Liquid drop behavior of low viscous liquids

Samip Karki

5/19/2021

Abstract

Studying the pinch off behavior of fluids out of a small nozzle can be done with a high speed camera. According to theory, the minimum neck radius of a liquid droplet, R_{min} will decrease according to a $t^{2/3}$ power law, with time t. By using a high speed camera, we explored the extent of the $t^{2/3}$ power law for solutions with $\sim 5\%, \sim 20\%, \sim 39\%, \sim 60\%$, and $\sim 83\%$ glycerol near pinch off. Analysis was done in two ways. One way was by collapsing and log-ing all the data sets and comparing their slope to a line with 2/3 slope. Another way was by finding the mean and standard deviation of various slopes along the log-ed curve of each dataset.

INTRODUCTION

A theoretical argument finds that the minimum neck radius, R_{min} of a liquid droplet follows $t^{2/3}$ power law moments before pinch off from a nozzle. This power law comes from a heuristic solution of the Navier-Stokes Equation:

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \left(\mathbf{u} \cdot \nabla \right) = -\nabla p$$

for density ρ pressure p and velocity $\mathbf{u}$. The Navier-Stokes Equation describes the motion of fluids and can be interpreted as

Interial Stress Term + Viscous Term + Pressure Term =0.

We will consider an inertially dominated case where the Viscous Term = 0. If we consider the gradient of p to be in one dimension, and rewrite the derivative of u as acceleration a, then we have

$$-\frac{ma}{V} = \frac{d}{dx} p$$

where m is mass and V is volume. Integrating with respect to dx on both sides gives p on the right hand side. Noting $-\int (ma)dx = KE$, kinetic energy, the left handside become $\frac{KE}{V}$. From now on, '=' will be replaced by '≈' because exact solutions are not needed for this heuristic argument. Pressure p can be rewritten as $\frac{\gamma}{R_{min}}$, where γ is the surface tension with unites $\frac{N}{m}$ and R_{min} is the minimum neck radius of our liquid. v^2 can be can be rewritten as $\left(\frac{dR_{min}}{dt}\right)^2$. Thus we have

$$\rho \left(\frac{dR_{min}}{dt} \right)^2 \approx \frac{\gamma}{R_{min}}.$$

This is an ordinary differential equation which can be solved using separation of variables. The result is as desired, that

$$R_{min} \propto t^{2/3}.$$

Using a high speed camera, our experiment investigated the behavior of R_{min} for various mixtures of water and glycerol to see if the theoretical $t^{2/3}$ power law is consistent for these liquids.

DISCUSSION

High speed camera footage of liquid pinch off was taken for $\sim 5\%, \sim 20\%, \sim 39\%, \sim 0\%$, and $\sim 83\%$ glycerol mixtures. Raw liquid drop datasets can be seen back to back in figure 1. Both R_{min} and time t are in arbitrary units, where t is time before pinch off. Just from looking at the shape of the raw data, we can postulate that R_{min} is proportional to t^a, where $0 < a < 1$. We can find the value for a by further analyzing our liquid drop curves. Our group used two different methods to validate whether or not this exponent was 2/3.

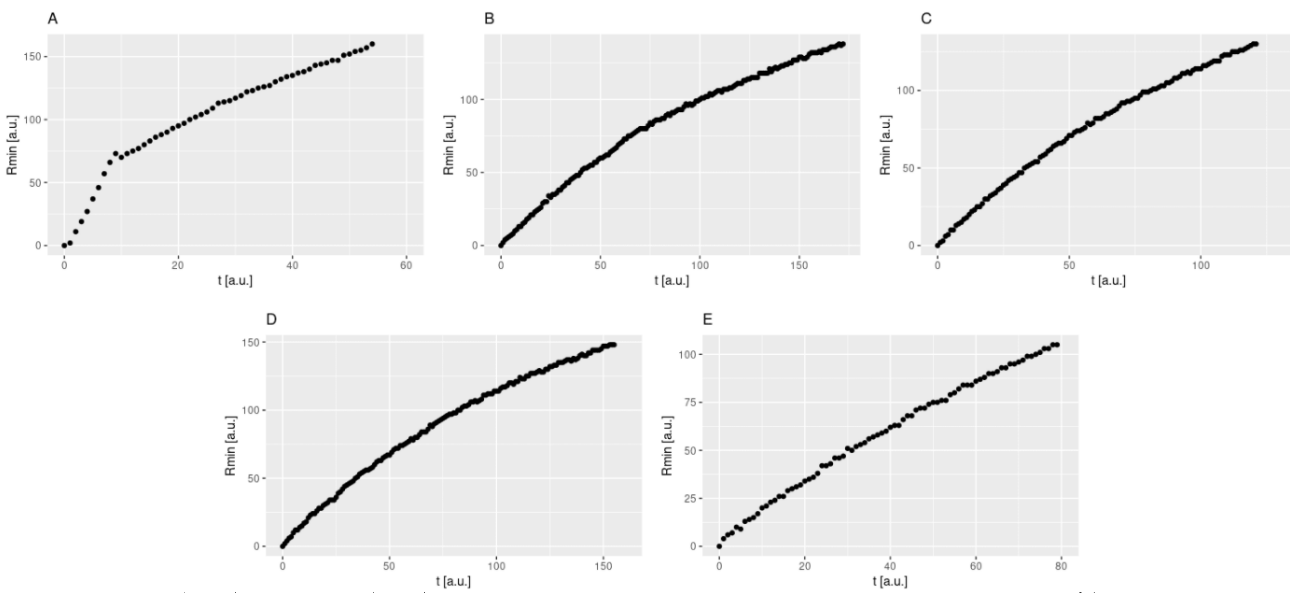

FIG. 1: R_{min} (a.u.) by time (a.u.) of raw data of various liquid concentrations. A: 5% glycerol, B: 20% glycerol; C: 39% glycerol, D: 60% glycerol; E: 83% glycerol.

Method 1

Functions can be transformed by applying log to both sides of the equation. If the function is a monomial like $y = x^a$, then applying log to both sides will transform the curve to a straight line with slope a, $log(y) = a * log(x)$. With this method, we can find the exponent a of our liquid drop datasets.

Before log-ing our data, we can collapse it to the same curve by finding some characteristic coordinate (T_c, R_c) and scaling our t and R_{min} values with that characteristic coordinate. The values for T_c and R_c are arbitrary and different for each curve, but they all must have the same relationship to their respective curves.

Here is our method for collapsing our data. First, we find the best fit line,R_{Line} ,which fits the first 20

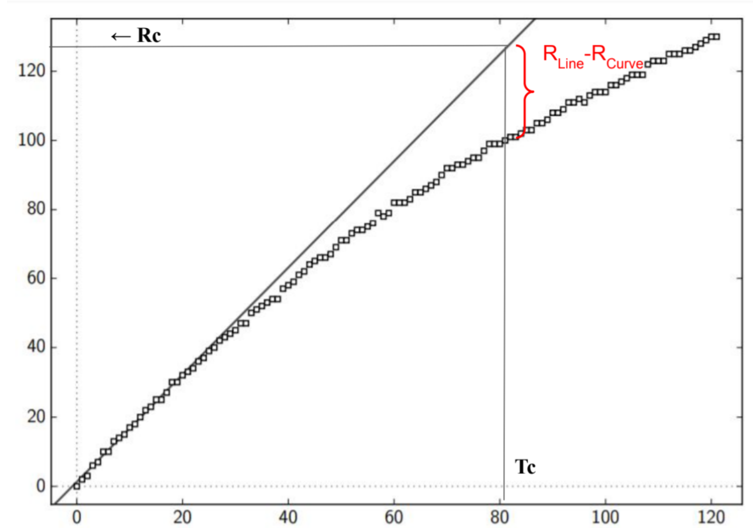

FIG. 2: R_{min} (a.u.) by time (a.u) of raw data of 39% glycerol liquid drop. Best fit line plotted from closest 20 points from pinch off. The difference between the line and the data at (Tc, Rc) is in red.

or so data points of one of our data sets. Then we find at what t satisfies $\frac{(R_{Line}-R_{Curve})}{R_{Curve}} = .3$, and call this time T_c. R_c is the value of R_{Line} at T_c. This process is illustrated in figure 2. Now, $(R_{min}/R_c, t/T_c)$ will collapse all the data sets into one curve. The collapsed and log-ed data for all the datasets along with a line with slope of 2/3 is shown in figure 3.

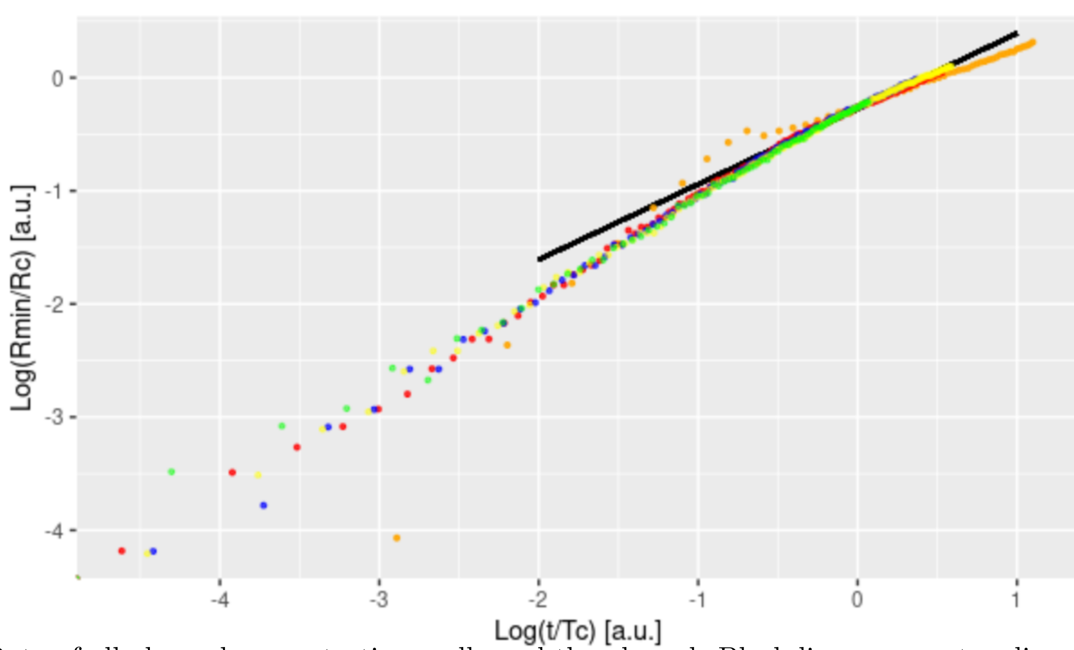

FIG. 3: Data of all glycerol concentrations collapsed then log-ed. Black line represents a line with slope of . Different colors represent data from different liquid concentrations. 5% (orange), 20% (red), 39% (blue), 60% (yellow), and 83% (green).

The figure 3 gives a qualitative look at the extent that our liquid drop data follows the 2/3 power law. Our method for collapsing the data was effective for all the datasets except for the 5% dataset where a knee in the data made it difficult to find appropriate T_c and R_c. Around $t/T_c = -0.05$, and we can see that all of the data sets follow the 2/3 line very well. The high density of data points in this region tells us that for most of the time the liquids are following the 2/3 power law. Once time gets to $t/T_c = -2$ until the pinch off, the slopes of all of the datasets become greater than 2/3, and the theorized result becomes less strong. What this tells us is that our liquids obey the $t^{2/3}$ theory until moments right before pinchoff when R_{min} decreases faster than the $t^{2/3}$ theory suggests.

Method 2

The second method to investigate the power law of R_{min} started with log-ing the raw data for each liquid drop. Because the log-ed curve seemed to have different slopes in different regions of time as opposed to one consistent slope, we decided to find various slopes for each dataset. Each dataset was grouped into fourths, and the slope was found for each grouping (figure4). We then took the mean and standard deviation of those four slopes. This processes is summarized in figure 5.

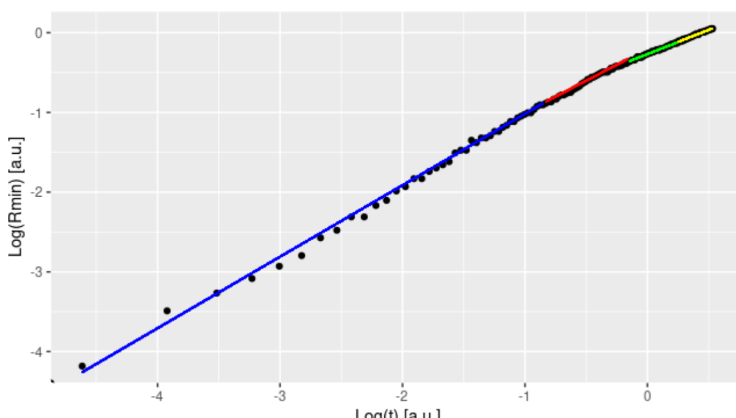

FIG. 4: Log-ed curve of collapsed 20% glycerol data. Lines indicate best fit for their respective regions. Regions are divided such that each has an equal amount of data points

Figure 5 provides a quantitative analysis of the pinch off phenomenon that supports the analysis of the collapsed data in method 1. Points closest to the pinch off have a higher slope than 2/3 and points farther away have a slope closer to 2/3. For all datasets except 83% glycerol our theorized slope of 2/3 was contained within one standard deviation of the average slope. The reason for this might be that our assumption of low viscosity breaks down near liquids with viscosity similar to the 83% glycerol liquid. Just like in method 1, the knee in the 5% glycerol liquid makes analysis difficult. The high variability in the slope make it difficult to say that this dataset is truly following a 2/3 power law.

CONCLUSION

The $t^{2/3}$ power law for the decrease of the minimum neck radius, R_{min}, held for most liquids in our experiment. However, some changes could have been made to get stronger results. The time frame for

Concentration	Slope1	Slope2	Slope3	Slope4	Avg Slope	SD
5g	1.063	0.4996	0.4899	0.5525	0.6513	0.276
20g	0.8967	0.8047	0.6028	0.6073	0.728	0.146
39g	0.9287	0.8364	0.697	0.6454	0.777	0.129
60g	0.889	0.7909	0.6769	0.573	0.732	0.137
83g	0.7511	0.87	0.8066	0.7308	0.790	0.062

FIG. 5: For each log-ed dataset, four slopes are included above along with their average and standard deviation.

all datasets in this experiment was small compared to the complete formation of the liquid droplet. This experiment could be refined simply by starting our data collection to include some of the droplet formation as opposed to only times near the pinch off. Having a larger time frame of the liquid drop process would let us see convergence to the 2/3 power law. Having a larger range of data would also help to resolve analysis issues caused by the knee in the 5% glycerol liquid drop. Also, including more liquids of higher concentrations, possibly 90% or 95% glycerol, could be noteworthy to see if these high viscosity liquids obey some other power law instead of the 2/3 power law of low viscosity liquids.

References

1. Kryzer, Bryce. Droplet Analysis: Using High-Speed Imaging to Capture Pinch- Off, Lab Manual, Physics 385, Spring 2021, St. Olaf College