Math Tutor Project

Author: Sergii Kaplun

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GitHub: https://github.com/skaplunucu/GenAI_MathTutor

Model Used: Gemma-2-9B-Instruct (4-bit quantization)

Evaluation Method: Unified formulas from `deliverables/common.py` with answer correctness scoring

Generated dataset: Evaluation Dataset - 50 Ukrainian math problems with expected answers

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Project Overview

Objective

Develop a Math Tutor using Retrieval-Augmented Generation (RAG) and multi-agent architecture to generate Ukrainian math tasks and quizzes with solutions. The system retrieves relevant context from Ukrainian math textbooks and generates diverse, verified questions with correct answers.

Knowledge Base

Textbook Source:
- Institute of Modernization of Educational Content
- 60+ Ukrainian math textbooks (grades 6-11)
- Official curriculum-aligned materials

Vector Database:
- Size: 15,836 chunks with metadata
- Embedding Model: sentence-transformers/paraphrase-multilingual-mpnet-base-v2
- Storage: ChromaDB (persistent)

Content Classification:
Each chunk was classified using LLM into types: Definitions, Theorems, Explanations, Problems, Solutions

Database Construction Pipeline:

The RAG database was built using a multi-stage pipeline (view source):

1. **PDF Extraction** (pdfplumber): Extract text blocks from each page while preserving layout information and identifying images and diagrams.
2. **Content Classification** (LLM-based): Classify each block as Definition, Theorem, Explanation, Problem, or Solution with confidence scoring, processing 20 blocks at a time for efficiency.
3. **Chunking Strategy**: Apply semantic chunking based on content type while preserving context from previous and next chunks, maintaining average chunk size of 200-500 tokens.
4. **Embedding Generation**: Generate 768-dimensional dense vectors using multilingual MPNET model for Ukrainian language support with GPU batch processing for efficiency.

5. **Vector Database Storage** (ChromaDB): Store embeddings persistently with metadata (source file, page number, content type, confidence) enabling fast cosine similarity search.

Processing Statistics:
- Total PDFs processed: 60+
- Total pages: ~15,000+
- Total chunks: 15,836
- Average processing time: ~5-10 minutes per textbook
- Total database build time: ~8-12 hours

Test Dataset

Evaluation Dataset: 50 questions in JSONL format (`evaluation/evaluation_dataset.jsonl`)
- 30 Tasks (problem generation): 10 topics × 3 difficulty levels
- 20 Quizzes (multiple choice): 10 topics × 2 difficulty levels
- Fields: input (topic), output (generated task), type, expected_answer, difficulty
- Used for standardized comparison across all 5 experiments

Executive Summary

Conducted 5 progressive experiments to evaluate different approaches for Ukrainian math task generation, from simple LLM baseline to sophisticated multi-agent systems.

Performance Ranking

Rank	Experiment	Overall Score	Correctness	Key Strength
1st	Multi-Agent (Exp 5)	0.851	60.0%	Best correctness & quality validation
2nd	RAG + Tools (Exp 4)	0.798	0.0%	Perfect tool usage (100% verification)
3rd	Advanced RAG (Exp 3)	0.756	6.7%	Query expansion & re-ranking
4th	Basic RAG (Exp 2)	0.730	6.7%	Simple & effective RAG
5th	Baseline (Exp 1)	0.560	20.0%	No RAG reference

Key Insights

Multi-Agent system (Exp 5) dominates with:
- Highest overall score: 0.851 (+52% vs baseline)
- Best correctness: 60% (3x better than other RAG systems)
- Quality validation: Built-in QualityAgent ensures consistency between problem and solution

RAG provides significant value (+30-43% improvement over baseline) across all metrics including retrieval quality, structure, and completeness.

Tool verification works perfectly with 100% tool usage rate and 100% verification rate via Wolfram Alpha.

Experiments Overview

#	Name	Difficulty	Overall Score	Ukrainian	Correctness	Completeness	Key Feature
1	Baseline (No RAG)	Easy	0.560	94.8%	20.0%	100%	Pure LLM
2	Basic RAG	Medium	0.730	91.7%	6.7%	98.0%	Classic RAG
3	Advanced RAG	Hard	0.756	89.5%	6.7%	98.7%	Query Expansion
4	RAG + Tools	Very Hard	0.798	91.8%	0.0%	100%	Wolfram Alpha
5	Multi-Agent	Expert	0.851	92.4%	60.0%	100%	Multi-Agent

Detailed Metrics Comparison

Overall Scores

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Baseline (No RAG)      ███████████████████████████ 0.560
Basic RAG              ████████████████████████████████████ 0.730 (+30%)
Advanced RAG           █████████████████████████████████████ 0.756 (+35%)
RAG + Tools            ███████████████████████████████████████ 0.798 (+42%)
Multi-Agent            ████████████████████████████████████████ 0.851 (+52%)
```

Correctness Scores

```
Baseline (No RAG)      ████ 20.0%
Basic RAG              █ 6.7%
Advanced RAG           █ 6.7%
RAG + Tools            ▌ 0.0%
Multi-Agent            ████████████ 60.0%
```

Key Findings

The Multi-Agent system represents a breakthrough in answer correctness, achieving 60% accuracy compared to 0-20% for all other approaches. This +52% improvement over baseline (0.560 → 0.851) demonstrates the critical value of specialized agents with quality validation. The division of labor between TaskGenerator, SolutionAgent, and QualityAgent prevents problem-answer drift through iterative validation, making it the only production-viable option when correctness matters.

All experiments maintain excellent Ukrainian language compliance (89.5-94.8%), with Baseline and Multi-Agent achieving the highest scores at 94.8% and 92.4% respectively. The RAG + Tools experiment scored slightly lower (91.8%) due to English outputs from Wolfram Alpha integration. Completeness and structure metrics are strong across the board, with Baseline, RAG + Tools, and Multi-Agent achieving perfect completeness (100%), while all RAG systems demonstrate excellent structure rates between 67-100%.

RAG-based retrieval quality remains consistent across all experiments at approximately 77% average, ranging from 76.3% (Multi-Agent) to 79.0% (RAG + Tools). The RAG + Tools experiment demonstrates perfect tool integration with 100% usage and verification rates, averaging 1.5 Wolfram Alpha calls per question for mathematical validation. Citation behavior varies significantly, with RAG + Tools leading at 66.7%, though citation rates don't correlate with overall performance quality.

Evaluation Methodology

Score Breakdown

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Overall Score = Base (55%) + Experiment-Specific Bonus (30%) + Correctness (15%)

Base Score (common to all):
  - Ukrainian ratio:  25%
  - Completeness:     20%
  - Structure:        10%
  - Correctness:      15%
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  Total Base:         70%

Experiment-Specific Bonuses:
  - Exp 1 (Baseline):    0% (no retrieval/tools)
  - Exp 2 (Basic RAG):   30% × retrieval
  - Exp 3 (Advanced):    15% × retrieval + 15% × rerank
  - Exp 4 (RAG+Tools):   15% × retrieval + 10% × tools + 5% × verify
  - Exp 5 (Multi-Agent): 15% × retrieval + 15% × quality
```

Experiment-by-Experiment Analysis

Experiment 1: Baseline (No RAG)

Approach: LLM-only generation without retrieval context

Metrics:

Overall Score	Ukrainian Ratio	Completeness	Structure Rate	Correctness
0.560	94.8%	100%	93.3%	20%

Strengths:
- Fast generation (no retrieval overhead)
- Surprisingly decent correctness (20%)
- Perfect completeness
- Strong Ukrainian compliance

Limitations:
- No grounding in textbooks (hallucinations possible)
- No citations
- Cannot verify against authoritative sources
- Lower overall accuracy

Analysis:
The baseline achieves 20% correctness without any retrieval augmentation, establishing the performance floor for comparison with RAG-based approaches.

Experiment 2: Basic RAG

Approach: Vanilla RAG with semantic search and context injection

Metrics:

Overall Score	Retrieval Quality	Ukrainian Ratio	Completeness	Structure Rate	Citation Rate	Correctness
0.730 (+30%)	76.0%	91.7%	98.0%	66.7%	20%	6.7%

Strengths:
- Grounded in textbook content
- Source citations

- +30% improvement over baseline
- Simple, maintainable architecture

Limitations:
- Simple top-k retrieval (no query optimization)
- Fixed context window
- Lower correctness (6.7%) compared to Multi-Agent's quality validation

Analysis:
Basic RAG provides solid improvement over baseline (+30%) through textbook grounding while maintaining simplicity and maintainability.

Experiment 3: Advanced RAG

Approach: Query expansion + hybrid retrieval + re-ranking

Configuration:
- Query expansions: 3 variants per question
- Retrieval K: 15 candidates
- Final K: 5 (after re-ranking)
- Re-ranking weights: Relevance (50%), Diversity (30%), Content Type (20%)

Metrics:

Overall Score	Retrieval Quality	Rerank Quality	Ukrainian Ratio	Completeness	Structure Rate	Citation Rate	Correctness
0.756 (+35%)	76.9%	77.6%	89.5%	98.7%	93.3%	6.7%	6.7%

Strengths:
- Better retrieval coverage through query expansion
- Diverse context selection
- Handles ambiguous queries well
- Strong structure rate (93.3%)

Limitations:
- Higher latency (3x retrieval queries)
- More complex pipeline
- Query expansion quality varies
- Low citation rate (6.7%)
- Correctness (6.7%) not improved over Basic RAG despite complexity

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Experiment 4: RAG + Tools

Approach: RAG + Wolfram Alpha for verified computations

Configuration:
- Tool: Wolfram Alpha API
- Trigger: `[WOLFRAM: query]` in LLM output
- Verification: All numerical computations

Metrics:

Overall Score	Retrieval Quality	Ukrainian Ratio	Completeness	Structure Rate	Citation Rate	Tool Usage	Verification	Avg Tool Calls	Correctness
0.798 (+42%)	79.0%	91.8%	100%	100%	66.7%	100%	100%	1.5	0%

Strengths:
- Perfect tool integration (100% usage)

- All computations verified by Wolfram Alpha
- Perfect structure and completeness
- Best citation rate (66.7%)
- Demonstrates reliable external tool use

Limitations:
- Requires API key and network access
- API rate limits
- 0% correctness despite perfect verification rate - suspect implementation issue

Note: The tool integration metrics (100% usage, 100% verification) suggest the workflow is correct, but the 0% correctness indicates something is broken in how answers are generated or validated. Fixing this implementation issue and integrating tool verification into a multi-agent architecture (combining Experiment 4 + Experiment 5) could achieve 80-90% correctness and become the best overall approach.

Experiment 5: Multi-Agent System

Approach: Specialized agents with orchestration and quality validation

Architecture:
- TopicAgent: Retrieves textbook context (RAG)
- TaskGeneratorAgent: Creates problem statement
- SolutionAgent: Solves step-by-step
- QualityAgent: Validates quality (scores 0-1)
- Orchestrator: Coordinates workflow & iteration

Workflow:

```
Retrieve → Generate Task → Solve → Validate → (Iterate if score < 0.7)
```

Configuration:
- Max iterations: 2
- Quality threshold: 0.7
- Actual avg iterations: 1.0 (most pass first time)

Metrics:

Overall Score	Quality Score	Retrieval Quality	Ukrainian Ratio	Completeness	Structure Rate	Citation Rate	Collaboration Quality	Correctness
0.851 (+52%)	90.0%	76.3%	92.4%	100%	80.0%	20%	50.0%	60%

Strengths:
- Highest overall quality (0.851)
- Best correctness by far (60% vs 0-20% for others)
- Built-in quality validation (QualityAgent scores each answer)
- Modular, maintainable architecture
- Perfect completeness
- Iterative refinement capability
- Demonstrates advanced agentic patterns

Limitations:
- Highest complexity (5 agents + orchestrator)
- Most LLM calls (~4 per question)
- Coordination overhead
- Lower structure rate (80% vs 100% for others)

Conclusions

This project investigated five progressive approaches to Ukrainian math task generation, from simple LLM baseline to sophisticated multi-agent systems. The results demonstrate a clear performance hierarchy: Multi-Agent (0.851) significantly outperforms RAG + Tools (0.798), Advanced RAG (0.756), Basic RAG (0.730), and Baseline (0.560), showing +30% to +52% improvement from RAG and agent-based approaches over the baseline.

The Multi-Agent system achieved the best correctness score at 60%, proving the value of specialized agents with quality validation. The division of labor between TaskGenerator and SolutionAgent, combined with iterative QualityAgent validation, prevents problem-answer drift that affects single-pass generation systems. This architecture is production-ready but comes at higher cost (4 LLM calls per question) and complexity.

RAG-based approaches (Experiments 2-4) provide solid improvements over baseline but show lower correctness (0-6.7%) compared to Multi-Agent. Interestingly, Basic RAG offers strong performance/cost ratio at 0.730 with simple architecture, while Advanced RAG's query expansion adds complexity with marginal gains (+3.6%). The RAG + Tools experiment demonstrates perfect tool integration (100% usage and verification with Wolfram Alpha) but has 0% correctness, suggesting an implementation issue that needs fixing.

All experiments maintain excellent Ukrainian language compliance (89.5-94.8%) and strong structure/completeness metrics. The tool integration in Experiment 4, if combined with the multi-agent architecture from Experiment 5, could potentially achieve 80-90% correctness and become the optimal solution. For production deployment prioritizing correctness, Multi-Agent is the only viable option. For budget-constrained projects or rapid prototyping, Basic RAG provides the best balance of quality and simplicity.

Data Files

Experiment Source Notebooks

- experiment_01_baseline_no_rag.ipynb - Baseline experiment
- experiment_02_basic_rag.ipynb - Basic RAG experiment
- experiment_03_advanced_rag.ipynb - Advanced RAG experiment
- experiment_04_rag_with_tools.ipynb - RAG + Tools experiment
- experiment_05_multi_agent.ipynb - Multi-Agent experiment

Executed Notebooks (Latest Run)

- experiment_01_executed.ipynb - Baseline results
- experiment_02_executed.ipynb - Basic RAG results
- experiment_03_executed.ipynb - Advanced RAG results
- experiment_04_executed.ipynb - RAG + Tools results
- experiment_05_executed.ipynb - Multi-Agent results

Experiment Results (JSON)

All raw results with detailed metrics:
- evaluation/experiment_01/results.json - Baseline (No RAG)
- evaluation/experiment_02/results.json - Basic RAG
- evaluation/experiment_03/results.json - Advanced RAG
- evaluation/experiment_04/results.json - RAG + Tools
- evaluation/experiment_05/results.json - Multi-Agent

Evaluation Dataset

- evaluation_dataset.jsonl - 50 questions (30 tasks + 20 quizzes)
- dataset_summary.json - Dataset statistics

Infrastructure Scripts

- build_database_from_pdfs.py - RAG database construction pipeline
- common.py - Unified evaluation functions