Composite Functions:-

$f(x) = \sin x \quad , \quad g(z) = z^2 \implies g(f(x)) = ?$

$\therefore \; g(f(x)) = g(\sin x) = (\sin x)^2 = \sin^2 x$

How to differentiate composite functions:-

$h(x) = g(f(x)) = \sin^2 x$

$\frac{dh}{dx} = ?$ Let, $f(x) = z = \sin x$

Then, $h(x) = g(z) = z^2$

$\frac{dh}{dz} = \frac{d}{dz}(z^2) = 2z$

$\frac{dh}{dx} = \frac{dh}{dz} \cdot \frac{dz}{dx} = (2z) \cdot \frac{d}{dx}(\sin x) = 2 \sin x \cos x$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(x^2) = 2x$

$\boxed{\frac{d}{dx}(\sin^2 x) = 2 \sin x \cos x}$

Chain Rule of Differentiation

$f(x)$, $g(x)$, $h(x)$, ...

$$\frac{d}{dx}\left[h(g(f(x)))\right] = ?$$

$\underline{f(x) = w}$, $\underline{g(f(x)) = z}$

$\Rightarrow g(w) = z$

$$\frac{d}{dx}\left[h(z)\right] = \frac{dh}{dz} \cdot \frac{dz}{dx}$$

$$= \frac{dh}{dz} \cdot \frac{dz}{dw} \cdot \frac{dw}{dx}$$

$$\boxed{\frac{d}{dx}\left[h(g(f(x)))\right] = \frac{dh}{dg} \cdot \frac{dg}{df} \cdot \frac{df}{dx}}$$

Chain Rule of Differentiation

(Very important concept to learn machine learning & deep learning)

$$\frac{d}{dx}\left(\underbrace{\sqrt{\sin^3 x}}\right) = ?$$

$f(x) = \sin x$, $g(x) = x^3$, $h(x) = \sqrt{x}$

$$h(\underbrace{g(f(x))}) = \sqrt{(\sin x)^3} = \sqrt{\sin^3 x}$$

$$z = g(\underbrace{f(x)}_{w}) = g(w)$$

$$dy/dx =$$

$$\frac{dh}{dz} \cdot \frac{dz}{dw} \cdot \frac{dw}{dx}$$

$$\Rightarrow \frac{dh}{dx} = \frac{1}{2\sqrt{z}} \cdot 3w^2 \cdot \cos x$$

$$= \frac{1}{2\sqrt{z}} \cdot 3\sin^2 x \cdot \cos x$$

$$\boxed{\frac{dh}{dx} = \frac{1}{2\sqrt{\sin^3 x}} \cdot 3\sin^2 x \cos x}$$

$h(x) = \sqrt{x}$

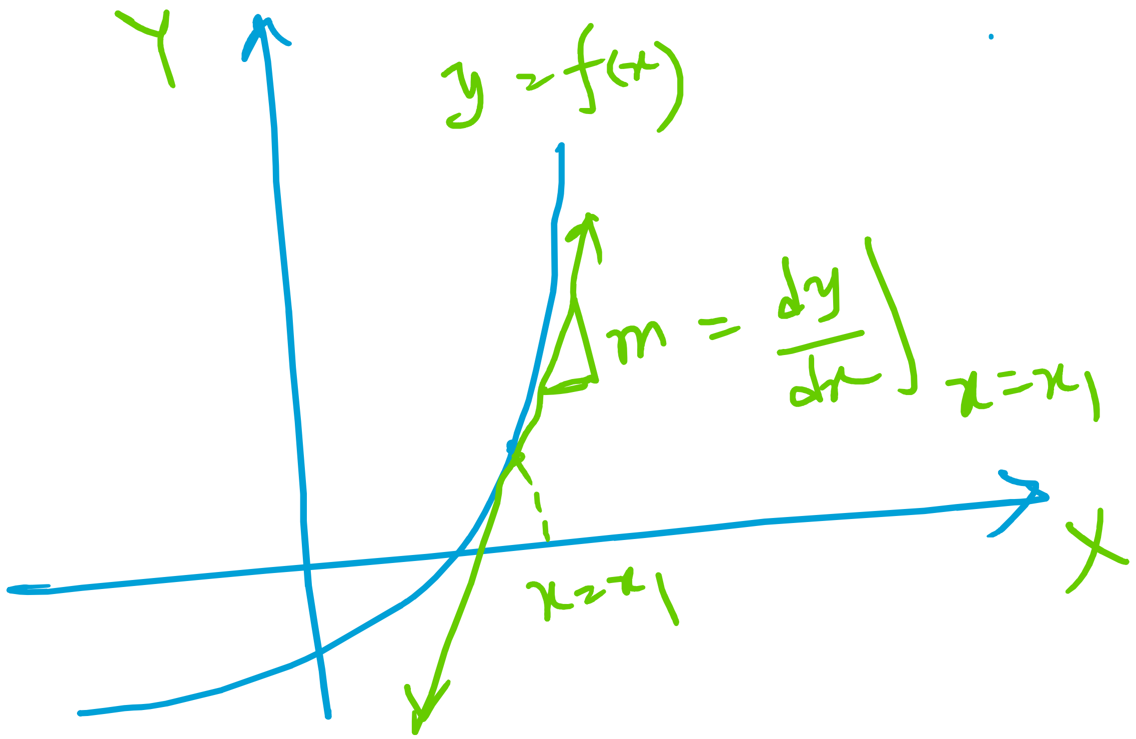
$$\Rightarrow \frac{dh}{dx} = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$$

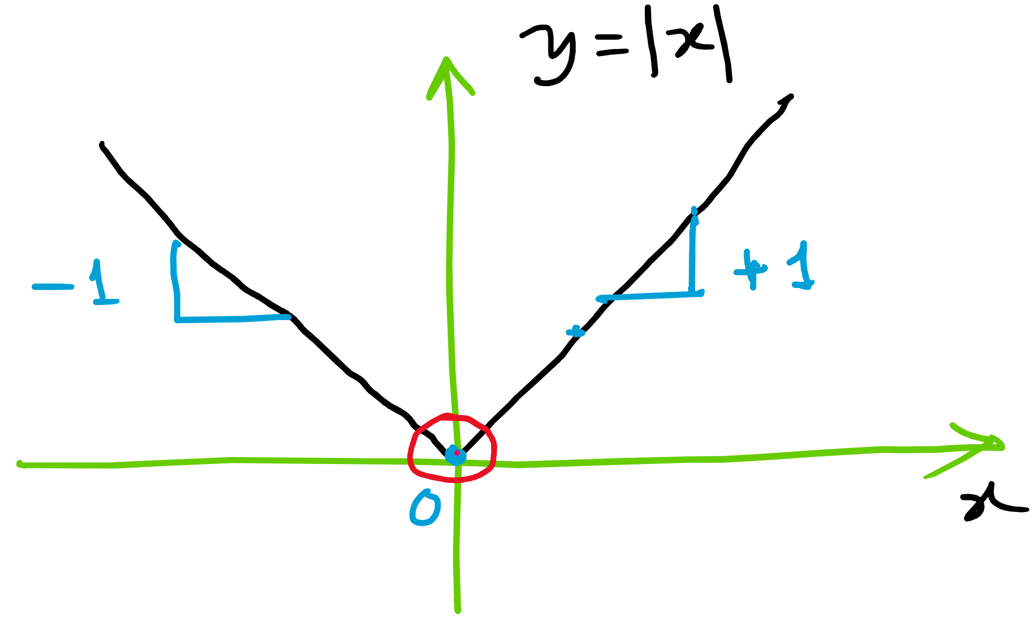
$g(x) = x^3$

$$\Rightarrow \frac{dg}{dx} = 3x^2$$

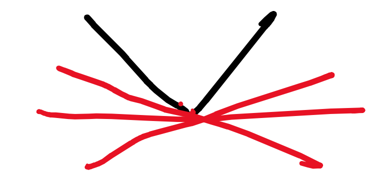
$f(x) = \sin x$

$$\Rightarrow \frac{df}{dx} = \cos x$$





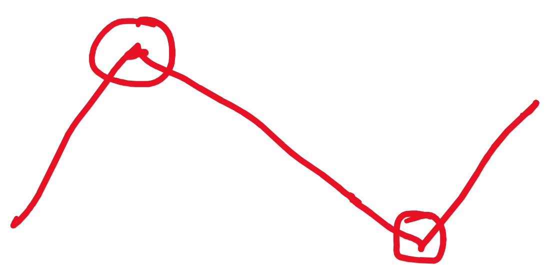
$$y = x \;;\; x \geq 0$$
$$= -x \;,\; x < 0$$

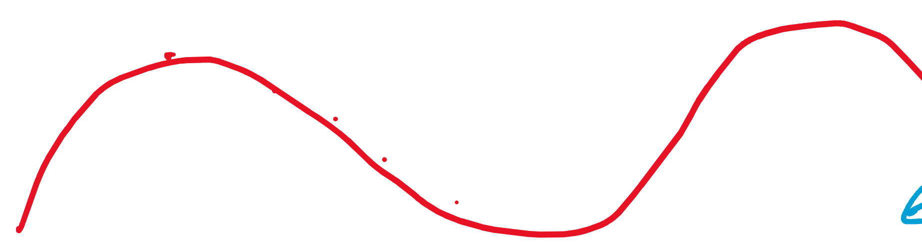


$\frac{dy}{dx}$ where $y = |x|$

$$\frac{dy}{dx} = +1 \quad x > 0$$
$$= -1 \quad x < 0$$

The derivative doesn't exist at $x = 0$



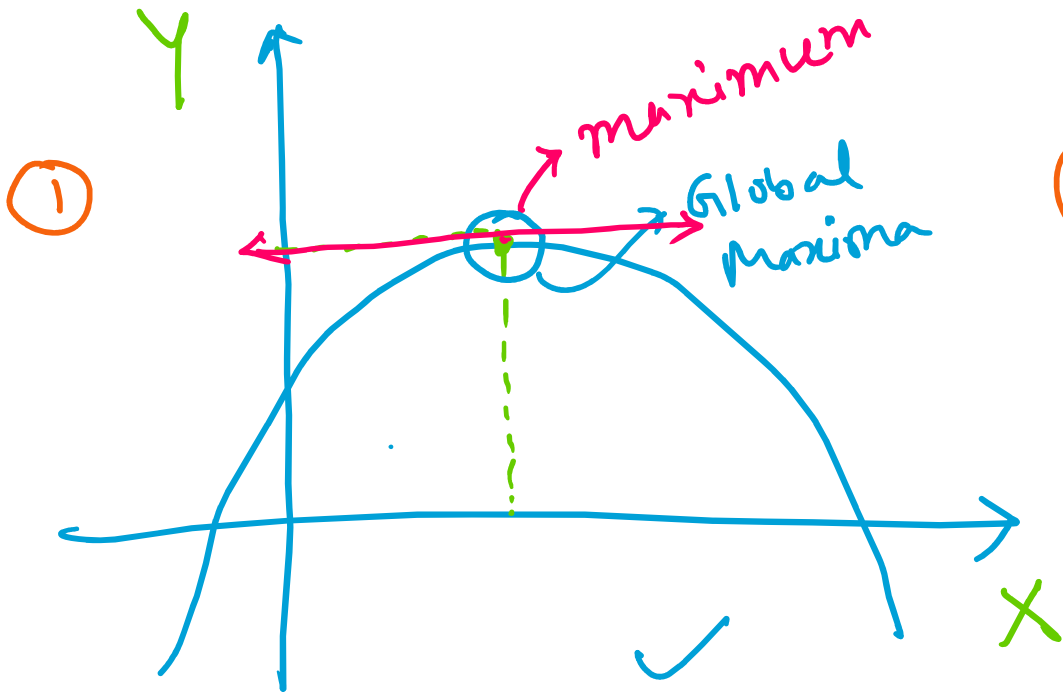


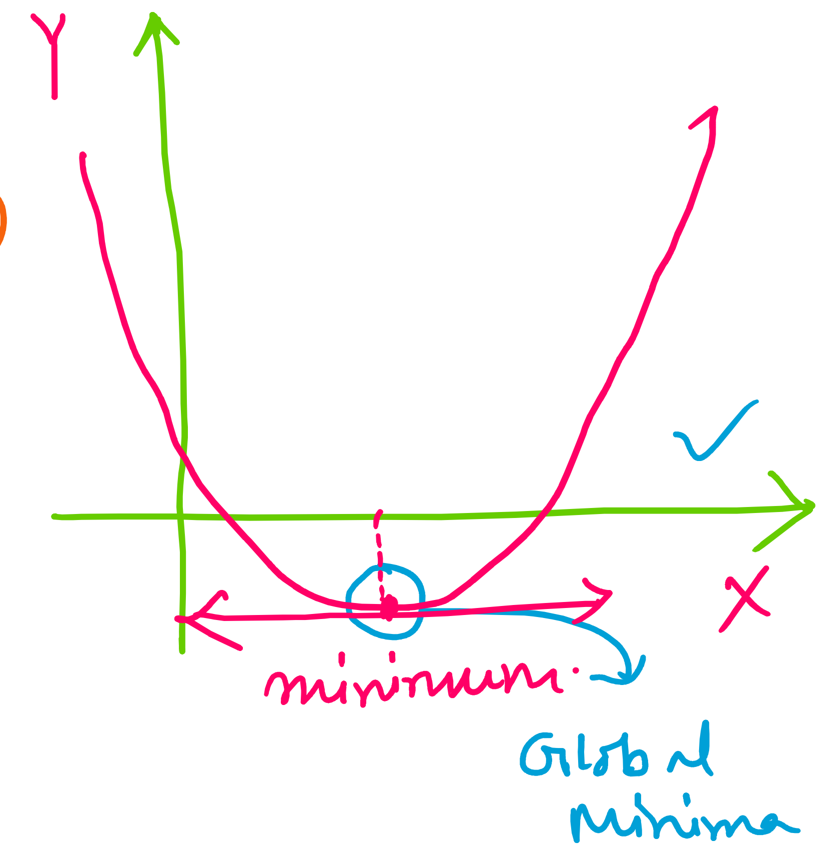
$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{df}{dx} \pm \frac{dg}{dx}$$

$$\frac{d}{dx}\left[f(x) \times g(x)\right]$$
$$= f\frac{dg}{dx} + g\frac{df}{dx}$$

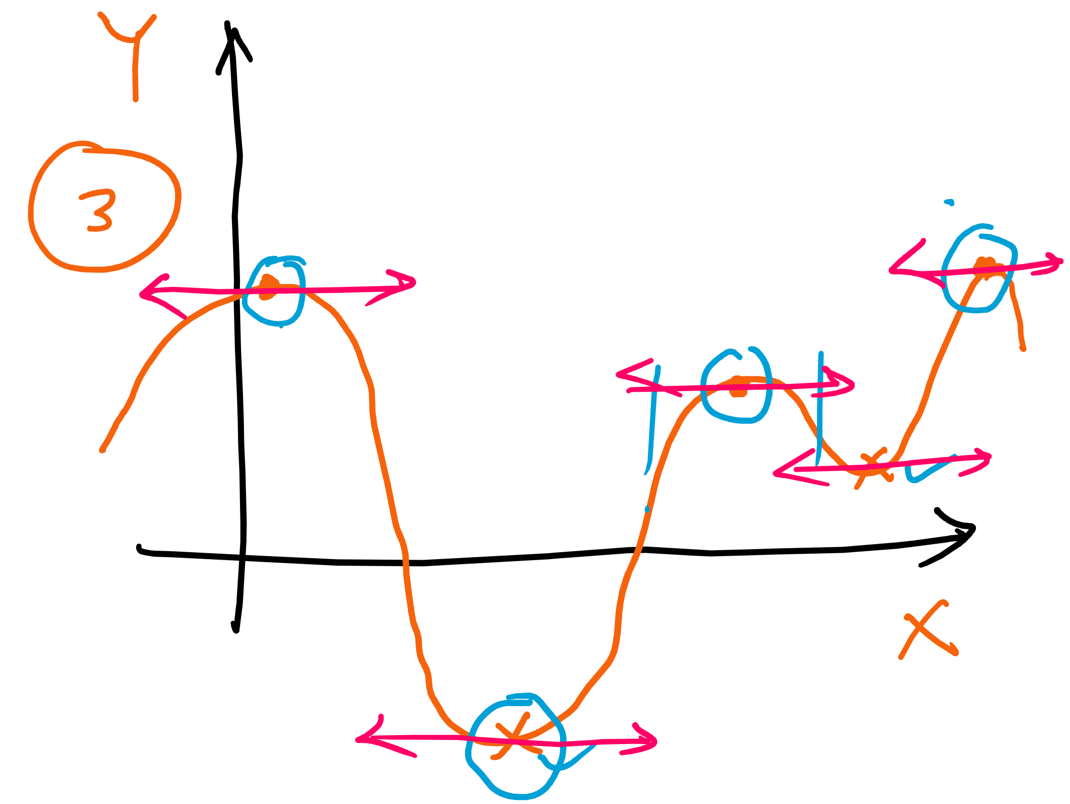
$$\frac{d}{dx}\left(x^2 \sin x\right) = x^2\frac{d}{dx}(\sin x) + \sin x \cdot \frac{d}{dx}(x^2)$$
$$= x^2 \cos x + 2x \sin x$$

Maxima & Minima (Finding Optimum) of a function :-





Convex



Non-convex

The tangents drawn on the maxima & minima of a differentiable function have zero slopes.

$y = f(x)$ $\quad \boxed{\dfrac{dy}{dx} = 0}$

We get maxima & minima when $\dfrac{dy}{dx} = 0$.

$y = 6x - 3x^2 + 2$

$$\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{d}{dx}(6x) - \frac{d}{dx}(3x^2) + \cancel{\frac{d}{dx}(2)}^{\,0} = 0$$

$$\Rightarrow 6 - 3 \times 2x = 0$$

$$\Rightarrow 6 = 6x$$

$$\Rightarrow x = 1.$$

$$y_{max} = 6 - 3 + 2 = 5$$

$$\frac{d}{dx}[f(x)]$$

$$\frac{d}{dx}[k f(x)] = k \frac{df(x)}{dx}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$y = x^3 e^{-x} \quad (x > 0) \quad \Rightarrow$$

$z = -x.$

$$\frac{dz}{dx} = -1$$

$$\frac{d}{dx}(e^{-x}) = \frac{d}{dz}(e^z) \cdot \frac{dz}{dx}$$

$$= e^z(-1) = -e^{-x}$$

$$y = x^3 e^{-x}$$

$$\frac{d}{dx}\left[f(x) \cdot g(x)\right] = f(x) \cdot \frac{d}{dx} g(x) + g(x) \cdot \frac{d}{dx} f(x)$$

$$\Rightarrow \frac{dy}{dx} = x^3 \cdot \frac{d}{dx}\left(e^{-x}\right) + e^{-x} \cdot \frac{d}{dx}\left(x^3\right)$$

$$\Rightarrow \frac{dy}{dx} = -e^{-x} \cdot x^3 + 3x^2 e^{-x}$$

$$\frac{dy}{dx} = 0 \Rightarrow -e^{-x} x^3 + 3x^2 e^{-x} = 0$$

$$\Rightarrow e^{-x}\left(3x^2 - x^3\right) = 0$$

$$\Rightarrow 3x^2 - x^3 = 0$$

$$\Rightarrow x^2 (3 - x) = 0$$

$$\Rightarrow x^2 = 0 \quad \text{or} \quad x = 3$$

$$y_{max} = 3^3 \cdot e^{-3} = \frac{27}{e^3}$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{df(x)}{dx} - f(x)\frac{dg(x)}{dx}}{(g(x))^2}$$

$$= \frac{\text{Denominator} \times \text{Differentiation of Numerator} - \text{Numerator} \times \text{Differentiation of denominator}}{(\text{denominator})^2}$$

$$\frac{d}{dx}\left(\frac{\sin x}{x^2}\right) = \frac{x^2 \frac{d}{dx}(\sin x) - \sin x \cdot \frac{d}{dx}(x^2)}{x^4}$$

$$= \frac{x^2 \cos x - 2x \sin x}{x^4}$$

$$\frac{d}{dx}(\tan x) = ?$$

$$\frac{d}{dx}(\sin x) = \cos x \quad , \quad \frac{d}{dx}(\cos x) = -\sin x$$

$$\Rightarrow \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x \cdot \frac{d}{dx}(\sin x) - \sin x \frac{d}{dx}(\cos x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \left(\frac{1}{\cos x}\right)^2 = (\sec x)^2$$

$$= \sec^2 x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$y = 3x^2 - 2x + 6$$

Find where y attend <u>minima</u>.

$$\frac{dy}{dx} = 3\cdot(2x) - 2 = 6x - 2$$

$$\frac{dy}{dx} = 0$$

$$\Rightarrow 6x - 2 = 0$$

$$\Rightarrow x = \frac{2}{6} = \frac{1}{3}$$

$$y_{min} = 3\cdot\frac{1}{3^2} - \frac{2}{3} + 6 = \frac{1}{3} - \frac{2}{3} + 6 = 6 - \frac{1}{3} = 5.67$$