Backpropagation za logističku regresiju

May 2, 2023

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Metoda optimizacije je stohastički gradijentni spust:

$$w_i' = w_i - \eta \frac{\partial \mathbb{E}}{\partial w_i}$$

Osnovna pravila deriviranja

1. LD:
$$[f(x)a + g(x)b]' = f(x)'a + g(x)'b$$
2. REC:
$$[\frac{1}{f(x)}]' = -\frac{f'(x)}{f(x)^2}$$
3. CONST:
$$c' = 0$$
4. VAR:
$$\frac{dy}{dz}z = 1$$
5. EXP:
$$[f(x)^n]' = n \cdot f(x)^{n-1} \cdot f'(x)$$
6. CEXP:
$$[e^{f(x)}]' = e^{f(x)} \cdot f'(x)$$
7. CHAIN:
$$\frac{dx}{dz} = \frac{dx}{dy}\frac{dy}{dz}$$

Weight update za logističku regresiju

Za puni GD:

$$w_i' = w_i - \eta \frac{\partial \mathbb{E}}{\partial w_i} = w_i + \sum_n \eta x_i^{(n)} (t^{(n)} - \hat{y}^{(n)})$$

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No zašto $\frac{\partial \mathbb{E}}{\partial w_i} = -x_i(t - \hat{y})$?

Za početak, CHAIN...

Prvo s CHAIN raskomadamo:

$$\frac{\partial \mathbb{E}}{\partial w_i} = \frac{\partial \hat{y}}{\partial w_i} \frac{d\mathbb{E}}{d\hat{y}}$$

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$$\frac{d\hat{y}}{dz}\frac{1}{1+e^{-z}} = -\frac{\frac{d\hat{y}}{dz}(1+e^{-z})}{(1+e^{-z})^2} \qquad (REC)$$

$$= -\frac{\frac{d\hat{y}}{dz}1 + \frac{d\hat{y}}{dz}e^{-z}}{(1+e^{-z})^2} \qquad (LD)$$

$$= -\frac{0 + \frac{d\hat{y}}{dz}(e^{-z})}{(1+e^{-z})^2} \qquad (CONST)$$

$$= -\frac{e^{-z} \cdot \frac{d\hat{y}}{dz}(-z)}{(1+e^{-z})^2} \qquad (CEXP)$$

$$= -\frac{e^{-z} \cdot \frac{d\hat{y}}{dz}(z) \cdot (-1)}{(1+e^{-z})^2} \qquad (LD)$$

$$= -\frac{e^{-z} \cdot (-1)}{(1+e^{-z})^2} = \frac{e^{-z}}{(1+e^{-z})^2} \qquad (VAR)$$

Ajmo ovo faktorizirati na sljedeći način:

$$\frac{d\hat{y}}{dz} = \frac{e^{-z}}{(1+e^{-z})^2} = \frac{1}{1+e^{-z}} \cdot \frac{e^{-z}}{1+e^{-z}}$$

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Prisjetimo se da koristimo SSE, i to u varijanti za stohastički gradijentni spust, $\mathbb{E} = \frac{1}{2}(t - y)^2$ (za običan gradijentni spust ide još suma po svim predikcijama na datasetu).

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$$\frac{d\mathbb{E}}{d\hat{y}}[\frac{1}{2}(t-\hat{y})^2] =_{LD} \frac{1}{2}\frac{d\mathbb{E}}{d\hat{y}}(t-\hat{y})^2 =_{EXP} \frac{1}{2} \cdot 2 \cdot (t-\hat{y}) \cdot \frac{d\mathbb{E}}{d\hat{y}}(t-\hat{y})$$

$$=_{LD} (t-\hat{y})[\frac{d\mathbb{E}}{d\hat{y}}t - \frac{d\mathbb{E}}{d\hat{y}}y] =_{CONST,VAR} (t-\hat{y}) \cdot (0-1) = -1 \cdot (t-\hat{y})$$

Konačni weight update za logističku regresiju prema specifikacijama (SSE, Logistička, SGD)

Prisjetimo se:

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Sve sto nam preostaje jest sastaviti weight update formulu:

$$w_i' = w_i - \eta \frac{\partial \mathbb{E}}{\partial w_i} = w_i - \eta \frac{\partial z}{\partial w_i} \frac{d\hat{y}}{dz} \frac{d\mathbb{E}}{d\hat{y}} = w_i - \eta(x_i \cdot \hat{y}(1 - \hat{y}) \cdot -(t - \hat{y})) =$$

$$= w_i + \eta x_i \hat{y}(1 - \hat{y})(t - \hat{y})$$