to any given state is never
better than the same path with the loop removed.

REDUNDANT PATH   Loopy paths are a special case of the more general concept of **redundant paths**, which
exist whenever there is more than one way to get from one state to another. Consider the paths
Arad–Sibiu (140 km long) and Arad–Zerind–Oradea–Sibiu (297 km long). Obviously, the
second path is redundant—it's just a worse way to get to the same state. If you are concerned
about reaching the goal, there's never any reason to keep more than one path to any given
state, because any goal state that is reachable by extending one path is also reachable by
extending the other.

In some cases, it is possible to define the problem itself so as to eliminate redundant
paths. For example, if we formulate the 8-queens problem (page 71) so that a queen can be
placed in any column, then each state with n queens can be reached by $n!$ different paths; but
if we reformulate the problem so that each new queen is placed in the leftmost empty column,
then each state can be reached only through one path.

Figure 3.6   Partial search trees for finding a route from Arad to Bucharest. Nodes that
have been expanded are shaded; nodes that have been generated but not yet expanded are
outlined in bold; nodes that have not yet been generated are shown in faint dashed lines.

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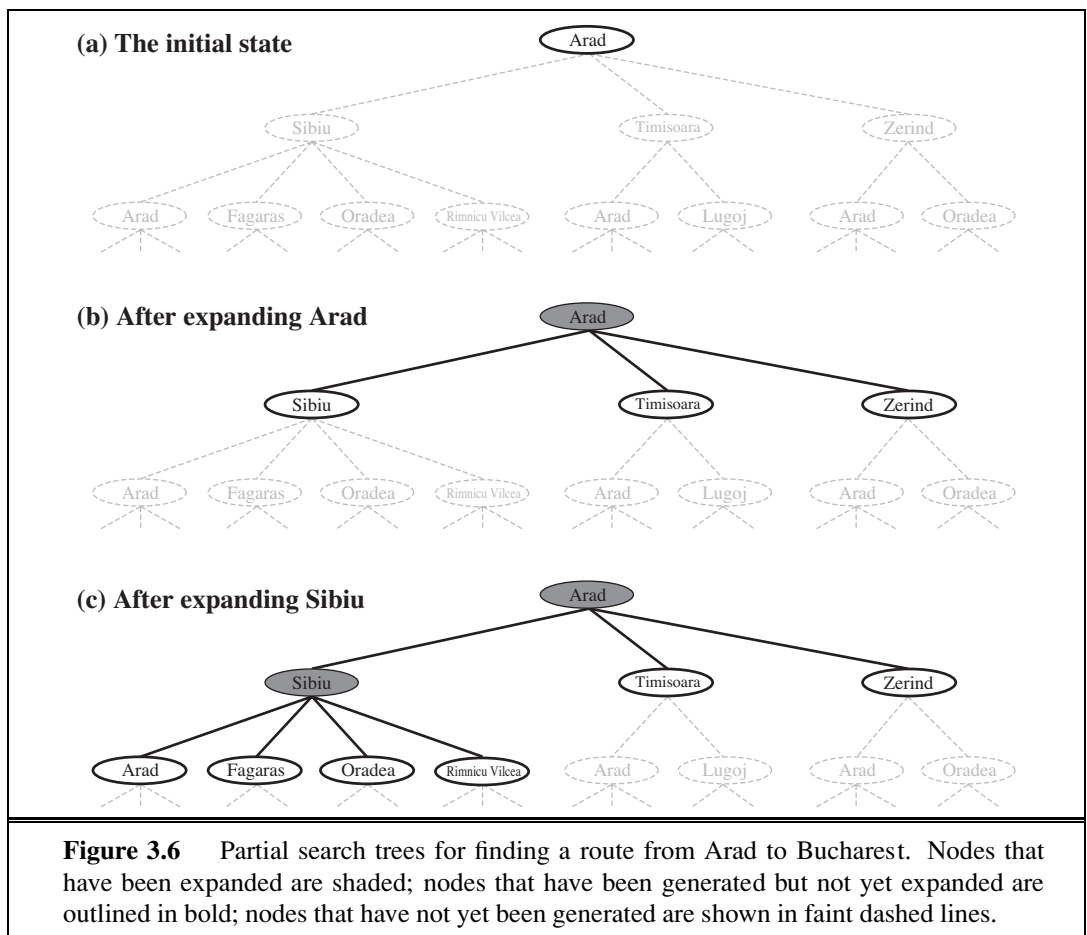
function TREE-SEARCH(*problem*) **returns** a solution, or failure
   initialize the frontier using the initial state of *problem*
   loop do
      if the frontier is empty **then return** failure
      choose a leaf node and remove it from the frontier
      if the node contains a goal state **then return** the corresponding solution
      expand the chosen node, adding the resulting nodes to the frontier

---

function GRAPH-SEARCH(*problem*) **returns** a solution, or failure
   initialize the frontier using the initial state of *problem*
   initialize the explored set to be empty
   loop do
      if the frontier is empty **then return** failure
      choose a leaf node and remove it from the frontier
      if the node contains a goal state **then return** the corresponding solution
      add the node to the explored set
      expand the chosen node, adding the resulting nodes to the frontier
         only if not in the frontier or explored set

Figure 3.7   An informal description of the general tree-search and graph-search algorithms. The parts of GRAPH-SEARCH marked in bold italic are the additions needed to handle repeated states.

RECTANGULAR GRID

EXPLORED SET
CLOSED LIST

SEPARATOR

In other cases, redundant paths are unavoidable. This includes all problems where the actions are reversible, such as route-finding problems and sliding-block puzzles. Route-finding on a **rectangular grid** (like the one used later for Figure 3.9) is a particularly important example in computer games. In such a grid, each state has four successors, so a search tree of depth d that includes repeated states has 4^d leaves; but there are only about $2d^2$ distinct states within d steps of any given state. For $d = 20$, this means about a trillion nodes but only about 800 distinct states. Thus, following redundant paths can cause a tractable problem to become intractable. This is true even for algorithms that know how to avoid infinite loops.

As the saying goes, *algorithms that forget their history are doomed to repeat it.* The way to avoid exploring redundant paths is to remember where one has been. To do this, we augment the TREE-SEARCH algorithm with a data structure called the **explored set** (also known as the **closed list**), which remembers every expanded node. Newly generated nodes that match previously generated nodes—ones in the explored set or the frontier—can be discarded instead of being added to the frontier. The new algorithm, called GRAPH-SEARCH, is shown informally in Figure 3.7. The specific algorithms in this chapter draw on this general design.

Clearly, the search tree constructed by the GRAPH-SEARCH algorithm contains at most one copy of each state, so we can think of it as growing a tree directly on the state-space graph, as shown in Figure 3.8. The algorithm has another nice property: the frontier **separates** the state-space graph into the explored region and the unexplored region, so that every path from

Figure 3.8   A sequence of search trees generated by a graph search on the Romania problem of Figure 3.2. At each stage, we have extended each path by one step. Notice that at the third stage, the northernmost city (Oradea) has become a dead end: both of its successors are already explored via other paths.

        (a)   (b)   (c)

Figure 3.9   The separation property of GRAPH-SEARCH, illustrated on a rectangular-grid problem. The frontier (white nodes) always separates the explored region of the state space (black nodes) from the unexplored region (gray nodes). In (a), just the root has been expanded. In (b), one leaf node has been expanded. In (c), the remaining successors of the root have been expanded in clockwise order.

the initial state to an unexplored state has to pass through a state in the frontier. (If this seems completely obvious, try Exercise 3.13 now.) This property is illustrated in Figure 3.9. As every step moves a state from the frontier into the explored region while moving some states from the unexplored region into the frontier, we see that the algorithm is *systematically* examining the states in the state space, one by one, until it finds a solution.

3.3.1   Infrastructure for search algorithms

Search algorithms require a data structure to keep track of the search tree that is being constructed. For each node n of the tree, we have a structure that contains four components:

- n.STATE: the state in the state space to which the node corresponds;
- n.PARENT: the node in the search tree that generated this node;
- n.ACTION: the action that was applied to the parent to generate the node;
- n.PATH-COST: the cost, traditionally denoted by $g(n)$, of the path from the initial state to the node, as indicated by the parent pointers.

Figure 3.10   Nodes are the data structures from which the search tree is constructed. Each has a parent, a state, and various bookkeeping fields. Arrows point from child to parent.
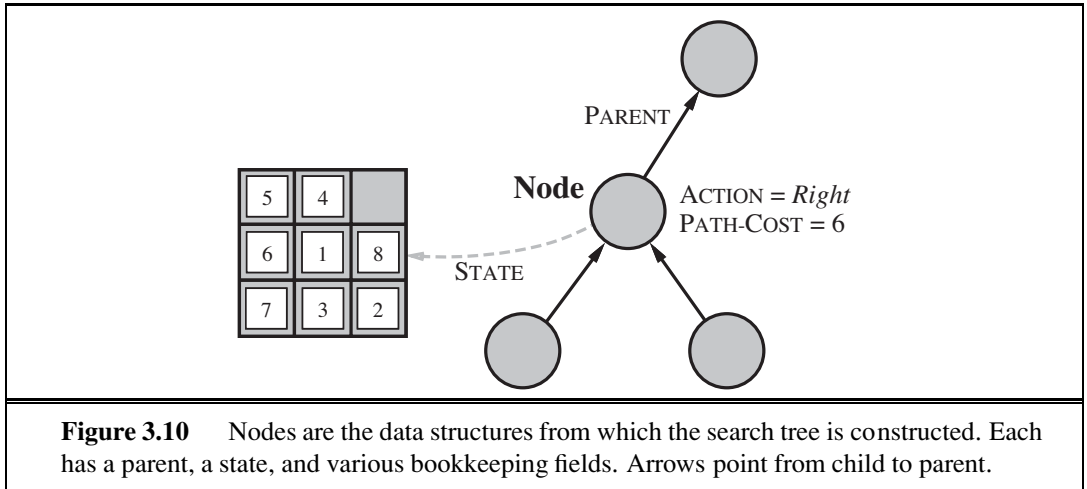
Given the components for a parent node, it is easy to see how to compute the necessary components for a child node. The function CHILD-NODE takes a parent node and an action and returns the resulting child node:

---

function CHILD-NODE(*problem*, *parent*, *action*) **returns** a node
   return a node with
      STATE = *problem*.RESULT(*parent*.STATE, *action*),
      PARENT = *parent*, ACTION = *action*,
      PATH-COST = *parent*.PATH-COST + *problem*.STEP-COST(*parent*.STATE, *action*)

The node data structure is depicted in Figure 3.10. Notice how the PARENT pointers string the nodes together into a tree structure. These pointers also allow the solution path to be extracted when a goal node is found; we use the SOLUTION function to return the sequence of actions obtained by following parent pointers back to the root.

Up to now, we have not been very careful to distinguish between nodes and states, but in writing detailed algorithms it's important to make that distinction. A node is a bookkeeping data structure used to represent the search tree. A state corresponds to a configuration of the world. Thus, nodes are on particular paths, as defined by PARENT pointers, whereas states are not. Furthermore, two different nodes can contain the same world state if that state is generated via two different search paths.

Now that we have nodes, we need somewhere to put them. The frontier needs to be stored in such a way that the search algorithm can easily choose the next node to expand according to its preferred strategy. The appropriate data structure for this is a **queue**. The operations on a queue are as follows:

QUEUE

- EMPTY?(*queue*) returns true only if there are no more elements in the queue.
- POP(*queue*) removes the first element of the queue and returns it.
- INSERT(*element*, *queue*) inserts an element and returns the resulting queue.

Queues are characterized by the *order* in which they store the inserted nodes. Three common

variants are the first-in, first-out or **FIFO queue**, which pops the *oldest* element of the queue;

the last-in, first-out or **LIFO queue** (also known as a **stack**), which pops the *newest* element

of the queue; and the **priority queue**, which pops the element of the queue with the highest
priority according to some ordering function.

The explored set can be implemented with a hash table to allow efficient checking for
repeated states. With a good implementation, insertion and lookup can be done in roughly
constant time no matter how many states are stored. One must take care to implement the
hash table with the right notion of equality between states. For example, in the traveling
salesperson problem (page 74), the hash table needs to know that the set of visited cities
{Bucharest,Urziceni,Vaslui} is the same as {Urziceni,Vaslui,Bucharest}. Sometimes this can
be achieved most easily by insisting that the data structures for states be in some **canonical**

form; that is, logically equivalent states should map to the same data structure. In the case
of states described by sets, for example, a bit-vector representation or a sorted list without
repetition would be canonical, whereas an unsorted list would not.

3.3.2   Measuring problem-solving performance

Before we get into the design of specific search algorithms, we need to consider the criteria
that might be used to choose among them. We can evaluate an algorithm's performance in
four ways:

- **Completeness**: Is the algorithm guaranteed to find a solution when there is one?

- **Optimality**: Does the strategy find the optimal solution, as defined on page 68?

- **Time complexity**: How long does it take to find a solution?

- **Space complexity**: How much memory is needed to perform the search?

Time and space complexity are always considered with respect to some measure of the problem difficulty. In theoretical computer science, the typical measure is the size of the state
space graph, $|V| + |E|$, where V is the set of vertices (nodes) of the graph and E is the set
of edges (links). This is appropriate when the graph is an explicit data structure that is input
to the search program. (The map of Romania is an example of this.) In AI, the graph is often
represented *implicitly* by the initial state, actions, and transition model and is frequently infinite. For these reasons, complexity is expressed in terms of three quantities: b, the **branching**

factor or maximum number of successors of any node; d, the **depth** of the shallowest goal

node (i.e., the number of steps along the path from the root); and m, the maximum length of
any path in the state space. Time is often measured in terms of the number of nodes generated
during the search, and space in terms of the maximum number of nodes stored in memory.
For the most part, we describe time and space complexity for search on a tree; for a graph,
the answer depends on how "redundant" the paths in the state space are.

To assess the effectiveness of a search algorithm, we can consider just the **search cost**—
which typically depends on the time complexity but can also include a term for memory

usage—or we can use the **total cost**, which combines the search cost and the path cost of the
solution found. For the problem of finding a route from Arad to Bucharest, the search cost
is the amount of time taken by the search and the solution cost is the total length of the path

in kilometers. Thus, to compute the total cost, we have to add milliseconds and kilometers. There is no "official exchange rate" between the two, but it might be reasonable in this case to convert kilometers into milliseconds by using an estimate of the car's average speed (because time is what the agent cares about). This enables the agent to find an optimal tradeoff point at which further computation to find a shorter path becomes counterproductive. The more general problem of tradeoffs between different goods is taken up in Chapter 16.

3.4   UNINFORMED SEARCH STRATEGIES

UNINFORMED
SEARCH
BLIND SEARCH

INFORMED SEARCH

HEURISTIC SEARCH

This section covers several search strategies that come under the heading of **uninformed search** (also called **blind search**). The term means that the strategies have no additional information about states beyond that provided in the problem definition. All they can do is generate successors and distinguish a goal state from a non-goal state. All search strategies are distinguished by the *order* in which nodes are expanded. Strategies that know whether one non-goal state is "more promising" than another are called **informed search** or **heuristic search** strategies; they are covered in Section 3.5.

3.4.1   Breadth-first search

BREADTH-FIRST
SEARCH

Breadth-first search is a simple strategy in which the root node is expanded first, then all the successors of the root node are expanded next, then *their* successors, and so on. In general, all the nodes are expanded at a given depth in the search tree before any nodes at the next level are expanded.

Breadth-first search is an instance of the general graph-search algorithm (Figure 3.7) in which the *shallowest* unexpanded node is chosen for expansion. This is achieved very simply by using a FIFO queue for the frontier. Thus, new nodes (which are always deeper than their parents) go to the back of the queue, and old nodes, which are shallower than the new nodes, get expanded first. There is one slight tweak on the general graph-search algorithm, which is that the goal test is applied to each node when it is *generated* rather than when it is selected for expansion. This decision is explained below, where we discuss time complexity. Note also that the algorithm, following the general template for graph search, discards any new path to a state already in the frontier or explored set; it is easy to see that any such path must be at least as deep as the one already found. Thus, breadth-first search always has the shallowest path to every node on the frontier.

Pseudocode is given in Figure 3.11. Figure 3.12 shows the progress of the search on a simple binary tree.

How does breadth-first search rate according to the four criteria from the previous section? We can easily see that it is *complete*—if the shallowest goal node is at some finite depth d, breadth-first search will eventually find it after generating all shallower nodes (provided the branching factor b is finite). Note that as soon as a goal node is generated, we know it is the shallowest goal node because all shallower nodes must have been generated already and failed the goal test. Now, the *shallowest* goal node is not necessarily the *optimal* one;

---

function BREADTH-FIRST-SEARCH(*problem*) **returns** a solution, or failure

   node ← a node with STATE = *problem*.INITIAL-STATE, PATH-COST = 0
   if *problem*.GOAL-TEST(*node*.STATE) **then return** SOLUTION(*node*)
   frontier ← a FIFO queue with *node* as the only element
   explored ← an empty set
   loop do
      if EMPTY?(*frontier*) **then return** failure
      node ← POP(*frontier*)   /* chooses the shallowest node in *frontier* */
      add *node*.STATE to *explored*
      for each *action* **in** *problem*.ACTIONS(*node*.STATE) **do**
         child ← CHILD-NODE(*problem*, *node*, *action*)
         if *child*.STATE is not in *explored* or *frontier* **then**
            if *problem*.GOAL-TEST(*child*.STATE) **then return** SOLUTION(*child*)
            frontier ← INSERT(*child*, *frontier*)

Figure 3.11   Breadth-first search on a graph.

technically, breadth-first search is optimal if the path cost is a nondecreasing function of the depth of the node. The most common such scenario is that all actions have the same cost.

So far, the news about breadth-first search has been good. The news about time and space is not so good. Imagine searching a uniform tree where every state has b successors. The root of the search tree generates b nodes at the first level, each of which generates b more nodes, for a total of b^2 at the second level. Each of *these* generates b more nodes, yielding b^3 nodes at the third level, and so on. Now suppose that the solution is at depth d. In the worst case, it is the last node generated at that level. Then the total number of nodes generated is

$$b + b^2 + b^3 + \cdots + b^d = O(b^d) \,.$$

(If the algorithm were to apply the goal test to nodes when selected for expansion, rather than when generated, the whole layer of nodes at depth d would be expanded before the goal was detected and the time complexity would be $O(b^{d+1})$.)

As for space complexity: for any kind of graph search, which stores every expanded node in the *explored* set, the space complexity is always within a factor of b of the time complexity. For breadth-first graph search in particular, every node generated remains in memory. There will be $O(b^{d-1})$ nodes in the *explored* set and $O(b^d)$ nodes in the frontier,
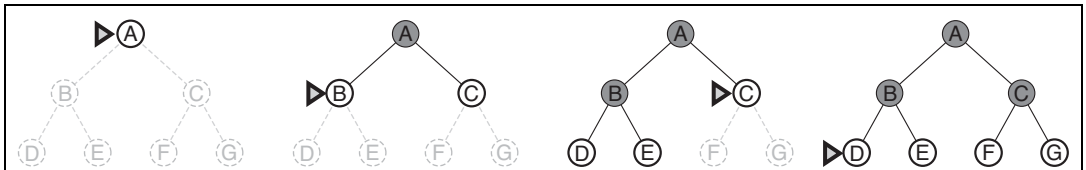
Figure 3.12   Breadth-first search on a simple binary tree. At each stage, the node to be expanded next is indicated by a marker.

so the space complexity is $O(b^d)$, i.e., it is dominated by the size of the frontier. Switching to a tree search would not save much space, and in a state space with many redundant paths, switching could cost a great deal of time.

An exponential complexity bound such as $O(b^d)$ is scary. Figure 3.13 shows why. It lists, for various values of the solution depth d, the time and memory required for a breadth-first search with branching factor $b = 10$. The table assumes that 1 million nodes can be generated per second and that a node requires 1000 bytes of storage. Many search problems fit roughly within these assumptions (give or take a factor of 100) when run on a modern personal computer.

Depth	Nodes	Time		Memory	
2	110	.11	milliseconds	107	kilobytes
4	11,110	11	milliseconds	10.6	megabytes
6	10^6	1.1	seconds	1	gigabyte
8	10^8	2	minutes	103	gigabytes
10	10^{10}	3	hours	10	terabytes
12	10^{12}	13	days	1	petabyte
14	10^{14}	3.5	years	99	petabytes
16	10^{16}	350	years	10	exabytes

Figure 3.13   Time and memory requirements for breadth-first search. The numbers shown assume branching factor $b = 10$; 1 million nodes/second; 1000 bytes/node.

Two lessons can be learned from Figure 3.13. First, *the memory requirements are a bigger problem for breadth-first search than is the execution time.* One might wait 13 days for the solution to an important problem with search depth 12, but no personal computer has the petabyte of memory it would take. Fortunately, other strategies require less memory.

The second lesson is that time is still a major factor. If your problem has a solution at depth 16, then (given our assumptions) it will take about 350 years for breadth-first search (or indeed any uninformed search) to find it. In general, *exponential-complexity search problems cannot be solved by uninformed methods for any but the smallest instances.*

3.4.2 Uniform-cost search

UNIFORM-COST
SEARCH

When all step costs are equal, breadth-first search is optimal because it always expands the *shallowest* unexpanded node. By a simple extension, we can find an algorithm that is optimal with any step-cost function. Instead of expanding the shallowest node, **uniform-cost search** expands the node n with the *lowest path cost* $g(n)$. This is done by storing the frontier as a priority queue ordered by g. The algorithm is shown in Figure 3.14.

In addition to the ordering of the queue by path cost, there are two other significant differences from breadth-first search. The first is that the goal test is applied to a node when it is *selected for expansion* (as in the generic graph-search algorithm shown in Figure 3.7) rather than when it is first generated. The reason is that the first goal node that is *generated*

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function UNIFORM-COST-SEARCH(*problem*) **returns** a solution, or failure

   node ← a node with STATE = *problem*.INITIAL-STATE, PATH-COST = 0
   frontier ← a priority queue ordered by PATH-COST, with *node* as the only element
   explored ← an empty set
   loop do
      if EMPTY?(*frontier*) **then return** failure
      node ← POP(*frontier*)   /* chooses the lowest-cost node in *frontier* */
      if *problem*.GOAL-TEST(*node*.STATE) **then return** SOLUTION(*node*)
      add *node*.STATE to *explored*
      for each *action* **in** *problem*.ACTIONS(*node*.STATE) **do**
         child ← CHILD-NODE(*problem*, *node*, *action*)
         if *child*.STATE is not in *explored* or *frontier* **then**
            frontier ← INSERT(*child*, *frontier*)
         else if *child*.STATE is in *frontier* with higher PATH-COST **then**
            replace that *frontier* node with *child*

Figure 3.14   Uniform-cost search on a graph. The algorithm is identical to the general graph search algorithm in Figure 3.7, except for the use of a priority queue and the addition of an extra check in case a shorter path to a frontier state is discovered. The data structure for *frontier* needs to support efficient membership testing, so it should combine the capabilities of a priority queue and a hash table.
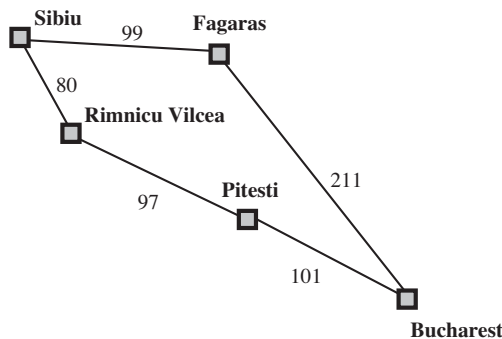
Figure 3.15   Part of the Romania state space, selected to illustrate uniform-cost search.

may be on a suboptimal path. The second difference is that a test is added in case a better path is found to a node currently on the frontier.

    Both of these modifications come into play in the example shown in Figure 3.15, where the problem is to get from Sibiu to Bucharest. The successors of Sibiu are Rimnicu Vilcea and Fagaras, with costs 80 and 99, respectively. The least-cost node, Rimnicu Vilcea, is expanded next, adding Pitesti with cost $80 + 97 = 177$. The least-cost node is now Fagaras, so it is expanded, adding Bucharest with cost $99 + 211 = 310$. Now a goal node has been generated, but uniform-cost search keeps going, choosing Pitesti for expansion and adding a second path

to Bucharest with cost $80 + 97 + 101 = 278$. Now the algorithm checks to see if this new path is better than the old one; it is, so the old one is discarded. Bucharest, now with g-cost 278, is selected for expansion and the solution is returned.

It is easy to see that uniform-cost search is optimal in general. First, we observe that whenever uniform-cost search selects a node n for expansion, the optimal path to that node has been found. (Were this not the case, there would have to be another frontier node n' on the optimal path from the start node to n, by the graph separation property of Figure 3.9; by definition, n' would have lower g-cost than n and would have been selected first.) Then, because step costs are nonnegative, paths never get shorter as nodes are added. These two facts together imply that *uniform-cost search expands nodes in order of their optimal path cost.* Hence, the first goal node selected for expansion must be the optimal solution.

Uniform-cost search does not care about the *number* of steps a path has, but only about their total cost. Therefore, it will get stuck in an infinite loop if there is a path with an infinite sequence of zero-cost actions—for example, a sequence of $NoOp$ actions.[6] Completeness is guaranteed provided the cost of every step exceeds some small positive constant ϵ.

Uniform-cost search is guided by path costs rather than depths, so its complexity is not easily characterized in terms of b and d. Instead, let C^* be the cost of the optimal solution,[7] and assume that every action costs at least ϵ. Then the algorithm's worst-case time and space complexity is $O(b^{1+\lfloor C^*/\epsilon \rfloor})$, which can be much greater than b^d. This is because uniform-cost search can explore large trees of small steps before exploring paths involving large and perhaps useful steps. When all step costs are equal, $b^{1+\lfloor C^*/\epsilon \rfloor}$ is just b^{d+1}. When all step costs are the same, uniform-cost search is similar to breadth-first search, except that the latter stops as soon as it generates a goal, whereas uniform-cost search examines all the nodes at the goal's depth to see if one has a lower cost; thus uniform-cost search does strictly more work by expanding nodes at depth d unnecessarily.

3.4.3   Depth-first search

DEPTH-FIRST
SEARCH

Depth-first search always expands the *deepest* node in the current frontier of the search tree. The progress of the search is illustrated in Figure 3.16. The search proceeds immediately to the deepest level of the search tree, where the nodes have no successors. As those nodes are expanded, they are dropped from the frontier, so then the search "backs up" to the next deepest node that still has unexplored successors.

The depth-first search algorithm is an instance of the graph-search algorithm in Figure 3.7; whereas breadth-first-search uses a FIFO queue, depth-first search uses a LIFO queue. A LIFO queue means that the most recently generated node is chosen for expansion. This must be the deepest unexpanded node because it is one deeper than its parent—which, in turn, was the deepest unexpanded node when it was selected.

As an alternative to the GRAPH-SEARCH-style implementation, it is common to implement depth-first search with a recursive function that calls itself on each of its children in turn. (A recursive depth-first algorithm incorporating a depth limit is shown in Figure 3.17.)

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[6] $NoOp$, or "no operation," is the name of an assembly language instruction that does nothing.

[7]   Here, and throughout the book, the "star" in C^* means an optimal value for C.

Figure 3.16   Depth-first search on a binary tree. The unexplored region is shown in light gray. Explored nodes with no descendants in the frontier are removed from memory. Nodes at depth 3 have no successors and M is the only goal node.
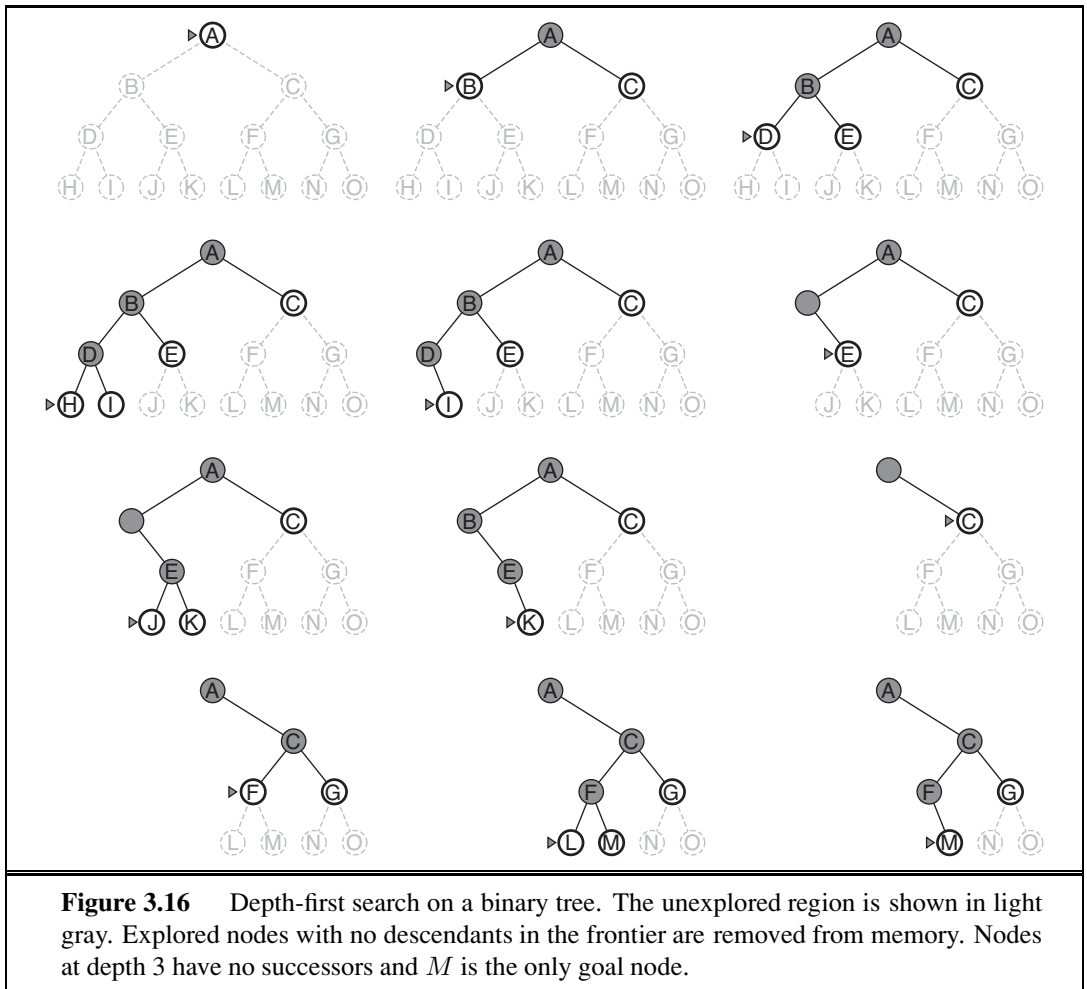
The properties of depth-first search depend strongly on whether the graph-search or tree-search version is used. The graph-search version, which avoids repeated states and redundant paths, is complete in finite state spaces because it will eventually expand every node. The tree-search version, on the other hand, is *not* complete—for example, in Figure 3.6 the algorithm will follow the Arad–Sibiu–Arad–Sibiu loop forever. Depth-first tree search can be modified at no extra memory cost so that it checks new states against those on the path from the root to the current node; this avoids infinite loops in finite state spaces but does not avoid the proliferation of redundant paths. In infinite state spaces, both versions fail if an infinite non-goal path is encountered. For example, in Knuth's 4 problem, depth-first search would keep applying the factorial operator forever.

For similar reasons, both versions are nonoptimal. For example, in Figure 3.16, depth-first search will explore the entire left subtree even if node C is a goal node. If node J were also a goal node, then depth-first search would return it as a solution instead of C, which would be a better solution; hence, depth-first search is not optimal.

The time complexity of depth-first graph search is bounded by the size of the state space (which may be infinite, of course). A depth-first tree search, on the other hand, may generate all of the $O(b^m)$ nodes in the search tree, where m is the maximum depth of any node; this can be much greater than the size of the state space. Note that m itself can be much larger than d (the depth of the shallowest solution) and is infinite if the tree is unbounded.

So far, depth-first search seems to have no clear advantage over breadth-first search, so why do we include it? The reason is the space complexity. For a graph search, there is no advantage, but a depth-first tree search needs to store only a single path from the root to a leaf node, along with the remaining unexpanded sibling nodes for each node on the path. Once a node has been expanded, it can be removed from memory as soon as all its descendants have been fully explored. (See Figure 3.16.) For a state space with branching factor b and maximum depth m, depth-first search requires storage of only $O(bm)$ nodes. Using the same assumptions as for Figure 3.13 and assuming that nodes at the same depth as the goal node have no successors, we find that depth-first search would require 156 kilobytes instead of 10 exabytes at depth $d = 16$, a factor of 7 trillion times less space. This has led to the adoption of depth-first tree search as the basic workhorse of many areas of AI, including constraint satisfaction (Chapter 6), propositional satisfiability (Chapter 7), and logic programming (Chapter 9). For the remainder of this section, we focus primarily on the tree-search version of depth-first search.

BACKTRACKING
SEARCH

A variant of depth-first search called **backtracking search** uses still less memory. (See Chapter 6 for more details.) In backtracking, only one successor is generated at a time rather than all successors; each partially expanded node remembers which successor to generate next. In this way, only $O(m)$ memory is needed rather than $O(bm)$. Backtracking search facilitates yet another memory-saving (and time-saving) trick: the idea of generating a successor by *modifying* the current state description directly rather than copying it first. This reduces the memory requirements to just one state description and $O(m)$ actions. For this to work, we must be able to undo each modification when we go back to generate the next successor. For problems with large state descriptions, such as robotic assembly, these techniques are critical to success.

3.4.4   Depth-limited search

DEPTH-LIMITED
SEARCH

The embarrassing failure of depth-first search in infinite state spaces can be alleviated by supplying depth-first search with a predetermined depth limit ℓ. That is, nodes at depth ℓ are treated as if they have no successors. This approach is called **depth-limited search**. The depth limit solves the infinite-path problem. Unfortunately, it also introduces an additional source of incompleteness if we choose $\ell < d$, that is, the shallowest goal is beyond the depth limit. (This is likely when d is unknown.) Depth-limited search will also be nonoptimal if we choose $\ell > d$. Its time complexity is $O(b^\ell)$ and its space complexity is $O(b\ell)$. Depth-first search can be viewed as a special case of depth-limited search with $\ell = \infty$.

Sometimes, depth limits can be based on knowledge of the problem. For example, on the map of Romania there are 20 cities. Therefore, we know that if there is a solution, it must be of length 19 at the longest, so $\ell = 19$ is a possible choice. But in fact if we studied the

---

function DEPTH-LIMITED-SEARCH(*problem*, *limit*) **returns** a solution, or failure/cutoff
    return RECURSIVE-DLS(MAKE-NODE(*problem*.INITIAL-STATE), *problem*, *limit*)

function RECURSIVE-DLS(*node*, *problem*, *limit*) **returns** a solution, or failure/cutoff
    if *problem*.GOAL-TEST(*node*.STATE) **then return** SOLUTION(*node*)
    else if *limit* = 0 **then return** *cutoff*
    else
        cutoff_occurred? ← false
        for each *action* **in** *problem*.ACTIONS(*node*.STATE) **do**
            child ← CHILD-NODE(*problem*, *node*, *action*)
            result ← RECURSIVE-DLS(*child*, *problem*, *limit* − 1)
            if *result* = *cutoff* **then** *cutoff_occurred?* ← true
            else if *result* ≠ *failure* **then return** *result*
        if *cutoff_occurred?* **then return** *cutoff* **else return** *failure*

Figure 3.17   A recursive implementation of depth-limited tree search.

DIAMETER

map carefully, we would discover that any city can be reached from any other city in at most
9 steps. This number, known as the **diameter** of the state space, gives us a better depth limit,
which leads to a more efficient depth-limited search. For most problems, however, we will
not know a good depth limit until we have solved the problem.

Depth-limited search can be implemented as a simple modification to the general tree-
or graph-search algorithm. Alternatively, it can be implemented as a simple recursive al-
gorithm as shown in Figure 3.17. Notice that depth-limited search can terminate with two
kinds of failure: the standard *failure* value indicates no solution; the *cutoff* value indicates
no solution within the depth limit.

3.4.5   Iterative deepening depth-first search

ITERATIVE
DEEPENING SEARCH

Iterative deepening search (or iterative deepening depth-first search) is a general strategy,
often used in combination with depth-first tree search, that finds the best depth limit. It does
this by gradually increasing the limit—first 0, then 1, then 2, and so on—until a goal is found.
This will occur when the depth limit reaches d, the depth of the shallowest goal node. The
algorithm is shown in Figure 3.18. Iterative deepening combines the benefits of depth-first
and breadth-first search. Like depth-first search, its memory requirements are modest: $O(bd)$
to be precise. Like breadth-first search, it is complete when the branching factor is finite and
optimal when the path cost is a nondecreasing function of the depth of the node. Figure 3.19
shows four iterations of ITERATIVE-DEEPENING-SEARCH on a binary search tree, where the
solution is found on the fourth iteration.

Iterative deepening search may seem wasteful because states are generated multiple
times. It turns out this is not too costly. The reason is that in a search tree with the same (or
nearly the same) branching factor at each level, most of the nodes are in the bottom level,
so it does not matter much that the upper levels are generated multiple times. In an iterative
deepening search, the nodes on the bottom level (depth d) are generated once, those on the

function ITERATIVE-DEEPENING-SEARCH(*problem*) **returns** a solution, or failure
  for *depth* = 0 **to** ∞ **do**
    result ← DEPTH-LIMITED-SEARCH(*problem*, *depth*)
    if *result* ≠ cutoff **then return** *result*

Figure 3.18   The iterative deepening search algorithm, which repeatedly applies depth-limited search with increasing limits. It terminates when a solution is found or if the depth-limited search returns *failure*, meaning that no solution exists.

Figure 3.19   Four iterations of iterative deepening search on a binary tree.
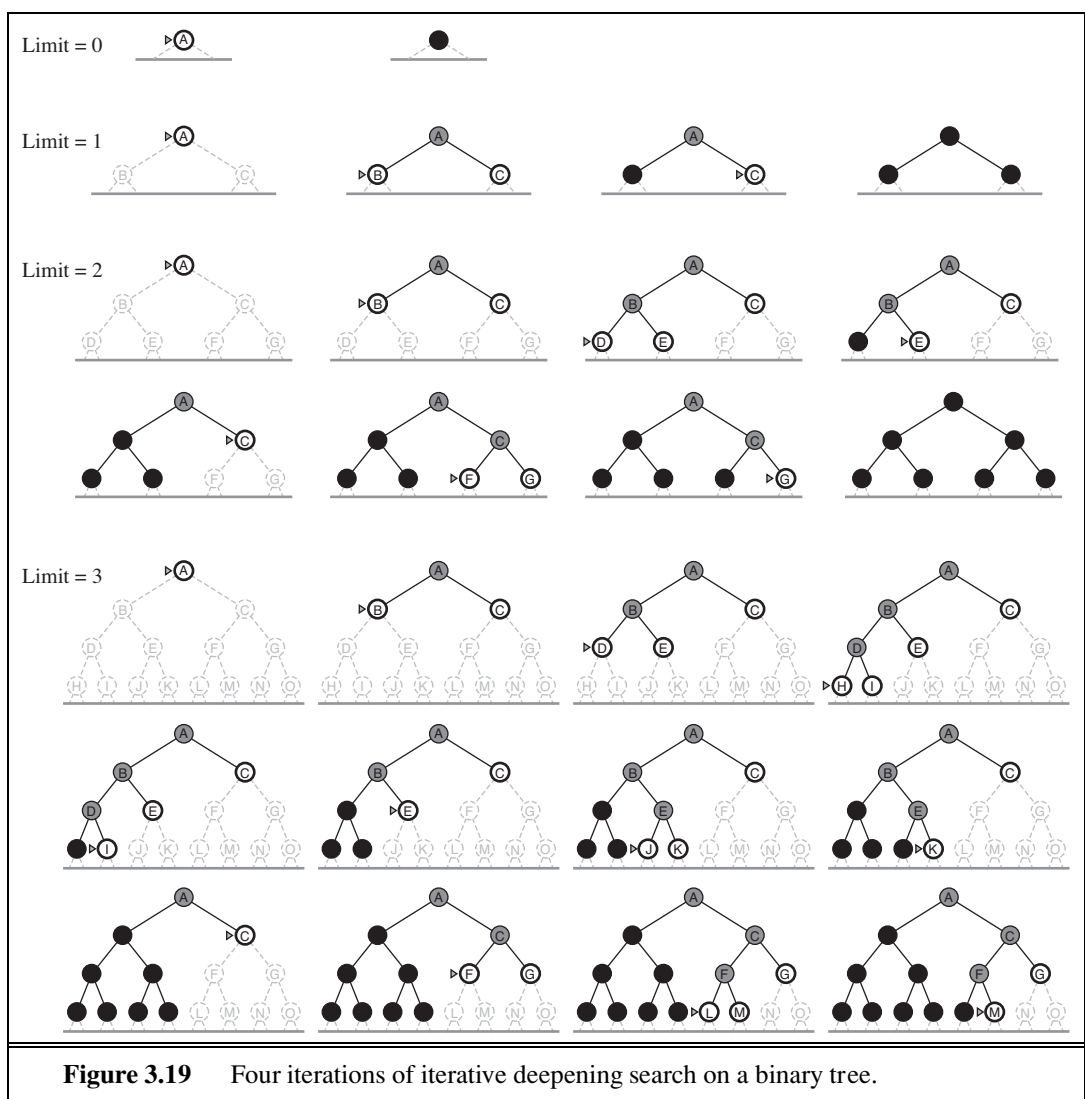
next-to-bottom level are generated twice, and so on, up to the children of the root, which are generated d times. So the total number of nodes generated in the worst case is

$$N(\text{IDS}) = (d)b + (d-1)b^2 + \cdots + (1)b^d \ ,$$

which gives a time complexity of $O(b^d)$—asymptotically the same as breadth-first search. There is some extra cost for generating the upper levels multiple times, but it is not large. For example, if $b = 10$ and $d = 5$, the numbers are

$$N(\text{IDS}) \ = \ 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450$$
$$N(\text{BFS}) \ = \ 10 + 100 + 1,000 + 10,000 + 100,000 = 111,110 \ .$$

If you are really concerned about repeating the repetition, you can use a hybrid approach that runs breadth-first search until almost all the available memory is consumed, and then runs iterative deepening from all the nodes in the frontier. *In general, iterative deepening is the preferred uninformed search method when the search space is large and the depth of the solution is not known.*

Iterative deepening search is analogous to breadth-first search in that it explores a complete layer of new nodes at each iteration before going on to the next layer. It would seem worthwhile to develop an iterative analog to uniform-cost search, inheriting the latter algorithm's optimality guarantees while avoiding its memory requirements. The idea is to use increasing path-cost limits instead of increasing depth limits. The resulting algorithm, called **iterative lengthening search**, is explored in Exercise 3.17. It turns out, unfortunately, that iterative lengthening incurs substantial overhead compared to uniform-cost search.

ITERATIVE
LENGTHENING
SEARCH

3.4.6   Bidirectional search

The idea behind bidirectional search is to run two simultaneous searches—one forward from the initial state and the other backward from the goal—hoping that the two searches meet in the middle (Figure 3.20). The motivation is that $b^{d/2} + b^{d/2}$ is much less than b^d, or in the figure, the area of the two small circles is less than the area of one big circle centered on the start and reaching to the goal.

Bidirectional search is implemented by replacing the goal test with a check to see whether the frontiers of the two searches intersect; if they do, a solution has been found. (It is important to realize that the first such solution found may not be optimal, even if the two searches are both breadth-first; some additional search is required to make sure there isn't another short-cut across the gap.) The check can be done when each node is generated or selected for expansion and, with a hash table, will take constant time. For example, if a problem has solution depth $d = 6$, and each direction runs breadth-first search one node at a time, then in the worst case the two searches meet when they have generated all of the nodes at depth 3. For $b = 10$, this means a total of 2,220 node generations, compared with 1,111,110 for a standard breadth-first search. Thus, the time complexity of bidirectional search using breadth-first searches in both directions is $O(b^{d/2})$. The space complexity is also $O(b^{d/2})$. We can reduce this by roughly half if one of the two searches is done by iterative deepening, but at least one of the frontiers must be kept in memory so that the intersection check can be done. This space requirement is the most significant weakness of bidirectional search.

Figure 3.20   A schematic view of a bidirectional search that is about to succeed when a branch from the start node meets a branch from the goal node.
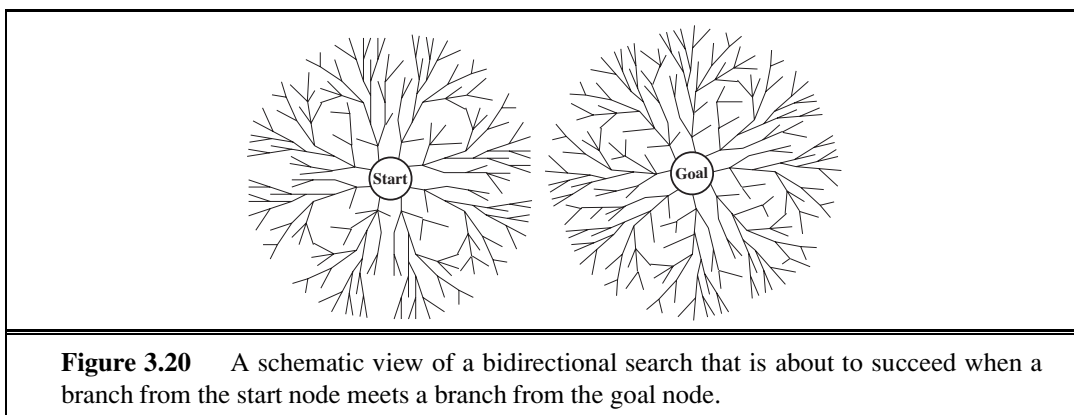
PREDECESSOR

The reduction in time complexity makes bidirectional search attractive, but how do we search backward? This is not as easy as it sounds. Let the **predecessors** of a state x be all those states that have x as a successor. Bidirectional search requires a method for computing predecessors. When all the actions in the state space are reversible, the predecessors of x are just its successors. Other cases may require substantial ingenuity.

Consider the question of what we mean by "the goal" in searching "backward from the goal." For the 8-puzzle and for finding a route in Romania, there is just one goal state, so the backward search is very much like the forward search. If there are several *explicitly listed* goal states—for example, the two dirt-free goal states in Figure 3.3—then we can construct a new dummy goal state whose immediate predecessors are all the actual goal states. But if the goal is an abstract description, such as the goal that "no queen attacks another queen" in the n-queens problem, then bidirectional search is difficult to use.

3.4.7   Comparing uninformed search strategies

Figure 3.21 compares search strategies in terms of the four evaluation criteria set forth in Section 3.3.2. This comparison is for tree-search versions. For graph searches, the main differences are that depth-first search is complete for finite state spaces and that the space and time complexities are bounded by the size of the state space.

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Complete?	Yes[a]	Yes[a,b]	No	No	Yes[a]	Yes[a,d]
Time	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(b^m)$	$O(b^\ell)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(bm)$	$O(b\ell)$	$O(bd)$	$O(b^{d/2})$
Optimal?	Yes[c]	Yes	No	No	Yes[c]	Yes[c,d]

Figure 3.21   Evaluation of tree-search strategies. b is the branching factor; d is the depth of the shallowest solution; m is the maximum depth of the search tree; l is the depth limit. Superscript caveats are as follows: [a] complete if b is finite; [b] complete if step costs $\geq \epsilon$ for positive ϵ; [c] optimal if step costs are all identical; [d] if both directions use breadth-first search.

3.5   INFORMED (HEURISTIC) SEARCH STRATEGIES

INFORMED SEARCH   This section shows how an **informed search** strategy—one that uses problem-specific knowl-
edge beyond the definition of the problem itself—can find solutions more efficiently than can
an uninformed strategy.

BEST-FIRST SEARCH   The general approach we consider is called **best-first search**. Best-first search is an
instance of the general TREE-SEARCH or GRAPH-SEARCH algorithm in which a node is
EVALUATION
FUNCTION   selected for expansion based on an **evaluation function**, $f(n)$. The evaluation function is
construed as a cost estimate, so the node with the *lowest* evaluation is expanded first. The
implementation of best-first graph search is identical to that for uniform-cost search (Fig-
ure 3.14), except for the use of f instead of g to order the priority queue.

The choice of f determines the search strategy. (For example, as Exercise 3.21 shows,
best-first tree search includes depth-first search as a special case.) Most best-first algorithms
HEURISTIC
FUNCTION   include as a component of f a **heuristic function**, denoted $h(n)$:

$h(n) = $ estimated cost of the cheapest path from the state at node *n* to a goal state.

(Notice that $h(n)$ takes a *node* as input, but, unlike $g(n)$, it depends only on the *state* at that
node.) For example, in Romania, one might estimate the cost of the cheapest path from Arad
to Bucharest via the straight-line distance from Arad to Bucharest.

Heuristic functions are the most common form in which additional knowledge of the
problem is imparted to the search algorithm. We study heuristics in more depth in Section 3.6.
For now, we consider them to be arbitrary, nonnegative, problem-specific functions, with one
constraint: if n is a goal node, then $h(n) = 0$. The remainder of this section covers two ways
to use heuristic information to guide search.

3.5.1   Greedy best-first search

GREEDY BEST-FIRST
SEARCH   **Greedy best-first search**[8] tries to expand the node that is closest to the goal, on the grounds
that this is likely to lead to a solution quickly. Thus, it evaluates nodes by using just the
heuristic function; that is, $f(n) = h(n)$.

Let us see how this works for route-finding problems in Romania; we use the **straight-**
STRAIGHT-LINE
DISTANCE   **line distance** heuristic, which we will call h_{SLD}. If the goal is Bucharest, we need to
know the straight-line distances to Bucharest, which are shown in Figure 3.22. For exam-
ple, $h_{SLD}(In(Arad)) = 366$. Notice that the values of h_{SLD} cannot be computed from the
problem description itself. Moreover, it takes a certain amount of experience to know that
h_{SLD} is correlated with actual road distances and is, therefore, a useful heuristic.

Figure 3.23 shows the progress of a greedy best-first search using h_{SLD} to find a path
from Arad to Bucharest. The first node to be expanded from Arad will be Sibiu because it
is closer to Bucharest than either Zerind or Timisoara. The next node to be expanded will
be Fagaras because it is closest. Fagaras in turn generates Bucharest, which is the goal. For
this particular problem, greedy best-first search using h_{SLD} finds a solution without ever

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[8] Our first edition called this **greedy search**; other authors have called it **best-first search**. Our more general
usage of the latter term follows Pearl (1984).

Arad	366	Mehadia	241
Bucharest	0	Neamt	234
Craiova	160	Oradea	380
Drobeta	242	Pitesti	100
Eforie	161	Rimnicu Vilcea	193
Fagaras	176	Sibiu	253
Giurgiu	77	Timisoara	329
Hirsova	151	Urziceni	80
Iasi	226	Vaslui	199
Lugoj	244	Zerind	374

Figure 3.22   Values of h_{SLD}—straight-line distances to Bucharest.

expanding a node that is not on the solution path; hence, its search cost is minimal. It is not optimal, however: the path via Sibiu and Fagaras to Bucharest is 32 kilometers longer than the path through Rimnicu Vilcea and Pitesti. This shows why the algorithm is called "greedy"—at each step it tries to get as close to the goal as it can.

Greedy best-first tree search is also incomplete even in a finite state space, much like depth-first search. Consider the problem of getting from Iasi to Fagaras. The heuristic suggests that Neamt be expanded first because it is closest to Fagaras, but it is a dead end. The solution is to go first to Vaslui—a step that is actually farther from the goal according to the heuristic—and then to continue to Urziceni, Bucharest, and Fagaras. The algorithm will never find this solution, however, because expanding Neamt puts Iasi back into the frontier, Iasi is closer to Fagaras than Vaslui is, and so Iasi will be expanded again, leading to an infinite loop. (The graph search version *is* complete in finite spaces, but not in infinite ones.) The worst-case time and space complexity for the tree version is $O(b^m)$, where m is the maximum depth of the search space. With a good heuristic function, however, the complexity can be reduced substantially. The amount of the reduction depends on the particular problem and on the quality of the heuristic.

3.5.2   A* search: Minimizing the total estimated solution cost

A* SEARCH

The most widely known form of best-first search is called **A* search** (pronounced "A-star search"). It evaluates nodes by combining $g(n)$, the cost to reach the node, and $h(n)$, the cost to get from the node to the goal:

$$f(n) = g(n) + h(n) .$$

Since $g(n)$ gives the path cost from the start node to node n, and $h(n)$ is the estimated cost of the cheapest path from n to the goal, we have

$$f(n) = \text{ estimated cost of the cheapest solution through } n .$$

Thus, if we are trying to find the cheapest solution, a reasonable thing to try first is the node with the lowest value of $g(n) + h(n)$. It turns out that this strategy is more than just reasonable: provided that the heuristic function $h(n)$ satisfies certain conditions, A* search is both complete and optimal. The algorithm is identical to UNIFORM-COST-SEARCH except that A* uses $g + h$ instead of g.

Figure 3.23 Stages in a greedy best-first tree search for Bucharest with the straight-line distance heuristic h_{SLD}. Nodes are labeled with their h-values.
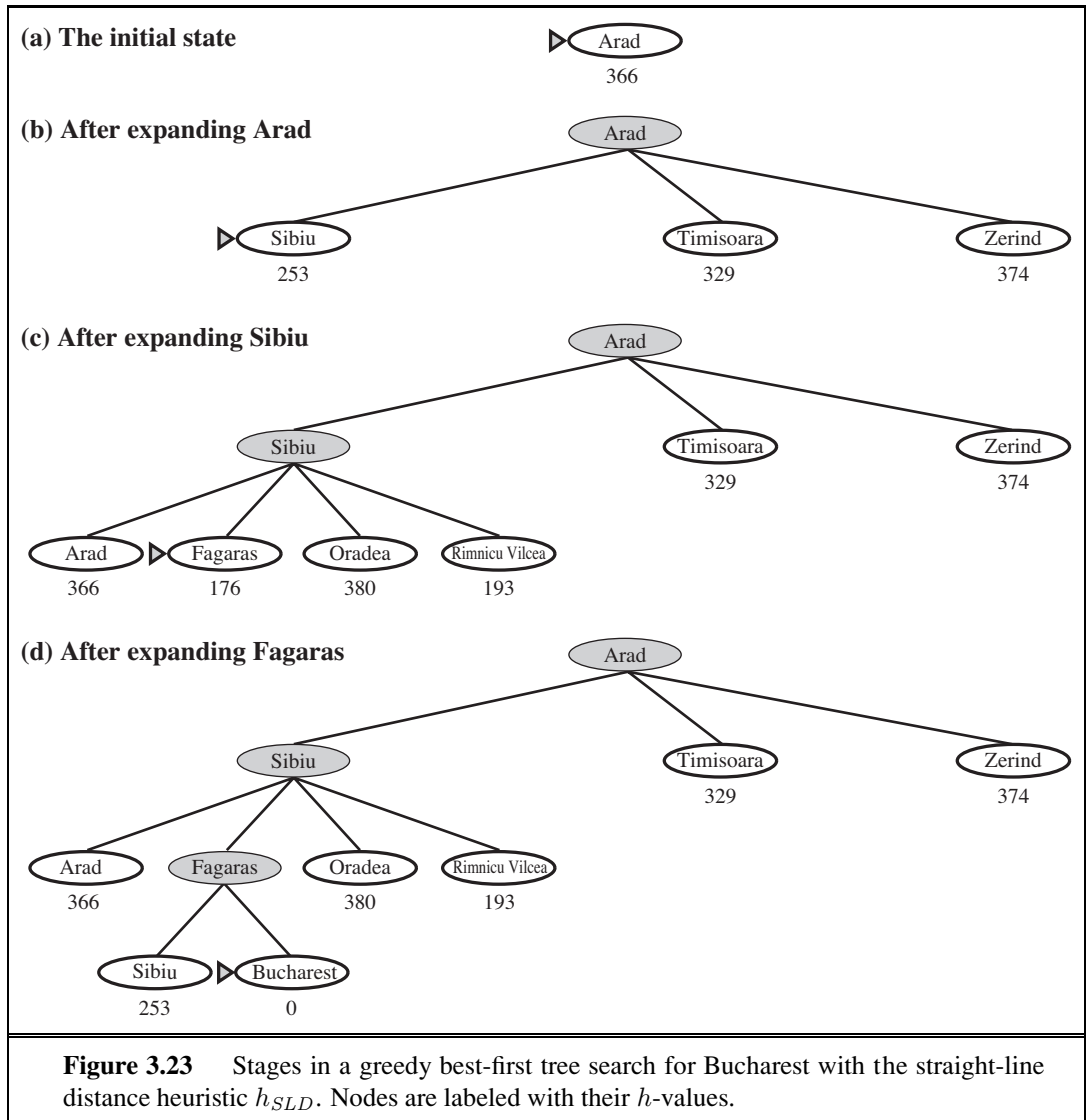
Conditions for optimality: Admissibility and consistency

ADMISSIBLE
HEURISTIC

The first condition we require for optimality is that $h(n)$ be an **admissible heuristic**. An admissible heuristic is one that *never overestimates* the cost to reach the goal. Because $g(n)$ is the actual cost to reach n along the current path, and $f(n) = g(n) + h(n)$, we have as an immediate consequence that $f(n)$ never overestimates the true cost of a solution along the current path through n.

Admissible heuristics are by nature optimistic because they think the cost of solving the problem is less than it actually is. An obvious example of an admissible heuristic is the straight-line distance h_{SLD} that we used in getting to Bucharest. Straight-line distance is admissible because the shortest path between any two points is a straight line, so the straight

line cannot be an overestimate. In Figure 3.24, we show the progress of an A* tree search for Bucharest. The values of g are computed from the step costs in Figure 3.2, and the values of h_{SLD} are given in Figure 3.22. Notice in particular that Bucharest first appears on the frontier at step (e), but it is not selected for expansion because its f-cost (450) is higher than that of Pitesti (417). Another way to say this is that there *might* be a solution through Pitesti whose cost is as low as 417, so the algorithm will not settle for a solution that costs 450.

CONSISTENCY

MONOTONICITY

A second, slightly stronger condition called **consistency** (or sometimes **monotonicity**) is required only for applications of A* to graph search.[9] A heuristic $h(n)$ is consistent if, for every node n and every successor n' of n generated by any action a, the estimated cost of reaching the goal from n is no greater than the step cost of getting to n' plus the estimated cost of reaching the goal from n':

$$h(n) \leq c(n, a, n') + h(n') .$$

TRIANGLE
INEQUALITY

This is a form of the general **triangle inequality**, which stipulates that each side of a triangle cannot be longer than the sum of the other two sides. Here, the triangle is formed by n, n', and the goal G_n closest to n. For an admissible heuristic, the inequality makes perfect sense: if there were a route from n to G_n via n' that was cheaper than $h(n)$, that would violate the property that $h(n)$ is a lower bound on the cost to reach G_n.

It is fairly easy to show (Exercise 3.29) that every consistent heuristic is also admissible. Consistency is therefore a stricter requirement than admissibility, but one has to work quite hard to concoct heuristics that are admissible but not consistent. All the admissible heuristics we discuss in this chapter are also consistent. Consider, for example, h_{SLD}. We know that the general triangle inequality is satisfied when each side is measured by the straight-line distance and that the straight-line distance between n and n' is no greater than $c(n, a, n')$. Hence, h_{SLD} is a consistent heuristic.

Optimality of A*

As we mentioned earlier, A* has the following properties: *the tree-search version of A^* is optimal if $h(n)$ is admissible, while the graph-search version is optimal if $h(n)$ is consistent.*

We show the second of these two claims since it is more useful. The argument essentially mirrors the argument for the optimality of uniform-cost search, with g replaced by f—just as in the A* algorithm itself.

The first step is to establish the following: *if $h(n)$ is consistent, then the values of $f(n)$ along any path are nondecreasing.* The proof follows directly from the definition of consistency. Suppose n' is a successor of n; then $g(n') = g(n) + c(n, a, n')$ for some action a, and we have

$$f(n') = g(n') + h(n') = g(n) + c(n, a, n') + h(n') \geq g(n) + h(n) = f(n) .$$

The next step is to prove that *whenever A^* selects a node n for expansion, the optimal path to that node has been found.* Were this not the case, there would have to be another frontier node n' on the optimal path from the start node to n, by the graph separation property of

---

[9]   With an admissible but inconsistent heuristic, A* requires some extra bookkeeping to ensure optimality.

(a) The initial state

Arad
366=0+366

(b) After expanding Arad

Arad

Sibiu   Timisoara   Zerind
393=140+253   447=118+329   449=75+374

(c) After expanding Sibiu

Arad

Sibiu   Timisoara   Zerind
                       447=118+329   449=75+374

Arad   Fagaras   Oradea   Rimnicu Vilcea
646=280+366 415=239+176 671=291+380 413=220+193

(d) After expanding Rimnicu Vilcea

Arad

Sibiu   Timisoara   Zerind
                       447=118+329   449=75+374

Arad   Fagaras   Oradea   Rimnicu Vilcea
646=280+366 415=239+176 671=291+380

                            Craiova   Pitesti   Sibiu
                            526=366+160 417=317+100 553=300+253

(e) After expanding Fagaras

Arad

Sibiu   Timisoara   Zerind
                       447=118+329   449=75+374

Arad   Fagaras   Oradea   Rimnicu Vilcea
646=280+366   671=291+380

        Sibiu   Bucharest   Craiova   Pitesti   Sibiu
        591=338+253 450=450+0   526=366+160 417=317+100 553=300+253

(f) After expanding Pitesti

Arad

Sibiu   Timisoara   Zerind
                       447=118+329   449=75+374

Arad   Fagaras   Oradea   Rimnicu Vilcea
646=280+366   671=291+380

        Sibiu   Bucharest   Craiova   Pitesti   Sibiu
        591=338+253 450=450+0   526=366+160   553=300+253

                                        Bucharest   Craiova   Rimnicu Vilcea
                                        418=418+0   615=455+160 607=414+193

Figure 3.24   Stages in an A* search for Bucharest. Nodes are labeled with $f = g + h$. The h values are the straight-line distances to Bucharest taken from Figure 3.22.

Figure 3.25   Map of Romania showing contours at $f = 380$, $f = 400$, and $f = 420$, with Arad as the start state. Nodes inside a given contour have f-costs less than or equal to the contour value.
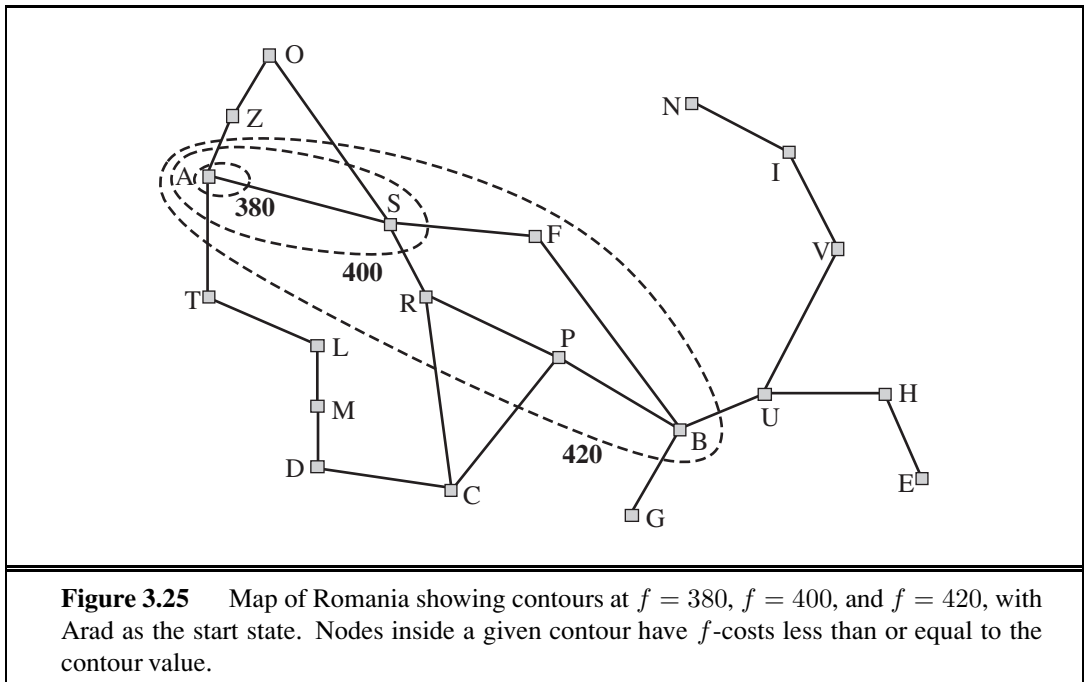
Figure 3.9; because f is nondecreasing along any path, n' would have lower f-cost than n and would have been selected first.

From the two preceding observations, it follows that the sequence of nodes expanded by A* using GRAPH-SEARCH is in nondecreasing order of $f(n)$. Hence, the first goal node selected for expansion must be an optimal solution because f is the true cost for goal nodes (which have $h = 0$) and all later goal nodes will be at least as expensive.

The fact that f-costs are nondecreasing along any path also means that we can draw
CONTOUR   **contours** in the state space, just like the contours in a topographic map. Figure 3.25 shows an example. Inside the contour labeled 400, all nodes have $f(n)$ less than or equal to 400, and so on. Then, because A* expands the frontier node of lowest f-cost, we can see that an A* search fans out from the start node, adding nodes in concentric bands of increasing f-cost.

With uniform-cost search (A* search using $h(n) = 0$), the bands will be "circular" around the start state. With more accurate heuristics, the bands will stretch toward the goal state and become more narrowly focused around the optimal path. If C^* is the cost of the optimal solution path, then we can say the following:

- A* expands all nodes with $f(n) < C^*$.
- A* might then expand some of the nodes right on the "goal contour" (where $f(n) = C^*$) before selecting a goal node.

Completeness requires that there be only finitely many nodes with cost less than or equal to C^*, a condition that is true if all step costs exceed some finite ϵ and if b is finite.

Notice that A* expands no nodes with $f(n) > C^*$—for example, Timisoara is not expanded in Figure 3.24 even though it is a child of the root. We say that the subtree below

PRUNING

Timisoara is **pruned**; because h_{SLD} is admissible, the algorithm can safely ignore this subtree while still guaranteeing optimality. The concept of pruning—eliminating possibilities from consideration without having to examine them—is important for many areas of AI.

OPTIMALLY
EFFICIENT

One final observation is that among optimal algorithms of this type—algorithms that extend search paths from the root and use the same heuristic information—A* is **optimally efficient** for any given consistent heuristic. That is, no other optimal algorithm is guaranteed to expand fewer nodes than A* (except possibly through tie-breaking among nodes with $f(n) = C^*$). This is because any algorithm that *does not* expand all nodes with $f(n) < C^*$ runs the risk of missing the optimal solution.

That A* search is complete, optimal, and optimally efficient among all such algorithms is rather satisfying. Unfortunately, it does not mean that A* is the answer to all our searching needs. The catch is that, for most problems, the number of states within the goal contour search space is still exponential in the length of the solution. The details of the analysis are beyond the scope of this book, but the basic results are as follows. For problems with constant step costs, the growth in run time as a function of the optimal solution depth d is analyzed in

ABSOLUTE ERROR

terms of the the **absolute error** or the **relative error** of the heuristic. The absolute error is

RELATIVE ERROR

defined as $\Delta \equiv h^* - h$, where h^* is the actual cost of getting from the root to the goal, and the relative error is defined as $\epsilon \equiv (h^* - h)/h^*$.

The complexity results depend very strongly on the assumptions made about the state space. The simplest model studied is a state space that has a single goal and is essentially a tree with reversible actions. (The 8-puzzle satisfies the first and third of these assumptions.) In this case, the time complexity of A* is exponential in the maximum absolute error, that is, $O(b^{\Delta})$. For constant step costs, we can write this as $O(b^{\epsilon d})$, where d is the solution depth. For almost all heuristics in practical use, the absolute error is at least proportional to the path cost h^*, so ϵ is constant or growing and the time complexity is exponential in d. We can also see the effect of a more accurate heuristic: $O(b^{\epsilon d}) = O((b^{\epsilon})^d)$, so the effective branching factor (defined more formally in the next section) is b^{ϵ}.

When the state space has many goal states—particularly *near-optimal* goal states—the search process can be led astray from the optimal path and there is an extra cost proportional to the number of goals whose cost is within a factor ϵ of the optimal cost. Finally, in the general case of a graph, the situation is even worse. There can be exponentially many states with $f(n) < C^*$ even if the absolute error is bounded by a constant. For example, consider a version of the vacuum world where the agent can clean up any square for unit cost without even having to visit it: in that case, squares can be cleaned in any order. With N initially dirty squares, there are 2^N states where some subset has been cleaned and all of them are on an optimal solution path—and hence satisfy $f(n) < C^*$—even if the heuristic has an error of 1.

The complexity of A* often makes it impractical to insist on finding an optimal solution. One can use variants of A* that find suboptimal solutions quickly, or one can sometimes design heuristics that are more accurate but not strictly admissible. In any case, the use of a good heuristic still provides enormous savings compared to the use of an uninformed search. In Section 3.6, we look at the question of designing good heuristics.

Computation time is not, however, A*'s main drawback. Because it keeps all generated nodes in memory (as do all GRAPH-SEARCH algorithms), A* usually runs out of space long

---

function RECURSIVE-BEST-FIRST-SEARCH(*problem*) **returns** a solution, or failure
  return RBFS(*problem*, MAKE-NODE(*problem*.INITIAL-STATE), ∞)

function RBFS(*problem*, *node*, *f_limit*) **returns** a solution, or failure and a new *f*-cost limit
  if *problem*.GOAL-TEST(*node*.STATE) **then return** SOLUTION(*node*)
  successors ← [ ]
  for each *action* **in** *problem*.ACTIONS(*node*.STATE) **do**
    add CHILD-NODE(*problem*, *node*, *action*) into *successors*
  if *successors* is empty **then return** *failure*, ∞
  for each *s* **in** *successors* **do** /* update *f* with value from previous search, if any */
    $s.f \leftarrow \max(s.g + s.h, node.f))$
  loop do
    best ← the lowest *f*-value node in *successors*
    if *best.f* > *f_limit* **then return** *failure*, *best.f*
    alternative ← the second-lowest *f*-value among *successors*
    result, *best.f* ← RBFS(*problem*, *best*, min(*f_limit*, *alternative*))
    if *result* ≠ *failure* **then return** *result*

Figure 3.26   The algorithm for recursive best-first search.

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before it runs out of time. For this reason, A* is not practical for many large-scale problems. There are, however, algorithms that overcome the space problem without sacrificing optimality or completeness, at a small cost in execution time. We discuss these next.

3.5.3   Memory-bounded heuristic search

ITERATIVE-DEEPENING A*

The simplest way to reduce memory requirements for A* is to adapt the idea of iterative deepening to the heuristic search context, resulting in the **iterative-deepening A*** (IDA*) algorithm. The main difference between IDA* and standard iterative deepening is that the cutoff used is the *f*-cost ($g + h$) rather than the depth; at each iteration, the cutoff value is the smallest *f*-cost of any node that exceeded the cutoff on the previous iteration. IDA* is practical for many problems with unit step costs and avoids the substantial overhead associated with keeping a sorted queue of nodes. Unfortunately, it suffers from the same difficulties with real-valued costs as does the iterative version of uniform-cost search described in Exercise 3.17. This section briefly examines two other memory-bounded algorithms, called RBFS and MA*.

RECURSIVE BEST-FIRST SEARCH

Recursive best-first search (RBFS) is a simple recursive algorithm that attempts to mimic the operation of standard best-first search, but using only linear space. The algorithm is shown in Figure 3.26. Its structure is similar to that of a recursive depth-first search, but rather than continuing indefinitely down the current path, it uses the *f_limit* variable to keep track of the *f*-value of the best *alternative* path available from any ancestor of the current node. If the current node exceeds this limit, the recursion unwinds back to the alternative path. As the recursion unwinds, RBFS replaces the *f*-value of each node along the path

BACKED-UP VALUE

with a **backed-up value**—the best *f*-value of its children. In this way, RBFS remembers the *f*-value of the best leaf in the forgotten subtree and can therefore decide whether it's worth
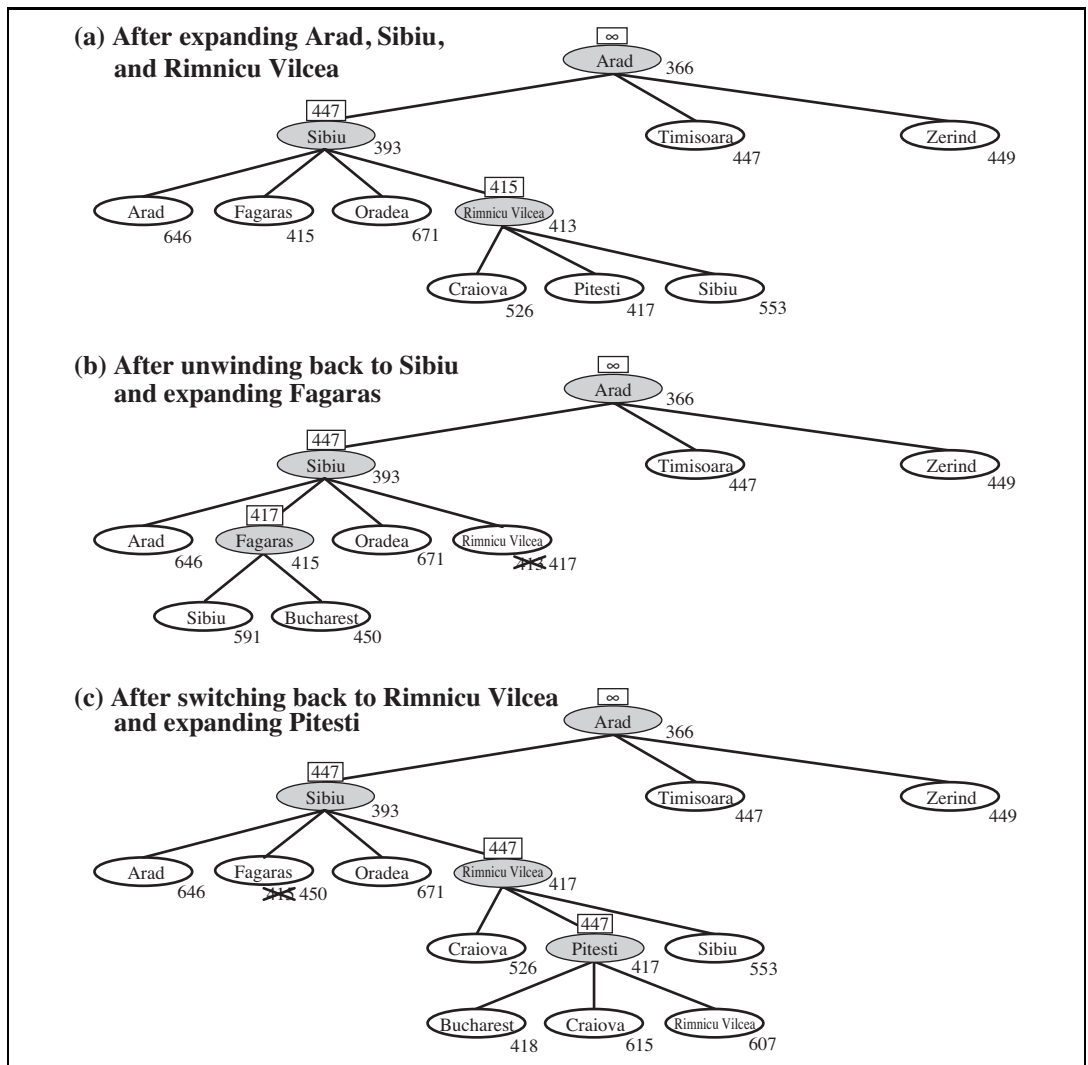
Figure 3.27   Stages in an RBFS search for the shortest route to Bucharest. The f-limit value for each recursive call is shown on top of each current node, and every node is labeled with its f-cost. (a) The path via Rimnicu Vilcea is followed until the current best leaf (Pitesti) has a value that is worse than the best alternative path (Fagaras). (b) The recursion unwinds and the best leaf value of the forgotten subtree (417) is backed up to Rimnicu Vilcea; then Fagaras is expanded, revealing a best leaf value of 450. (c) The recursion unwinds and the best leaf value of the forgotten subtree (450) is backed up to Fagaras; then Rimnicu Vilcea is expanded. This time, because the best alternative path (through Timisoara) costs at least 447, the expansion continues to Bucharest.

reexpanding the subtree at some later time. Figure 3.27 shows how RBFS reaches Bucharest.

RBFS is somewhat more efficient than IDA*, but still suffers from excessive node regeneration. In the example in Figure 3.27, RBFS follows the path via Rimnicu Vilcea, then

"changes its mind" and tries Fagaras, and then changes its mind back again. These mind changes occur because every time the current best path is extended, its f-value is likely to increase—h is usually less optimistic for nodes closer to the goal. When this happens, the second-best path might become the best path, so the search has to backtrack to follow it. Each mind change corresponds to an iteration of IDA* and could require many reexpansions of forgotten nodes to recreate the best path and extend it one more node.

Like A* tree search, RBFS is an optimal algorithm if the heuristic function $h(n)$ is admissible. Its space complexity is linear in the depth of the deepest optimal solution, but its time complexity is rather difficult to characterize: it depends both on the accuracy of the heuristic function and on how often the best path changes as nodes are expanded.

IDA* and RBFS suffer from using *too little* memory. Between iterations, IDA* retains only a single number: the current f-cost limit. RBFS retains more information in memory, but it uses only linear space: even if more memory were available, RBFS has no way to make use of it. Because they forget most of what they have done, both algorithms may end up reexpanding the same states many times over. Furthermore, they suffer the potentially exponential increase in complexity associated with redundant paths in graphs (see Section 3.3).

It seems sensible, therefore, to use all available memory. Two algorithms that do this are **MA*** (memory-bounded A*) and **SMA*** (simplified MA*). SMA* is—well—simpler, so we will describe it. SMA* proceeds just like A*, expanding the best leaf until memory is full. At this point, it cannot add a new node to the search tree without dropping an old one. SMA* always drops the *worst* leaf node—the one with the highest f-value. Like RBFS, SMA* then backs up the value of the forgotten node to its parent. In this way, the ancestor of a forgotten subtree knows the quality of the best path in that subtree. With this information, SMA* regenerates the subtree only when all other paths have been shown to look worse than the path it has forgotten. Another way of saying this is that, if all the descendants of a node n are forgotten, then we will not know which way to go from n, but we will still have an idea of how worthwhile it is to go anywhere from n.

The complete algorithm is too complicated to reproduce here,[10] but there is one subtlety worth mentioning. We said that SMA* expands the best leaf and deletes the worst leaf. What if *all* the leaf nodes have the same f-value? To avoid selecting the same node for deletion and expansion, SMA* expands the *newest* best leaf and deletes the *oldest* worst leaf. These coincide when there is only one leaf, but in that case, the current search tree must be a single path from root to leaf that fills all of memory. If the leaf is not a goal node, then *even if it is on an optimal solution path*, that solution is not reachable with the available memory. Therefore, the node can be discarded exactly as if it had no successors.

SMA* is complete if there is any reachable solution—that is, if d, the depth of the shallowest goal node, is less than the memory size (expressed in nodes). It is optimal if any optimal solution is reachable; otherwise, it returns the best reachable solution. In practical terms, SMA* is a fairly robust choice for finding optimal solutions, particularly when the state space is a graph, step costs are not uniform, and node generation is expensive compared to the overhead of maintaining the frontier and the explored set.

<div style="margin-left: 2em">

MA*

SMA*

</div>

---

[10] A rough sketch appeared in the first edition of this book.

THRASHING

On very hard problems, however, it will often be the case that SMA* is forced to switch back and forth continually among many candidate solution paths, only a small subset of which can fit in memory. (This resembles the problem of **thrashing** in disk paging systems.) Then the extra time required for repeated regeneration of the same nodes means that problems that would be practically solvable by A*, given unlimited memory, become intractable for SMA*. That is to say, *memory limitations can make a problem intractable from the point of view of computation time.* Although no current theory explains the tradeoff between time and memory, it seems that this is an inescapable problem. The only way out is to drop the optimality requirement.

3.5.4 Learning to search better

METALEVEL STATE SPACE

OBJECT-LEVEL STATE SPACE

We have presented several fixed strategies—breadth-first, greedy best-first, and so on—that have been designed by computer scientists. Could an agent *learn* how to search better? The answer is yes, and the method rests on an important concept called the **metalevel state space**. Each state in a metalevel state space captures the internal (computational) state of a program that is searching in an **object-level state space** such as Romania. For example, the internal state of the A* algorithm consists of the current search tree. Each action in the metalevel state space is a computation step that alters the internal state; for example, each computation step in A* expands a leaf node and adds its successors to the tree. Thus, Figure 3.24, which shows a sequence of larger and larger search trees, can be seen as depicting a path in the metalevel state space where each state on the path is an object-level search tree.

METALEVEL LEARNING

Now, the path in Figure 3.24 has five steps, including one step, the expansion of Fagaras, that is not especially helpful. For harder problems, there will be many such missteps, and a **metalevel learning** algorithm can learn from these experiences to avoid exploring unpromising subtrees. The techniques used for this kind of learning are described in Chapter 21. The goal of learning is to minimize the **total cost** of problem solving, trading off computational expense and path cost.

3.6 Heuristic Functions

In this section, we look at heuristics for the 8-puzzle, in order to shed light on the nature of heuristics in general.

The 8-puzzle was one of the earliest heuristic search problems. As mentioned in Section 3.2, the object of the puzzle is to slide the tiles horizontally or vertically into the empty space until the configuration matches the goal configuration (Figure 3.28).

The average solution cost for a randomly generated 8-puzzle instance is about 22 steps. The branching factor is about 3. (When the empty tile is in the middle, four moves are possible; when it is in a corner, two; and when it is along an edge, three.) This means that an exhaustive tree search to depth 22 would look at about $3^{22} \approx 3.1 \times 10^{10}$ states. A graph search would cut this down by a factor of about 170,000 because only $9!/2 = 181,440$ distinct states are reachable. (See Exercise 3.4.) This is a manageable number, but

Figure 3.28   A typical instance of the 8-puzzle. The solution is 26 steps long.
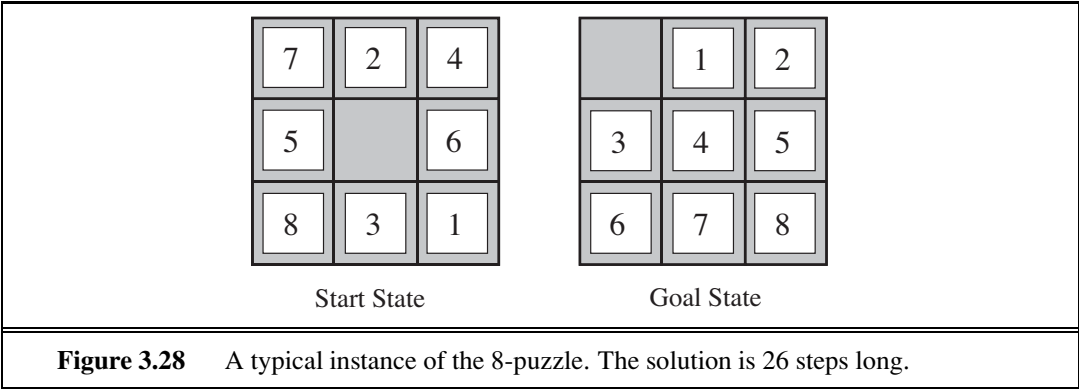
the corresponding number for the 15-puzzle is roughly 10^{13}, so the next order of business is to find a good heuristic function. If we want to find the shortest solutions by using A^*, we need a heuristic function that never overestimates the number of steps to the goal. There is a long history of such heuristics for the 15-puzzle; here are two commonly used candidates:

- h_1 = the number of misplaced tiles. For Figure 3.28, all of the eight tiles are out of position, so the start state would have $h_1 = 8$. h_1 is an admissible heuristic because it is clear that any tile that is out of place must be moved at least once.

- h_2 = the sum of the distances of the tiles from their goal positions. Because tiles cannot move along diagonals, the distance we will count is the sum of the horizontal and vertical distances. This is sometimes called the **city block distance** or **Manhattan distance**. h_2 is also admissible because all any move can do is move one tile one step closer to the goal. Tiles 1 to 8 in the start state give a Manhattan distance of

$$h_2 = 3 + 1 + 2 + 2 + 2 + 3 + 3 + 2 = 18 .$$

As expected, neither of these overestimates the true solution cost, which is 26.

3.6.1   The effect of heuristic accuracy on performance

One way to characterize the quality of a heuristic is the **effective branching factor** b^*. If the total number of nodes generated by A^* for a particular problem is N and the solution depth is d, then b^* is the branching factor that a uniform tree of depth d would have to have in order to contain $N + 1$ nodes. Thus,

$$N + 1 = 1 + b^* + (b^*)^2 + \cdots + (b^*)^d .$$

For example, if A^* finds a solution at depth 5 using 52 nodes, then the effective branching factor is 1.92. The effective branching factor can vary across problem instances, but usually it is fairly constant for sufficiently hard problems. (The existence of an effective branching factor follows from the result, mentioned earlier, that the number of nodes expanded by A^* grows exponentially with solution depth.) Therefore, experimental measurements of b^* on a small set of problems can provide a good guide to the heuristic's overall usefulness. A well-designed heuristic would have a value of b^* close to 1, allowing fairly large problems to be solved at reasonable computational cost.

To test the heuristic functions h_1 and h_2, we generated 1200 random problems with solution lengths from 2 to 24 (100 for each even number) and solved them with iterative deepening search and with A* tree search using both h_1 and h_2. Figure 3.29 gives the average number of nodes generated by each strategy and the effective branching factor. The results suggest that h_2 is better than h_1, and is far better than using iterative deepening search. Even for small problems with $d = 12$, A* with h_2 is 50,000 times more efficient than uninformed iterative deepening search.

	Search Cost (nodes generated)			Effective Branching Factor		
d	IDS	A*(h_1)	A*(h_2)	IDS	A*(h_1)	A*(h_2)
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	3644035	227	73	2.78	1.42	1.24
14	–	539	113	–	1.44	1.23
16	–	1301	211	–	1.45	1.25
18	–	3056	363	–	1.46	1.26
20	–	7276	676	–	1.47	1.27
22	–	18094	1219	–	1.48	1.28
24	–	39135	1641	–	1.48	1.26

Figure 3.29   Comparison of the search costs and effective branching factors for the ITERATIVE-DEEPENING-SEARCH and A* algorithms with h_1, h_2. Data are averaged over 100 instances of the 8-puzzle for each of various solution lengths d.

One might ask whether h_2 is *always* better than h_1. The answer is "Essentially, yes." It is easy to see from the definitions of the two heuristics that, for any node n, $h_2(n) \geq h_1(n)$. We thus say that h_2 **dominates** h_1. Domination translates directly into efficiency: A* using h_2 will never expand more nodes than A* using h_1 (except possibly for some nodes with $f(n) = C^*$). The argument is simple. Recall the observation on page 97 that every node with $f(n) < C^*$ will surely be expanded. This is the same as saying that every node with $h(n) < C^* - g(n)$ will surely be expanded. But because h_2 is at least as big as h_1 for all nodes, every node that is surely expanded by A* search with h_2 will also surely be expanded with h_1, and h_1 might cause other nodes to be expanded as well. Hence, it is generally better to use a heuristic function with higher values, provided it is consistent and that the computation time for the heuristic is not too long.

DOMINATION

3.6.2   Generating admissible heuristics from relaxed problems

We have seen that both h_1 (misplaced tiles) and h_2 (Manhattan distance) are fairly good heuristics for the 8-puzzle and that h_2 is better. How might one have come up with h_2? Is it possible for a computer to invent such a heuristic mechanically?

h_1 and h_2 are estimates of the remaining path length for the 8-puzzle, but they are also perfectly accurate path lengths for *simplified* versions of the puzzle. If the rules of the puzzle

were changed so that a tile could move anywhere instead of just to the adjacent empty square, then h_1 would give the exact number of steps in the shortest solution. Similarly, if a tile could move one square in any direction, even onto an occupied square, then h_2 would give the exact number of steps in the shortest solution. A problem with fewer restrictions on the actions is called a **relaxed problem**. The state-space graph of the relaxed problem is a *supergraph* of the original state space because the removal of restrictions creates added edges in the graph.

RELAXED PROBLEM

Because the relaxed problem adds edges to the state space, any optimal solution in the original problem is, by definition, also a solution in the relaxed problem; but the relaxed problem may have *better* solutions if the added edges provide short cuts. Hence, *the cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem.* Furthermore, because the derived heuristic is an exact cost for the relaxed problem, it must obey the triangle inequality and is therefore **consistent** (see page 95).

If a problem definition is written down in a formal language, it is possible to construct relaxed problems automatically.[11] For example, if the 8-puzzle actions are described as

A tile can move from square A to square B if
   A is horizontally or vertically adjacent to B **and** B is blank,

we can generate three relaxed problems by removing one or both of the conditions:

(a) A tile can move from square A to square B if A is adjacent to B.
(b) A tile can move from square A to square B if B is blank.
(c) A tile can move from square A to square B.

From (a), we can derive h_2 (Manhattan distance). The reasoning is that h_2 would be the proper score if we moved each tile in turn to its destination. The heuristic derived from (b) is discussed in Exercise 3.31. From (c), we can derive h_1 (misplaced tiles) because it would be the proper score if tiles could move to their intended destination in one step. Notice that it is crucial that the relaxed problems generated by this technique can be solved essentially *without search*, because the relaxed rules allow the problem to be decomposed into eight independent subproblems. If the relaxed problem is hard to solve, then the values of the corresponding heuristic will be expensive to obtain.[12]

A program called ABSOLVER can generate heuristics automatically from problem definitions, using the "relaxed problem" method and various other techniques (Prieditis, 1993). ABSOLVER generated a new heuristic for the 8-puzzle that was better than any preexisting heuristic and found the first useful heuristic for the famous Rubik's Cube puzzle.

One problem with generating new heuristic functions is that one often fails to get a single "clearly best" heuristic. If a collection of admissible heuristics $h_1 \ldots h_m$ is available for a problem and none of them dominates any of the others, which should we choose? As it turns out, we need not make a choice. We can have the best of all worlds, by defining

$$h(n) = \max\{h_1(n), \ldots, h_m(n)\} .$$

---

[11] In Chapters 8 and 10, we describe formal languages suitable for this task; with formal descriptions that can be manipulated, the construction of relaxed problems can be automated. For now, we use English.

[12] Note that a perfect heuristic can be obtained simply by allowing h to run a full breadth-first search "on the sly." Thus, there is a tradeoff between accuracy and computation time for heuristic functions.

Figure 3.30   A subproblem of the 8-puzzle instance given in Figure 3.28. The task is to get tiles 1, 2, 3, and 4 into their correct positions, without worrying about what happens to the other tiles.
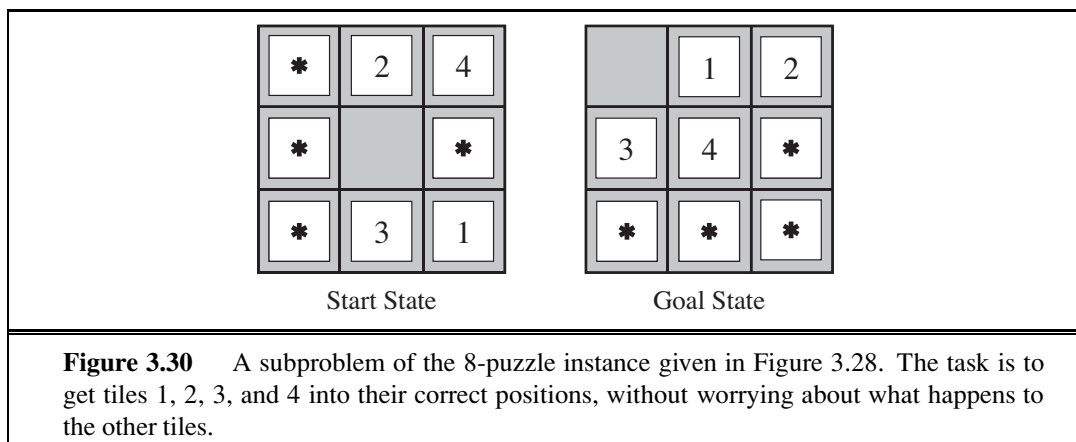
This composite heuristic uses whichever function is most accurate on the node in question. Because the component heuristics are admissible, h is admissible; it is also easy to prove that h is consistent. Furthermore, h dominates all of its component heuristics.

3.6.3   Generating admissible heuristics from subproblems: Pattern databases

SUBPROBLEM

Admissible heuristics can also be derived from the solution cost of a **subproblem** of a given problem. For example, Figure 3.30 shows a subproblem of the 8-puzzle instance in Figure 3.28. The subproblem involves getting tiles 1, 2, 3, 4 into their correct positions. Clearly, the cost of the optimal solution of this subproblem is a lower bound on the cost of the complete problem. It turns out to be more accurate than Manhattan distance in some cases.

PATTERN DATABASE

The idea behind **pattern databases** is to store these exact solution costs for every possible subproblem instance—in our example, every possible configuration of the four tiles and the blank. (The locations of the other four tiles are irrelevant for the purposes of solving the subproblem, but moves of those tiles do count toward the cost.) Then we compute an admissible heuristic h_{DB} for each complete state encountered during a search simply by looking up the corresponding subproblem configuration in the database. The database itself is constructed by searching back[13] from the goal and recording the cost of each new pattern encountered; the expense of this search is amortized over many subsequent problem instances.

The choice of 1-2-3-4 is fairly arbitrary; we could also construct databases for 5-6-7-8, for 2-4-6-8, and so on. Each database yields an admissible heuristic, and these heuristics can be combined, as explained earlier, by taking the maximum value. A combined heuristic of this kind is much more accurate than the Manhattan distance; the number of nodes generated when solving random 15-puzzles can be reduced by a factor of 1000.

One might wonder whether the heuristics obtained from the 1-2-3-4 database and the 5-6-7-8 could be *added*, since the two subproblems seem not to overlap. Would this still give an admissible heuristic? The answer is no, because the solutions of the 1-2-3-4 subproblem and the 5-6-7-8 subproblem for a given state will almost certainly share some moves—it is

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[13] By working backward from the goal, the exact solution cost of every instance encountered is immediately available. This is an example of **dynamic programming**, which we discuss further in Chapter 17.

unlikely that 1-2-3-4 can be moved into place without touching 5-6-7-8, and vice versa. But what if we don't count those moves? That is, we record not the total cost of solving the 1-2-3-4 subproblem, but just the number of moves involving 1-2-3-4. Then it is easy to see that the sum of the two costs is still a lower bound on the cost of solving the entire problem. This is the idea behind **disjoint pattern databases**. With such databases, it is possible to solve random 15-puzzles in a few milliseconds—the number of nodes generated is reduced by a factor of 10,000 compared with the use of Manhattan distance. For 24-puzzles, a speedup of roughly a factor of a million can be obtained.

DISJOINT PATTERN
DATABASES

Disjoint pattern databases work for sliding-tile puzzles because the problem can be divided up in such a way that each move affects only one subproblem—because only one tile is moved at a time. For a problem such as Rubik's Cube, this kind of subdivision is difficult because each move affects 8 or 9 of the 26 cubies. More general ways of defining additive, admissible heuristics have been proposed that do apply to Rubik's cube (Yang *et al.*, 2008), but they have not yielded a heuristic better than the best nonadditive heuristic for the problem.

3.6.4   Learning heuristics from experience

A heuristic function $h(n)$ is supposed to estimate the cost of a solution beginning from the state at node n. How could an agent construct such a function? One solution was given in the preceding sections—namely, to devise relaxed problems for which an optimal solution can be found easily. Another solution is to learn from experience. "Experience" here means solving lots of 8-puzzles, for instance. Each optimal solution to an 8-puzzle problem provides examples from which $h(n)$ can be learned. Each example consists of a state from the solution path and the actual cost of the solution from that point. From these examples, a learning algorithm can be used to construct a function $h(n)$ that can (with luck) predict solution costs for other states that arise during search. Techniques for doing just this using neural nets, decision trees, and other methods are demonstrated in Chapter 18. (The reinforcement learning methods described in Chapter 21 are also applicable.)

FEATURE

Inductive learning methods work best when supplied with **features** of a state that are relevant to predicting the state's value, rather than with just the raw state description. For example, the feature "number of misplaced tiles" might be helpful in predicting the actual distance of a state from the goal. Let's call this feature $x_1(n)$. We could take 100 randomly generated 8-puzzle configurations and gather statistics on their actual solution costs. We might find that when $x_1(n)$ is 5, the average solution cost is around 14, and so on. Given these data, the value of x_1 can be used to predict $h(n)$. Of course, we can use several features. A second feature $x_2(n)$ might be "number of pairs of adjacent tiles that are not adjacent in the goal state." How should $x_1(n)$ and $x_2(n)$ be combined to predict $h(n)$? A common approach is to use a linear combination:

$$h(n) = c_1 x_1(n) + c_2 x_2(n) \ .$$

The constants c_1 and c_2 are adjusted to give the best fit to the actual data on solution costs. One expects both c_1 and c_2 to be positive because misplaced tiles and incorrect adjacent pairs make the problem harder to solve. Notice that this heuristic does satisfy the condition that $h(n) = 0$ for goal states, but it is not necessarily admissible or consistent.

3.7   SUMMARY

This chapter has introduced methods that an agent can use to select actions in environments that are deterministic, observable, static, and completely known. In such cases, the agent can construct sequences of actions that achieve its goals; this process is called **search**.

- Before an agent can start searching for solutions, a **goal** must be identified and a well-defined **problem** must be formulated.

- A problem consists of five parts: the **initial state**, a set of **actions**, a **transition model** describing the results of those actions, a **goal test** function, and a **path cost** function. The environment of the problem is represented by a **state space**. A **path** through the state space from the initial state to a goal state is a **solution**.

- Search algorithms treat states and actions as **atomic**: they do not consider any internal structure they might possess.

- A general TREE-SEARCH algorithm considers all possible paths to find a solution, whereas a GRAPH-SEARCH algorithm avoids consideration of redundant paths.

- Search algorithms are judged on the basis of **completeness**, **optimality**, **time complexity**, and **space complexity**. Complexity depends on b, the branching factor in the state space, and d, the depth of the shallowest solution.

- **Uninformed search** methods have access only to the problem definition. The basic algorithms are as follows:

    - **Breadth-first search** expands the shallowest nodes first; it is complete, optimal for unit step costs, but has exponential space complexity.

    - **Uniform-cost search** expands the node with lowest path cost, $g(n)$, and is optimal for general step costs.

    - **Depth-first search** expands the deepest unexpanded node first. It is neither complete nor optimal, but has linear space complexity. **Depth-limited search** adds a depth bound.

    - **Iterative deepening search** calls depth-first search with increasing depth limits until a goal is found. It is complete, optimal for unit step costs, has time complexity comparable to breadth-first search, and has linear space complexity.

    - **Bidirectional search** can enormously reduce time complexity, but it is not always applicable and may require too much space.

- **Informed search** methods may have access to a **heuristic** function $h(n)$ that estimates the cost of a solution from n.

    - The generic **best-first search** algorithm selects a node for expansion according to an **evaluation function**.

    - **Greedy best-first search** expands nodes with minimal $h(n)$. It is not optimal but is often efficient.

- **A* search** expands nodes with minimal $f(n) = g(n) + h(n)$. A* is complete and optimal, provided that $h(n)$ is admissible (for TREE-SEARCH) or consistent (for GRAPH-SEARCH). The space complexity of A* is still prohibitive.

- **RBFS** (recursive best-first search) and **SMA*** (simplified memory-bounded A*) are robust, optimal search algorithms that use limited amounts of memory; given enough time, they can solve problems that A* cannot solve because it runs out of memory.

- The performance of heuristic search algorithms depends on the quality of the heuristic function. One can sometimes construct good heuristics by relaxing the problem definition, by storing precomputed solution costs for subproblems in a pattern database, or by learning from experience with the problem class.

BIBLIOGRAPHICAL AND HISTORICAL NOTES

The topic of state-space search originated in more or less its current form in the early years of AI. Newell and Simon's work on the Logic Theorist (1957) and GPS (1961) led to the establishment of search algorithms as the primary weapons in the armory of 1960s AI researchers and to the establishment of problem solving as the canonical AI task. Work in operations research by Richard Bellman (1957) showed the importance of additive path costs in simplifying optimization algorithms. The text on *Automated Problem Solving* by Nils Nilsson (1971) established the area on a solid theoretical footing.

Most of the state-space search problems analyzed in this chapter have a long history in the literature and are less trivial than they might seem. The missionaries and cannibals problem used in Exercise 3.9 was analyzed in detail by Amarel (1968). It had been considered earlier—in AI by Simon and Newell (1961) and in operations research by Bellman and Dreyfus (1962).

The 8-puzzle is a smaller cousin of the 15-puzzle, whose history is recounted at length by Slocum and Sonneveld (2006). It was widely believed to have been invented by the famous American game designer Sam Loyd, based on his claims to that effect from 1891 onward (Loyd, 1959). Actually it was invented by Noyes Chapman, a postmaster in Canastota, New York, in the mid-1870s. (Chapman was unable to patent his invention, as a generic patent covering sliding blocks with letters, numbers, or pictures was granted to Ernest Kinsey in 1878.) It quickly attracted the attention of the public and of mathematicians (Johnson and Story, 1879; Tait, 1880). The editors of the *American Journal of Mathematics* stated, "The '15' puzzle for the last few weeks has been prominently before the American public, and may safely be said to have engaged the attention of nine out of ten persons of both sexes and all ages and conditions of the community." Ratner and Warmuth (1986) showed that the general $n \times n$ version of the 15-puzzle belongs to the class of NP-complete problems.

The 8-queens problem was first published anonymously in the German chess magazine *Schach* in 1848; it was later attributed to one Max Bezzel. It was republished in 1850 and at that time drew the attention of the eminent mathematician Carl Friedrich Gauss, who

attempted to enumerate all possible solutions; initially he found only 72, but eventually he found the correct answer of 92, although Nauck published all 92 solutions first, in 1850. Netto (1901) generalized the problem to n queens, and Abramson and Yung (1989) found an $O(n)$ algorithm.

Each of the real-world search problems listed in the chapter has been the subject of a good deal of research effort. Methods for selecting optimal airline flights remain proprietary for the most part, but Carl de Marcken (personal communication) has shown that airline ticket pricing and restrictions have become so convoluted that the problem of selecting an optimal flight is formally *undecidable*. The traveling-salesperson problem is a standard combinatorial problem in theoretical computer science (Lawler *et al.*, 1992). Karp (1972) proved the TSP to be NP-hard, but effective heuristic approximation methods were developed (Lin and Kernighan, 1973). Arora (1998) devised a fully polynomial approximation scheme for Euclidean TSPs. VLSI layout methods are surveyed by Shahookar and Mazumder (1991), and many layout optimization papers appear in VLSI journals. Robotic navigation and assembly problems are discussed in Chapter 25.

Uninformed search algorithms for problem solving are a central topic of classical computer science (Horowitz and Sahni, 1978) and operations research (Dreyfus, 1969). Breadth-first search was formulated for solving mazes by Moore (1959). The method of **dynamic programming** (Bellman, 1957; Bellman and Dreyfus, 1962), which systematically records solutions for all subproblems of increasing lengths, can be seen as a form of breadth-first search on graphs. The two-point shortest-path algorithm of Dijkstra (1959) is the origin of uniform-cost search. These works also introduced the idea of explored and frontier sets (closed and open lists).

A version of iterative deepening designed to make efficient use of the chess clock was first used by Slate and Atkin (1977) in the CHESS 4.5 game-playing program. Martelli's algorithm B (1977) includes an iterative deepening aspect and also dominates A*'s worst-case performance with admissible but inconsistent heuristics. The iterative deepening technique came to the fore in work by Korf (1985a). Bidirectional search, which was introduced by Pohl (1971), can also be effective in some cases.

The use of heuristic information in problem solving appears in an early paper by Simon and Newell (1958), but the phrase "heuristic search" and the use of heuristic functions that estimate the distance to the goal came somewhat later (Newell and Ernst, 1965; Lin, 1965). Doran and Michie (1966) conducted extensive experimental studies of heuristic search. Although they analyzed path length and "penetrance" (the ratio of path length to the total number of nodes examined so far), they appear to have ignored the information provided by the path cost $g(n)$. The A* algorithm, incorporating the current path cost into heuristic search, was developed by Hart, Nilsson, and Raphael (1968), with some later corrections (Hart *et al.*, 1972). Dechter and Pearl (1985) demonstrated the optimal efficiency of A*.

The original A* paper introduced the consistency condition on heuristic functions. The monotone condition was introduced by Pohl (1977) as a simpler replacement, but Pearl (1984) showed that the two were equivalent.

Pohl (1977) pioneered the study of the relationship between the error in heuristic functions and the time complexity of A*. Basic results were obtained for tree search with unit step

costs and a single goal node (Pohl, 1977; Gaschnig, 1979; Huyn *et al.*, 1980; Pearl, 1984) and with multiple goal nodes (Dinh *et al.*, 2007). The "effective branching factor" was proposed by Nilsson (1971) as an empirical measure of the efficiency; it is equivalent to assuming a time cost of $O((b^*)^d)$. For tree search applied to a graph, Korf *et al.* (2001) argue that the time cost is better modeled as $O(b^{d-k})$, where k depends on the heuristic accuracy; this analysis has elicited some controversy, however. For graph search, Helmert and Röger (2008) noted that several well-known problems contained exponentially many nodes on optimal solution paths, implying exponential time complexity for A* even with constant absolute error in h.

There are many variations on the A* algorithm. Pohl (1973) proposed the use of *dynamic weighting*, which uses a weighted sum $f_w(n) = w_g g(n) + w_h h(n)$ of the current path length and the heuristic function as an evaluation function, rather than the simple sum $f(n) = g(n) + h(n)$ used in A*. The weights w_g and w_h are adjusted dynamically as the search progresses. Pohl's algorithm can be shown to be ϵ-admissible—that is, guaranteed to find solutions within a factor $1 + \epsilon$ of the optimal solution, where ϵ is a parameter supplied to the algorithm. The same property is exhibited by the A_ϵ^* algorithm (Pearl, 1984), which can select any node from the frontier provided its f-cost is within a factor $1 + \epsilon$ of the lowest-f-cost frontier node. The selection can be done so as to minimize search cost.

Bidirectional versions of A* have been investigated; a combination of bidirectional A* and known landmarks was used to efficiently find driving routes for Microsoft's online map service (Goldberg *et al.*, 2006). After caching a set of paths between landmarks, the algorithm can find an optimal path between any pair of points in a 24 million point graph of the United States, searching less than 0.1% of the graph. Others approaches to bidirectional search include a breadth-first search backward from the goal up to a fixed depth, followed by a forward IDA* search (Dillenburg and Nelson, 1994; Manzini, 1995).

A* and other state-space search algorithms are closely related to the *branch-and-bound* techniques that are widely used in operations research (Lawler and Wood, 1966). The relationships between state-space search and branch-and-bound have been investigated in depth (Kumar and Kanal, 1983; Nau *et al.*, 1984; Kumar *et al.*, 1988). Martelli and Montanari (1978) demonstrate a connection between dynamic programming (see Chapter 17) and certain types of state-space search. Kumar and Kanal (1988) attempt a "grand unification" of heuristic search, dynamic programming, and branch-and-bound techniques under the name of CDP—the "composite decision process."

Because computers in the late 1950s and early 1960s had at most a few thousand words of main memory, memory-bounded heuristic search was an early research topic. The Graph Traverser (Doran and Michie, 1966), one of the earliest search programs, commits to an operator after searching best-first up to the memory limit. IDA* (Korf, 1985a, 1985b) was the first widely used optimal, memory-bounded heuristic search algorithm, and a large number of variants have been developed. An analysis of the efficiency of IDA* and of its difficulties with real-valued heuristics appears in Patrick *et al.* (1992).

RBFS (Korf, 1993) is actually somewhat more complicated than the algorithm shown in Figure 3.26, which is closer to an independently developed algorithm called **iterative expansion** (Russell, 1992). RBFS uses a lower bound as well as the upper bound; the two algorithms behave identically with admissible heuristics, but RBFS expands nodes in best-first

ITERATIVE
EXPANSION

order even with an inadmissible heuristic. The idea of keeping track of the best alternative path appeared earlier in Bratko's (1986) elegant Prolog implementation of A* and in the DTA* algorithm (Russell and Wefald, 1991). The latter work also discusses metalevel state spaces and metalevel learning.

The MA* algorithm appeared in Chakrabarti *et al.* (1989). SMA*, or Simplified MA*, emerged from an attempt to implement MA* as a comparison algorithm for IE (Russell, 1992). Kaindl and Khorsand (1994) have applied SMA* to produce a bidirectional search algorithm that is substantially faster than previous algorithms. Korf and Zhang (2000) describe a divide-and-conquer approach, and Zhou and Hansen (2002) introduce memory-bounded A* graph search and a strategy for switching to breadth-first search to increase memory-efficiency (Zhou and Hansen, 2006). Korf (1995) surveys memory-bounded search techniques.

The idea that admissible heuristics can be derived by problem relaxation appears in the seminal paper by Held and Karp (1970), who used the minimum-spanning-tree heuristic to solve the TSP. (See Exercise 3.30.)

The automation of the relaxation process was implemented successfully by Prieditis (1993), building on earlier work with Mostow (Mostow and Prieditis, 1989). Holte and Hernadvolgyi (2001) describe more recent steps towards automating the process. The use of pattern databases to derive admissible heuristics is due to Gasser (1995) and Culberson and Schaeffer (1996, 1998); disjoint pattern databases are described by Korf and Felner (2002); a similar method using symbolic patterns is due to Edelkamp (2009). Felner *et al.* (2007) show how to compress pattern databases to save space. The probabilistic interpretation of heuristics was investigated in depth by Pearl (1984) and Hansson and Mayer (1989).

By far the most comprehensive source on heuristics and heuristic search algorithms is Pearl's (1984) *Heuristics* text. This book provides especially good coverage of the wide variety of offshoots and variations of A*, including rigorous proofs of their formal properties. Kanal and Kumar (1988) present an anthology of important articles on heuristic search, and Rayward-Smith *et al.* (1996) cover approaches from Operations Research. Papers about new search algorithms—which, remarkably, continue to be discovered—appear in journals such as *Artificial Intelligence* and *Journal of the ACM*.

PARALLEL SEARCH   The topic of **parallel search** algorithms was not covered in the chapter, partly because it requires a lengthy discussion of parallel computer architectures. Parallel search became a popular topic in the 1990s in both AI and theoretical computer science (Mahanti and Daniels, 1993; Grama and Kumar, 1995; Crauser *et al.*, 1998) and is making a comeback in the era of new multicore and cluster architectures (Ralphs *et al.*, 2004; Korf and Schultze, 2005). Also of increasing importance are search algorithms for very large graphs that require disk storage (Korf, 2008).

EXERCISES

3.1   Explain why problem formulation must follow goal formulation.

3.2   Your goal is to navigate a robot out of a maze. The robot starts in the center of the maze

facing north. You can turn the robot to face north, east, south, or west. You can direct the robot to move forward a certain distance, although it will stop before hitting a wall.

    a. Formulate this problem. How large is the state space?

    b. In navigating a maze, the only place we need to turn is at the intersection of two or more corridors. Reformulate this problem using this observation. How large is the state space now?

    c. From each point in the maze, we can move in any of the four directions until we reach a turning point, and this is the only action we need to do. Reformulate the problem using these actions. Do we need to keep track of the robot's orientation now?

    d. In our initial description of the problem we already abstracted from the real world, restricting actions and removing details. List three such simplifications we made.

3.3   Suppose two friends live in different cities on a map, such as the Romania map shown in Figure 3.2. On every turn, we can simultaneously move each friend to a neighboring city on the map. The amount of time needed to move from city i to neighbor j is equal to the road distance $d(i, j)$ between the cities, but on each turn the friend that arrives first must wait until the other one arrives (and calls the first on his/her cell phone) before the next turn can begin. We want the two friends to meet as quickly as possible.

    a. Write a detailed formulation for this search problem. (You will find it helpful to define some formal notation here.)

    b. Let $D(i, j)$ be the straight-line distance between cities i and j. Which of the following heuristic functions are admissible? (i) $D(i, j)$; (ii) $2 \cdot D(i, j)$; (iii) $D(i, j)/2$.

    c. Are there completely connected maps for which no solution exists?

    d. Are there maps in which all solutions require one friend to visit the same city twice?

3.4   Show that the 8-puzzle states are divided into two disjoint sets, such that any state is reachable from any other state in the same set, while no state is reachable from any state in the other set. (*Hint:* See Berlekamp *et al.* (1982).) Devise a procedure to decide which set a given state is in, and explain why this is useful for generating random states.

3.5   Consider the n-queens problem using the "efficient" incremental formulation given on page 72. Explain why the state space has at least $\sqrt[3]{n!}$ states and estimate the largest n for which exhaustive exploration is feasible. (*Hint*: Derive a lower bound on the branching factor by considering the maximum number of squares that a queen can attack in any column.)

3.6   Give a complete problem formulation for each of the following. Choose a formulation that is precise enough to be implemented.

    a. Using only four colors, you have to color a planar map in such a way that no two adjacent regions have the same color.

    b. A 3-foot-tall monkey is in a room where some bananas are suspended from the 8-foot ceiling. He would like to get the bananas. The room contains two stackable, movable, climbable 3-foot-high crates.

Figure 3.31   A scene with polygonal obstacles. S and G are the start and goal states.
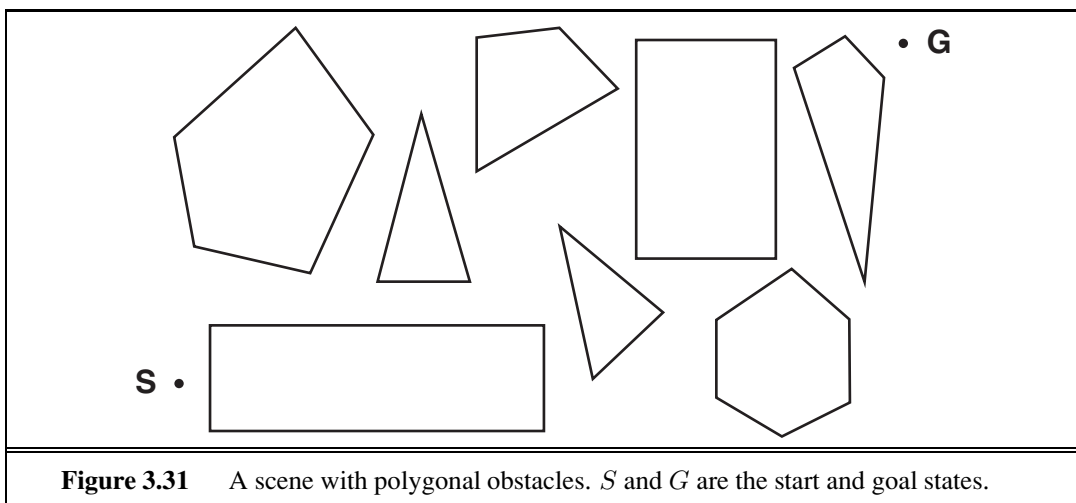
   c. You have a program that outputs the message "illegal input record" when fed a certain
file of input records. You know that processing of each record is independent of the
other records. You want to discover what record is illegal.

   d. You have three jugs, measuring 12 gallons, 8 gallons, and 3 gallons, and a water faucet.
You can fill the jugs up or empty them out from one to another or onto the ground. You
need to measure out exactly one gallon.

3.7   Consider the problem of finding the shortest path between two points on a plane that has
convex polygonal obstacles as shown in Figure 3.31. This is an idealization of the problem
that a robot has to solve to navigate in a crowded environment.

   a. Suppose the state space consists of all positions (x, y) in the plane. How many states
are there? How many paths are there to the goal?

   b. Explain briefly why the shortest path from one polygon vertex to any other in the scene
must consist of straight-line segments joining some of the vertices of the polygons.
Define a good state space now. How large is this state space?

   c. Define the necessary functions to implement the search problem, including an ACTIONS
function that takes a vertex as input and returns a set of vectors, each of which maps the
current vertex to one of the vertices that can be reached in a straight line. (Do not forget
the neighbors on the same polygon.) Use the straight-line distance for the heuristic
function.

   d. Apply one or more of the algorithms in this chapter to solve a range of problems in the
domain, and comment on their performance.

3.8   On page 68, we said that we would not consider problems with negative path costs. In
this exercise, we explore this decision in more depth.

   a. Suppose that actions can have arbitrarily large negative costs; explain why this possi-
bility would force any optimal algorithm to explore the entire state space.

b. Does it help if we insist that step costs must be greater than or equal to some negative constant c? Consider both trees and graphs.

c. Suppose that a set of actions forms a loop in the state space such that executing the set in some order results in no net change to the state. If all of these actions have negative cost, what does this imply about the optimal behavior for an agent in such an environment?

d. One can easily imagine actions with high negative cost, even in domains such as route finding. For example, some stretches of road might have such beautiful scenery as to far outweigh the normal costs in terms of time and fuel. Explain, in precise terms, within the context of state-space search, why humans do not drive around scenic loops indefinitely, and explain how to define the state space and actions for route finding so that artificial agents can also avoid looping.

e. Can you think of a real domain in which step costs are such as to cause looping?

3.9 The **missionaries and cannibals** problem is usually stated as follows. Three missionaries and three cannibals are on one side of a river, along with a boat that can hold one or two people. Find a way to get everyone to the other side without ever leaving a group of missionaries in one place outnumbered by the cannibals in that place. This problem is famous in AI because it was the subject of the first paper that approached problem formulation from an analytical viewpoint (Amarel, 1968).

a. Formulate the problem precisely, making only those distinctions necessary to ensure a valid solution. Draw a diagram of the complete state space.

b. Implement and solve the problem optimally using an appropriate search algorithm. Is it a good idea to check for repeated states?

c. Why do you think people have a hard time solving this puzzle, given that the state space is so simple?

3.10 Define in your own words the following terms: state, state space, search tree, search node, goal, action, transition model, and branching factor.

3.11 What's the difference between a world state, a state description, and a search node? Why is this distinction useful?

3.12 An action such as *Go(Sibiu)* really consists of a long sequence of finer-grained actions: turn on the car, release the brake, accelerate forward, etc. Having composite actions of this kind reduces the number of steps in a solution sequence, thereby reducing the search time. Suppose we take this to the logical extreme, by making super-composite actions out of every possible sequence of *Go* actions. Then every problem instance is solved by a single super-composite action, such as *Go(Sibiu)Go(Rimnicu Vilcea)Go(Pitesti)Go(Bucharest)*. Explain how search would work in this formulation. Is this a practical approach for speeding up problem solving?

3.13 Prove that GRAPH-SEARCH satisfies the graph separation property illustrated in Figure 3.9. (*Hint*: Begin by showing that the property holds at the start, then show that if it holds before an iteration of the algorithm, it holds afterwards.) Describe a search algorithm that violates the property.

Figure 3.32   The track pieces in a wooden railway set; each is labeled with the number of copies in the set. Note that curved pieces and "fork" pieces ("switches" or "points") can be flipped over so they can curve in either direction. Each curve subtends 45 degrees.
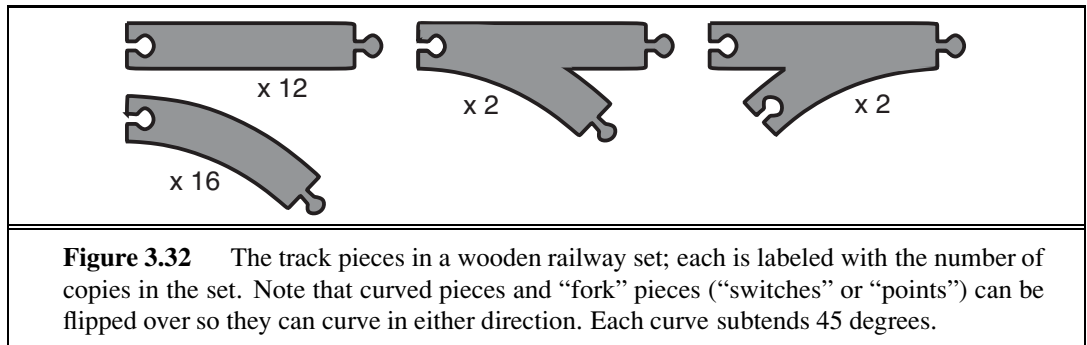
3.14   Which of the following are true and which are false? Explain your answers.

   a. Depth-first search always expands at least as many nodes as A* search with an admissible heuristic.

   b. $h(n) = 0$ is an admissible heuristic for the 8-puzzle.

   c. A* is of no use in robotics because percepts, states, and actions are continuous.

   d. Breadth-first search is complete even if zero step costs are allowed.

   e. Assume that a rook can move on a chessboard any number of squares in a straight line, vertically or horizontally, but cannot jump over other pieces. Manhattan distance is an admissible heuristic for the problem of moving the rook from square A to square B in the smallest number of moves.

3.15   Consider a state space where the start state is number 1 and each state k has two successors: numbers $2k$ and $2k + 1$.

   a. Draw the portion of the state space for states 1 to 15.

   b. Suppose the goal state is 11. List the order in which nodes will be visited for breadth-first search, depth-limited search with limit 3, and iterative deepening search.

   c. How well would bidirectional search work on this problem? What is the branching factor in each direction of the bidirectional search?

   d. Does the answer to (c) suggest a reformulation of the problem that would allow you to solve the problem of getting from state 1 to a given goal state with almost no search?

   e. Call the action going from k to $2k$ Left, and the action going to $2k + 1$ Right. Can you find an algorithm that outputs the solution to this problem without any search at all?

3.16   A basic wooden railway set contains the pieces shown in Figure 3.32. The task is to connect these pieces into a railway that has no overlapping tracks and no loose ends where a train could run off onto the floor.

   a. Suppose that the pieces fit together *exactly* with no slack. Give a precise formulation of the task as a search problem.

   b. Identify a suitable uninformed search algorithm for this task and explain your choice.

   c. Explain why removing any one of the "fork" pieces makes the problem unsolvable.

d. Give an upper bound on the total size of the state space defined by your formulation. (*Hint*: think about the maximum branching factor for the construction process and the maximum depth, ignoring the problem of overlapping pieces and loose ends. Begin by pretending that every piece is unique.)
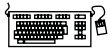
3.17   On page 90, we mentioned **iterative lengthening search**, an iterative analog of uniform cost search. The idea is to use increasing limits on path cost. If a node is generated whose path cost exceeds the current limit, it is immediately discarded. For each new iteration, the limit is set to the lowest path cost of any node discarded in the previous iteration.

   a. Show that this algorithm is optimal for general path costs.
   b. Consider a uniform tree with branching factor b, solution depth d, and unit step costs. How many iterations will iterative lengthening require?
   c. Now consider step costs drawn from the continuous range $[\epsilon, 1]$, where $0 < \epsilon < 1$. How many iterations are required in the worst case?
   d. Implement the algorithm and apply it to instances of the 8-puzzle and traveling salesperson problems. Compare the algorithm's performance to that of uniform-cost search, and comment on your results.

3.18   Describe a state space in which iterative deepening search performs much worse than depth-first search (for example, $O(n^2)$ vs. $O(n)$).

3.19   Write a program that will take as input two Web page URLs and find a path of links from one to the other. What is an appropriate search strategy? Is bidirectional search a good idea? Could a search engine be used to implement a predecessor function?

3.20   Consider the vacuum-world problem defined in Figure 2.2.

   a. Which of the algorithms defined in this chapter would be appropriate for this problem? Should the algorithm use tree search or graph search?
   b. Apply your chosen algorithm to compute an optimal sequence of actions for a 3×3 world whose initial state has dirt in the three top squares and the agent in the center.
   c. Construct a search agent for the vacuum world, and evaluate its performance in a set of 3×3 worlds with probability 0.2 of dirt in each square. Include the search cost as well as path cost in the performance measure, using a reasonable exchange rate.
   d. Compare your best search agent with a simple randomized reflex agent that sucks if there is dirt and otherwise moves randomly.
   e. Consider what would happen if the world were enlarged to $n \times n$. How does the performance of the search agent and of the reflex agent vary with n?

3.21   Prove each of the following statements, or give a counterexample:

   a. Breadth-first search is a special case of uniform-cost search.
   b. Depth-first search is a special case of best-first tree search.
   c. Uniform-cost search is a special case of A* search.
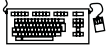
3.22   Compare the performance of A* and RBFS on a set of randomly generated problems in the 8-puzzle (with Manhattan distance) and TSP (with MST—see Exercise 3.30) domains. Discuss your results. What happens to the performance of RBFS when a small random number is added to the heuristic values in the 8-puzzle domain?

3.23   Trace the operation of A* search applied to the problem of getting to Bucharest from Lugoj using the straight-line distance heuristic. That is, show the sequence of nodes that the algorithm will consider and the f, g, and h score for each node.
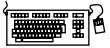
3.24   Devise a state space in which A* using GRAPH-SEARCH returns a suboptimal solution with an $h(n)$ function that is admissible but inconsistent.

HEURISTIC PATH
ALGORITHM

3.25   The **heuristic path algorithm** (Pohl, 1977) is a best-first search in which the evaluation function is $f(n) = (2 - w)g(n) + wh(n)$. For what values of w is this complete? For what values is it optimal, assuming that h is admissible? What kind of search does this perform for $w = 0$, $w = 1$, and $w = 2$?

3.26   Consider the unbounded version of the regular 2D grid shown in Figure 3.9. The start state is at the origin, (0,0), and the goal state is at (x, y).

   a. What is the branching factor b in this state space?
   b. How many distinct states are there at depth k (for $k > 0$)?
   c. What is the maximum number of nodes expanded by breadth-first tree search?
   d. What is the maximum number of nodes expanded by breadth-first graph search?
   e. Is $h = |u - x| + |v - y|$ an admissible heuristic for a state at (u, v)? Explain.
   **f**. How many nodes are expanded by A* graph search using h?
   g. Does h remain admissible if some links are removed?
   h. Does h remain admissible if some links are added between nonadjacent states?

3.27 n vehicles occupy squares $(1, 1)$ through $(n, 1)$ (i.e., the bottom row) of an $n \times n$ grid. The vehicles must be moved to the top row but in reverse order; so the vehicle i that starts in $(i, 1)$ must end up in $(n - i + 1, n)$. On each time step, every one of the n vehicles can move one square up, down, left, or right, or stay put; but if a vehicle stays put, one other adjacent vehicle (but not more than one) can hop over it. Two vehicles cannot occupy the same square.

   a. Calculate the size of the state space as a function of n.
   b. Calculate the branching factor as a function of n.
   c. Suppose that vehicle i is at (x_i, y_i); write a nontrivial admissible heuristic h_i for the number of moves it will require to get to its goal location $(n - i + 1, n)$, assuming no other vehicles are on the grid.
   d. Which of the following heuristics are admissible for the problem of moving all n vehicles to their destinations? Explain.
       (i) $\sum_{i=1}^{n} h_i$.
       (ii) $\max\{h_1, \ldots, h_n\}$.
       (iii) $\min\{h_1, \ldots, h_n\}$.

3.28 Invent a heuristic function for the 8-puzzle that sometimes overestimates, and show how it can lead to a suboptimal solution on a particular problem. (You can use a computer to help if you want.) Prove that if h never overestimates by more than c, A* using h returns a solution whose cost exceeds that of the optimal solution by no more than c.

3.29 Prove that if a heuristic is consistent, it must be admissible. Construct an admissible heuristic that is not consistent.
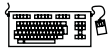
3.30 The traveling salesperson problem (TSP) can be solved with the minimum-spanning-tree (MST) heuristic, which estimates the cost of completing a tour, given that a partial tour has already been constructed. The MST cost of a set of cities is the smallest sum of the link costs of any tree that connects all the cities.

   a. Show how this heuristic can be derived from a relaxed version of the TSP.
   b. Show that the MST heuristic dominates straight-line distance.
   c. Write a problem generator for instances of the TSP where cities are represented by random points in the unit square.
   d. Find an efficient algorithm in the literature for constructing the MST, and use it with A* graph search to solve instances of the TSP.

3.31 On page 105, we defined the relaxation of the 8-puzzle in which a tile can move from square A to square B if B is blank. The exact solution of this problem defines **Gaschnig's heuristic** (Gaschnig, 1979). Explain why Gaschnig's heuristic is at least as accurate as h_1 (misplaced tiles), and show cases where it is more accurate than both h_1 and h_2 (Manhattan distance). Explain how to calculate Gaschnig's heuristic efficiently.
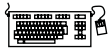
3.32 We gave two simple heuristics for the 8-puzzle: Manhattan distance and misplaced tiles. Several heuristics in the literature purport to improve on this—see, for example, Nilsson (1971), Mostow and Prieditis (1989), and Hansson *et al.* (1992). Test these claims by implementing the heuristics and comparing the performance of the resulting algorithms.

4 BEYOND CLASSICAL SEARCH

In which we relax the simplifying assumptions of the previous chapter, thereby getting closer to the real world.

Chapter 3 addressed a single category of problems: observable, deterministic, known environments where the solution is a sequence of actions. In this chapter, we look at what happens when these assumptions are relaxed. We begin with a fairly simple case: Sections 4.1 and 4.2 cover algorithms that perform purely **local search** in the state space, evaluating and modifying one or more current states rather than systematically exploring paths from an initial state. These algorithms are suitable for problems in which all that matters is the solution state, not the path cost to reach it. The family of local search algorithms includes methods inspired by statistical physics (**simulated annealing**) and evolutionary biology (**genetic algorithms**).

Then, in Sections 4.3–4.4, we examine what happens when we relax the assumptions of determinism and observability. The key idea is that if an agent cannot predict exactly what percept it will receive, then it will need to consider what to do under each **contingency** that its percepts may reveal. With partial observability, the agent will also need to keep track of the states it might be in.

Finally, Section 4.5 investigates **online search**, in which the agent is faced with a state space that is initially unknown and must be explored.

4.1 LOCAL SEARCH ALGORITHMS AND OPTIMIZATION PROBLEMS

The search algorithms that we have seen so far are designed to explore search spaces systematically. This systematicity is achieved by keeping one or more paths in memory and by recording which alternatives have been explored at each point along the path. When a goal is found, the *path* to that goal also constitutes a *solution* to the problem. In many problems, however, the path to the goal is irrelevant. For example, in the 8-queens problem (see page 71), what matters is the final configuration of queens, not the order in which they are added. The same general property holds for many important applications such as integrated-circuit design, factory-floor layout, job-shop scheduling, automatic programming, telecommunications network optimization, vehicle routing, and portfolio management.

If the path to the goal does not matter, we might consider a different class of algorithms, ones that do not worry about paths at all. **Local search** algorithms operate using a single **current node** (rather than multiple paths) and generally move only to neighbors of that node. Typically, the paths followed by the search are not retained. Although local search algorithms are not systematic, they have two key advantages: (1) they use very little memory—usually a constant amount; and (2) they can often find reasonable solutions in large or infinite (continuous) state spaces for which systematic algorithms are unsuitable.

In addition to finding goals, local search algorithms are useful for solving pure **optimization problems**, in which the aim is to find the best state according to an **objective function**. Many optimization problems do not fit the "standard" search model introduced in Chapter 3. For example, nature provides an objective function—reproductive fitness—that Darwinian evolution could be seen as attempting to optimize, but there is no "goal test" and no "path cost" for this problem.

To understand local search, we find it useful to consider the **state-space landscape** (as in Figure 4.1). A landscape has both "location" (defined by the state) and "elevation" (defined by the value of the heuristic cost function or objective function). If elevation corresponds to cost, then the aim is to find the lowest valley—a **global minimum**; if elevation corresponds to an objective function, then the aim is to find the highest peak—a **global maximum**. (You can convert from one to the other just by inserting a minus sign.) Local search algorithms explore this landscape. A **complete** local search algorithm always finds a goal if one exists; an **optimal** algorithm always finds a global minimum/maximum.

LOCAL SEARCH
CURRENT NODE

OPTIMIZATION PROBLEM
OBJECTIVE FUNCTION

STATE-SPACE LANDSCAPE

GLOBAL MINIMUM

GLOBAL MAXIMUM
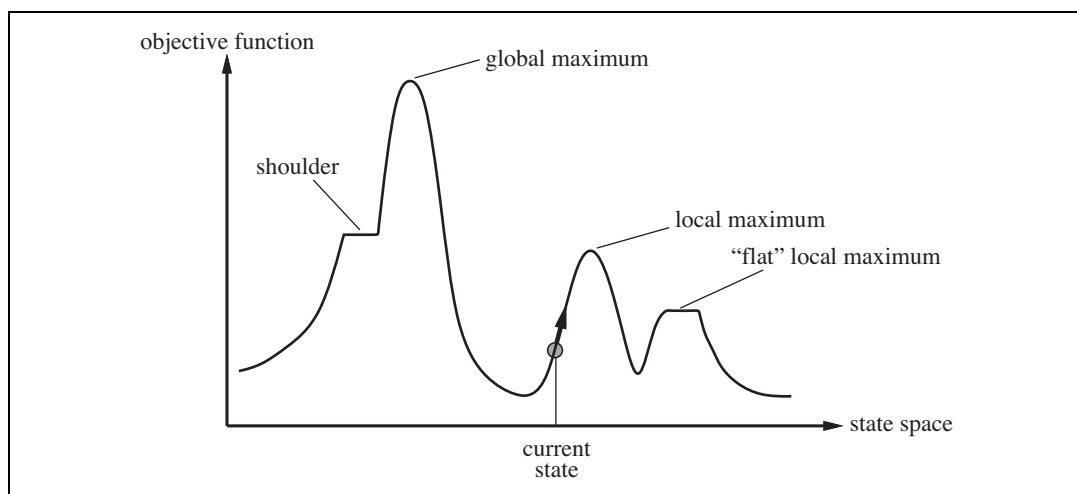
Figure 4.1   A one-dimensional state-space landscape in which elevation corresponds to the objective function. The aim is to find the global maximum. Hill-climbing search modifies the current state to try to improve it, as shown by the arrow. The various topographic features are defined in the text.

---

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum

　　current ← MAKE-NODE(*problem*.INITIAL-STATE)
　　loop do
　　　　neighbor ← a highest-valued successor of *current*
　　　　if neighbor.VALUE ≤ current.VALUE **then return** *current*.STATE
　　　　current ← *neighbor*

Figure 4.2　　The hill-climbing search algorithm, which is the most basic local search technique. At each step the current node is replaced by the best neighbor; in this version, that means the neighbor with the highest VALUE, but if a heuristic cost estimate h is used, we would find the neighbor with the lowest h.

4.1.1 Hill-climbing search

HILL CLIMBING

STEEPEST ASCENT

The **hill-climbing** search algorithm (**steepest-ascent** version) is shown in Figure 4.2. It is simply a loop that continually moves in the direction of increasing value—that is, uphill. It terminates when it reaches a "peak" where no neighbor has a higher value. The algorithm does not maintain a search tree, so the data structure for the current node need only record the state and the value of the objective function. Hill climbing does not look ahead beyond the immediate neighbors of the current state. This resembles trying to find the top of Mount Everest in a thick fog while suffering from amnesia.

To illustrate hill climbing, we will use the **8-queens problem** introduced on page 71. Local search algorithms typically use a **complete-state formulation**, where each state has 8 queens on the board, one per column. The successors of a state are all possible states generated by moving a single queen to another square in the same column (so each state has $8 \times 7 = 56$ successors). The heuristic cost function h is the number of pairs of queens that are attacking each other, either directly or indirectly. The global minimum of this function is zero, which occurs only at perfect solutions. Figure 4.3(a) shows a state with $h = 17$. The figure also shows the values of all its successors, with the best successors having $h = 12$. Hill-climbing algorithms typically choose randomly among the set of best successors if there is more than one.

GREEDY LOCAL
SEARCH

Hill climbing is sometimes called **greedy local search** because it grabs a good neighbor state without thinking ahead about where to go next. Although greed is considered one of the seven deadly sins, it turns out that greedy algorithms often perform quite well. Hill climbing often makes rapid progress toward a solution because it is usually quite easy to improve a bad state. For example, from the state in Figure 4.3(a), it takes just five steps to reach the state in Figure 4.3(b), which has $h = 1$ and is very nearly a solution. Unfortunately, hill climbing often gets stuck for the following reasons:

LOCAL MAXIMUM

- **Local maxima**: a local maximum is a peak that is higher than each of its neighboring states but lower than the global maximum. Hill-climbing algorithms that reach the vicinity of a local maximum will be drawn upward toward the peak but will then be stuck with nowhere else to go. Figure 4.1 illustrates the problem schematically. More

18	12	14	13	13	12	14	14
14	16	13	15	12	14	12	16
14	12	18	13	15	12	14	14
15	14	14	♛	13	16	13	16
♛	14	17	15	♛	14	16	16
17	♛	16	18	15	♛	15	♛
18	14	♛	15	15	14	♛	16
14	14	13	17	12	14	12	18

(a)   (b)

Figure 4.3   (a) An 8-queens state with heuristic cost estimate $h = 17$, showing the value of h for each possible successor obtained by moving a queen within its column. The best moves are marked. (b) A local minimum in the 8-queens state space; the state has $h = 1$ but every successor has a higher cost.

concretely, the state in Figure 4.3(b) is a local maximum (i.e., a local minimum for the cost h); every move of a single queen makes the situation worse.

<div style="margin-left:0;">

RIDGE

PLATEAU
SHOULDER

SIDEWAYS MOVE

</div>

- **Ridges**: a ridge is shown in Figure 4.4. Ridges result in a sequence of local maxima that is very difficult for greedy algorithms to navigate.

- **Plateaux**: a plateau is a flat area of the state-space landscape. It can be a flat local maximum, from which no uphill exit exists, or a **shoulder**, from which progress is possible. (See Figure 4.1.) A hill-climbing search might get lost on the plateau.

In each case, the algorithm reaches a point at which no progress is being made. Starting from a randomly generated 8-queens state, steepest-ascent hill climbing gets stuck 86% of the time, solving only 14% of problem instances. It works quickly, taking just 4 steps on average when it succeeds and 3 when it gets stuck—not bad for a state space with $8^8 \approx 17$ million states.

The algorithm in Figure 4.2 halts if it reaches a plateau where the best successor has the same value as the current state. Might it not be a good idea to keep going—to allow a **sideways move** in the hope that the plateau is really a shoulder, as shown in Figure 4.1? The answer is usually yes, but we must take care. If we always allow sideways moves when there are no uphill moves, an infinite loop will occur whenever the algorithm reaches a flat local maximum that is not a shoulder. One common solution is to put a limit on the number of consecutive sideways moves allowed. For example, we could allow up to, say, 100 consecutive sideways moves in the 8-queens problem. This raises the percentage of problem instances solved by hill climbing from 14% to 94%. Success comes at a cost: the algorithm averages roughly 21 steps for each successful instance and 64 for each failure.
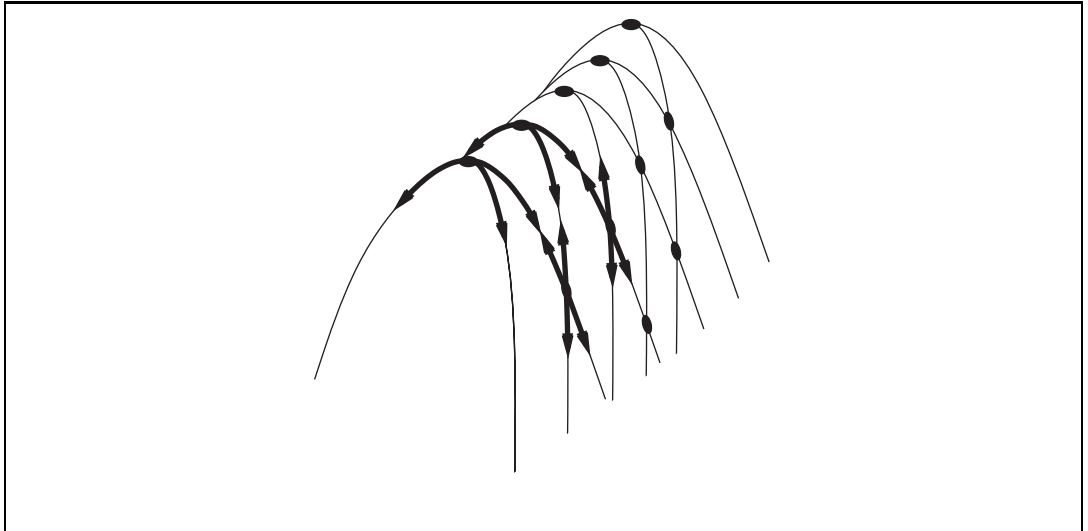
Figure 4.4   Illustration of why ridges cause difficulties for hill climbing. The grid of states (dark circles) is superimposed on a ridge rising from left to right, creating a sequence of local maxima that are not directly connected to each other. From each local maximum, all the available actions point downhill.

STOCHASTIC HILL
CLIMBING

FIRST-CHOICE HILL
CLIMBING

RANDOM-RESTART
HILL CLIMBING

Many variants of hill climbing have been invented. **Stochastic hill climbing** chooses at random from among the uphill moves; the probability of selection can vary with the steepness of the uphill move. This usually converges more slowly than steepest ascent, but in some state landscapes, it finds better solutions. **First-choice hill climbing** implements stochastic hill climbing by generating successors randomly until one is generated that is better than the current state. This is a good strategy when a state has many (e.g., thousands) of successors.

The hill-climbing algorithms described so far are incomplete—they often fail to find a goal when one exists because they can get stuck on local maxima. **Random-restart hill climbing** adopts the well-known adage, "If at first you don't succeed, try, try again." It conducts a series of hill-climbing searches from randomly generated initial states,[1] until a goal is found. It is trivially complete with probability approaching 1, because it will eventually generate a goal state as the initial state. If each hill-climbing search has a probability p of success, then the expected number of restarts required is $1/p$. For 8-queens instances with no sideways moves allowed, $p \approx 0.14$, so we need roughly 7 iterations to find a goal (6 failures and 1 success). The expected number of steps is the cost of one successful iteration plus $(1-p)/p$ times the cost of failure, or roughly 22 steps in all. When we allow sideways moves, $1/0.94 \approx 1.06$ iterations are needed on average and $(1 \times 21) + (0.06/0.94) \times 64 \approx 25$ steps. For 8-queens, then, random-restart hill climbing is very effective indeed. Even for three million queens, the approach can find solutions in under a minute.[2]

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[1]   Generating a *random* state from an implicitly specified state space can be a hard problem in itself.

[2]   Luby *et al.* (1993) prove that it is best, in some cases, to restart a randomized search algorithm after a particular, fixed amount of time and that this can be *much* more efficient than letting each search continue indefinitely. Disallowing or limiting the number of sideways moves is an example of this idea.

The success of hill climbing depends very much on the shape of the state-space landscape: if there are few local maxima and plateaux, random-restart hill climbing will find a good solution very quickly. On the other hand, many real problems have a landscape that looks more like a widely scattered family of balding porcupines on a flat floor, with miniature porcupines living on the tip of each porcupine needle, *ad infinitum*. NP-hard problems typically have an exponential number of local maxima to get stuck on. Despite this, a reasonably good local maximum can often be found after a small number of restarts.

4.1.2   Simulated annealing

A hill-climbing algorithm that *never* makes "downhill" moves toward states with lower value (or higher cost) is guaranteed to be incomplete, because it can get stuck on a local maximum. In contrast, a purely random walk—that is, moving to a successor chosen uniformly at random from the set of successors—is complete but extremely inefficient. Therefore, it seems reasonable to try to combine hill climbing with a random walk in some way that yields both efficiency and completeness. **Simulated annealing** is such an algorithm. In metallurgy, **annealing** is the process used to temper or harden metals and glass by heating them to a high temperature and then gradually cooling them, thus allowing the material to reach a low-energy crystalline state. To explain simulated annealing, we switch our point of view from hill climbing to **gradient descent** (i.e., minimizing cost) and imagine the task of getting a ping-pong ball into the deepest crevice in a bumpy surface. If we just let the ball roll, it will come to rest at a local minimum. If we shake the surface, we can bounce the ball out of the local minimum. The trick is to shake just hard enough to bounce the ball out of local minima but not hard enough to dislodge it from the global minimum. The simulated-annealing solution is to start by shaking hard (i.e., at a high temperature) and then gradually reduce the intensity of the shaking (i.e., lower the temperature).

The innermost loop of the simulated-annealing algorithm (Figure 4.5) is quite similar to hill climbing. Instead of picking the *best* move, however, it picks a *random* move. If the move improves the situation, it is always accepted. Otherwise, the algorithm accepts the move with some probability less than 1. The probability decreases exponentially with the "badness" of the move—the amount ΔE by which the evaluation is worsened. The probability also decreases as the "temperature" T goes down: "bad" moves are more likely to be allowed at the start when T is high, and they become more unlikely as T decreases. If the *schedule* lowers T slowly enough, the algorithm will find a global optimum with probability approaching 1.

Simulated annealing was first used extensively to solve VLSI layout problems in the early 1980s. It has been applied widely to factory scheduling and other large-scale optimization tasks. In Exercise 4.4, you are asked to compare its performance to that of random-restart hill climbing on the 8-queens puzzle.

4.1.3   Local beam search

Keeping just one node in memory might seem to be an extreme reaction to the problem of memory limitations. The **local beam search** algorithm[3] keeps track of k states rather than

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[3]   Local beam search is an adaptation of **beam search**, which is a path-based algorithm.

SIMULATED
ANNEALING

GRADIENT DESCENT

LOCAL BEAM
SEARCH

---

function SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state
    inputs: *problem*, a problem
              schedule, a mapping from time to "temperature"

    current ← MAKE-NODE(*problem*.INITIAL-STATE)
    for *t* = 1 **to** ∞ **do**
        T ← *schedule*(*t*)
        if *T* = 0 **then return** *current*
        next ← a randomly selected successor of *current*
        ΔE ← *next*.VALUE − *current*.VALUE
        if $\Delta E > 0$ **then** *current* ← *next*
        else *current* ← *next* only with probability $e^{\Delta E/T}$

Figure 4.5   The simulated annealing algorithm, a version of stochastic hill climbing where
some downhill moves are allowed. Downhill moves are accepted readily early in the anneal-
ing schedule and then less often as time goes on. The *schedule* input determines the value of
the temperature T as a function of time.

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just one. It begins with k randomly generated states. At each step, all the successors of all k
states are generated. If any one is a goal, the algorithm halts. Otherwise, it selects the k best
successors from the complete list and repeats.

At first sight, a local beam search with k states might seem to be nothing more than
running k random restarts in parallel instead of in sequence. In fact, the two algorithms
are quite different. In a random-restart search, each search process runs independently of
the others. *In a local beam search, useful information is passed among the parallel search
threads.* In effect, the states that generate the best successors say to the others, "Come over
here, the grass is greener!" The algorithm quickly abandons unfruitful searches and moves
its resources to where the most progress is being made.

In its simplest form, local beam search can suffer from a lack of diversity among the
k states—they can quickly become concentrated in a small region of the state space, making
the search little more than an expensive version of hill climbing. A variant called **stochastic
beam search**, analogous to stochastic hill climbing, helps alleviate this problem. Instead
of choosing the best k from the the pool of candidate successors, stochastic beam search
chooses k successors at random, with the probability of choosing a given successor being
an increasing function of its value. Stochastic beam search bears some resemblance to the
process of natural selection, whereby the "successors" (offspring) of a "state" (organism)
populate the next generation according to its "value" (fitness).

STOCHASTIC BEAM
SEARCH

4.1.4   Genetic algorithms
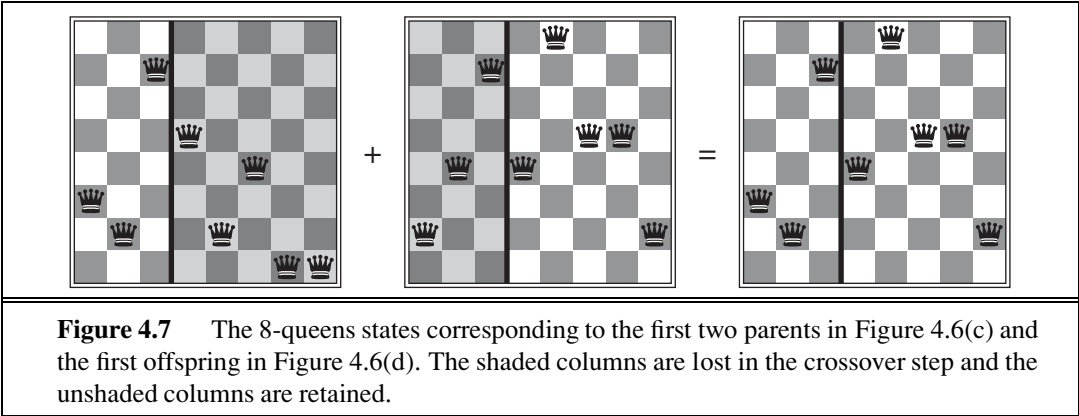
GENETIC
ALGORITHM

A **genetic algorithm** (or **GA**) is a variant of stochastic beam search in which successor states
are generated by combining *two* parent states rather than by modifying a single state. The
analogy to natural selection is the same as in stochastic beam search, except that now we are
dealing with sexual rather than asexual reproduction.

24748552	**24** **31%**	32752411		32748552	→	3274815̲2
32752411	**23** **29%**	24748552		24752411	→	24752411
24415124	**20** **26%**	32752411		32752124	→	32̲252124
32543213	**11** **14%**	24415124		24415411	→	244154̲17
(a)	(b)	(c)		(d)		(e)
Initial Population	Fitness Function	Selection		Crossover		Mutation

Figure 4.6   The genetic algorithm, illustrated for digit strings representing 8-queens states. The initial population in (a) is ranked by the fitness function in (b), resulting in pairs for mating in (c). They produce offspring in (d), which are subject to mutation in (e).

Figure 4.7   The 8-queens states corresponding to the first two parents in Figure 4.6(c) and the first offspring in Figure 4.6(d). The shaded columns are lost in the crossover step and the unshaded columns are retained.

Like beam searches, GAs begin with a set of k randomly generated states, called the **population**. Each state, or **individual**, is represented as a string over a finite alphabet—most commonly, a string of 0s and 1s. For example, an 8-queens state must specify the positions of 8 queens, each in a column of 8 squares, and so requires $8 \times \log_2 8 = 24$ bits. Alternatively, the state could be represented as 8 digits, each in the range from 1 to 8. (We demonstrate later that the two encodings behave differently.) Figure 4.6(a) shows a population of four 8-digit strings representing 8-queens states.

The production of the next generation of states is shown in Figure 4.6(b)–(e). In (b), each state is rated by the objective function, or (in GA terminology) the **fitness function**. A fitness function should return higher values for better states, so, for the 8-queens problem we use the number of *nonattacking* pairs of queens, which has a value of 28 for a solution. The values of the four states are 24, 23, 20, and 11. In this particular variant of the genetic algorithm, the probability of being chosen for reproducing is directly proportional to the fitness score, and the percentages are shown next to the raw scores.

In (c), two pairs are selected at random for reproduction, in accordance with the prob-

POPULATION
INDIVIDUAL

FITNESS FUNCTION

CROSSOVER

abilities in (b). Notice that one individual is selected twice and one not at all.[4] For each pair to be mated, a **crossover** point is chosen randomly from the positions in the string. In Figure 4.6, the crossover points are after the third digit in the first pair and after the fifth digit in the second pair.[5]

In (d), the offspring themselves are created by crossing over the parent strings at the crossover point. For example, the first child of the first pair gets the first three digits from the first parent and the remaining digits from the second parent, whereas the second child gets the first three digits from the second parent and the rest from the first parent. The 8-queens states involved in this reproduction step are shown in Figure 4.7. The example shows that when two parent states are quite different, the crossover operation can produce a state that is a long way from either parent state. It is often the case that the population is quite diverse early on in the process, so crossover (like simulated annealing) frequently takes large steps in the state space early in the search process and smaller steps later on when most individuals are quite similar.

MUTATION

Finally, in (e), each location is subject to random **mutation** with a small independent probability. One digit was mutated in the first, third, and fourth offspring. In the 8-queens problem, this corresponds to choosing a queen at random and moving it to a random square in its column. Figure 4.8 describes an algorithm that implements all these steps.

Like stochastic beam search, genetic algorithms combine an uphill tendency with random exploration and exchange of information among parallel search threads. The primary advantage, if any, of genetic algorithms comes from the crossover operation. Yet it can be shown mathematically that, if the positions of the genetic code are permuted initially in a random order, crossover conveys no advantage. Intuitively, the advantage comes from the ability of crossover to combine large blocks of letters that have evolved independently to perform useful functions, thus raising the level of granularity at which the search operates. For example, it could be that putting the first three queens in positions 2, 4, and 6 (where they do not attack each other) constitutes a useful block that can be combined with other blocks to construct a solution.

SCHEMA

INSTANCE

The theory of genetic algorithms explains how this works using the idea of a **schema**, which is a substring in which some of the positions can be left unspecified. For example, the schema 246***** describes all 8-queens states in which the first three queens are in positions 2, 4, and 6, respectively. Strings that match the schema (such as 24613578) are called **instances** of the schema. It can be shown that if the average fitness of the instances of a schema is above the mean, then the number of instances of the schema within the population will grow over time. Clearly, this effect is unlikely to be significant if adjacent bits are totally unrelated to each other, because then there will be few contiguous blocks that provide a consistent benefit. Genetic algorithms work best when schemata correspond to meaningful components of a solution. For example, if the string is a representation of an antenna, then the schemata may represent components of the antenna, such as reflectors and deflectors. A good

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[4] There are many variants of this selection rule. The method of **culling**, in which all individuals below a given threshold are discarded, can be shown to converge faster than the random version (Baum *et al.*, 1995).

[5] It is here that the encoding matters. If a 24-bit encoding is used instead of 8 digits, then the crossover point has a 2/3 chance of being in the middle of a digit, which results in an essentially arbitrary mutation of that digit.

---

function GENETIC-ALGORITHM(*population*, FITNESS-FN) **returns** an individual
  inputs: *population*, a set of individuals
          FITNESS-FN, a function that measures the fitness of an individual

  repeat
     new_population ← empty set
     for $i = 1$ **to** SIZE(*population*) **do**
        $x \leftarrow$ RANDOM-SELECTION(*population*, FITNESS-FN)
        $y \leftarrow$ RANDOM-SELECTION(*population*, FITNESS-FN)
        child ← REPRODUCE(x, y)
        if (small random probability) **then** *child* ← MUTATE(*child*)
        add *child* to *new_population*
     population ← *new_population*
  until some individual is fit enough, or enough time has elapsed
  return the best individual in *population*, according to FITNESS-FN

---

function REPRODUCE(x, y) **returns** an individual
  inputs: x, y, parent individuals

  $n \leftarrow$ LENGTH(x); $c \leftarrow$ random number from 1 to n
  return APPEND(SUBSTRING($x, 1, c$), SUBSTRING($y, c + 1, n$))

Figure 4.8   A genetic algorithm. The algorithm is the same as the one diagrammed in Figure 4.6, with one variation: in this more popular version, each mating of two parents produces only one offspring, not two.

component is likely to be good in a variety of different designs. This suggests that successful use of genetic algorithms requires careful engineering of the representation.

In practice, genetic algorithms have had a widespread impact on optimization problems, such as circuit layout and job-shop scheduling. At present, it is not clear whether the appeal of genetic algorithms arises from their performance or from their æsthetically pleasing origins in the theory of evolution. Much work remains to be done to identify the conditions under which genetic algorithms perform well.

4.2   LOCAL SEARCH IN CONTINUOUS SPACES

In Chapter 2, we explained the distinction between discrete and continuous environments, pointing out that most real-world environments are continuous. Yet none of the algorithms we have described (except for first-choice hill climbing and simulated annealing) can handle continuous state and action spaces, because they have infinite branching factors. This section provides a *very brief* introduction to some local search techniques for finding optimal solutions in continuous spaces. The literature on this topic is vast; many of the basic techniques

EVOLUTION AND SEARCH

The theory of **evolution** was developed in Charles Darwin's *On the Origin of Species by Means of Natural Selection* (1859) and independently by Alfred Russel Wallace (1858). The central idea is simple: variations occur in reproduction and will be preserved in successive generations approximately in proportion to their effect on reproductive fitness.

Darwin's theory was developed with no knowledge of how the traits of organisms can be inherited and modified. The probabilistic laws governing these processes were first identified by Gregor Mendel (1866), a monk who experimented with sweet peas. Much later, Watson and Crick (1953) identified the structure of the DNA molecule and its alphabet, AGTC (adenine, guanine, thymine, cytosine). In the standard model, variation occurs both by point mutations in the letter sequence and by "crossover" (in which the DNA of an offspring is generated by combining long sections of DNA from each parent).

The analogy to local search algorithms has already been described; the principal difference between stochastic beam search and evolution is the use of *sexual* reproduction, wherein successors are generated from *multiple* organisms rather than just one. The actual mechanisms of evolution are, however, far richer than most genetic algorithms allow. For example, mutations can involve reversals, duplications, and movement of large chunks of DNA; some viruses borrow DNA from one organism and insert it in another; and there are transposable genes that do nothing but copy themselves many thousands of times within the genome. There are even genes that poison cells from potential mates that do not carry the gene, thereby increasing their own chances of replication. Most important is the fact that the *genes themselves encode the mechanisms* whereby the genome is reproduced and translated into an organism. In genetic algorithms, those mechanisms are a separate program that is not represented within the strings being manipulated.

Darwinian evolution may appear inefficient, having generated blindly some 10^{45} or so organisms without improving its search heuristics one iota. Fifty years before Darwin, however, the otherwise great French naturalist Jean Lamarck (1809) proposed a theory of evolution whereby traits *acquired by adaptation during an organism's lifetime* would be passed on to its offspring. Such a process would be effective but does not seem to occur in nature. Much later, James Baldwin (1896) proposed a superficially similar theory: that behavior learned during an organism's lifetime could accelerate the rate of evolution. Unlike Lamarck's, Baldwin's theory is entirely consistent with Darwinian evolution because it relies on selection pressures operating on individuals that have found local optima among the set of possible behaviors allowed by their genetic makeup. Computer simulations confirm that the "Baldwin effect" is real, once "ordinary" evolution has created organisms whose internal performance measure correlates with actual fitness.

originated in the 17th century, after the development of calculus by Newton and Leibniz.[6] We find uses for these techniques at several places in the book, including the chapters on learning, vision, and robotics.

We begin with an example. Suppose we want to place three new airports anywhere in Romania, such that the sum of squared distances from each city on the map (Figure 3.2) to its nearest airport is minimized. The state space is then defined by the coordinates of the airports: (x_1, y_1), (x_2, y_2), and (x_3, y_3). This is a *six-dimensional* space; we also say

VARIABLE

that states are defined by six **variables**. (In general, states are defined by an n-dimensional vector of variables, $\mathbf{x}$.) Moving around in this space corresponds to moving one or more of the airports on the map. The objective function $f(x_1, y_1, x_2, y_2, x_3, y_3)$ is relatively easy to compute for any particular state once we compute the closest cities. Let C_i be the set of cities whose closest airport (in the current state) is airport i. Then, *in the neighborhood of the current state*, where the C_is remain constant, we have

$$f(x_1, y_1, x_2, y_2, x_3, y_3) = \sum_{i=1}^{3} \sum_{c \in C_i} (x_i - x_c)^2 + (y_i - y_c)^2 \ . \tag{4.1}$$

This expression is correct *locally*, but not globally because the sets C_i are (discontinuous) functions of the state.

DISCRETIZATION

One way to avoid continuous problems is simply to **discretize** the neighborhood of each state. For example, we can move only one airport at a time in either the x or y direction by a fixed amount $\pm\delta$. With 6 variables, this gives 12 possible successors for each state. We can then apply any of the local search algorithms described previously. We could also apply stochastic hill climbing and simulated annealing directly, without discretizing the space. These algorithms choose successors randomly, which can be done by generating random vectors of length δ.

GRADIENT

Many methods attempt to use the **gradient** of the landscape to find a maximum. The gradient of the objective function is a vector ∇f that gives the magnitude and direction of the steepest slope. For our problem, we have

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3} \right) \ .$$

In some cases, we can find a maximum by solving the equation $\nabla f = 0$. (This could be done, for example, if we were placing just one airport; the solution is the arithmetic mean of all the cities' coordinates.) In many cases, however, this equation cannot be solved in closed form. For example, with three airports, the expression for the gradient depends on what cities are closest to each airport in the current state. This means we can compute the gradient *locally* (but not *globally*); for example,

$$\frac{\partial f}{\partial x_1} = 2 \sum_{c \in C_1} (x_i - x_c) \ . \tag{4.2}$$

Given a locally correct expression for the gradient, we can perform steepest-ascent hill climb-

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[6]   A basic knowledge of multivariate calculus and vector arithmetic is useful for reading this section.

ing by updating the current state according to the formula

$$\mathbf{x} \leftarrow \mathbf{x} + \alpha \nabla f(\mathbf{x}) \ ,$$

STEP SIZE

where α is a small constant often called the **step size**. In other cases, the objective function might not be available in a differentiable form at all—for example, the value of a particular set of airport locations might be determined by running some large-scale economic simulation package. In those cases, we can calculate a so-called **empirical gradient** by evaluating the response to small increments and decrements in each coordinate. Empirical gradient search is the same as steepest-ascent hill climbing in a discretized version of the state space.

EMPIRICAL GRADIENT

Hidden beneath the phrase "α is a small constant" lies a huge variety of methods for adjusting α. The basic problem is that, if α is too small, too many steps are needed; if α is too large, the search could overshoot the maximum. The technique of **line search** tries to overcome this dilemma by extending the current gradient direction—usually by repeatedly doubling α—until f starts to decrease again. The point at which this occurs becomes the new current state. There are several schools of thought about how the new direction should be chosen at this point.

LINE SEARCH

NEWTON–RAPHSON

For many problems, the most effective algorithm is the venerable **Newton–Raphson** method. This is a general technique for finding roots of functions—that is, solving equations of the form $g(x) = 0$. It works by computing a new estimate for the root x according to Newton's formula

$$x \leftarrow x - g(x)/g'(x) \ .$$

To find a maximum or minimum of f, we need to find $\mathbf{x}$ such that the *gradient* is zero (i.e., $\nabla f(\mathbf{x}) = \mathbf{0}$). Thus, $g(x)$ in Newton's formula becomes $\nabla f(\mathbf{x})$, and the update equation can be written in matrix–vector form as

$$\mathbf{x} \leftarrow \mathbf{x} - \mathbf{H}_f^{-1}(\mathbf{x})\nabla f(\mathbf{x}) \ ,$$

HESSIAN

where $\mathbf{H}_f(\mathbf{x})$ is the **Hessian** matrix of second derivatives, whose elements H_{ij} are given by $\partial^2 f / \partial x_i \partial x_j$. For our airport example, we can see from Equation (4.2) that $\mathbf{H}_f(\mathbf{x})$ is particularly simple: the off-diagonal elements are zero and the diagonal elements for airport i are just twice the number of cities in C_i. A moment's calculation shows that one step of the update moves airport i directly to the centroid of C_i, which is the minimum of the local expression for f from Equation (4.1).[7] For high-dimensional problems, however, computing the n^2 entries of the Hessian and inverting it may be expensive, so many approximate versions of the Newton–Raphson method have been developed.

Local search methods suffer from local maxima, ridges, and plateaux in continuous state spaces just as much as in discrete spaces. Random restarts and simulated annealing can be used and are often helpful. High-dimensional continuous spaces are, however, big places in which it is easy to get lost.

CONSTRAINED OPTIMIZATION

A final topic with which a passing acquaintance is useful is **constrained optimization**. An optimization problem is constrained if solutions must satisfy some hard constraints on the values of the variables. For example, in our airport-siting problem, we might constrain sites

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[7] In general, the Newton–Raphson update can be seen as fitting a quadratic surface to f at $\mathbf{x}$ and then moving directly to the minimum of that surface—which is also the minimum of f if f is quadratic.

<div style="float:left">LINEAR
PROGRAMMING</div>

<div style="float:left">CONVEX SET</div>

to be inside Romania and on dry land (rather than in the middle of lakes). The difficulty of constrained optimization problems depends on the nature of the constraints and the objective function. The best-known category is that of **linear programming** problems, in which constraints must be linear inequalities forming a **convex set** [8] and the objective function is also linear. The time complexity of linear programming is polynomial in the number of variables.

<div style="float:left">CONVEX
OPTIMIZATION</div>

Linear programming is probably the most widely studied and broadly useful class of optimization problems. It is a special case of the more general problem of **convex optimization**, which allows the constraint region to be any convex region and the objective to be any function that is convex within the constraint region. Under certain conditions, convex optimization problems are also polynomially solvable and may be feasible in practice with thousands of variables. Several important problems in machine learning and control theory can be formulated as convex optimization problems (see Chapter 20).

4.3   SEARCHING WITH NONDETERMINISTIC ACTIONS

In Chapter 3, we assumed that the environment is fully observable and deterministic and that the agent knows what the effects of each action are. Therefore, the agent can calculate exactly which state results from any sequence of actions and always knows which state it is in. Its percepts provide no new information after each action, although of course they tell the agent the initial state.

When the environment is either partially observable or nondeterministic (or both), percepts become useful. In a partially observable environment, every percept helps narrow down the set of possible states the agent might be in, thus making it easier for the agent to achieve its goals. When the environment is nondeterministic, percepts tell the agent which of the possible outcomes of its actions has actually occurred. In both cases, the future percepts cannot be determined in advance and the agent's future actions will depend on those future percepts.

<div style="float:left">CONTINGENCY PLAN</div>

<div style="float:left">STRATEGY</div>

So the solution to a problem is not a sequence but a **contingency plan** (also known as a **strategy**) that specifies what to do depending on what percepts are received. In this section, we examine the case of nondeterminism, deferring partial observability to Section 4.4.

4.3.1   The erratic vacuum world

As an example, we use the vacuum world, first introduced in Chapter 2 and defined as a search problem in Section 3.2.1. Recall that the state space has eight states, as shown in Figure 4.9. There are three actions—*Left*, *Right*, and *Suck*—and the goal is to clean up all the dirt (states 7 and 8). If the environment is observable, deterministic, and completely known, then the problem is trivially solvable by any of the algorithms in Chapter 3 and the solution is an action sequence. For example, if the initial state is 1, then the action sequence [*Suck*,*Right*,*Suck*] will reach a goal state, 8.

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[8]   A set of points $\mathcal{S}$ is convex if the line joining any two points in $\mathcal{S}$ is also contained in $\mathcal{S}$. A **convex function** is one for which the space "above" it forms a convex set; by definition, convex functions have no local (as opposed to global) minima.

Figure 4.9   The eight possible states of the vacuum world; states 7 and 8 are goal states.
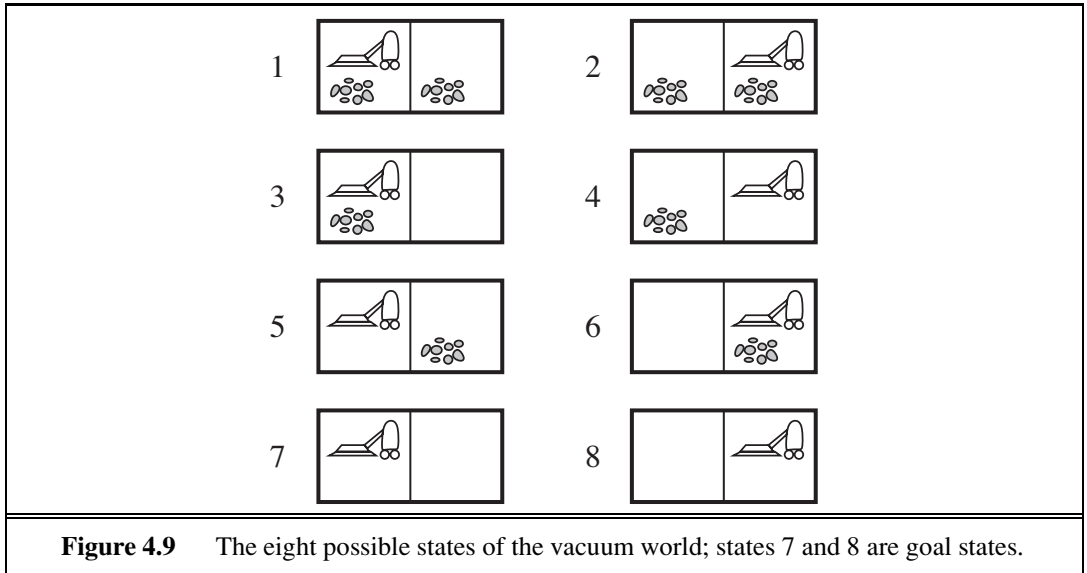
Now suppose that we introduce nondeterminism in the form of a powerful but erratic vacuum cleaner. In the **erratic vacuum world**, the *Suck* action works as follows:

- When applied to a dirty square the action cleans the square and sometimes cleans up dirt in an adjacent square, too.

- When applied to a clean square the action sometimes deposits dirt on the carpet.[9]

To provide a precise formulation of this problem, we need to generalize the notion of a **transition model** from Chapter 3. Instead of defining the transition model by a RESULT function that returns a single state, we use a RESULTS function that returns a *set* of possible outcome states. For example, in the erratic vacuum world, the *Suck* action in state 1 leads to a state in the set $\{5, 7\}$—the dirt in the right-hand square may or may not be vacuumed up.

We also need to generalize the notion of a **solution** to the problem. For example, if we start in state 1, there is no single *sequence* of actions that solves the problem. Instead, we need a contingency plan such as the following:

$$[Suck, \textbf{if } State = 5 \textbf{ then } [Right, Suck] \textbf{ else } []] .\qquad\qquad (4.3)$$

Thus, solutions for nondeterministic problems can contain nested **if–then–else** statements; this means that they are *trees* rather than sequences. This allows the selection of actions based on contingencies arising during execution. Many problems in the real, physical world are contingency problems because exact prediction is impossible. For this reason, many people keep their eyes open while walking around or driving.

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[9]   We assume that most readers face similar problems and can sympathize with our agent. We apologize to owners of modern, efficient home appliances who cannot take advantage of this pedagogical device.

4.3.2   AND–OR **search trees**

The next question is how to find contingent solutions to nondeterministic problems. As in Chapter 3, we begin by constructing search trees, but here the trees have a different character. In a deterministic environment, the only branching is introduced by the agent's own choices in each state. We call these nodes **OR nodes**. In the vacuum world, for example, at an OR node the agent chooses *Left or Right or Suck*. In a nondeterministic environment, branching is also introduced by the *environment's* choice of outcome for each action. We call these nodes **AND nodes**. For example, the *Suck* action in state 1 leads to a state in the set $\{5, 7\}$, so the agent would need to find a plan for state 5 *and* for state 7. These two kinds of nodes alternate, leading to an AND–OR **tree** as illustrated in Figure 4.10.

OR NODE

AND NODE

AND–OR TREE

A solution for an AND–OR search problem is a subtree that (1) has a goal node at every leaf, (2) specifies one action at each of its OR nodes, and (3) includes every outcome branch at each of its AND nodes. The solution is shown in bold lines in the figure; it corresponds to the plan given in Equation (4.3). (The plan uses if–then–else notation to handle the AND branches, but when there are more than two branches at a node, it might be better to use a **case**
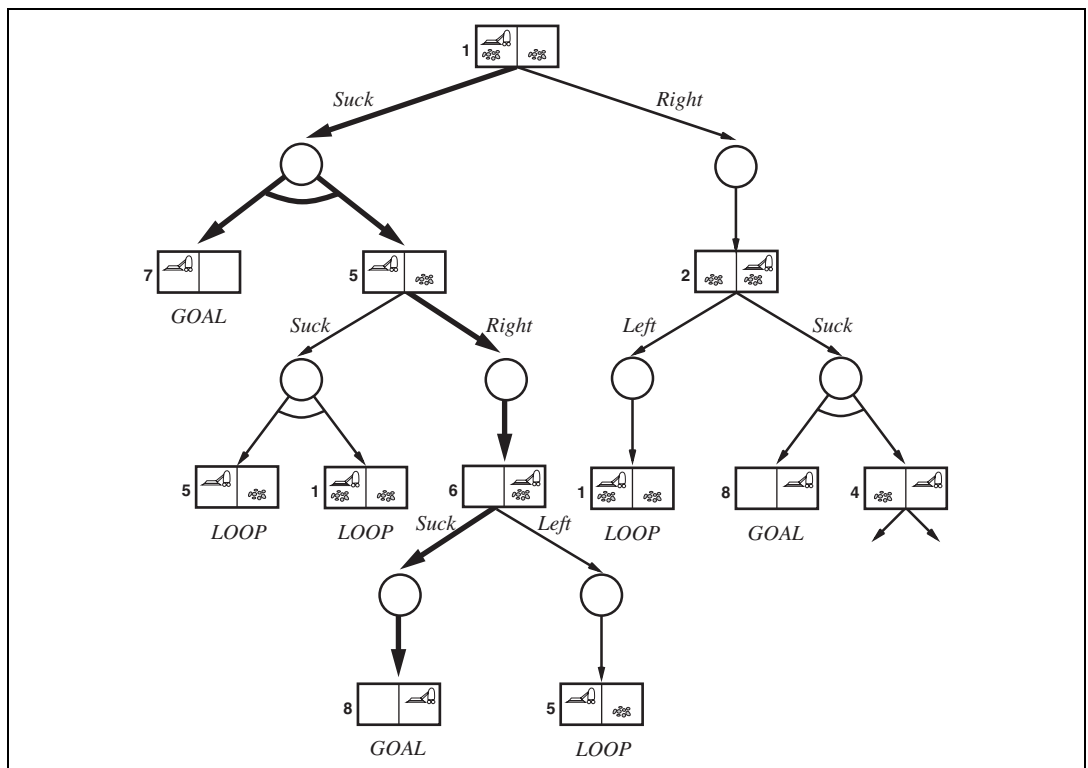
Figure 4.10   The first two levels of the search tree for the erratic vacuum world. State nodes are OR nodes where some action must be chosen. At the AND nodes, shown as circles, every outcome must be handled, as indicated by the arc linking the outgoing branches. The solution found is shown in bold lines.

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function AND-OR-GRAPH-SEARCH(*problem*) **returns** *a conditional plan*, *or failure*
  OR-SEARCH(*problem*.INITIAL-STATE, *problem*, [ ])

---

function OR-SEARCH(*state*, *problem*, *path*) **returns** *a conditional plan*, *or failure*
  if *problem*.GOAL-TEST(*state*) **then return** the empty plan
  if *state* is on *path* **then return** *failure*
  for each *action* **in** *problem*.ACTIONS(*state*) **do**
    plan ← AND-SEARCH(RESULTS(*state*, *action*), *problem*, [*state* | *path*])
    if *plan* ≠ *failure* **then return** [*action* | *plan*]
  return *failure*

---

function AND-SEARCH(*states*, *problem*, *path*) **returns** *a conditional plan*, *or failure*
  for each s_i **in** *states* **do**
    $plan_i$ ← OR-SEARCH(s_i, *problem*, *path*)
    if $plan_i$ = *failure* **then return** *failure*
  return [**if** s_1 **then** $plan_1$ **else if** s_2 **then** $plan_2$ **else** … **if** s_{n-1} **then** $plan_{n-1}$ **else** $plan_n$]
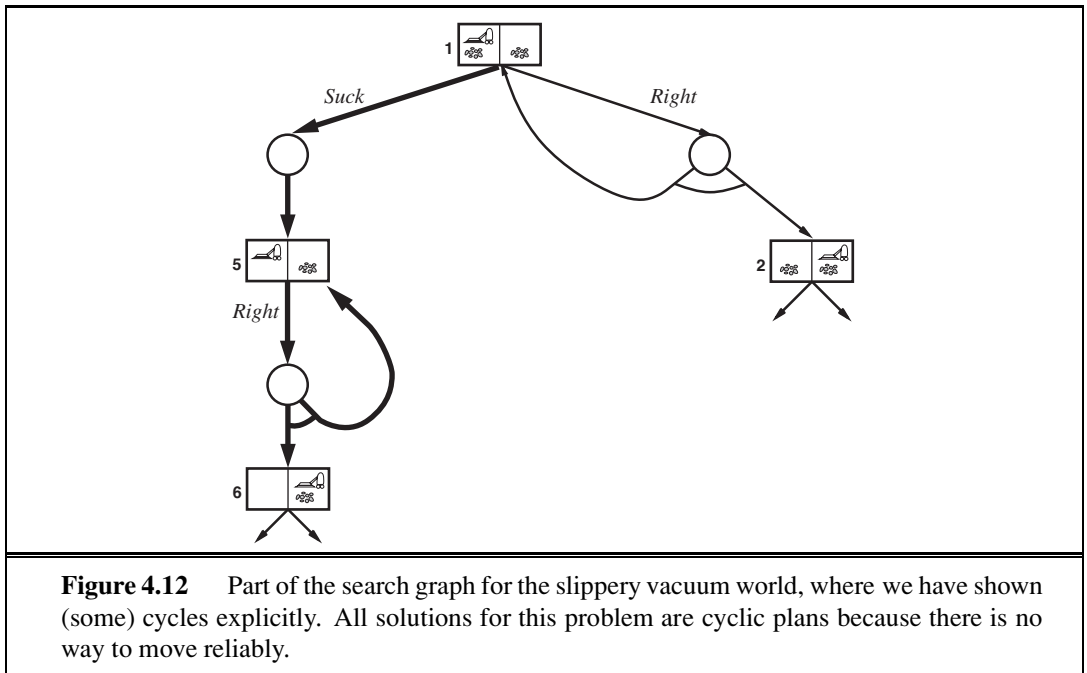

Figure 4.11   An algorithm for searching AND–OR graphs generated by nondeterministic environments. It returns a conditional plan that reaches a goal state in all circumstances. (The notation [*x* | *l*] refers to the list formed by adding object *x* to the front of list *l*.)

construct.) Modifying the basic problem-solving agent shown in Figure 3.1 to execute contingent solutions of this kind is straightforward. One may also consider a somewhat different agent design, in which the agent can act *before* it has found a guaranteed plan and deals with some contingencies only as they arise during execution. This type of **interleaving** of search and execution is also useful for exploration problems (see Section 4.5) and for game playing (see Chapter 5).

    Figure 4.11 gives a recursive, depth-first algorithm for AND–OR graph search. One key aspect of the algorithm is the way in which it deals with cycles, which often arise in nondeterministic problems (e.g., if an action sometimes has no effect or if an unintended effect can be corrected). If the current state is identical to a state on the path from the root, then it returns with failure. This doesn't mean that there is *no* solution from the current state; it simply means that if there *is* a noncyclic solution, it must be reachable from the earlier incarnation of the current state, so the new incarnation can be discarded. With this check, we ensure that the algorithm terminates in every finite state space, because every path must reach a goal, a dead end, or a repeated state. Notice that the algorithm does not check whether the current state is a repetition of a state on some *other* path from the root, which is important for efficiency. Exercise 4.5 investigates this issue.

    AND–OR graphs can also be explored by breadth-first or best-first methods. The concept of a heuristic function must be modified to estimate the cost of a contingent solution rather than a sequence, but the notion of admissibility carries over and there is an analog of the A* algorithm for finding optimal solutions. Pointers are given in the bibliographical notes at the end of the chapter.

INTERLEAVING

Figure 4.12   Part of the search graph for the slippery vacuum world, where we have shown (some) cycles explicitly. All solutions for this problem are cyclic plans because there is no way to move reliably.

4.3.3   Try, try again

Consider the slippery vacuum world, which is identical to the ordinary (non-erratic) vacuum world except that movement actions sometimes fail, leaving the agent in the same location. For example, moving *Right* in state 1 leads to the state set $\{1, 2\}$. Figure 4.12 shows part of the search graph; clearly, there are no longer any acyclic solutions from state 1, and AND-OR-GRAPH-SEARCH would return with failure. There is, however, a **cyclic solution**, which is to keep trying *Right* until it works. We can express this solution by adding a **label** to denote some portion of the plan and using that label later instead of repeating the plan itself. Thus, our cyclic solution is

CYCLIC SOLUTION

LABEL

$$[Suck, L_1 : \ Right, \textbf{if } State = 5 \textbf{ then } L_1 \textbf{ else } Suck] \ .$$

(A better syntax for the looping part of this plan would be "**while** $State = 5$ **do** $Right$.") In general a cyclic plan may be considered a solution provided that every leaf is a goal state and that a leaf is reachable from every point in the plan. The modifications needed to AND-OR-GRAPH-SEARCH are covered in Exercise 4.6. The key realization is that a loop in the state space back to a state L translates to a loop in the plan back to the point where the subplan for state L is executed.

Given the definition of a cyclic solution, an agent executing such a solution will eventually reach the goal *provided that each outcome of a nondeterministic action eventually occurs*. Is this condition reasonable? It depends on the reason for the nondeterminism. If the action rolls a die, then it's reasonable to suppose that eventually a six will be rolled. If the action is to insert a hotel card key into the door lock, but it doesn't work the first time, then perhaps it will eventually work, or perhaps one has the wrong key (or the wrong room!). After seven or

eight tries, most people will assume the problem is with the key and will go back to the front
desk to get a new one. One way to understand this decision is to say that the initial problem
formulation (observable, nondeterministic) is abandoned in favor of a different formulation
(partially observable, deterministic) where the failure is attributed to an unobservable prop-
erty of the key. We have more to say on this issue in Chapter 13.

4.4   SEARCHING WITH PARTIAL OBSERVATIONS

We now turn to the problem of partial observability, where the agent's percepts do not suf-
fice to pin down the exact state. As noted at the beginning of the previous section, if the
agent is in one of several possible states, then an action may lead to one of several possible
outcomes—*even if the environment is deterministic*. The key concept required for solving
BELIEF STATE   partially observable problems is the **belief state**, representing the agent's current belief about
the possible physical states it might be in, given the sequence of actions and percepts up to
that point. We begin with the simplest scenario for studying belief states, which is when the
agent has no sensors at all; then we add in partial sensing as well as nondeterministic actions.

4.4.1   Searching with no observation

When the agent's percepts provide *no information at all*, we have what is called a **sensor-**
SENSORLESS   **less** problem or sometimes a **conformant** problem. At first, one might think the sensorless
CONFORMANT   agent has no hope of solving a problem if it has no idea what state it's in; in fact, sensorless
problems are quite often solvable. Moreover, sensorless agents can be surprisingly useful,
primarily because they *don't* rely on sensors working properly. In manufacturing systems,
for example, many ingenious methods have been developed for orienting parts correctly from
an unknown initial position by using a sequence of actions with no sensing at all. The high
cost of sensing is another reason to avoid it: for example, doctors often prescribe a broad-
spectrum antibiotic rather than using the contingent plan of doing an expensive blood test,
then waiting for the results to come back, and then prescribing a more specific antibiotic and
perhaps hospitalization because the infection has progressed too far.

We can make a sensorless version of the vacuum world. Assume that the agent knows
the geography of its world, but doesn't know its location or the distribution of dirt. In that
case, its initial state could be any element of the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$. Now, consider what
happens if it tries the action *Right*. This will cause it to be in one of the states $\{2, 4, 6, 8\}$—the
agent now has more information! Furthermore, the action sequence [*Right*,*Suck*] will always
end up in one of the states $\{4, 8\}$. Finally, the sequence [*Right*,*Suck*,*Left*,*Suck*] is guaranteed
COERCION   to reach the goal state 7 no matter what the start state. We say that the agent can **coerce** the
world into state 7.

To solve sensorless problems, we search in the space of belief states rather than physical
states.[10] Notice that in belief-state space, the problem is *fully observable* because the agent

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[10] In a fully observable environment, each belief state contains one physical state. Thus, we can view the algo-
rithms in Chapter 3 as searching in a belief-state space of singleton belief states.

always knows its own belief state. Furthermore, the solution (if any) is always a sequence of actions. This is because, as in the ordinary problems of Chapter 3, the percepts received after each action are completely predictable—they're always empty! So there are no contingencies to plan for. This is true *even if the environment is nondeterminstic.*

It is instructive to see how the belief-state search problem is constructed. Suppose the underlying physical problem P is defined by ACTIONS$_P$, RESULT$_P$, GOAL-TEST$_P$, and STEP-COST$_P$. Then we can define the corresponding sensorless problem as follows:

- **Belief states**: The entire belief-state space contains every possible set of physical states. If P has N states, then the sensorless problem has up to 2^N states, although many may be unreachable from the initial state.

- **Initial state**: Typically the set of all states in P, although in some cases the agent will have more knowledge than this.

- **Actions**: This is slightly tricky. Suppose the agent is in belief state $b = \{s_1, s_2\}$, but ACTIONS$_P(s_1) \neq$ ACTIONS$_P(s_2)$; then the agent is unsure of which actions are legal. If we assume that illegal actions have no effect on the environment, then it is safe to take the *union* of all the actions in any of the physical states in the current belief state b:
  $$\text{ACTIONS}(b) = \bigcup_{s \in b} \text{ACTIONS}_P(s) \ .$$
  On the other hand, if an illegal action might be the end of the world, it is safer to allow only the *intersection*, that is, the set of actions legal in *all* the states. For the vacuum world, every state has the same legal actions, so both methods give the same result.

- **Transition model**: The agent doesn't know which state in the belief state is the right one; so as far as it knows, it might get to any of the states resulting from applying the action to one of the physical states in the belief state. For deterministic actions, the set of states that might be reached is
  $$b' = \text{RESULT}(b, a) = \{s' : s' = \text{RESULT}_P(s, a) \text{ and } s \in b\} \ . \tag{4.4}$$
  With deterministic actions, b' is never larger than b. With nondeterminism, we have
  $$\begin{aligned} b' = \text{RESULT}(b, a) &= \{s' : s' \in \text{RESULTS}_P(s, a) \text{ and } s \in b\} \\ &= \bigcup_{s \in b} \text{RESULTS}_P(s, a) \ , \end{aligned}$$
  which may be larger than b, as shown in Figure 4.13. The process of generating the new belief state after the action is called the **prediction** step; the notation $b' = \text{PREDICT}_P(b, a)$ will come in handy.

- **Goal test**: The agent wants a plan that is sure to work, which means that a belief state satisfies the goal only if *all* the physical states in it satisfy GOAL-TEST$_P$. The agent may *accidentally* achieve the goal earlier, but it won't *know* that it has done so.

- **Path cost**: This is also tricky. If the same action can have different costs in different states, then the cost of taking an action in a given belief state could be one of several values. (This gives rise to a new class of problems, which we explore in Exercise 4.9.) For now we assume that the cost of an action is the same in all states and so can be transferred directly from the underlying physical problem.

PREDICTION

Figure 4.13   (a) Predicting the next belief state for the sensorless vacuum world with a
deterministic action, $Right$. (b) Prediction for the same belief state and action in the slippery
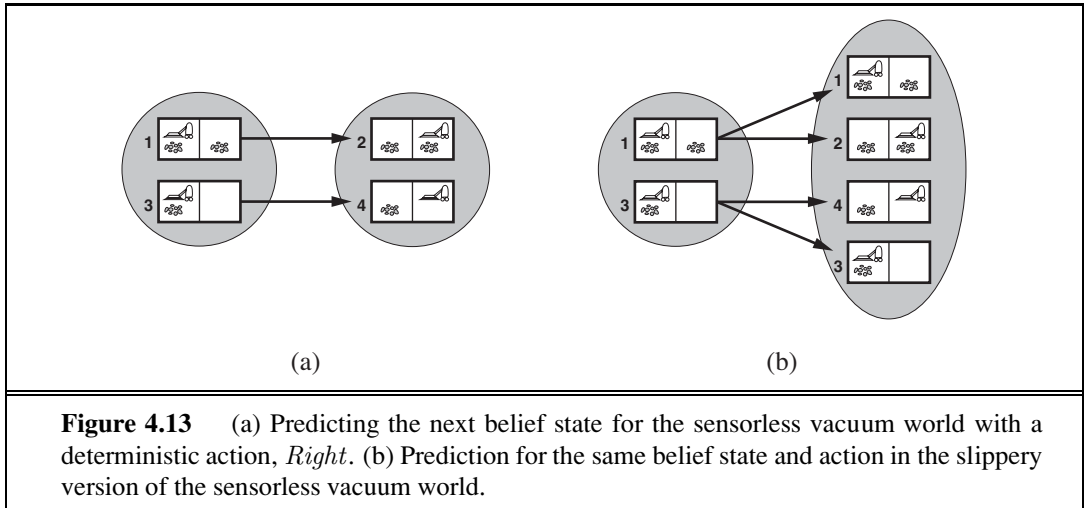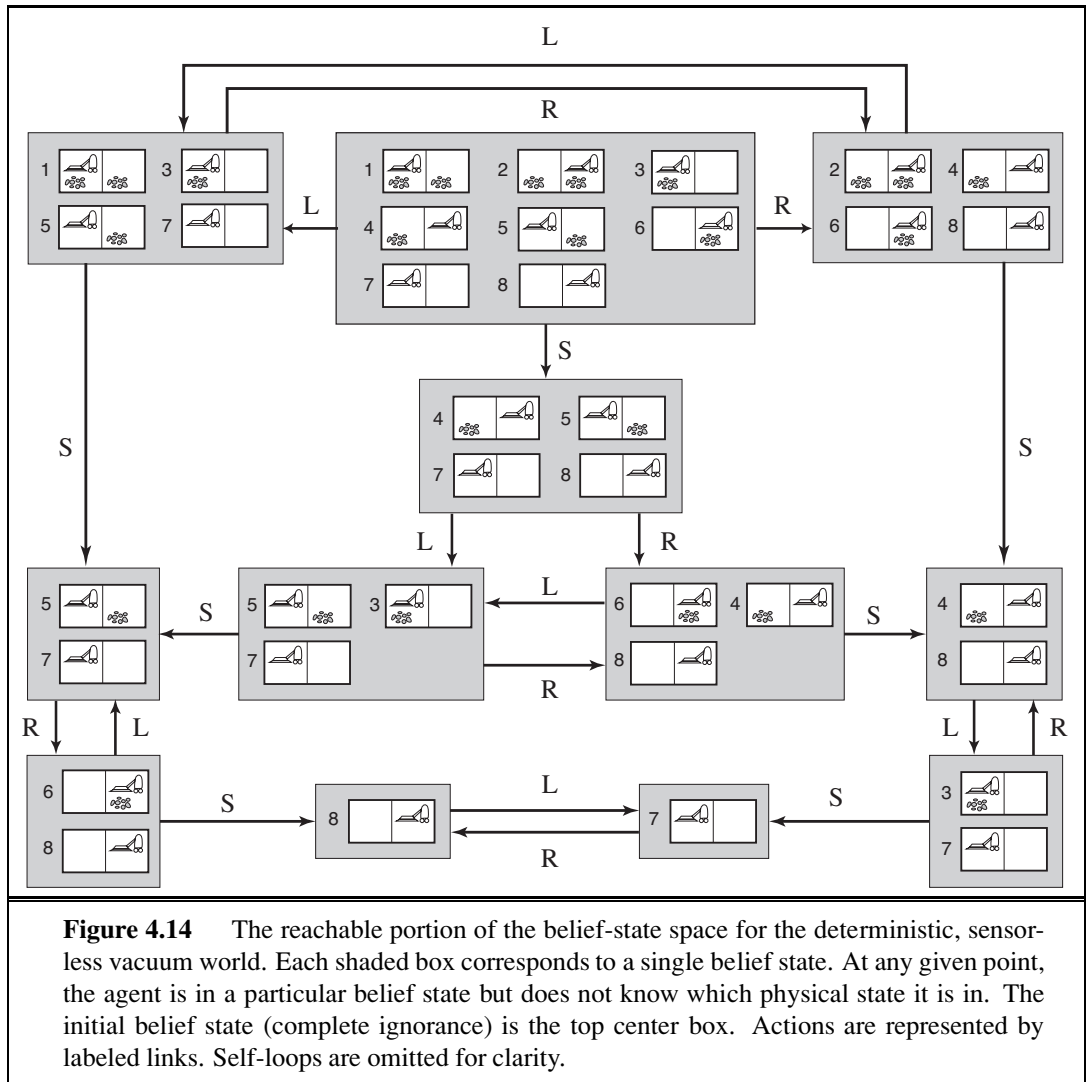version of the sensorless vacuum world.

Figure 4.14 shows the reachable belief-state space for the deterministic, sensorless vacuum
world. There are only 12 reachable belief states out of $2^8 = 256$ possible belief states.

    The preceding definitions enable the automatic construction of the belief-state problem
formulation from the definition of the underlying physical problem. Once this is done, we
can apply any of the search algorithms of Chapter 3. In fact, we can do a little bit more
than that. In "ordinary" graph search, newly generated states are tested to see if they are
identical to existing states. This works for belief states, too; for example, in Figure 4.14, the
action sequence [*Suck*,*Left*,*Suck*] starting at the initial state reaches the same belief state as
[*Right*,*Left*,*Suck*], namely, $\{5, 7\}$. Now, consider the belief state reached by [*Left*], namely,
$\{1, 3, 5, 7\}$. Obviously, this is not identical to $\{5, 7\}$, but it is a *superset*. It is easy to prove
(Exercise 4.8) that if an action sequence is a solution for a belief state b, it is also a solution for
any subset of b. Hence, we can discard a path reaching $\{1, 3, 5, 7\}$ if $\{5, 7\}$ has already been
generated. Conversely, if $\{1, 3, 5, 7\}$ has already been generated and found to be solvable,
then any *subset*, such as $\{5, 7\}$, is guaranteed to be solvable. This extra level of pruning may
dramatically improve the efficiency of sensorless problem solving.

    Even with this improvement, however, sensorless problem-solving as we have described
it is seldom feasible in practice. The difficulty is not so much the vastness of the belief-state
space—even though it is exponentially larger than the underlying physical state space; in
most cases the branching factor and solution length in the belief-state space and physical
state space are not so different. The real difficulty lies with the size of each belief state. For
example, the initial belief state for the 10×10 vacuum world contains 100×2^{100} or around
10^{32} physical states—far too many if we use the atomic representation, which is an explicit
list of states.

    One solution is to represent the belief state by some more compact description. In
English, we could say the agent knows "Nothing" in the initial state; after moving *Left*, we
could say, "Not in the rightmost column," and so on. Chapter 7 explains how to do this in a
formal representation scheme. Another approach is to avoid the standard search algorithms,
which treat belief states as black boxes just like any other problem state. Instead, we can look

Figure 4.14   The reachable portion of the belief-state space for the deterministic, sensorless vacuum world. Each shaded box corresponds to a single belief state. At any given point, the agent is in a particular belief state but does not know which physical state it is in. The initial belief state (complete ignorance) is the top center box. Actions are represented by labeled links. Self-loops are omitted for clarity.

INCREMENTAL
BELIEF-STATE
SEARCH

inside the belief states and develop **incremental belief-state search** algorithms that build up the solution one physical state at a time. For example, in the sensorless vacuum world, the initial belief state is $\{1, 2, 3, 4, 5, 6, 7, 8\}$, and we have to find an action sequence that works in all 8 states. We can do this by first finding a solution that works for state 1; then we check if it works for state 2; if not, go back and find a different solution for state 1, and so on. Just as an AND–OR search has to find a solution for every branch at an AND node, this algorithm has to find a solution for every state in the belief state; the difference is that AND–OR search can find a different solution for each branch, whereas an incremental belief-state search has to find *one* solution that works for *all* the states.

The main advantage of the incremental approach is that it is typically able to detect failure quickly—when a belief state is unsolvable, it is usually the case that a small subset of the belief state, consisting of the first few states examined, is also unsolvable. In some cases,

this leads to a speedup proportional to the size of the belief states, which may themselves be as large as the physical state space itself.

Even the most efficient solution algorithm is not of much use when no solutions exist. Many things just cannot be done without sensing. For example, the sensorless 8-puzzle is impossible. On the other hand, a little bit of sensing can go a long way. For example, every 8-puzzle instance is solvable if just one square is visible—the solution involves moving each tile in turn into the visible square and then keeping track of its location.

4.4.2   Searching with observations

For a general partially observable problem, we have to specify how the environment generates percepts for the agent. For example, we might define the local-sensing vacuum world to be one in which the agent has a position sensor and a local dirt sensor but has no sensor capable of detecting dirt in other squares. The formal problem specification includes a $\text{PERCEPT}(s)$ function that returns the percept received in a given state. (If sensing is nondeterministic, then we use a PERCEPTS function that returns a set of possible percepts.) For example, in the local-sensing vacuum world, the PERCEPT in state 1 is $[A, Dirty]$. Fully observable problems are a special case in which $\text{PERCEPT}(s) = s$ for every state s, while sensorless problems are a special case in which $\text{PERCEPT}(s) = null$.

When observations are partial, it will usually be the case that several states could have produced any given percept. For example, the percept $[A, Dirty]$ is produced by state 3 as well as by state 1. Hence, given this as the initial percept, the initial belief state for the local-sensing vacuum world will be $\{1, 3\}$. The ACTIONS, STEP-COST, and GOAL-TEST are constructed from the underlying physical problem just as for sensorless problems, but the transition model is a bit more complicated. We can think of transitions from one belief state to the next for a particular action as occurring in three stages, as shown in Figure 4.15:

- The **prediction** stage is the same as for sensorless problems: given the action a in belief state b, the predicted belief state is $\hat{b} = \text{PREDICT}(b, a)$.[11]

- The **observation prediction** stage determines the set of percepts o that could be observed in the predicted belief state:

$$\text{POSSIBLE-PERCEPTS}(\hat{b}) = \{o : o = \text{PERCEPT}(s) \text{ and } s \in \hat{b}\} .$$

- The **update** stage determines, for each possible percept, the belief state that would result from the percept. The new belief state b_o is just the set of states in $\hat{b}$ that could have produced the percept:

$$b_o = \text{UPDATE}(\hat{b}, o) = \{s : o = \text{PERCEPT}(s) \text{ and } s \in \hat{b}\} .$$

  Notice that each updated belief state b_o can be no larger than the predicted belief state $\hat{b}$; observations can only help reduce uncertainty compared to the sensorless case. Moreover, for deterministic sensing, the belief states for the different possible percepts will be disjoint, forming a *partition* of the original predicted belief state.

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[11] Here, and throughout the book, the "hat" in $\hat{b}$ means an estimated or predicted value for b.
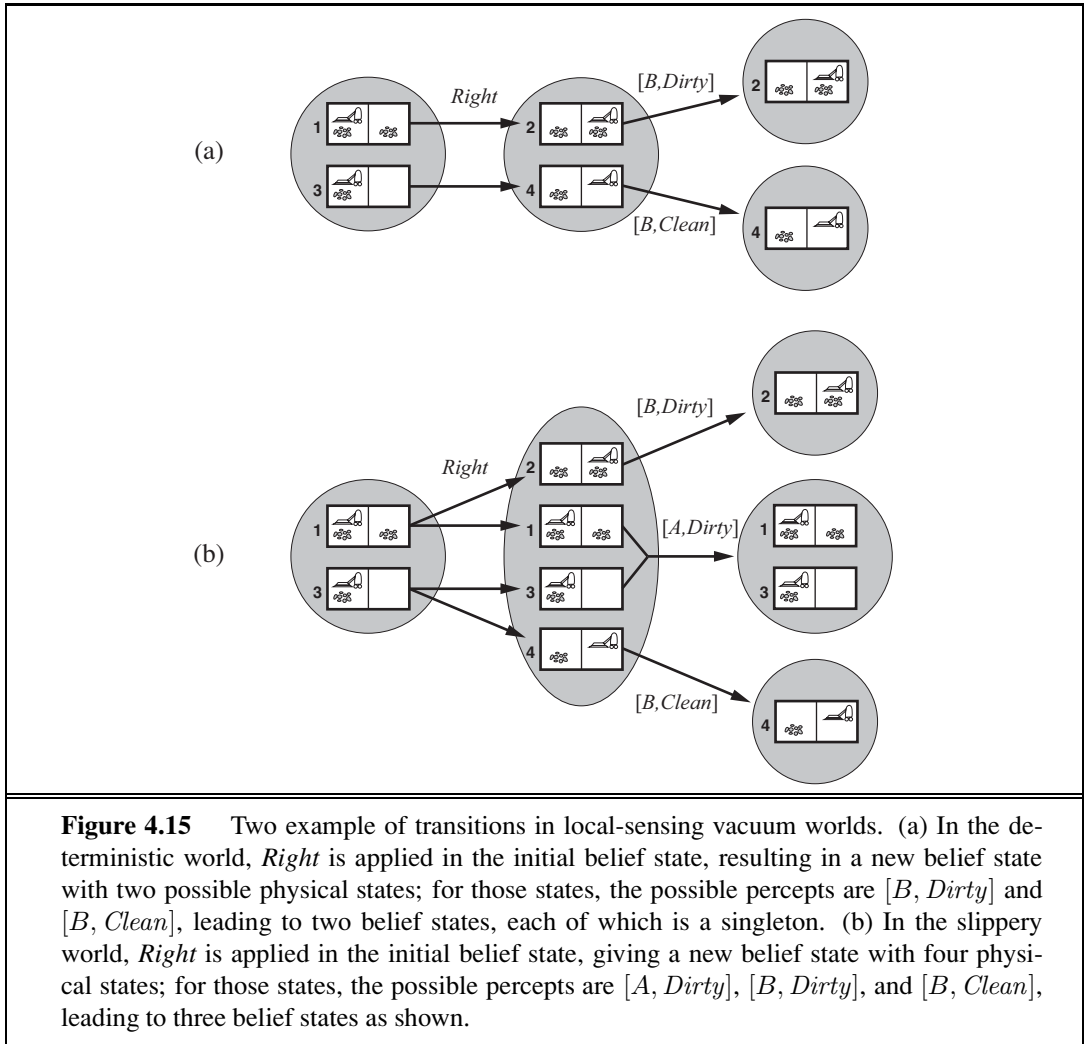
Figure 4.15   Two example of transitions in local-sensing vacuum worlds. (a) In the deterministic world, *Right* is applied in the initial belief state, resulting in a new belief state with two possible physical states; for those states, the possible percepts are $[B, Dirty]$ and $[B, Clean]$, leading to two belief states, each of which is a singleton. (b) In the slippery world, *Right* is applied in the initial belief state, giving a new belief state with four physical states; for those states, the possible percepts are $[A, Dirty]$, $[B, Dirty]$, and $[B, Clean]$, leading to three belief states as shown.
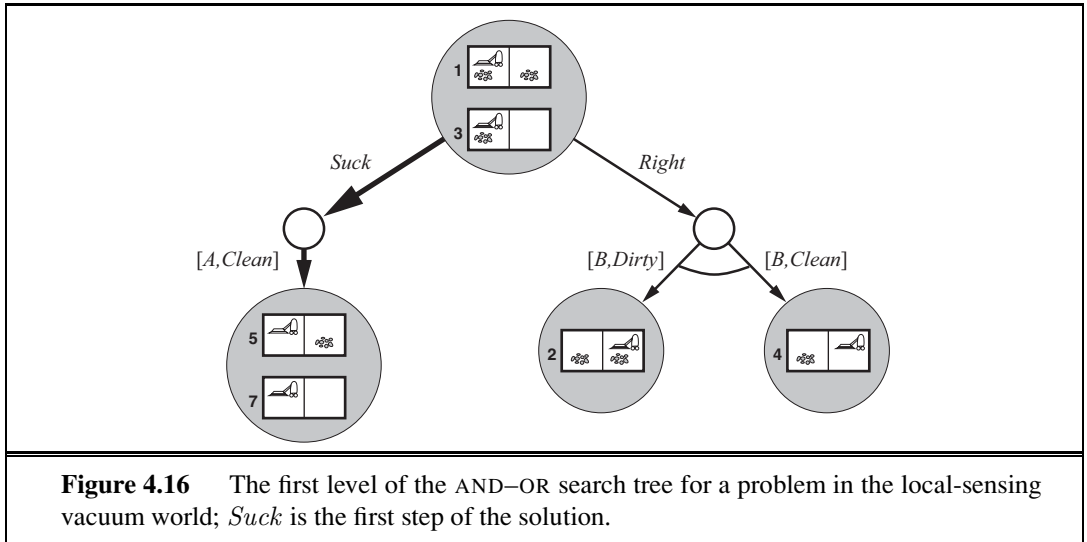
Putting these three stages together, we obtain the possible belief states resulting from a given action and the subsequent possible percepts:

$$\text{RESULTS}(b, a) = \{b_o : b_o = \text{UPDATE}(\text{PREDICT}(b, a), o) \text{ and}$$
$$o \in \text{POSSIBLE-PERCEPTS}(\text{PREDICT}(b, a))\} . \qquad (4.5)$$

Again, the nondeterminism in the partially observable problem comes from the inability to predict exactly which percept will be received after acting; underlying nondeterminism in the physical environment may *contribute* to this inability by enlarging the belief state at the prediction stage, leading to more percepts at the observation stage.

4.4.3   Solving partially observable problems

The preceding section showed how to derive the RESULTS function for a nondeterministic belief-state problem from an underlying physical problem and the PERCEPT function. Given

Figure 4.16   The first level of the AND–OR search tree for a problem in the local-sensing vacuum world; $Suck$ is the first step of the solution.

such a formulation, the AND–OR search algorithm of Figure 4.11 can be applied directly to derive a solution. Figure 4.16 shows part of the search tree for the local-sensing vacuum world, assuming an initial percept $[A, Dirty]$. The solution is the conditional plan

  [*Suck*, *Right*, **if** $Bstate = \{6\}$ **then** *Suck* **else** $[\,]$] .
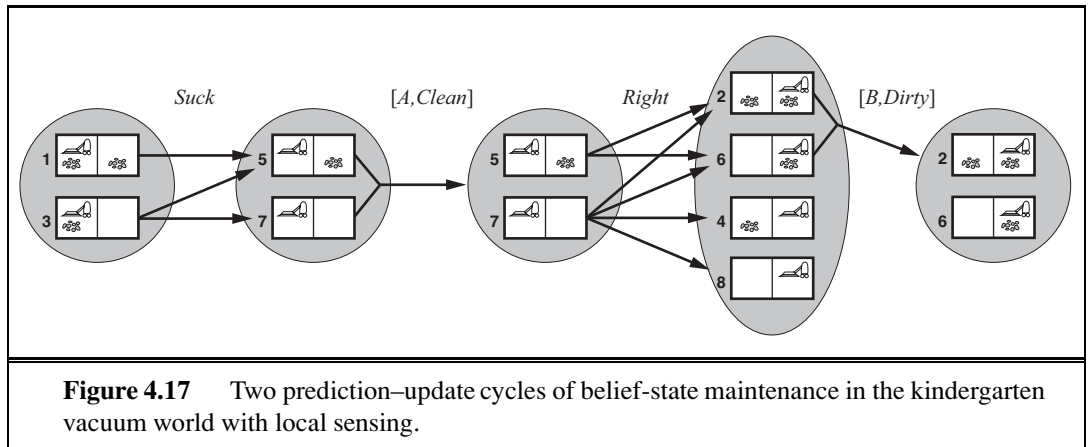
Notice that, because we supplied a belief-state problem to the AND–OR search algorithm, it returned a conditional plan that tests the belief state rather than the actual state. This is as it should be: in a partially observable environment the agent won't be able to execute a solution that requires testing the actual state.

  As in the case of standard search algorithms applied to sensorless problems, the AND–OR search algorithm treats belief states as black boxes, just like any other states. One can improve on this by checking for previously generated belief states that are subsets or supersets of the current state, just as for sensorless problems. One can also derive incremental search algorithms, analogous to those described for sensorless problems, that provide substantial speedups over the black-box approach.

4.4.4   An agent for partially observable environments

The design of a problem-solving agent for partially observable environments is quite similar to the simple problem-solving agent in Figure 3.1: the agent formulates a problem, calls a search algorithm (such as AND-OR-GRAPH-SEARCH) to solve it, and executes the solution. There are two main differences. First, the solution to a problem will be a conditional plan rather than a sequence; if the first step is an if–then–else expression, the agent will need to test the condition in the if-part and execute the then-part or the else-part accordingly. Second, the agent will need to maintain its belief state as it performs actions and receives percepts. This process resembles the prediction–observation–update process in Equation (4.5) but is actually simpler because the percept is given by the environment rather than calculated by the

Figure 4.17   Two prediction–update cycles of belief-state maintenance in the kindergarten vacuum world with local sensing.

agent. Given an initial belief state b, an action a, and a percept o, the new belief state is:

$$b' = \text{UPDATE}(\text{PREDICT}(b,a), o) . \tag{4.6}$$

Figure 4.17 shows the belief state being maintained in the *kindergarten* vacuum world with local sensing, wherein any square may become dirty at any time unless the agent is actively cleaning it at that moment.[12]

In partially observable environments—which include the vast majority of real-world environments—maintaining one's belief state is a core function of any intelligent system. This function goes under various names, including **monitoring**, **filtering** and **state estimation**. Equation (4.6) is called a **recursive** state estimator because it computes the new belief state from the previous one rather than by examining the entire percept sequence. If the agent is not to "fall behind," the computation has to happen as fast as percepts are coming in. As the environment becomes more complex, the exact update computation becomes infeasible and the agent will have to compute an approximate belief state, perhaps focusing on the implications of the percept for the aspects of the environment that are of current interest. Most work on this problem has been done for stochastic, continuous-state environments with the tools of probability theory, as explained in Chapter 15. Here we will show an example in a discrete environment with detrministic sensors and nondeterministic actions.

The example concerns a robot with the task of **localization**: working out where it is, given a map of the world and a sequence of percepts and actions. Our robot is placed in the maze-like environment of Figure 4.18. The robot is equipped with four sonar sensors that tell whether there is an obstacle—the outer wall or a black square in the figure—in each of the four compass directions. We assume that the sensors give perfectly correct data, and that the robot has a correct map of the enviornment. But unfortunately the robot's navigational system is broken, so when it executes a *Move* action, it moves randomly to one of the adjacent squares. The robot's task is to determine its current location.

Suppose the robot has just been switched on, so it does not know where it is. Thus its initial belief state b consists of the set of all locations. The the robot receives the percept

MONITORING
FILTERING
STATE ESTIMATION
RECURSIVE

LOCALIZATION

[12] The usual apologies to those who are unfamiliar with the effect of small children on the environment.
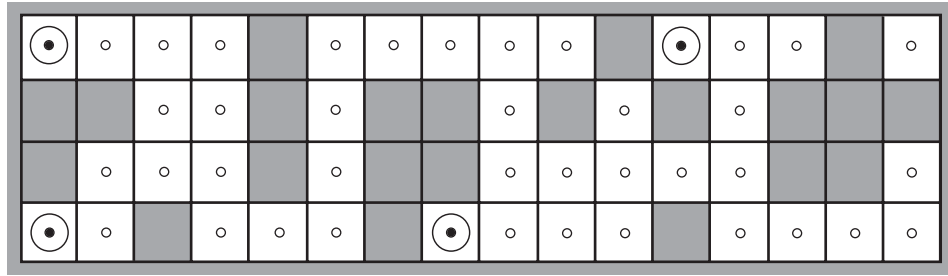
(a) Possible locations of robot after $E_1 = NSW$
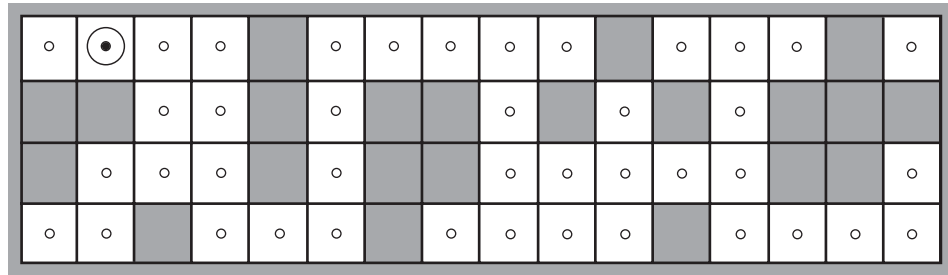
(b) Possible locations of robot After $E_1 = NSW, E_2 = NS$

Figure 4.18   Possible positions of the robot, $\odot$, (a) after one observation $E_1 = NSW$ and (b) after a second observation $E_2 = NS$. When sensors are noiseless and the transition model is accurate, there are no other possible locations for the robot consistent with this sequence of two observations.

NSW, meaning there are obstacles to the north, west, and south, and does an update using the equation $b_o = \text{UPDATE}(b)$, yielding the 4 locations shown in Figure 4.18(a). You can inspect the maze to see that those are the only four locations that yield the percept NWS.

Next the robot executes a *Move* action, but the result is nondeterministic. The new belief state, $b_a = \text{PREDICT}(b_o, Move)$, contains all the locations that are one step away from the locations in b_o. When the second percept, NS, arrives, the robot does $\text{UPDATE}(b_a, NS)$ and finds that the belief state has collapsed down to the single location shown in Figure 4.18(b). That's the only location that could be the result of

$$\text{UPDATE}(\text{PREDICT}(\text{UPDATE}(b, NSW), Move), NS) .$$

With nondetermnistic actions the PREDICT step grows the belief state, but the UPDATE step shrinks it back down—as long as the percepts provide some useful identifying information. Sometimes the percepts don't help much for localization: If there were one or more long east-west corridors, then a robot could receive a long sequence of NS percepts, but never know where in the corridor(s) it was.

4.5   ONLINE SEARCH AGENTS AND UNKNOWN ENVIRONMENTS

OFFLINE SEARCH

ONLINE SEARCH

So far we have concentrated on agents that use **offline search** algorithms. They compute a complete solution before setting foot in the real world and then execute the solution. In contrast, an **online search**[13] agent **interleaves** computation and action: first it takes an action, then it observes the environment and computes the next action. Online search is a good idea in dynamic or semidynamic domains—domains where there is a penalty for sitting around and computing too long. Online search is also helpful in nondeterministic domains because it allows the agent to focus its computational efforts on the contingencies that actually arise rather than those that *might* happen but probably won't. Of course, there is a tradeoff: the more an agent plans ahead, the less often it will find itself up the creek without a paddle.

EXPLORATION
PROBLEM

Online search is a *necessary* idea for unknown environments, where the agent does not know what states exist or what its actions do. In this state of ignorance, the agent faces an **exploration problem** and must use its actions as experiments in order to learn enough to make deliberation worthwhile.

The canonical example of online search is a robot that is placed in a new building and must explore it to build a map that it can use for getting from A to B. Methods for escaping from labyrinths—required knowledge for aspiring heroes of antiquity—are also examples of online search algorithms. Spatial exploration is not the only form of exploration, however. Consider a newborn baby: it has many possible actions but knows the outcomes of none of them, and it has experienced only a few of the possible states that it can reach. The baby's gradual discovery of how the world works is, in part, an online search process.
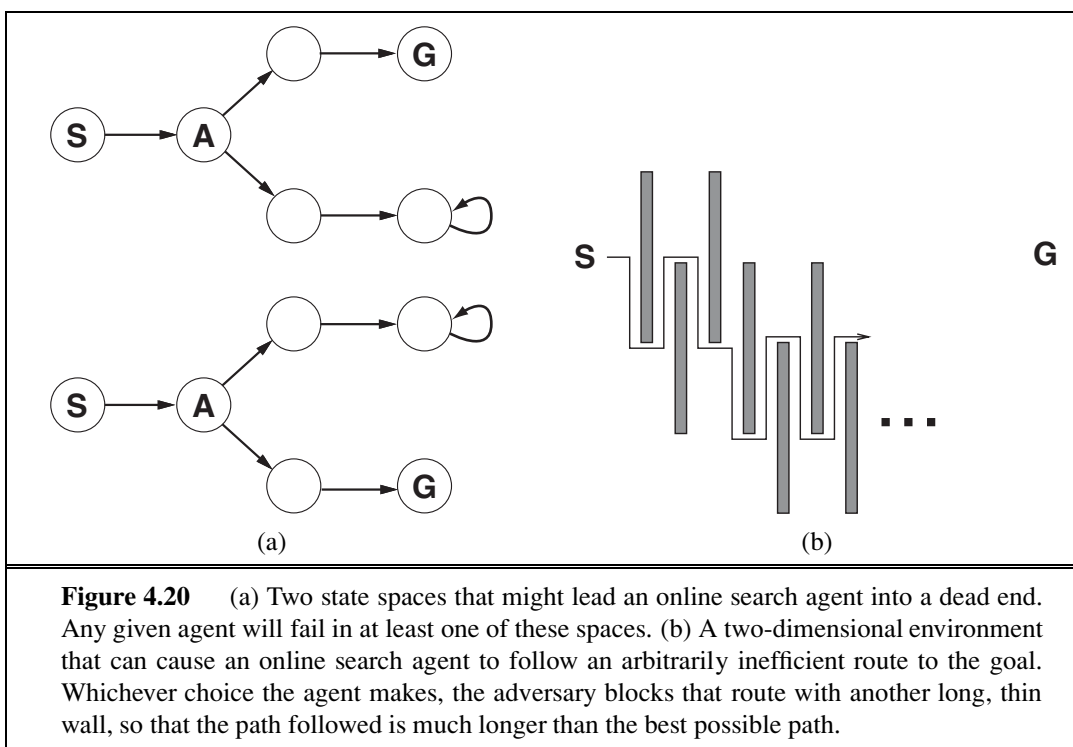
4.5.1   Online search problems

An online search problem must be solved by an agent executing actions, rather than by pure computation. We assume a deterministic and fully observable environment (Chapter 17 relaxes these assumptions), but we stipulate that the agent knows only the following:

- ACTIONS(s), which returns a list of actions allowed in state s;
- The step-cost function $c(s, a, s')$—note that this cannot be used until the agent knows that s' is the outcome; and
- GOAL-TEST(s).

Note in particular that the agent *cannot* determine RESULT(s, a) except by actually being in s and doing a. For example, in the maze problem shown in Figure 4.19, the agent does not know that going Up from (1,1) leads to (1,2); nor, having done that, does it know that going $Down$ will take it back to (1,1). This degree of ignorance can be reduced in some applications—for example, a robot explorer might know how its movement actions work and be ignorant only of the locations of obstacles.

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[13] The term "online" is commonly used in computer science to refer to algorithms that must process input data as they are received rather than waiting for the entire input data set to become available.

Figure 4.19   A simple maze problem. The agent starts at S and must reach G but knows nothing of the environment.

(a)   (b)

Figure 4.20   (a) Two state spaces that might lead an online search agent into a dead end. Any given agent will fail in at least one of these spaces. (b) A two-dimensional environment that can cause an online search agent to follow an arbitrarily inefficient route to the goal. Whichever choice the agent makes, the adversary blocks that route with another long, thin wall, so that the path followed is much longer than the best possible path.

Finally, the agent might have access to an admissible heuristic function $h(s)$ that estimates the distance from the current state to a goal state. For example, in Figure 4.19, the agent might know the location of the goal and be able to use the Manhattan-distance heuristic.

Typically, the agent's objective is to reach a goal state while minimizing cost. (Another possible objective is simply to explore the entire environment.) The cost is the total path cost of the path that the agent actually travels. It is common to compare this cost with the path cost of the path the agent would follow *if it knew the search space in advance*—that is, the actual shortest path (or shortest complete exploration). In the language of online algorithms, COMPETITIVE RATIO   this is called the **competitive ratio**; we would like it to be as small as possible.

Although this sounds like a reasonable request, it is easy to see that the best achievable competitive ratio is infinite in some cases. For example, if some actions are **irreversible**— i.e., they lead to a state from which no action leads back to the previous state—the online search might accidentally reach a **dead-end** state from which no goal state is reachable. Perhaps the term "accidentally" is unconvincing—after all, there might be an algorithm that happens not to take the dead-end path as it explores. Our claim, to be more precise, is that *no algorithm can avoid dead ends in all state spaces.* Consider the two dead-end state spaces in Figure 4.20(a). To an online search algorithm that has visited states S and A, the two state spaces look *identical*, so it must make the same decision in both. Therefore, it will fail in one of them. This is an example of an **adversary argument**—we can imagine an adversary constructing the state space while the agent explores it and putting the goals and dead ends wherever it chooses.

IRREVERSIBLE

DEAD END

ADVERSARY
ARGUMENT

Dead ends are a real difficulty for robot exploration—staircases, ramps, cliffs, one-way streets, and all kinds of natural terrain present opportunities for irreversible actions. To make progress, we simply assume that the state space is **safely explorable**—that is, some goal state is reachable from every reachable state. State spaces with reversible actions, such as mazes and 8-puzzles, can be viewed as undirected graphs and are clearly safely explorable.

SAFELY EXPLORABLE

Even in safely explorable environments, no bounded competitive ratio can be guaranteed if there are paths of unbounded cost. This is easy to show in environments with irreversible actions, but in fact it remains true for the reversible case as well, as Figure 4.20(b) shows. For this reason, it is common to describe the performance of online search algorithms in terms of the size of the entire state space rather than just the depth of the shallowest goal.

4.5.2   Online search agents

After each action, an online agent receives a percept telling it what state it has reached; from this information, it can augment its map of the environment. The current map is used to decide where to go next. This interleaving of planning and action means that online search algorithms are quite different from the offline search algorithms we have seen previously. For example, offline algorithms such as A* can expand a node in one part of the space and then immediately expand a node in another part of the space, because node expansion involves simulated rather than real actions. An online algorithm, on the other hand, can discover successors only for a node that it physically occupies. To avoid traveling all the way across the tree to expand the next node, it seems better to expand nodes in a *local* order. Depth-first search has exactly this property because (except when backtracking) the next node expanded is a child of the previous node expanded.

An online depth-first search agent is shown in Figure 4.21. This agent stores its map in a table, RESULT$[s, a]$, that records the state resulting from executing action a in state s. Whenever an action from the current state has not been explored, the agent tries that action. The difficulty comes when the agent has tried all the actions in a state. In offline depth-first search, the state is simply dropped from the queue; in an online search, the agent has to backtrack physically. In depth-first search, this means going back to the state from which the agent most recently entered the current state. To achieve that, the algorithm keeps a table that

---

function ONLINE-DFS-AGENT(s') **returns** an action
 inputs: s', a percept that identifies the current state
 persistent: $result$, a table indexed by state and action, initially empty
      $untried$, a table that lists, for each state, the actions not yet tried
      $unbacktracked$, a table that lists, for each state, the backtracks not yet tried
      s, a, the previous state and action, initially null

 if GOAL-TEST(s') **then return** $stop$
 if s' is a new state (not in $untried$) **then** $untried[s'] \leftarrow$ ACTIONS(s')
 if s is not null **then**
  $result[s, a] \leftarrow s'$
  add s to the front of $unbacktracked[s']$
 if $untried[s']$ is empty **then**
  if $unbacktracked[s']$ is empty **then return** $stop$
  else $a \leftarrow$ an action b such that $result[s', b] =$ POP($unbacktracked[s']$)
 else $a \leftarrow$ POP($untried[s']$)
 $s \leftarrow s'$
 return a

Figure 4.21 An online search agent that uses depth-first exploration. The agent is applicable only in state spaces in which every action can be "undone" by some other action.

lists, for each state, the predecessor states to which the agent has not yet backtracked. If the agent has run out of states to which it can backtrack, then its search is complete.

We recommend that the reader trace through the progress of ONLINE-DFS-AGENT when applied to the maze given in Figure 4.19. It is fairly easy to see that the agent will, in the worst case, end up traversing every link in the state space exactly twice. For exploration, this is optimal; for finding a goal, on the other hand, the agent's competitive ratio could be arbitrarily bad if it goes off on a long excursion when there is a goal right next to the initial state. An online variant of iterative deepening solves this problem; for an environment that is a uniform tree, the competitive ratio of such an agent is a small constant.

Because of its method of backtracking, ONLINE-DFS-AGENT works only in state spaces where the actions are reversible. There are slightly more complex algorithms that work in general state spaces, but no such algorithm has a bounded competitive ratio.

4.5.3   Online local search

Like depth-first search, **hill-climbing search** has the property of locality in its node expansions. In fact, because it keeps just one current state in memory, hill-climbing search is *already* an online search algorithm! Unfortunately, it is not very useful in its simplest form because it leaves the agent sitting at local maxima with nowhere to go. Moreover, random restarts cannot be used, because the agent cannot transport itself to a new state.

RANDOM WALK   Instead of random restarts, one might consider using a **random walk** to explore the environment. A random walk simply selects at random one of the available actions from the

Figure 4.22   An environment in which a random walk will take exponentially many steps to find the goal.
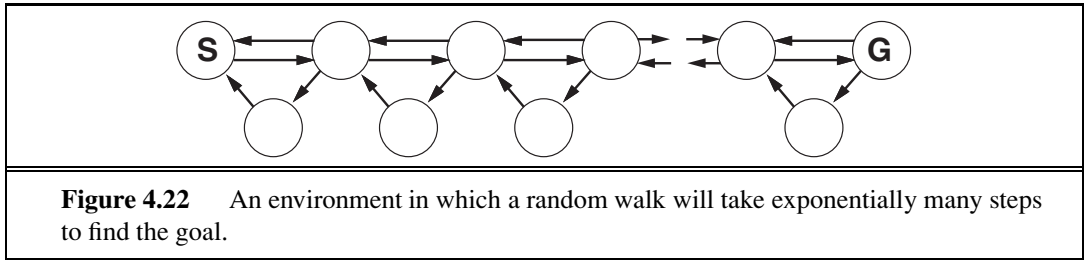
current state; preference can be given to actions that have not yet been tried. It is easy to prove that a random walk will *eventually* find a goal or complete its exploration, provided that the space is finite.[14] On the other hand, the process can be very slow. Figure 4.22 shows an environment in which a random walk will take exponentially many steps to find the goal because, at each step, backward progress is twice as likely as forward progress. The example is contrived, of course, but there are many real-world state spaces whose topology causes these kinds of "traps" for random walks.

Augmenting hill climbing with *memory* rather than randomness turns out to be a more effective approach. The basic idea is to store a "current best estimate" $H(s)$ of the cost to reach the goal from each state that has been visited. $H(s)$ starts out being just the heuristic estimate $h(s)$ and is updated as the agent gains experience in the state space. Figure 4.23 shows a simple example in a one-dimensional state space. In (a), the agent seems to be stuck in a flat local minimum at the shaded state. Rather than staying where it is, the agent should follow what seems to be the best path to the goal given the current cost estimates for its neighbors. The estimated cost to reach the goal through a neighbor s' is the cost to get to s' plus the estimated cost to get to a goal from there—that is, $c(s, a, s') + H(s')$. In the example, there are two actions, with estimated costs $1 + 9$ and $1 + 2$, so it seems best to move right. Now, it is clear that the cost estimate of 2 for the shaded state was overly optimistic. Since the best move cost 1 and led to a state that is at least 2 steps from a goal, the shaded state must be at least 3 steps from a goal, so its H should be updated accordingly, as shown in Figure 4.23(b). Continuing this process, the agent will move back and forth twice more, updating H each time and "flattening out" the local minimum until it escapes to the right.
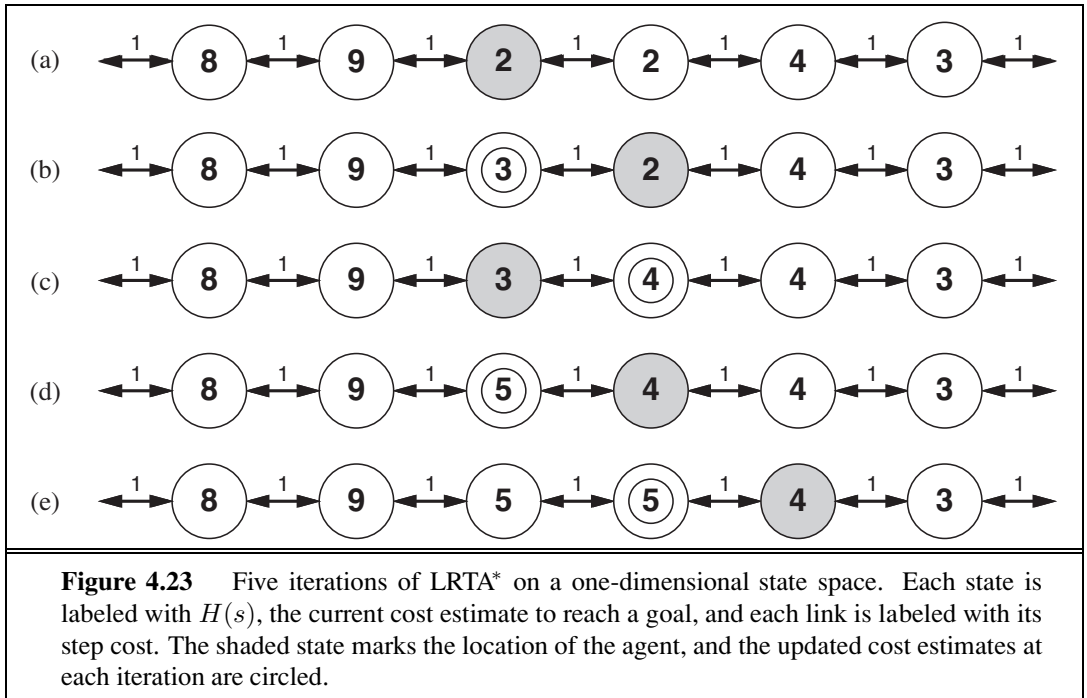
LRTA*

An agent implementing this scheme, which is called learning real-time A* (**LRTA***), is shown in Figure 4.24. Like ONLINE-DFS-AGENT, it builds a map of the environment in the *result* table. It updates the cost estimate for the state it has just left and then chooses the "apparently best" move according to its current cost estimates. One important detail is that actions that have not yet been tried in a state s are always assumed to lead immediately to the

OPTIMISM UNDER
UNCERTAINTY

goal with the least possible cost, namely $h(s)$. This **optimism under uncertainty** encourages the agent to explore new, possibly promising paths.

An LRTA* agent is guaranteed to find a goal in any finite, safely explorable environment. Unlike A*, however, it is not complete for infinite state spaces—there are cases where it can be led infinitely astray. It can explore an environment of n states in $O(n^2)$ steps in the worst case,

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[14] Random walks are complete on infinite one-dimensional and two-dimensional grids. On a three-dimensional grid, the probability that the walk ever returns to the starting point is only about 0.3405 (Hughes, 1995).

Figure 4.23   Five iterations of LRTA* on a one-dimensional state space. Each state is
labeled with $H(s)$, the current cost estimate to reach a goal, and each link is labeled with its
step cost. The shaded state marks the location of the agent, and the updated cost estimates at
each iteration are circled.

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function LRTA*-AGENT(s') **returns** an action
  inputs: s', a percept that identifies the current state
  persistent: $result$, a table, indexed by state and action, initially empty
             H, a table of cost estimates indexed by state, initially empty
             s, a, the previous state and action, initially null

  if GOAL-TEST(s') **then return** $stop$
  if s' is a new state (not in H) **then** $H[s'] \leftarrow h(s')$
  if s is not null
     $result[s, a] \leftarrow s'$
     $H[s] \leftarrow \min_{b \in \text{ACTIONS}(s)} \text{LRTA*-COST}(s, b, result[s, b], H)$
  $a \leftarrow$ an action b in ACTIONS(s') that minimizes LRTA*-COST($s', b, result[s', b], H$)
  $s \leftarrow s'$
  return a

function LRTA*-COST(s, a, s', H) **returns** a cost estimate
  if s' is undefined **then return** $h(s)$
  else return $c(s, a, s') + H[s']$

Figure 4.24   LRTA*-AGENT selects an action according to the values of neighboring
states, which are updated as the agent moves about the state space.

but often does much better. The LRTA* agent is just one of a large family of online agents that one can define by specifying the action selection rule and the update rule in different ways. We discuss this family, developed originally for stochastic environments, in Chapter 21.

4.5.4   Learning in online search

The initial ignorance of online search agents provides several opportunities for learning. First, the agents learn a "map" of the environment—more precisely, the outcome of each action in each state—simply by recording each of their experiences. (Notice that the assumption of deterministic environments means that one experience is enough for each action.) Second, the local search agents acquire more accurate estimates of the cost of each state by using local updating rules, as in LRTA*. In Chapter 21, we show that these updates eventually converge to *exact* values for every state, provided that the agent explores the state space in the right way. Once exact values are known, optimal decisions can be taken simply by moving to the lowest-cost successor—that is, pure hill climbing is then an optimal strategy.

If you followed our suggestion to trace the behavior of ONLINE-DFS-AGENT in the environment of Figure 4.19, you will have noticed that the agent is not very bright. For example, after it has seen that the Up action goes from (1,1) to (1,2), the agent still has no idea that the $Down$ action goes back to (1,1) or that the Up action also goes from (2,1) to (2,2), from (2,2) to (2,3), and so on. In general, we would like the agent to learn that Up increases the y-coordinate unless there is a wall in the way, that $Down$ reduces it, and so on. For this to happen, we need two things. First, we need a formal and explicitly manipulable representation for these kinds of general rules; so far, we have hidden the information inside the black box called the RESULT function. Part III is devoted to this issue. Second, we need algorithms that can construct suitable general rules from the specific observations made by the agent. These are covered in Chapter 18.

4.6   SUMMARY

This chapter has examined search algorithms for problems beyond the "classical" case of finding the shortest path to a goal in an observable, deterministic, discrete environment.

- *Local search* methods such as **hill climbing** operate on complete-state formulations, keeping only a small number of nodes in memory. Several stochastic algorithms have been developed, including **simulated annealing**, which returns optimal solutions when given an appropriate cooling schedule.

- Many local search methods apply also to problems in continuous spaces. **Linear programming** and **convex optimization** problems obey certain restrictions on the shape of the state space and the nature of the objective function, and admit polynomial-time algorithms that are often extremely efficient in practice.

- A **genetic algorithm** is a stochastic hill-climbing search in which a large population of states is maintained. New states are generated by **mutation** and by **crossover**, which combines pairs of states from the population.

- In **nondeterministic** environments, agents can apply AND–OR search to generate **contingent** plans that reach the goal regardless of which outcomes occur during execution.

- When the environment is partially observable, the **belief state** represents the set of possible states that the agent might be in.

- Standard search algorithms can be applied directly to belief-state space to solve **sensorless problems**, and belief-state AND–OR search can solve general partially observable problems. Incremental algorithms that construct solutions state-by-state within a belief state are often more efficient.

- **Exploration problems** arise when the agent has no idea about the states and actions of its environment. For safely explorable environments, **online search** agents can build a map and find a goal if one exists. Updating heuristic estimates from experience provides an effective method to escape from local minima.

BIBLIOGRAPHICAL AND HISTORICAL NOTES

Local search techniques have a long history in mathematics and computer science. Indeed, the Newton–Raphson method (Newton, 1671; Raphson, 1690) can be seen as a very efficient local search method for continuous spaces in which gradient information is available. Brent (1973) is a classic reference for optimization algorithms that do not require such information. Beam search, which we have presented as a local search algorithm, originated as a bounded-width variant of dynamic programming for speech recognition in the HARPY system (Lowerre, 1976). A related algorithm is analyzed in depth by Pearl (1984, Ch. 5).

The topic of local search was reinvigorated in the early 1990s by surprisingly good results for large constraint-satisfaction problems such as n-queens (Minton *et al.*, 1992) and logical reasoning (Selman *et al.*, 1992) and by the incorporation of randomness, multiple simultaneous searches, and other improvements. This renaissance of what Christos Papadimitriou has called "New Age" algorithms also sparked increased interest among theoretical computer scientists (Koutsoupias and Papadimitriou, 1992; Aldous and Vazirani, 1994). In the field of operations research, a variant of hill climbing called **tabu search** has gained popularity (Glover and Laguna, 1997). This algorithm maintains a tabu list of k previously visited states that cannot be revisited; as well as improving efficiency when searching graphs, this list can allow the algorithm to escape from some local minima. Another useful improvement on hill climbing is the STAGE algorithm (Boyan and Moore, 1998). The idea is to use the local maxima found by random-restart hill climbing to get an idea of the overall shape of the landscape. The algorithm fits a smooth surface to the set of local maxima and then calculates the global maximum of that surface analytically. This becomes the new restart point. The algorithm has been shown to work in practice on hard problems. Gomes *et al.* (1998) showed that the run times of systematic backtracking algorithms often have a **heavy-tailed distribution**, which means that the probability of a very long run time is more than would be predicted if the run times were exponentially distributed. When the run time distribution is heavy-tailed, random restarts find a solution faster, on average, than a single run to completion.

TABU SEARCH

HEAVY-TAILED
DISTRIBUTION

Simulated annealing was first described by Kirkpatrick *et al.* (1983), who borrowed directly from the **Metropolis algorithm** (which is used to simulate complex systems in physics (Metropolis *et al.*, 1953) and was supposedly invented at a Los Alamos dinner party). Simulated annealing is now a field in itself, with hundreds of papers published every year.

Finding optimal solutions in continuous spaces is the subject matter of several fields, including **optimization theory**, **optimal control theory**, and the **calculus of variations**. The basic techniques are explained well by Bishop (1995); Press *et al.* (2007) cover a wide range of algorithms and provide working software.

As Andrew Moore points out, researchers have taken inspiration for search and optimization algorithms from a wide variety of fields of study: metallurgy (simulated annealing), biology (genetic algorithms), economics (market-based algorithms), entomology (ant colony optimization), neurology (neural networks), animal behavior (reinforcement learning), mountaineering (hill climbing), and others.

Linear programming (LP) was first studied systematically by the Russian mathematician Leonid Kantorovich (1939). It was one of the first applications of computers; the **simplex algorithm** (Dantzig, 1949) is still used despite worst-case exponential complexity. Karmarkar (1984) developed the far more efficient family of **interior-point** methods, which was shown to have polynomial complexity for the more general class of convex optimization problems by Nesterov and Nemirovski (1994). Excellent introductions to convex optimization are provided by Ben-Tal and Nemirovski (2001) and Boyd and Vandenberghe (2004).

Work by Sewall Wright (1931) on the concept of a **fitness landscape** was an important precursor to the development of genetic algorithms. In the 1950s, several statisticians, including Box (1957) and Friedman (1959), used evolutionary techniques for optimization problems, but it wasn't until Rechenberg (1965) introduced **evolution strategies** to solve optimization problems for airfoils that the approach gained popularity. In the 1960s and 1970s, John Holland (1975) championed genetic algorithms, both as a useful tool and as a method to expand our understanding of adaptation, biological or otherwise (Holland, 1995). The **artificial life** movement (Langton, 1995) takes this idea one step further, viewing the products of genetic algorithms as *organisms* rather than solutions to problems. Work in this field by Hinton and Nowlan (1987) and Ackley and Littman (1991) has done much to clarify the implications of the Baldwin effect. For general background on evolution, we recommend Smith and Szathmáry (1999), Ridley (2004), and Carroll (2007).

EVOLUTION
STRATEGY

ARTIFICIAL LIFE

Most comparisons of genetic algorithms to other approaches (especially stochastic hill climbing) have found that the genetic algorithms are slower to converge (O'Reilly and Oppacher, 1994; Mitchell *et al.*, 1996; Juels and Wattenberg, 1996; Baluja, 1997). Such findings are not universally popular within the GA community, but recent attempts within that community to understand population-based search as an approximate form of Bayesian learning (see Chapter 20) might help close the gap between the field and its critics (Pelikan *et al.*, 1999). The theory of **quadratic dynamical systems** may also explain the performance of GAs (Rabani *et al.*, 1998). See Lohn *et al.* (2001) for an example of GAs applied to antenna design, and Renner and Ekart (2003) for an application to computer-aided design.

GENETIC
PROGRAMMING

The field of **genetic programming** is closely related to genetic algorithms. The principal difference is that the representations that are mutated and combined are programs rather

than bit strings. The programs are represented in the form of expression trees; the expressions can be in a standard language such as Lisp or can be specially designed to represent circuits, robot controllers, and so on. Crossover involves splicing together subtrees rather than sub-strings. This form of mutation guarantees that the offspring are well-formed expressions, which would not be the case if programs were manipulated as strings.

Interest in genetic programming was spurred by John Koza's work (Koza, 1992, 1994), but it goes back at least to early experiments with machine code by Friedberg (1958) and with finite-state automata by Fogel *et al.* (1966). As with genetic algorithms, there is debate about the effectiveness of the technique. Koza *et al.* (1999) describe experiments in the use of genetic programming to design circuit devices.

The journals *Evolutionary Computation* and *IEEE Transactions on Evolutionary Computation* cover genetic algorithms and genetic programming; articles are also found in *Complex Systems*, *Adaptive Behavior*, and *Artificial Life*. The main conference is the *Genetic and Evolutionary Computation Conference* (GECCO). Good overview texts on genetic algorithms are given by Mitchell (1996), Fogel (2000), and Langdon and Poli (2002), and by the free online book by Poli *et al.* (2008).

The unpredictability and partial observability of real environments were recognized early on in robotics projects that used planning techniques, including Shakey (Fikes *et al.*, 1972) and FREDDY (Michie, 1974). The problems received more attention after the publication of McDermott's (1978a) influential article, *Planning and Acting*.

The first work to make explicit use of AND–OR trees seems to have been Slagle's SAINT program for symbolic integration, mentioned in Chapter 1. Amarel (1967) applied the idea to propositional theorem proving, a topic discussed in Chapter 7, and introduced a search algorithm similar to AND-OR-GRAPH-SEARCH. The algorithm was further developed and formalized by Nilsson (1971), who also described AO*—which, as its name suggests, finds optimal solutions given an admissible heuristic. AO* was analyzed and improved by Martelli and Montanari (1973). AO* is a top-down algorithm; a bottom-up generalization of A* is A*LD, for A* Lightest Derivation (Felzenszwalb and McAllester, 2007). Interest in AND–OR search has undergone a revival in recent years, with new algorithms for finding cyclic solutions (Jimenez and Torras, 2000; Hansen and Zilberstein, 2001) and new techniques inspired by dynamic programming (Bonet and Geffner, 2005).

The idea of transforming partially observable problems into belief-state problems originated with Astrom (1965) for the much more complex case of probabilistic uncertainty (see Chapter 17). Erdmann and Mason (1988) studied the problem of robotic manipulation without sensors, using a continuous form of belief-state search. They showed that it was possible to orient a part on a table from an arbitrary initial position by a well-designed sequence of tilting actions. More practical methods, based on a series of precisely oriented diagonal barriers across a conveyor belt, use the same algorithmic insights (Wiegley *et al.*, 1996).

The belief-state approach was reinvented in the context of sensorless and partially observable search problems by Genesereth and Nourbakhsh (1993). Additional work was done on sensorless problems in the logic-based planning community (Goldman and Boddy, 1996; Smith and Weld, 1998). This work has emphasized concise representations for belief states, as explained in Chapter 11. Bonet and Geffner (2000) introduced the first effective heuristics

for belief-state search; these were refined by Bryce *et al.* (2006). The incremental approach to belief-state search, in which solutions are constructed incrementally for subsets of states within each belief state, was studied in the planning literature by Kurien *et al.* (2002); several new incremental algorithms were introduced for nondeterministic, partially observable problems by Russell and Wolfe (2005). Additional references for planning in stochastic, partially observable environments appear in Chapter 17.

EULERIAN GRAPH

Algorithms for exploring unknown state spaces have been of interest for many centuries. Depth-first search in a maze can be implemented by keeping one's left hand on the wall; loops can be avoided by marking each junction. Depth-first search fails with irreversible actions; the more general problem of exploring **Eulerian graphs** (i.e., graphs in which each node has equal numbers of incoming and outgoing edges) was solved by an algorithm due to Hierholzer (1873). The first thorough algorithmic study of the exploration problem for arbitrary graphs was carried out by Deng and Papadimitriou (1990), who developed a completely general algorithm but showed that no bounded competitive ratio is possible for exploring a general graph. Papadimitriou and Yannakakis (1991) examined the question of finding paths to a goal in geometric path-planning environments (where all actions are reversible). They showed that a small competitive ratio is achievable with square obstacles, but with general rectangular obstacles no bounded ratio can be achieved. (See Figure 4.20.)

REAL-TIME SEARCH

The LRTA* algorithm was developed by Korf (1990) as part of an investigation into **real-time search** for environments in which the agent must act after searching for only a fixed amount of time (a common situation in two-player games). LRTA* is in fact a special case of reinforcement learning algorithms for stochastic environments (Barto *et al.*, 1995). Its policy of optimism under uncertainty—always head for the closest unvisited state—can result in an exploration pattern that is less efficient in the uninformed case than simple depth-first search (Koenig, 2000). Dasgupta *et al.* (1994) show that online iterative deepening search is optimally efficient for finding a goal in a uniform tree with no heuristic information. Several informed variants on the LRTA* theme have been developed with different methods for searching and updating within the known portion of the graph (Pemberton and Korf, 1992). As yet, there is no good understanding of how to find goals with optimal efficiency when using heuristic information.
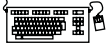
EXERCISES

4.1 Give the name of the algorithm that results from each of the following special cases:

a. Local beam search with $k = 1$.

b. Local beam search with one initial state and no limit on the number of states retained.

c. Simulated annealing with $T = 0$ at all times (and omitting the termination test).

d. Simulated annealing with $T = \infty$ at all times.

e. Genetic algorithm with population size $N = 1$.

4.2   Exercise 3.16 considers the problem of building railway tracks under the assumption that pieces fit exactly with no slack. Now consider the real problem, in which pieces don't fit exactly but allow for up to 10 degrees of rotation to either side of the "proper" alignment. Explain how to formulate the problem so it could be solved by simulated annealing.

4.3   In this exercise, we explore the use of local search methods to solve TSPs of the type defined in Exercise 3.30.

   a. Implement and test a hill-climbing method to solve TSPs. Compare the results with optimal solutions obtained from the A* algorithm with the MST heuristic (Exercise 3.30).

   b. Repeat part (a) using a genetic algorithm instead of hill climbing. You may want to consult Larrañaga *et al.* (1999) for some suggestions for representations.

4.4   Generate a large number of 8-puzzle and 8-queens instances and solve them (where possible) by hill climbing (steepest-ascent and first-choice variants), hill climbing with random restart, and simulated annealing. Measure the search cost and percentage of solved problems and graph these against the optimal solution cost. Comment on your results.

4.5   The AND-OR-GRAPH-SEARCH algorithm in Figure 4.11 checks for repeated states only on the path from the root to the current state. Suppose that, in addition, the algorithm were to store *every* visited state and check against that list. (See BREADTH-FIRST-SEARCH in Figure 3.11 for an example.) Determine the information that should be stored and how the algorithm should use that information when a repeated state is found. (*Hint*: You will need to distinguish at least between states for which a successful subplan was constructed previously and states for which no subplan could be found.) Explain how to use labels, as defined in Section 4.3.3, to avoid having multiple copies of subplans.
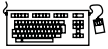
4.6   Explain precisely how to modify the AND-OR-GRAPH-SEARCH algorithm to generate a cyclic plan if no acyclic plan exists. You will need to deal with three issues: labeling the plan steps so that a cyclic plan can point back to an earlier part of the plan, modifying OR-SEARCH so that it continues to look for acyclic plans after finding a cyclic plan, and augmenting the plan representation to indicate whether a plan is cyclic. Show how your algorithm works on (a) the slippery vacuum world, and (b) the slippery, erratic vacuum world. You might wish to use a computer implementation to check your results.

4.7   In Section 4.4.1 we introduced belief states to solve sensorless search problems. A sequence of actions solves a sensorless problem if it maps every physical state in the initial belief state b to a goal state. Suppose the agent knows $h^*(s)$, the true optimal cost of solving the physical state s in the fully observable problem, for every state s in b. Find an admissible heuristic $h(b)$ for the sensorless problem in terms of these costs, and prove its admissibilty. Comment on the accuracy of this heuristic on the sensorless vacuum problem of Figure 4.14. How well does A* perform?

**4.8**   This exercise explores subset–superset relations between belief states in sensorless or partially observable environments.

   a. Prove that if an action sequence is a solution for a belief state b, it is also a solution for any subset of b. Can anything be said about supersets of b?

b. Explain in detail how to modify graph search for sensorless problems to take advantage of your answers in (a).

c. Explain in detail how to modify AND–OR search for partially observable problems, beyond the modifications you describe in (b).

4.9   On page 139 it was assumed that a given action would have the same cost when executed in any physical state within a given belief state. (This leads to a belief-state search problem with well-defined step costs.) Now consider what happens when the assumption does not hold. Does the notion of optimality still make sense in this context, or does it require modification? Consider also various possible definitions of the "cost" of executing an action in a belief state; for example, we could use the *minimum* of the physical costs; or the *maximum*; or a cost *interval* with the lower bound being the minimum cost and the upper bound being the maximum; or just keep the set of all possible costs for that action. For each of these, explore whether A* (with modifications if necessary) can return optimal solutions.

4.10   Consider the sensorless version of the erratic vacuum world. Draw the belief-state space reachable from the initial belief state $\{1, 2, 3, 4, 5, 6, 7, 8\}$, and explain why the problem is unsolvable.
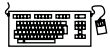
4.11   We can turn the navigation problem in Exercise 3.7 into an environment as follows:

- The percept will be a list of the positions, *relative to the agent*, of the visible vertices. The percept does *not* include the position of the robot! The robot must learn its own position from the map; for now, you can assume that each location has a different "view."

- Each action will be a vector describing a straight-line path to follow. If the path is unobstructed, the action succeeds; otherwise, the robot stops at the point where its path first intersects an obstacle. If the agent returns a zero motion vector and is at the goal (which is fixed and known), then the environment teleports the agent to a *random location* (not inside an obstacle).

- The performance measure charges the agent 1 point for each unit of distance traversed and awards 1000 points each time the goal is reached.

a. Implement this environment and a problem-solving agent for it. After each teleportation, the agent will need to formulate a new problem, which will involve discovering its current location.

b. Document your agent's performance (by having the agent generate suitable commentary as it moves around) and report its performance over 100 episodes.

c. Modify the environment so that 30% of the time the agent ends up at an unintended destination (chosen randomly from the other visible vertices if any; otherwise, no move at all). This is a crude model of the motion errors of a real robot. Modify the agent so that when such an error is detected, it finds out where it is and then constructs a plan to get back to where it was and resume the old plan. Remember that sometimes getting back to where it was might also fail! Show an example of the agent successfully overcoming two successive motion errors and still reaching the goal.

    d. Now try two different recovery schemes after an error: (1) head for the closest vertex on the original route; and (2) replan a route to the goal from the new location. Compare the performance of the three recovery schemes. Would the inclusion of search costs affect the comparison?

    e. Now suppose that there are locations from which the view is identical. (For example, suppose the world is a grid with square obstacles.) What kind of problem does the agent now face? What do solutions look like?

4.12   Suppose that an agent is in a 3×3 maze environment like the one shown in Figure 4.19. The agent knows that its initial location is (1,1), that the goal is at (3,3), and that the actions *Up*, *Down*, *Left*, *Right* have their usual effects unless blocked by a wall. The agent does *not* know where the internal walls are. In any given state, the agent perceives the set of legal actions; it can also tell whether the state is one it has visited before.

    a. Explain how this online search problem can be viewed as an offline search in belief-state space, where the initial belief state includes all possible environment configurations. How large is the initial belief state? How large is the space of belief states?

    b. How many distinct percepts are possible in the initial state?

    c. Describe the first few branches of a contingency plan for this problem. How large (roughly) is the complete plan?

Notice that this contingency plan is a solution for *every possible environment* fitting the given description. Therefore, interleaving of search and execution is not strictly necessary even in unknown environments.

4.13   In this exercise, we examine hill climbing in the context of robot navigation, using the environment in Figure 3.31 as an example.

    a. Repeat Exercise 4.11 using hill climbing. Does your agent ever get stuck in a local minimum? Is it *possible* for it to get stuck with convex obstacles?

    b. Construct a nonconvex polygonal environment in which the agent gets stuck.

    c. Modify the hill-climbing algorithm so that, instead of doing a depth-1 search to decide where to go next, it does a depth-k search. It should find the best k-step path and do one step along it, and then repeat the process.

    d. Is there some k for which the new algorithm is guaranteed to escape from local minima?

    e. Explain how LRTA* enables the agent to escape from local minima in this case.

4.14   Like DFS, online DFS is incomplete for reversible state spaces with infinite paths. For example, suppose that states are points on the infinite two-dimensional grid and actions are unit vectors $(1, 0)$, $(0, 1)$, $(-1, 0)$, $(0, -1)$, tried in that order. Show that online DFS starting at $(0, 0)$ will not reach $(1, -1)$. Suppose the agent can observe, in addition to its current state, all successor states and the actions that would lead to them. Write an algorithm that is complete even for bidirected state spaces with infinite paths. What states does it visit in reaching $(1, -1)$?

5 ADVERSARIAL SEARCH

In which we examine the problems that arise when we try to plan ahead in a world where other agents are planning against us.

5.1 GAMES

GAME

Chapter 2 introduced **multiagent environments**, in which each agent needs to consider the actions of other agents and how they affect its own welfare. The unpredictability of these other agents can introduce **contingencies** into the agent's problem-solving process, as discussed in Chapter 4. In this chapter we cover **competitive** environments, in which the agents' goals are in conflict, giving rise to **adversarial search** problems—often known as **games**.

Mathematical **game theory**, a branch of economics, views any multiagent environment as a game, provided that the impact of each agent on the others is "significant," regardless of whether the agents are cooperative or competitive.[1] In AI, the most common games are of a rather specialized kind—what game theorists call deterministic, turn-taking, two-player,

ZERO-SUM GAMES
PERFECT INFORMATION

zero-sum games of **perfect information** (such as chess). In our terminology, this means deterministic, fully observable environments in which two agents act alternately and in which the utility values at the end of the game are always equal and opposite. For example, if one player wins a game of chess, the other player necessarily loses. It is this opposition between the agents' utility functions that makes the situation adversarial.

Games have engaged the intellectual faculties of humans—sometimes to an alarming degree—for as long as civilization has existed. For AI researchers, the abstract nature of games makes them an appealing subject for study. The state of a game is easy to represent, and agents are usually restricted to a small number of actions whose outcomes are defined by precise rules. Physical games, such as croquet and ice hockey, have much more complicated descriptions, a much larger range of possible actions, and rather imprecise rules defining the legality of actions. With the exception of robot soccer, these physical games have not attracted much interest in the AI community.

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[1] Environments with very many agents are often viewed as **economies** rather than games.

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Games, unlike most of the toy problems studied in Chapter 3, are interesting *because* they are too hard to solve. For example, chess has an average branching factor of about 35, and games often go to 50 moves by each player, so the search tree has about 35^{100} or 10^{154} nodes (although the search graph has "only" about 10^{40} distinct nodes). Games, like the real world, therefore require the ability to make *some* decision even when calculating the *optimal* decision is infeasible. Games also penalize inefficiency severely. Whereas an implementation of A* search that is half as efficient will simply take twice as long to run to completion, a chess program that is half as efficient in using its available time probably will be beaten into the ground, other things being equal. Game-playing research has therefore spawned a number of interesting ideas on how to make the best possible use of time.

We begin with a definition of the optimal move and an algorithm for finding it. We then look at techniques for choosing a good move when time is limited. **Pruning** allows us to ignore portions of the search tree that make no difference to the final choice, and heuristic **evaluation functions** allow us to approximate the true utility of a state without doing a complete search. Section 5.5 discusses games such as backgammon that include an element of chance; we also discuss bridge, which includes elements of **imperfect information** because not all cards are visible to each player. Finally, we look at how state-of-the-art game-playing programs fare against human opposition and at directions for future developments.

We first consider games with two players, whom we call MAX and MIN for reasons that will soon become obvious. MAX moves first, and then they take turns moving until the game is over. At the end of the game, points are awarded to the winning player and penalties are given to the loser. A game can be formally defined as a kind of search problem with the following elements:

- S_0: The **initial state**, which specifies how the game is set up at the start.
- PLAYER(s): Defines which player has the move in a state.
- ACTIONS(s): Returns the set of legal moves in a state.
- RESULT(s, a): The **transition model**, which defines the result of a move.
- TERMINAL-TEST(s): A **terminal test**, which is true when the game is over and false otherwise. States where the game has ended are called **terminal states**.
- UTILITY(s, p): A **utility function** (also called an objective function or payoff function), defines the final numeric value for a game that ends in terminal state s for a player p. In chess, the outcome is a win, loss, or draw, with values $+1$, 0, or $\frac{1}{2}$. Some games have a wider variety of possible outcomes; the payoffs in backgammon range from 0 to $+192$. A **zero-sum game** is (confusingly) defined as one where the total payoff to all players is the same for every instance of the game. Chess is zero-sum because every game has payoff of either $0 + 1$, $1 + 0$ or $\frac{1}{2} + \frac{1}{2}$. "Constant-sum" would have been a better term, but zero-sum is traditional and makes sense if you imagine each player is charged an entry fee of $\frac{1}{2}$.
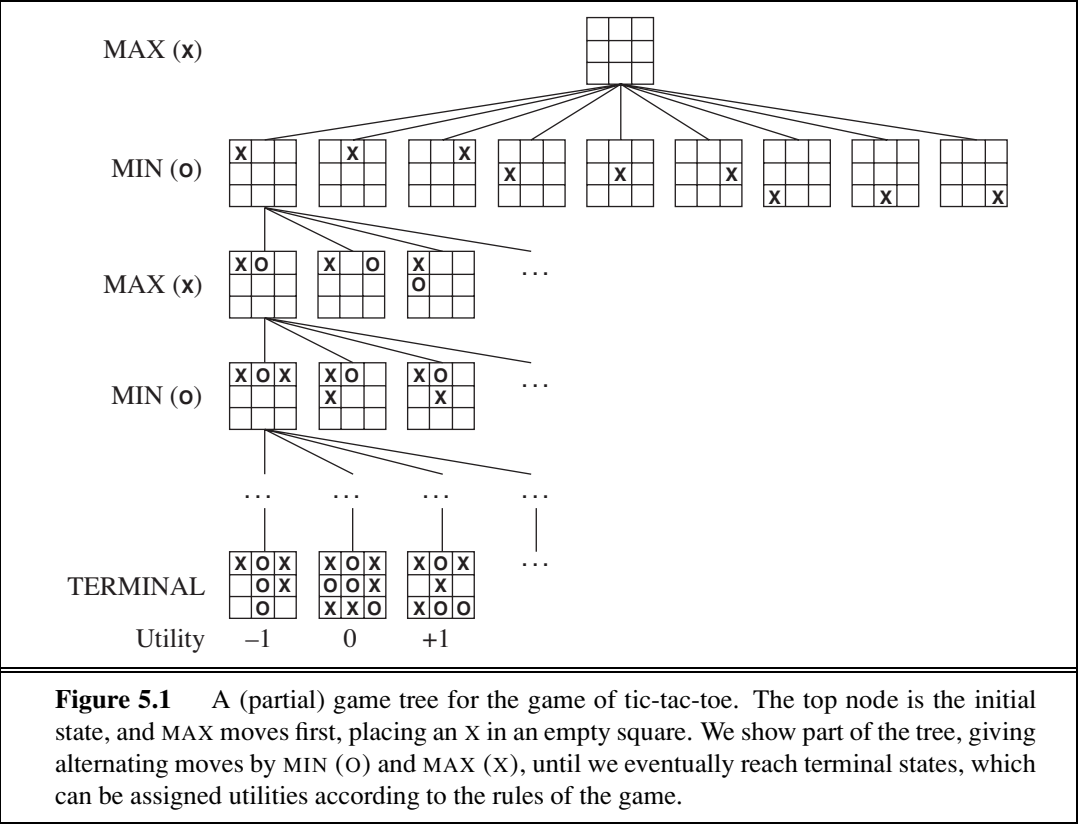
The initial state, ACTIONS function, and RESULT function define the **game tree** for the game—a tree where the nodes are game states and the edges are moves. Figure 5.1 shows part of the game tree for tic-tac-toe (noughts and crosses). From the initial state, MAX has nine possible moves. Play alternates between MAX's placing an X and MIN's placing an O

until we reach leaf nodes corresponding to terminal states such that one player has three in a row or all the squares are filled. The number on each leaf node indicates the utility value of the terminal state from the point of view of MAX; high values are assumed to be good for MAX and bad for MIN (which is how the players get their names).

For tic-tac-toe the game tree is relatively small—fewer than $9! = 362,880$ terminal nodes. But for chess there are over 10^{40} nodes, so the game tree is best thought of as a theoretical construct that we cannot realize in the physical world. But regardless of the size of the game tree, it is MAX's job to search for a good move. We use the term **search tree** for a tree that is superimposed on the full game tree, and examines enough nodes to allow a player to determine what move to make.

SEARCH TREE

Figure 5.1   A (partial) game tree for the game of tic-tac-toe. The top node is the initial state, and MAX moves first, placing an X in an empty square. We show part of the tree, giving alternating moves by MIN (O) and MAX (X), until we eventually reach terminal states, which can be assigned utilities according to the rules of the game.

5.2   OPTIMAL DECISIONS IN GAMES

In a normal search problem, the optimal solution would be a sequence of actions leading to a goal state—a terminal state that is a win. In adversarial search, MIN has something to say about it. MAX therefore must find a contingent **strategy**, which specifies MAX's move in the initial state, then MAX's moves in the states resulting from every possible response by

STRATEGY