Function of a single variable:-

$$x \longrightarrow \boxed{f(\ )} \longrightarrow y$$

$y = f(x) \rightarrow y$ is a function of x.

$f \hookrightarrow$ function.

$f(x) = x^2 \rightarrow$ a function that gives square of the input as output.

x	$f(x) = x^2$
1	1
2	4
2.5	6.25
9	81

$$f(x) = 2x + x^2$$

$$f(1) = f(x)\Big|_{x=1} = 2 \times 1 + 1^2 = 3$$

$$f(2) = 2 \times 2 + 2^2 = 8$$

$$f(3) = 2 \times 3 + 3^2 = 15$$

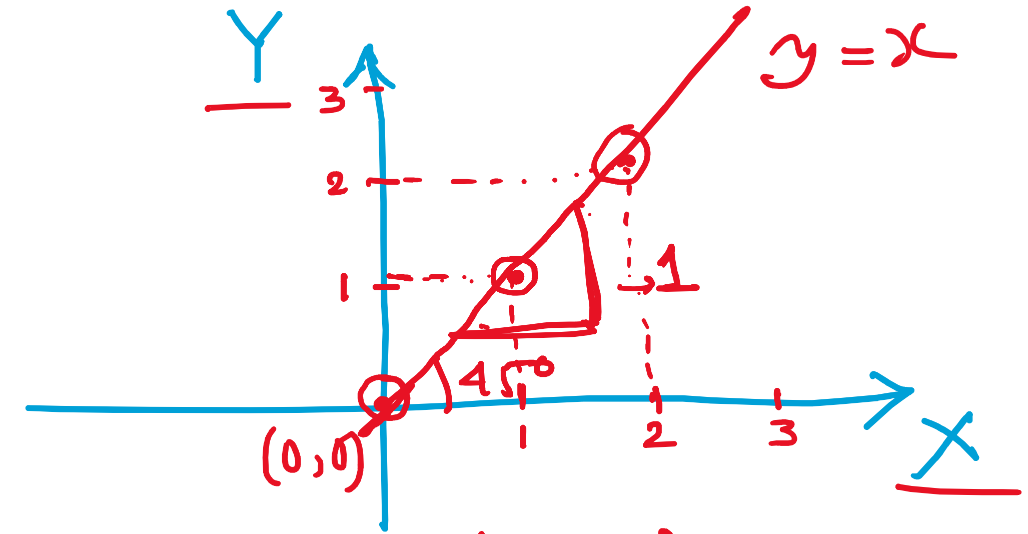
Linear function :–

$y = f(x) = ax + b$

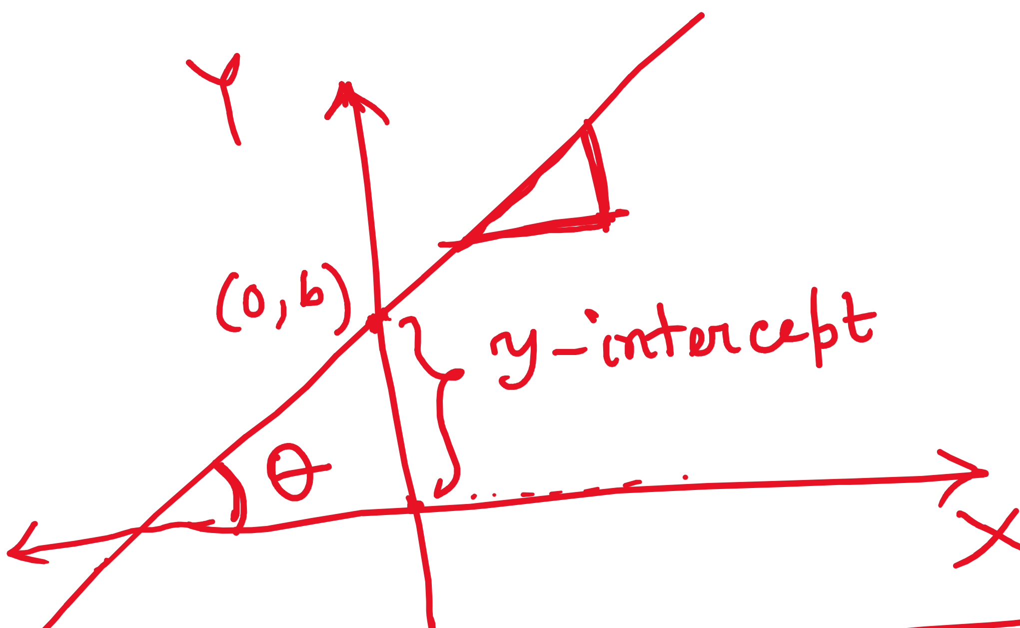
Where a & b are constants

$a = 1$
$b = 0$
$y = x$

x	y
1	1
2	2
3.5	3.5
0	0



$\tan 45^\circ = 1$



$\theta \rightarrow$ inclination.

$m = \tan\theta \rightarrow$ Slope of the straight

$y = 2x + 3$

x	y
0	3
1	5
2	7

$$\boxed{y = ax + b} \longrightarrow \text{It represents straight line in 2D}$$

$x = 0 \rightarrow$ y axis

$y = 0 \rightarrow$ x axis

$a =$ Slope of the straight line.

$b =$ y-intercept of the st. line.

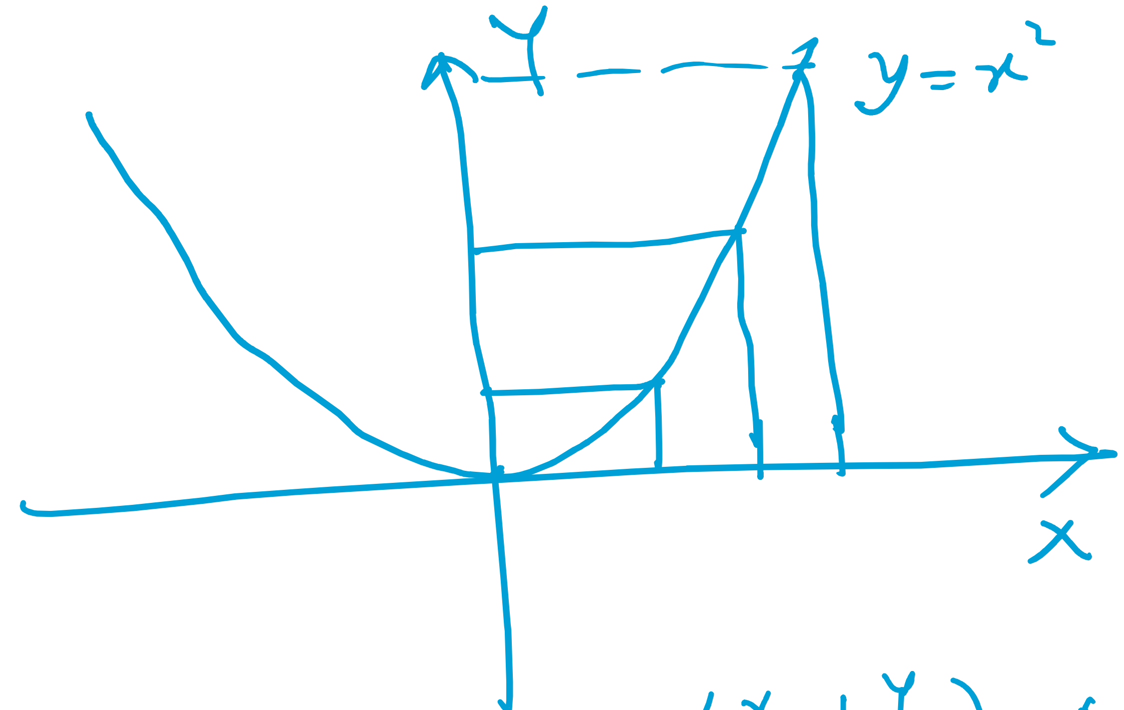
Quadratic function

$$y = f(x) = ax^2 + bx + c \quad (a \neq 0)$$

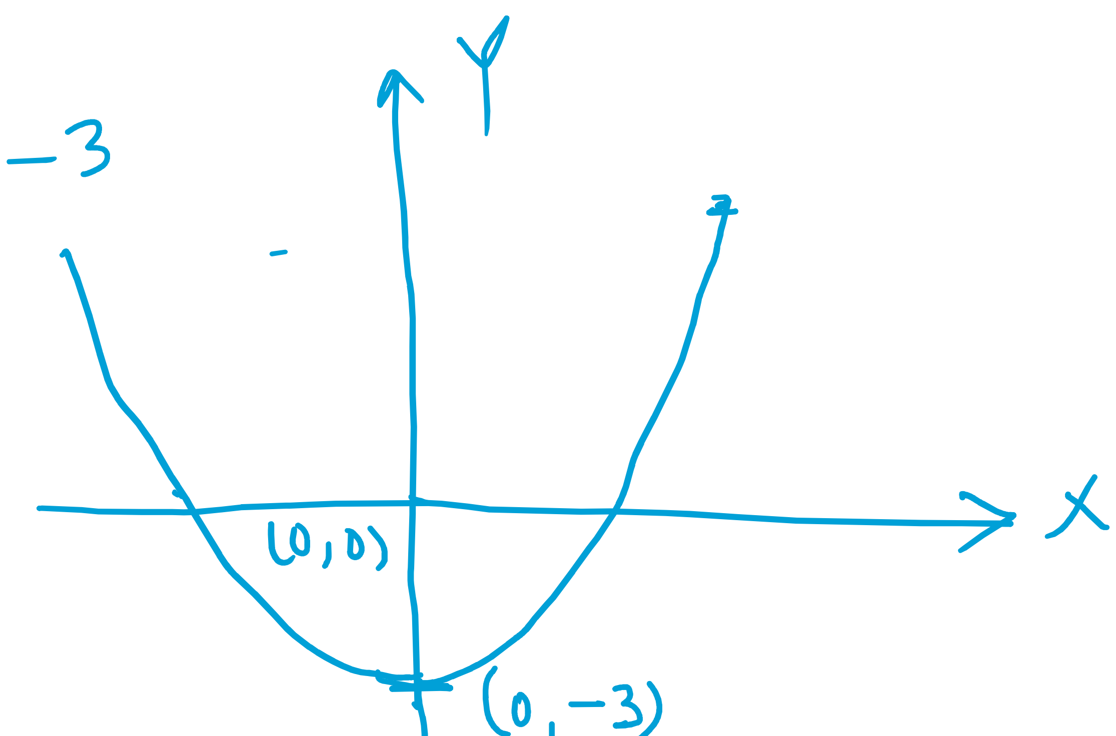
$y = 2x^2 - 3$, $y = x^2$, $y = -7x^2 + 3x - 5$

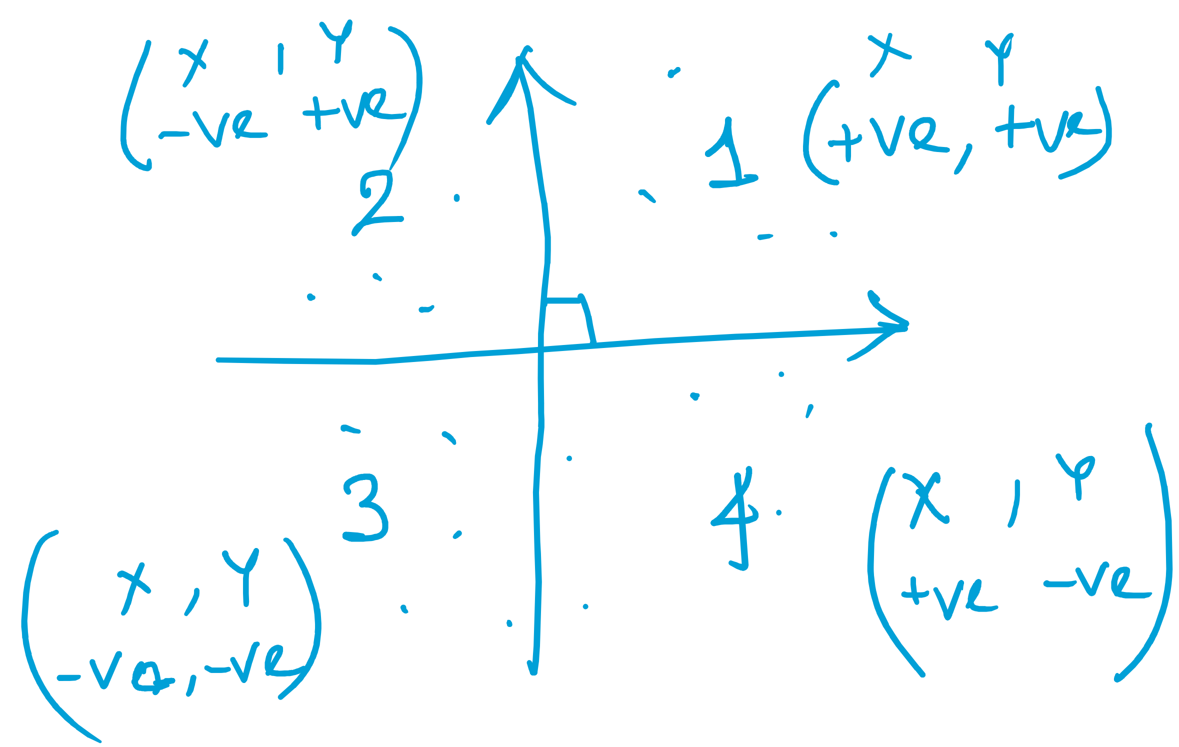
$y = x^2$

x	y
0	0
1	1
2	4
5	25
-5	25

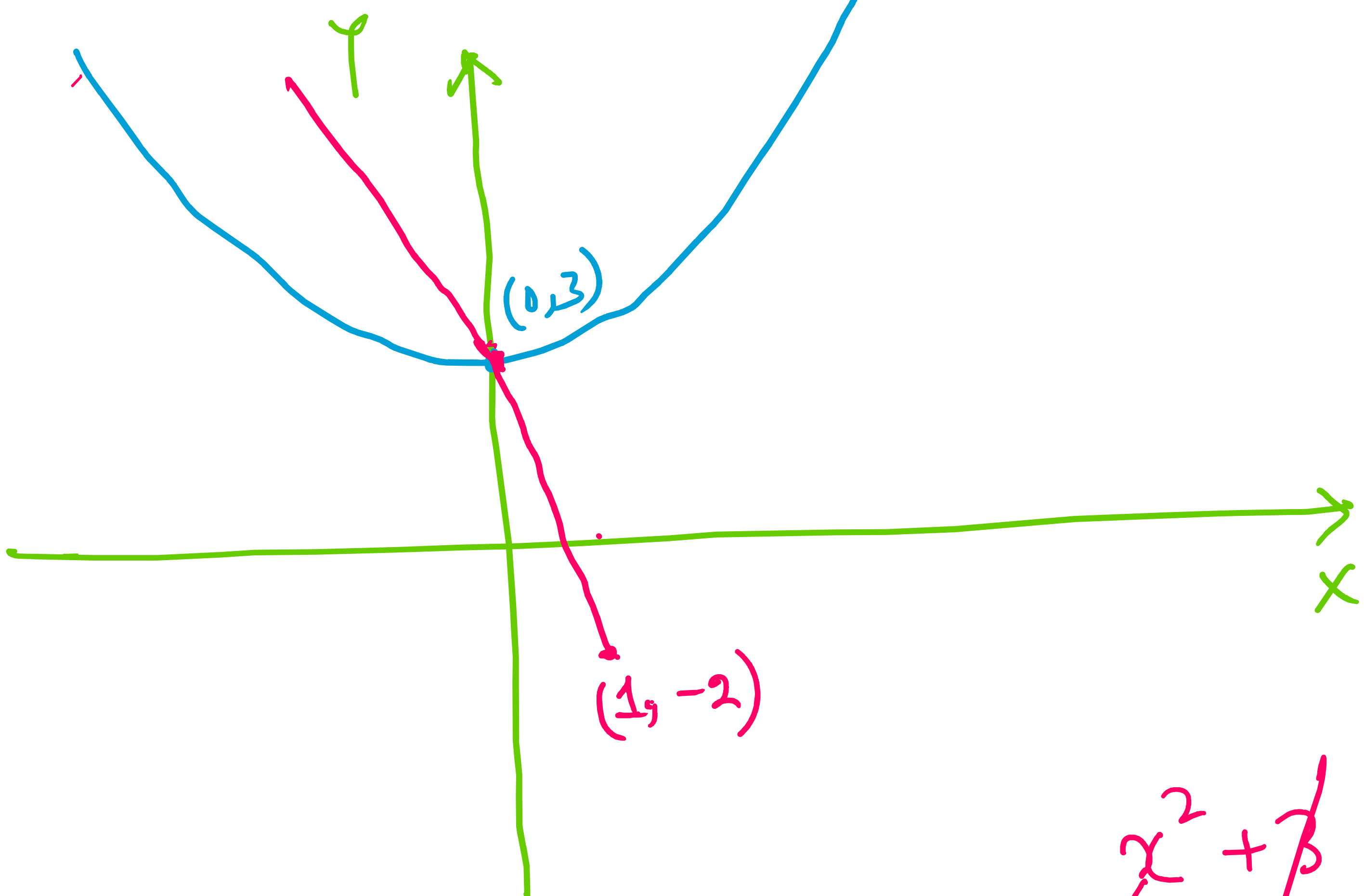


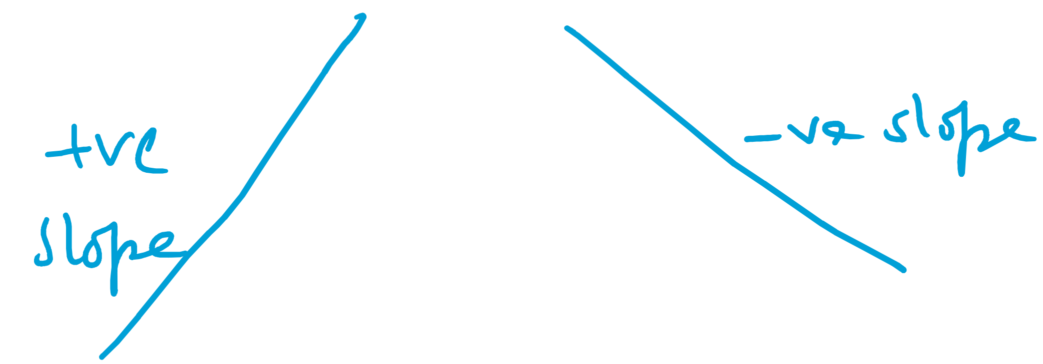
$y = 2x^2 - 3$





$f(x) = x^2 + 3$, $g(x) = -5x + 3$





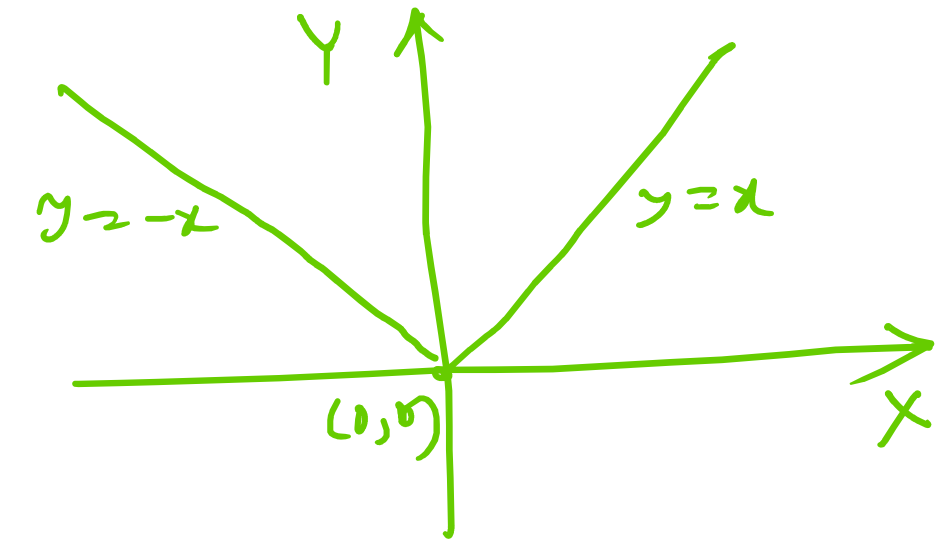
$$x^2 + \cancel{3} = -5x + \cancel{3}$$

$$\Rightarrow \quad x^2 + 5x = 0$$

$$\Rightarrow \quad x(x+5) = 0 \quad \Rightarrow \underline{x=0} \text{ or } \underline{x=-5}$$

Mod function (absolute value)

$y = |x| = ?$



when value of $x > 0$ $\quad y = x$

when " " $x = 0$ $\quad y = 0$

when " " $x < 0$ $\quad y = -x$

$y = |x + a|$ $\Rightarrow$ when $x + a > 0$, $y = x + a$

$\Rightarrow x > -a$, $y = x + a$

when $x + a = 0 \Rightarrow x = -a$, $y = 0$

when $x + a < 0 \Rightarrow x < -a$, $y = -(x + a)$

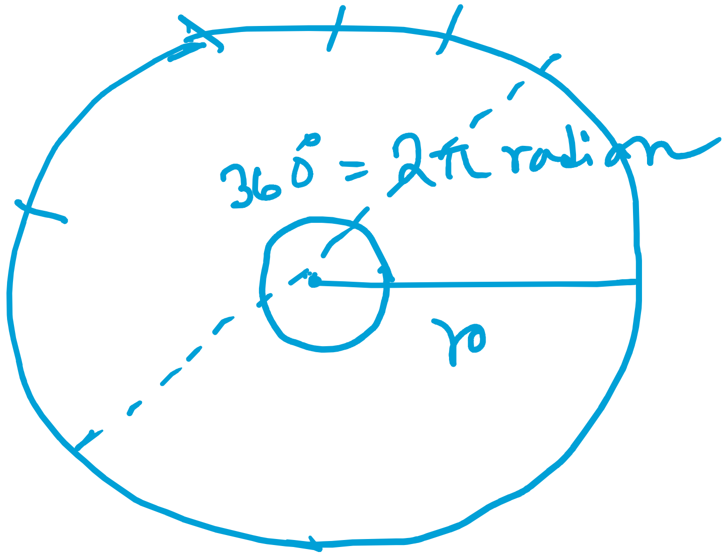
$y = |x+5|$

$y = x + 5$ for $x > -5$

$= 0$ for $x = 0$

$= -x - 5$ for $x < -5$

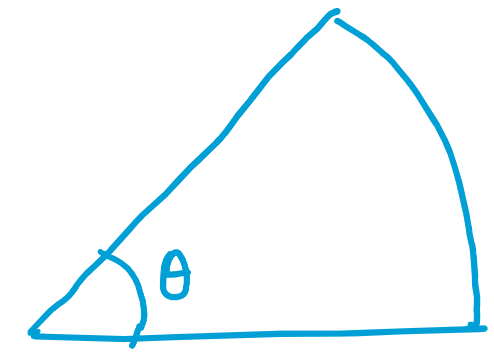
$y = \sin(x)$

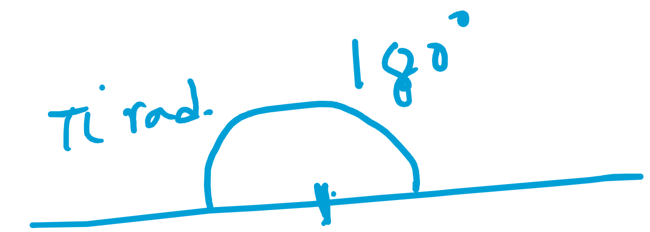


Perimeter $= 2\pi r$

$= \pi(2r)$

$= \pi d$





$d = 2r \rightarrow$ diameter.

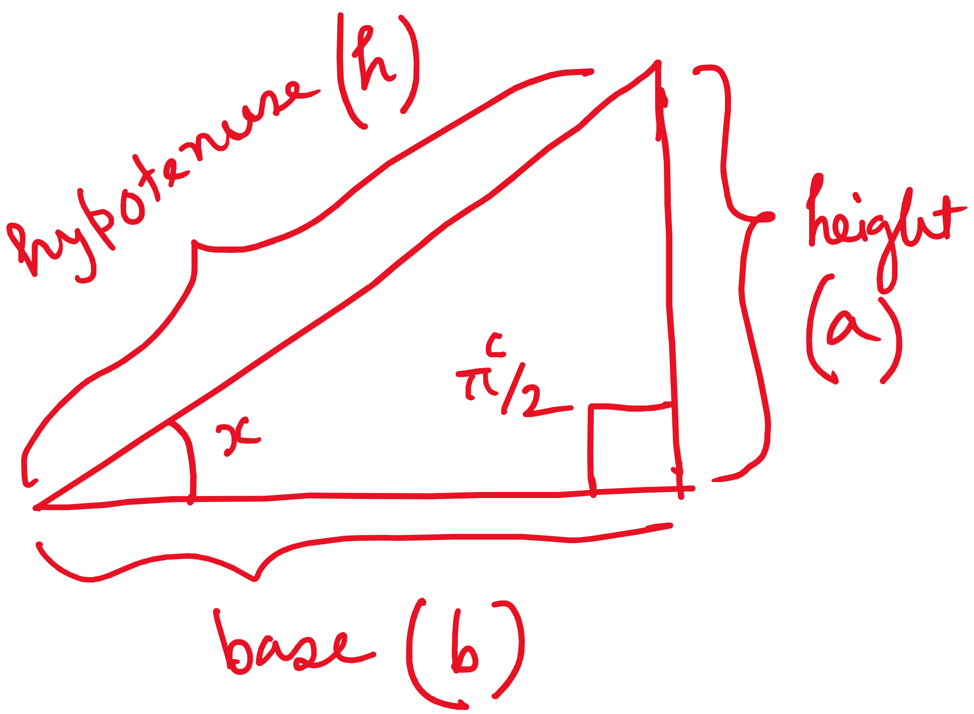
$360^\circ = 2\pi$ rad

$\Rightarrow \boxed{\pi \text{ rad} = 180^\circ}$

$\Rightarrow 1 \text{ rad} = ? = \left(\dfrac{180}{\pi}\right)^\circ$

$\pi = 3.14159\ldots\ldots\ldots$

$1 \text{ rad} = \left(\dfrac{180}{3.14159}\right)^\circ \approx 57^\circ$



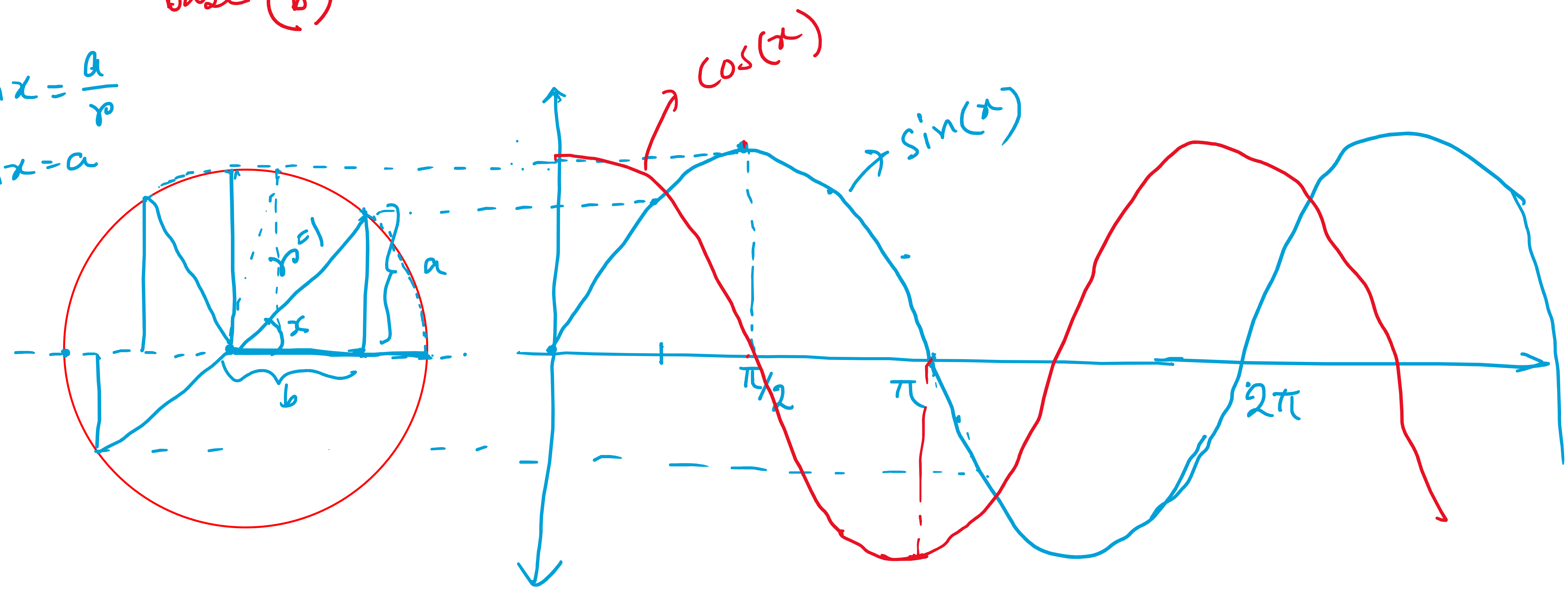
$$\sin(x) = \frac{a}{h}$$

$$\cos(x) = \frac{b}{h}$$

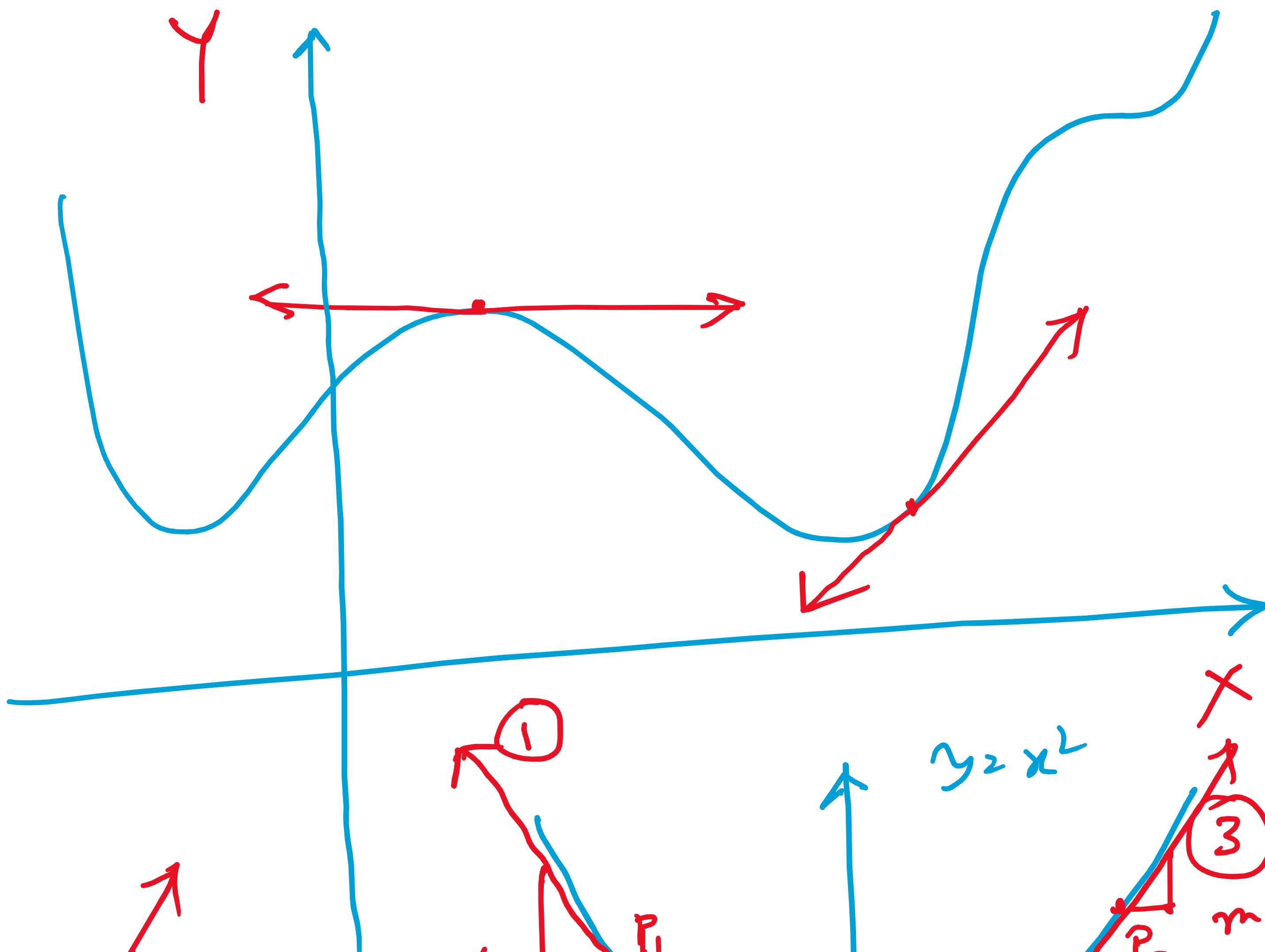
$$\tan(x) = \frac{a}{b} = \frac{\sin(x)}{\cos(x)}$$

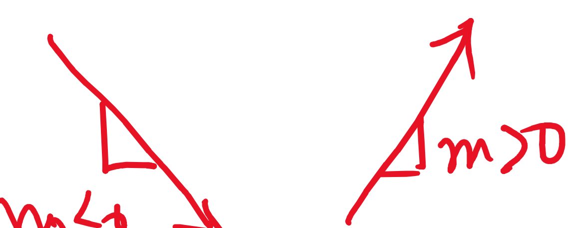
$$\sin x = \frac{a}{r}$$

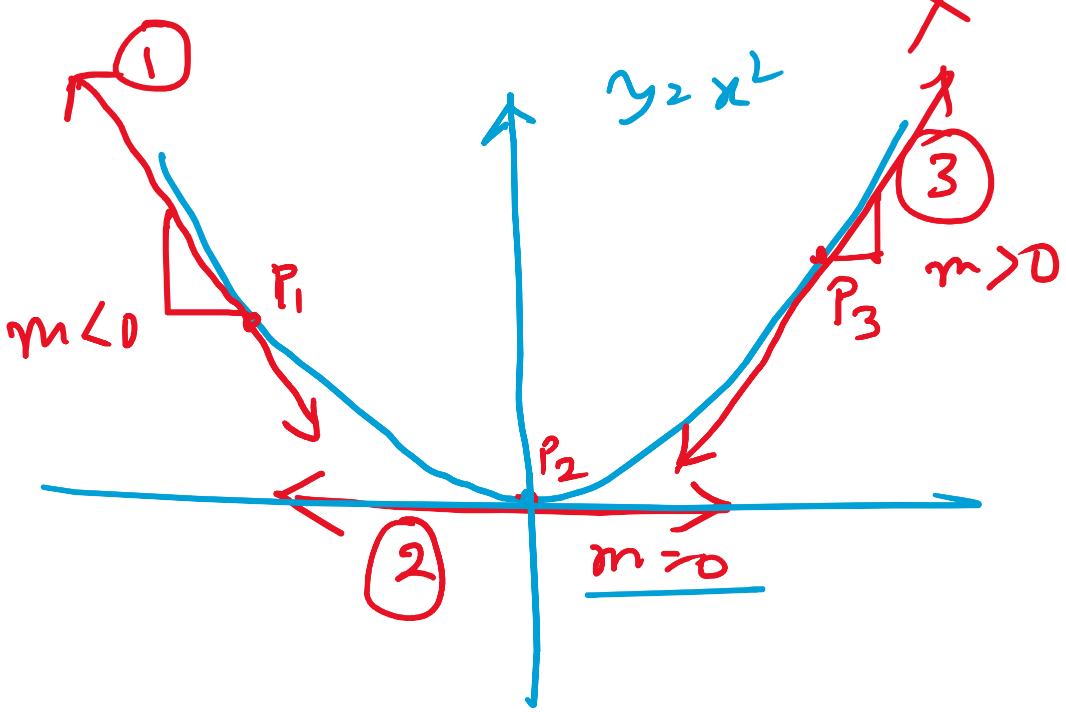
$$\Rightarrow \sin x = a$$



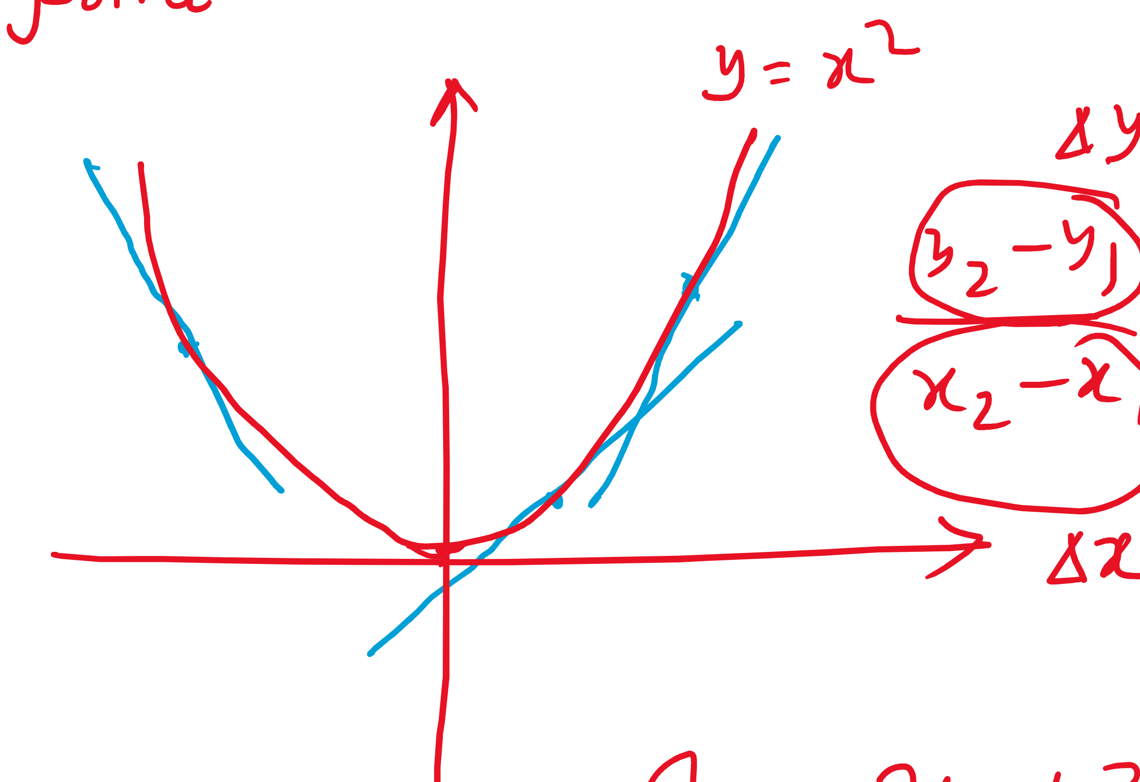
Differentiation of a function of single variable







Tangent is a st. line which touches the graph at a single point.



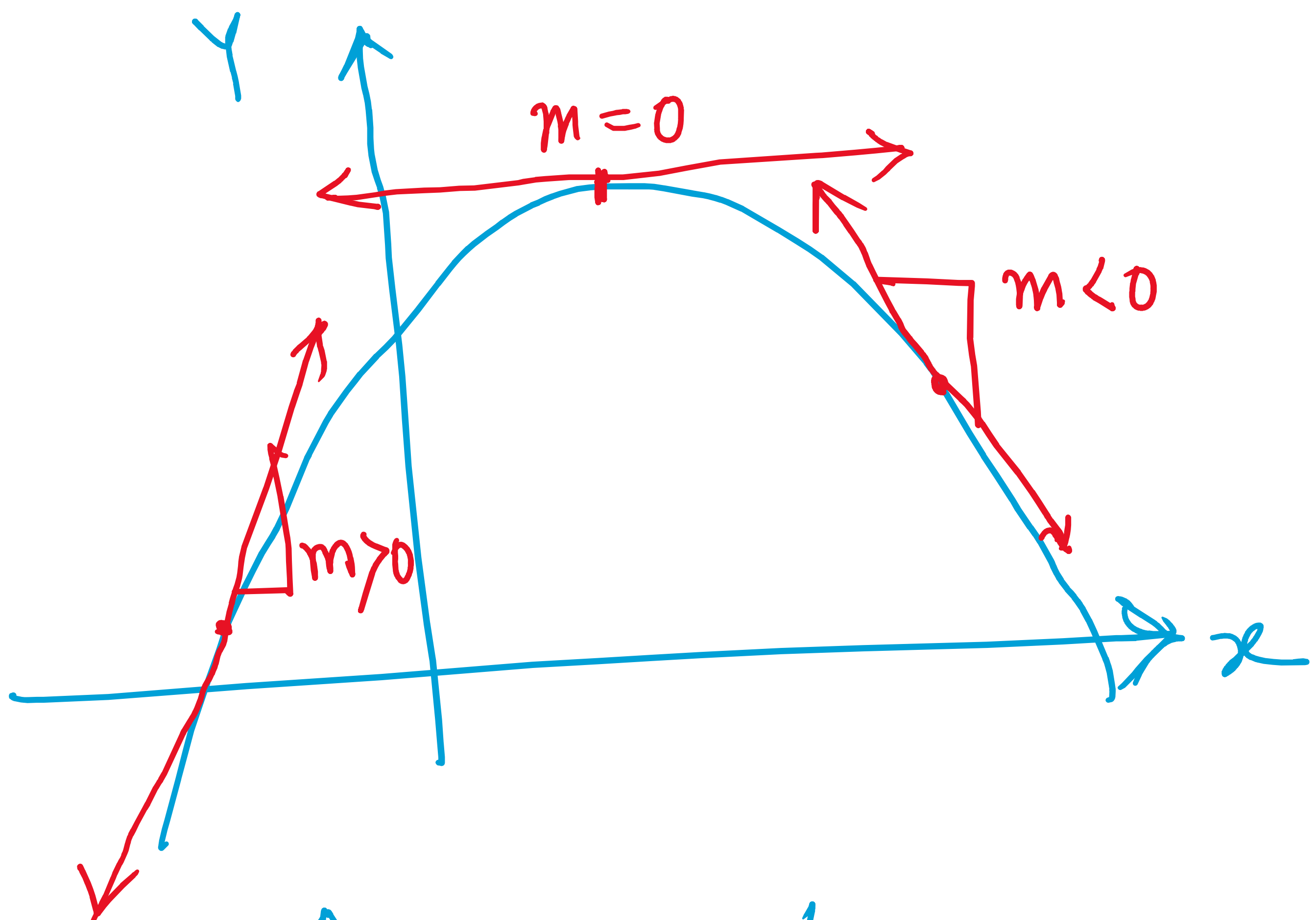
$$\frac{y_2 - y_1}{x_2 - x_1} = 2$$

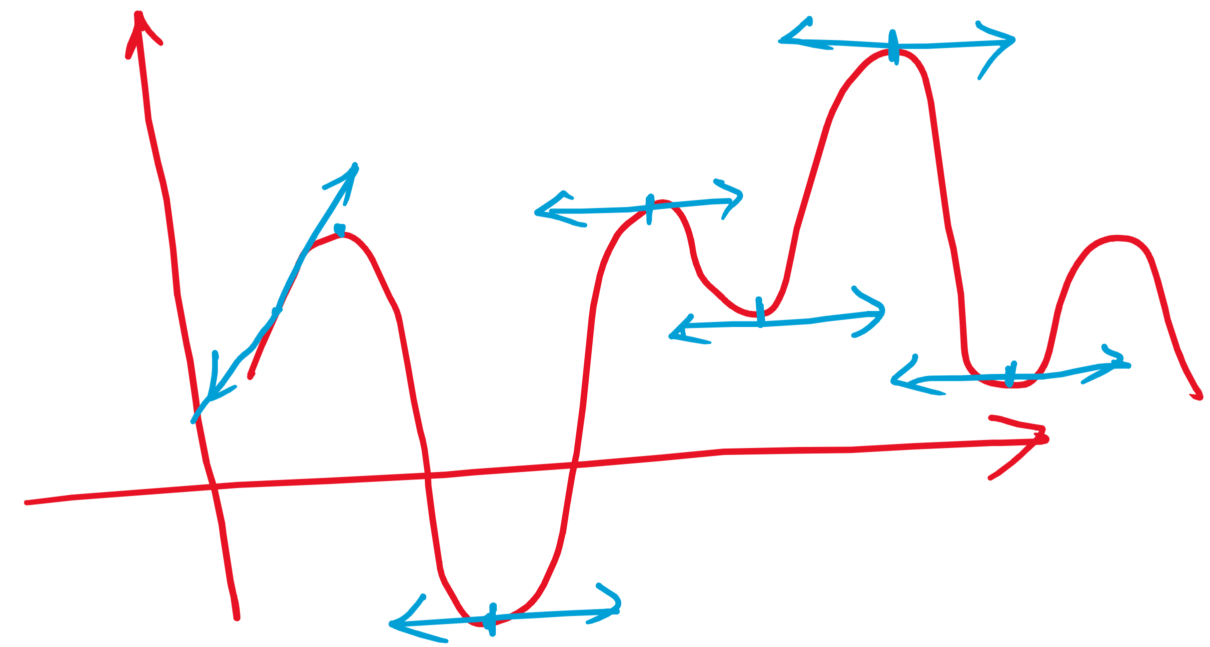
$$y_1 = 2x_1 + 3$$

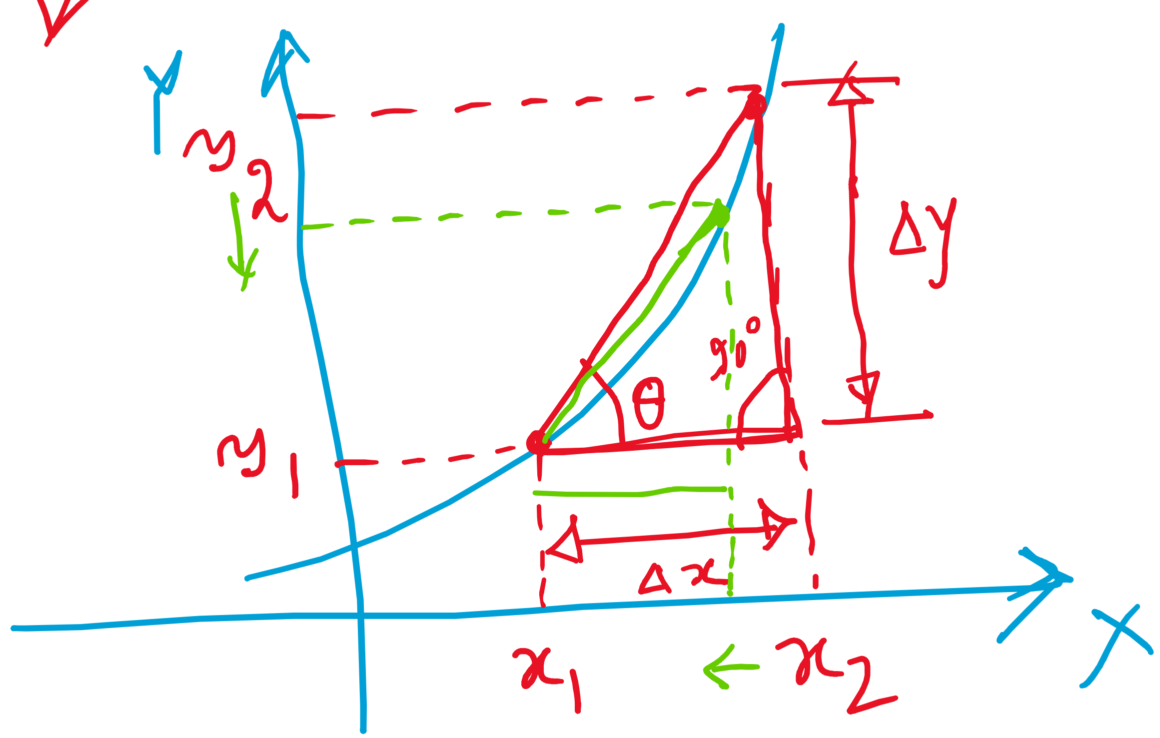
$$y_2 = 2x_2 + 3$$

$$y_2 - y_1 = (2x_2 + 3) - (2x_1 + 3)$$

$$= 2(x_2 - x_1)$$

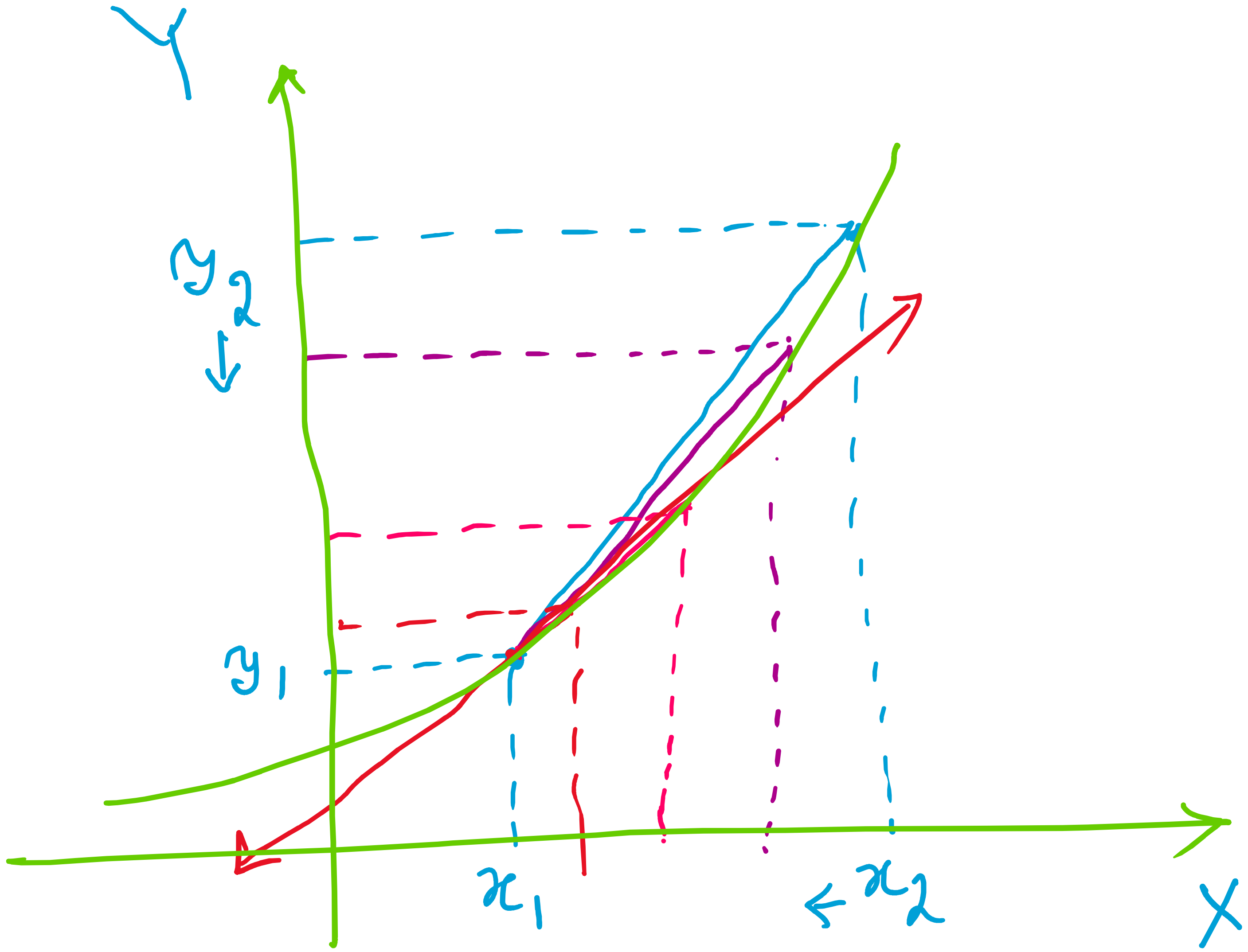


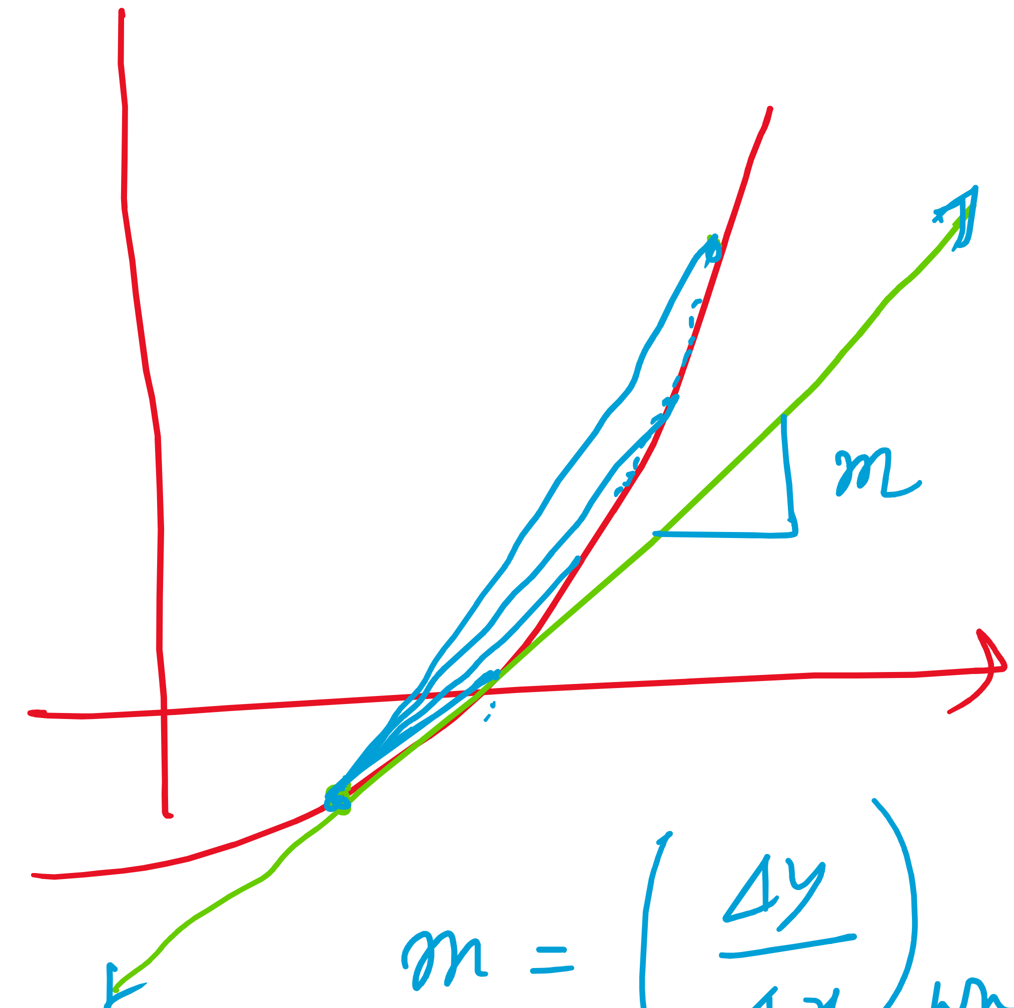




the

$$\tan\theta = \frac{\Delta y}{\Delta x}$$ = slope of a line





$$m = \left(\frac{\Delta y}{\Delta x}\right)$$ when Δx is very very very small

$$m = \lim_{\Delta x \to 0}\left(\frac{\Delta y}{\Delta x}\right) = \frac{dy}{dx} \longrightarrow$$ Differentiation of y wrt x.

When a function reaches its optimum point then at that point $\frac{dy}{dx} = 0$.

What does $\frac{dy}{dx}$ signify? What is the geometrical significance of $\frac{dy}{dx} = ?$

$\frac{dy}{dx} \longrightarrow$ Slope of the tangent drawn at any point on the curve.

$y = x^2$, Now how can we find $\frac{dy}{dx} = ?$

Given $y = f(x)$, how can we find $\frac{dy}{dx} = ?$

$$y = f(x) \Rightarrow \underbrace{\frac{dy}{dx}}_{\text{First Derivative}} = \frac{d}{dx} f(x) \Rightarrow \underbrace{\frac{d^2y}{dx^2}}_{\text{Second derivative}} = \frac{d}{dx}\left(\frac{dy}{dx}\right) \Rightarrow \underbrace{\frac{d^3y}{dx^3}}_{\text{Third Derivative}}$$

Now we will take a look at first derivative of different functions.

$y = f(x)$	$\frac{dy}{dx}$
x	1
c (c is any constant)	0
$ax+b$	a
$\boxed{x^n}$	$\boxed{nx^{n-1}}$

$y = f(x)$	$\frac{dy}{dx}$
x^2	$2x$
x^3	$3x^2$
x^5	$5x^4$
$x^{3.5}$	$3.5x^{2.5}$
x^{-1}	$(-1)\,x^{-2} = -\frac{1}{x^2}$

$x^{-1} = ?$ $\quad x^{-1} = \frac{1}{x}$

$x^{-n} = \frac{1}{x^n}$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

y	$\frac{dy}{dx}$
$\sin(x)$	$\cos(x)$
e^x	e^x
$\log x$	$\frac{1}{x}$
$\cos(x)$	$-\sin(x)$

$$2.7 < e < 2.8$$

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots = \sum_{k=0}^{\infty} \frac{1}{k!}$$

Composite functions

$f(x) = \sin x$ $\quad$ $g(x) = x^2$

$$f(g(x)) = f(x^2) = \sin(x^2)$$

$$g(f(x)) = g(\sin x) = (\sin x)^2 = \sin^2 x$$

$f(x) = x^2 + 2x \quad , \quad g(z) = \log z$

```
def f(x):
    return 2*x + x**2
```

$f(g(z)) = ?$ $\qquad g(f(x)) = ?$

$f(\log z) = 2\log z + (\log z)^2$ $\quad | \quad$ $g(\underbrace{x^2+2x}_{z}) = \log(x^2+2x)$

$f(x) = \sin x \quad , \quad g(x) = 2x \quad , \quad h(x) = \frac{1}{x^2}$

$h(g(f(x))) = ?$ $\qquad h(g(\sin x)) \Rightarrow h(2\sin x) = \frac{1}{(2\sin x)^2}$

$= \frac{1}{2^2 \sin^2 x} = \frac{1}{4\sin^2 x}$

$h(g(f(x))) = \frac{1}{4\sin^2 x}$

$$f(x) = \sqrt{x} \quad , \quad g(z) = z^2$$

$$g(f(x)) = ? \quad \Rightarrow \quad g(\sqrt{x}) = (\sqrt{x})^2 = x$$

$$g(f(x)) = x$$

$$x \longrightarrow \underbrace{\boxed{f(\ )} \longrightarrow \boxed{g(\ )}}_{\text{inverse of one another.}} \longrightarrow \text{output} = x$$

$$\underline{f(x)} = 10^x \quad , \quad \underline{g(z)} = \log_{10} z$$

$$g(f(x)) = g(10^x) = \log_{10}(10^x) = x$$

$$g(f(x)) = x$$

$f(x) \cdots\cdots$ $\quad \underline{f^{-1}(x)} \longrightarrow$ f inverse x.

<u>Inverse function</u>