Combination

r-permutation : Arranging r-objects from n-objects.
Order does matter.

$$^{n}P_{r} = \frac{n!}{(n-r)!} \qquad ^{5}P_{3} = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{120}{2} = 60$$

r-combination : Choosing r-objects from n-objects. $^{n}C_{r}$

Suppose I want select 3 students from a group of 10 students

✓ abc ✓ cab

abc → How many different arrangements I can form?

abc bac cab
acb bca cba

6 no. of arrangements.

→ $6 = 3!$

a b c d → How many different arrangements possible?

a b c d	b a c d	c b a d	d a b c
a b d c	b a d c	c b d a	d a c b
a c b d	b c a d	c a b d	d b a c
a c d b	b c d a	c a d b	d b c a
a d b c	b d a c	c d a b	d c a b
a d c b	b d c a	c d b a	d c b a

24 arrangements. $(4!)$

$6 \to 4$ $\quad {}^6P_4$

$4! \times {}^6C_4 = {}^6P_4$

$\underbrace{\bigcirc\ \bigcirc\ \cdots\cdots\ \bigcirc}_{r\text{-objects}} \longrightarrow r!$

$$({}^nC_r) \times r! = {}^nP_r \Rightarrow {}^nC_r = \frac{1}{r!}\,{}^nP_r = \frac{n!}{(n-r)!\,r!}$$

$$\boxed{{}^nC_r = \frac{n!}{r!\,(n-r)!}}$$

$$\boxed{{}^nC_r \times r! = {}^nP_r}$$

$${}^6C_2 = ? \qquad \frac{6!}{2!\,(6-2)!} = \frac{6!}{2!\,4!} = \frac{6\times5\times\cancel{4!}}{2!\times\cancel{4!}} = \frac{{}^{3}\cancel{6}\times5}{\cancel{2}} = 15$$

$${}^7C_3 = ? \qquad \frac{7!}{3!\,4!} = \frac{7\times6\times5\times\cancel{4!}}{3!\times\cancel{4!}} = \frac{7\times\cancel{6}\times5}{\cancel{6}} = 35$$

$${}^nC_r = \frac{n!}{r!\,(n-r)!}$$

$${}^nC_{n-r} = \frac{n!}{(n-r)!\,(n-(n-r))!}$$

$$= \frac{n!}{(n-r)!\,(\cancel{n}-\cancel{n}+r)!} = \frac{n!}{(n-r)!\,r!}$$

$$\boxed{{}^nC_r = {}^nC_{n-r}}$$

$${}^7C_3 = {}^7C_4 \qquad {}^6C_4 = {}^6C_2$$

$^nC_1 = n$

$$^nC_1 = \frac{n!}{1!\,(n-1)!} = \frac{n \times \cancel{(n-1)!}}{\cancel{(n-1)!}} = n$$

$^nC_n = ?$

$$^nC_n = 1 \Rightarrow \frac{\cancel{n!}}{0!\,\cancel{n!}} = 1 \Rightarrow \boxed{0! = 1}$$

$^nC_0 = 1$

$\underline{a+b}$

$$(1+x) = 1\cdot x^0 + 1\cdot x^1$$

$$(1+x)^2 = 1 + 2x + x^2$$

$$(1+x)^3 = 1 + 3x + 3x^2 + x^3$$

$$(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$$

$$(1+x)^2 \cdot (1+x)^2$$

$$(1+2x+x^2)(1+2x+x^2) = \left.\begin{array}{l} 1 + 2x + x^2 + \\ 2x + 4x^2 + 2x^3 + \\ x^2 + 2x^3 + x^4 \end{array}\right\} = 1 + 4x + 6x^2 + 4x^3 + x^4$$

$$\begin{array}{ccccccccc} & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

Pascal's Triangle

H.W. - Write Python code to print Pascal's triangle (no. of lines should be taken as user input)

$(1+x)^0$ — 1

$(1+x)^1$ — $({}^1C_0)\ 1 \quad 1\ ({}^1C_1)$

$(1+x)^2$ — $({}^2C_0)\ 1 \quad ({}^2C_1)\ 2 \quad 1\ ({}^2C_2)$

$(1+x)^3$ — $({}^3C_0)\ 1 \quad ({}^3C_1)\ 3 \quad ({}^3C_2)\ 3 \quad 1\ ({}^3C_3)$

$(1+x)^4$ — $({}^4C_0)\ 1 \quad ({}^4C_1)\ 4 \quad ({}^4C_2)\ 6 \quad ({}^4C_3)\ 4 \quad 1\ ({}^4C_4)$

$(1+x)^5$ — $({}^5C_0)\ 1 \quad 5\ ({}^5C_1) \quad 10\ ({}^5C_2) \quad 10\ ({}^5C_3) \quad 5\ ({}^5C_4) \quad 1\ ({}^5C_5)$

$$(1+x)^n = 1 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + {}^nC_4 x^4 + \dots + {}^nC_{n-1} x^{n-1} + {}^nC_n x^n$$

Binomial Theorem

${}^nC_r \rightarrow$ Binomial coefficients.

$$(1+x)^n = 1 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + \cdots + {}^nC_{n-1} x^{n-1} + {}^nC_n x^n$$

put $x = 1$ in the above equation —

$$(1+1)^n = 1 + {}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4 + \cdots + {}^nC_{n-1} + {}^nC_n$$

$$\Rightarrow 2^n = 1 + {}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4 + \cdots + {}^nC_{n-1} + {}^nC_n$$

$$\boxed{\sum_{r=0}^{n} {}^nC_r = 2^n}$$