AMATH 482/582: HOMEWORK 1: SUBMARINE

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Abstract. Fourier transform is a powerful tool used in signal processing. In this work, I demonstrate how a noisy dataset can be cleaned up by accessing the k-space and filtering out unwanted frequencies. With these techniques, I am able to find the exact trajectory of a submarine hidden in the Puget Sound area as it moves in a 24 hour time interval. I compare two methods, one which uses Fourier transforms, and another without using Fourier transforms and see which produces a clearer trajectory of the submarine.

1. Introduction and Overview

In this report, I explored signal processing techniques involving fast Fourier transform to denoise 3D time series data. The problem is as follows. I have obtained a 262144 by 49 data array which corresponds to acoustical measurements of the Puget Sound area over a 24 hour interval. Measurements where made every 30 minutes, corresponding to the 49 columns of the array. Each column has 262144 entries which is the flattened version of a 64 by 64 by 64 array of acoustical amplitudes in a cube with side length 20. Hidden is in this noisy data is a moving submarine. A preliminary plotting of the unfiltered data can be seen in figure.1 and a projection of the entire dataset onto the $x-y$ plane can be seen on the left in figure.2. As it can be seen, noise in the data obstructs the exact location of the submarine. In this report, I use signal processing techniques to filter the original data and locate the submarine at every point in time.

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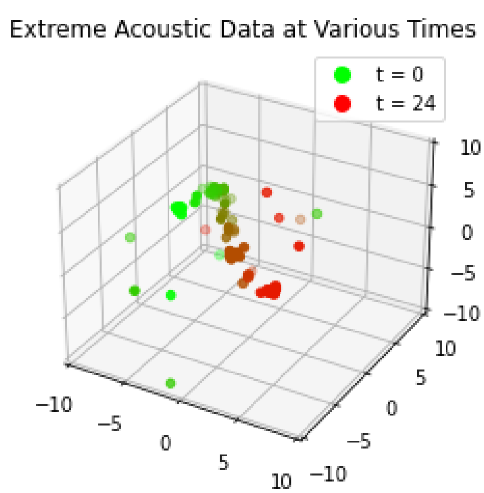

Figure 1. Plot of the largest absolute amplitudes of the original submarine data for all measurements.

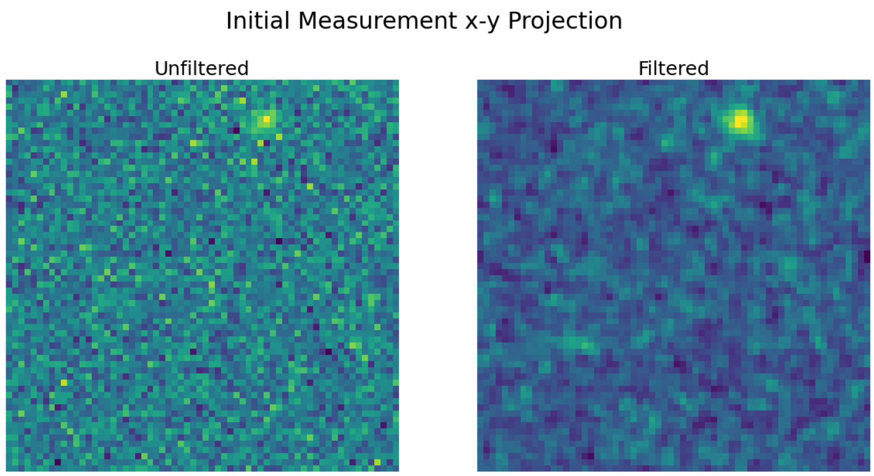

Figure 2. Projection onto x-y plane of unfiltered data (**left**) vs filtered data (**right**). These plots were created by taking the $64x64x64$ array of the first measurement and averaging across every entry with the same $x - y$ coordinate (averaging across the z-coordinates).

2. Theoretical Background

This project uses Fourier transform and inverse Fourier transform extensively. Fourier transform and its inverse are given by the following:

$$F(k) = \int_{-\infty}^{\infty} f(x)\, e^{-ikt}\, dx$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(k)\, e^{ikt}\, dk$$

The Fourier transform of a some signal, $F(k)$ at a frequency k represents the amplitude of a sinusoid with the frequency k in the original signal. In this way, we can say the the Fourier transform takes us from position-space (x-space) to frequency-space (k-space). In many different applications, it can be insightful to analyze a signal in k-space rather than x-space. In our case, as will be discussed further in this report, by filtering out noisy frequencies in k-space, we denoise the original signal in x-space.

The Fourier transform can be generalized to three dimensions. In this project, I do not compute explicitly compute the Fourier tranform with the integral above, but rather make use of the famous fast Fourier transform algorithm which is easily computed in python using `numpy.fft.fft()`, or in three dimensions `numpy.fft.fftn()`

3. Algorithm Implementation and Development

I was able to find the submarine's trajectory in two ways, which I will call method-A and method-B, repectfully. Method-A used extensive use of `numpy.fft.fftn()`, while method-B did not.

Method-A

First, we take the the Fourier transform of 49 measurements and average them. Doing this smooths out much of the noise of the original data while keeping the strong amplitude associated with the frequency of the submarine which is common in all the measurements. Figure.3

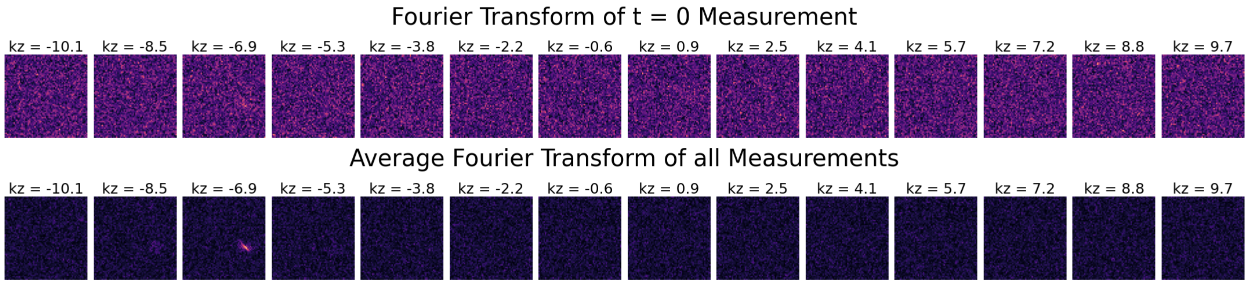

FIGURE 3. (**top**) Fourier transform of the first measurement vs (**bottom**) Average of all the Fourier Transformed Measurements. k_x and k_y are the horizontal and vertical axis of all plots.

shows the comparison between the Fourier transform of a single measurement with the average of all the Fourier transformed measurements. Then I am able to locate the exact frequency vector $\vec{k} = [k_x, k_y, k_z]$ which is associated with the submarine. I then multiply all the Fourier transformed measurements by a gaussian centered at $\vec{k}$. Finally, by performing the inverse Fourier transform, the filtering of the original data is complete. Figure.2 compares the $x - y$ projection of unfiltered data against the filtered data. Then, I simply find the largest amplitude in the filtered measurements, which are plotted on the right in figure.4. This method used `numpy` and `numpy.fft` packages.

Method-B

This method consists of simply taking the absolute value of each unfiltered measurement and then finding $[x, y, z]$ coordinates which had the largest amplitude. Figure.1 was created using this method. This method used basic `numpy` commands like `abs`, and `argmax`.

4. Computational Results

The effect of averaging the Fourier transformed measurements can be seen clearly in figure.3. Much of the noise that was in the top plot is gone in the bottom plot after averaging, and there is a clear bright spot in the data located at in the $k_z = -6.9$ plane that was difficult to see in the top plot. The exact location of this bright spot was $\vec{k} = [2.2, 5.3, -6.9]$. Likewise, after filtering with a gaussian centered at this frequency in k-space, we can see in figure.2 that the location of submarine became clearly visible as the only bright spot in the filtered data.

Comparison between the two different methods can be seen in figure.4. Method-A, involving the filtering of the noisy data in k-space resulted in a much more consistent trajectory of the submarine. However, except for a few points, method-B of simply finding the maximum value in all the measurement still was able to trace the submarine's trajectory successfully.

5. Summary and Conclusions

This was a simple yet instructive project in demonstrating how Fourier transforms can be used in signal processing. Not only that, this project was very much a learning by doing experience, in which I had to problem solve many little problems which arose in the coding process. Overall, I had a nice time doing this project.

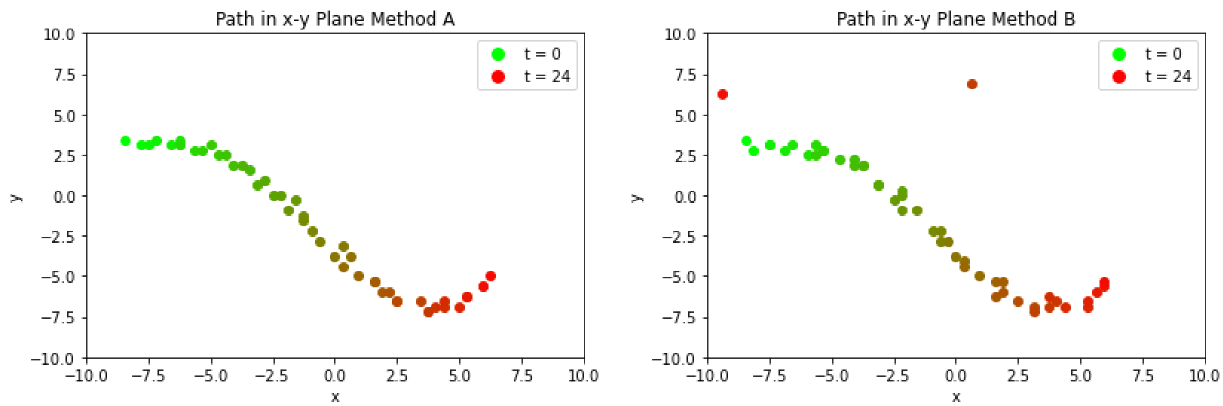

FIGURE 4. Comparison of the tracing of the submarine's trajectory by method-A (**left**) and method-B (**right**).