Function of several variable

$$f(x_1, x_2, \ldots\ldots, x_n)$$

$$f(x_1, x_2) = x_1 * x_2$$

$$f(2, 3) = 6$$

$$f(1, 0) = 0$$

$$f(x, y) = x^2 + 2xy^2 + \sin(x+y)$$

$$\underbrace{f(x, t) = A \sin(\omega t - kx)}_{\text{Wave function of single dimension}}$$

$$f(x, y) = \boxed{1000\,x + 500\,y + 10000}$$

$f(x, y) = xy^2 + x^2y = xy(x+y)$

Differentiation: $f(x,y)$

Differentiate w.r.t. x keep y constant.

$\dfrac{\partial f}{\partial x}$ [Del f Del x]

[Partial Differentiation of 'f' w.r.t. x]

Differentiate w.r.t. y keeping x constant.

$\dfrac{\partial f}{\partial y}$ [Partial Derivative of f w.r.t. y]

$f(x,y) = x^2y + y^2x \quad \therefore \dfrac{\partial f}{\partial x} = y(2x) + y^2 = 2xy + y^2$

$\dfrac{\partial f}{\partial y} = x^2 + 2yx$

$$\frac{\partial}{\partial x}\left[\sin\left(x^2+y^2\right)\right] = \cos\left(x^2+y^2\right)\cdot 2x$$

$$= 2x\cos\left(x^2+y^2\right).$$

Partial: $\frac{\partial}{\partial x}\left[x^2+y^2\right] = 2x + 0 = 2x$

Partial Differentiation:- $f(x_1, x_2, x_3, \ldots, x_n)$.

$$\frac{\partial f}{\partial x_1} \quad \frac{\partial f}{\partial x_2} \quad \frac{\partial f}{\partial x_3} \quad \cdots\cdots \quad \frac{\partial f}{\partial x_n}$$

Differentiation wrt x_1
keeping all other variable constant.

$$f(x,y) = \log_e(\underbrace{x^2 \sin y}_{z}) \qquad \text{find } \frac{\partial f}{\partial x} \;\&\; \frac{\partial f}{\partial y}$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial x} \qquad \frac{\partial}{\partial z}(\log z) = \frac{1}{z}$$

$$= \frac{1}{z}(2x \sin y)$$

$$= \frac{1}{x\cancel{x}\cancel{\sin y}} \cdot (2\cancel{x}\cancel{\sin y})$$

$$= \frac{2}{x}$$

$$\frac{\partial f}{\partial y} = \frac{1}{\cancel{x^2}\sin y}(\cancel{x^2}\cos y) = \cot y$$

$$f(x_1, x_2, x_3) = a(x_1 x_2 x_3 + x_2^2 + x_3) + b(x_3^2 + x_2 + 5x_1 x_2)$$

$$\frac{\partial f}{\partial x_2} = ? \qquad \frac{\partial f}{\partial x_2} = a(x_1 x_3 + 2x_2) + b(1 + 5x_1)$$

Maxima & Minima of a function with several variables:-

$$f(x_1, x_2, x_3, \dots, x_n)$$

$$\begin{cases} x+y = 20 \\ x-y = 10 \end{cases}$$

$x+y=30$

$$\left.\begin{aligned} \frac{\partial f}{\partial x_1} &= 0 \\ \frac{\partial f}{\partial x_2} &= 0 \\ &\vdots \\ \frac{\partial f}{\partial x_n} &= 0 \end{aligned}\right\} \underline{n\text{-equations.}}$$

($\underline{n \text{ variables}}$)

$$f(x,y) = \underline{x^2 y + x - y}$$

$$\frac{\partial f}{\partial x} = 2xy + 1 \quad , \quad \frac{\partial f}{\partial y} = x^2 - 1$$

$$\frac{\partial f}{\partial x} = 0$$

$$\Rightarrow 2xy + 1 = 0$$

$$\Rightarrow 2 \cdot (1) y = -1$$

$$\Rightarrow y = -\frac{1}{2}$$

$$2(-1) y = -1$$

$$\Rightarrow y = \frac{1}{2}$$

$$\frac{\partial f}{\partial y} = 0$$

$$\Rightarrow x^2 - 1 = 0$$

$$\Rightarrow x^2 = 1$$

$$\Rightarrow x = \pm 1$$

$$x = \pm 1, \quad y = \mp \frac{1}{2}$$

$$\left(1, -\frac{1}{2}\right) \checkmark$$

$$\left(-1, \frac{1}{2}\right) \checkmark$$