

Integral Calculus:-

$$\frac{d}{dx} F(x) = f(x)$$

$$\Rightarrow F(x) = \int f(x) dx + C$$

$$\int f(x) dx = F(x) + C$$

Indefinite Integral

$$\int_a^b f(x) dx = F(b) - F(a)$$

Definite Integral.

$$\frac{d}{dx}(x^2) = 2x$$

$$\frac{d}{dx}(x^2 + 3) = 2x$$

$$\int 2x dx = x^2 + C$$

(C is constant of Integration)

$$1) \int k f(x) dx = k \int f(x) dx$$

$$2) \int [a f(x) \pm b g(x)] dx = a \int f(x) dx \pm b \int g(x) dx \quad] \text{Linearity.}$$

$$3) \frac{d}{dx}(x^n) = nx^{n-1} \Rightarrow \int nx^{n-1} dx = x^n + C$$

$$\Rightarrow \int x^{n-1} dx = \frac{x^n}{n} + C$$

$$\text{Let } n = k+1$$

$$\Rightarrow \boxed{\int x^k dx = \frac{x^{k+1}}{k+1} + C}$$

$$(\text{When } k \neq -1)$$

for $k=0$

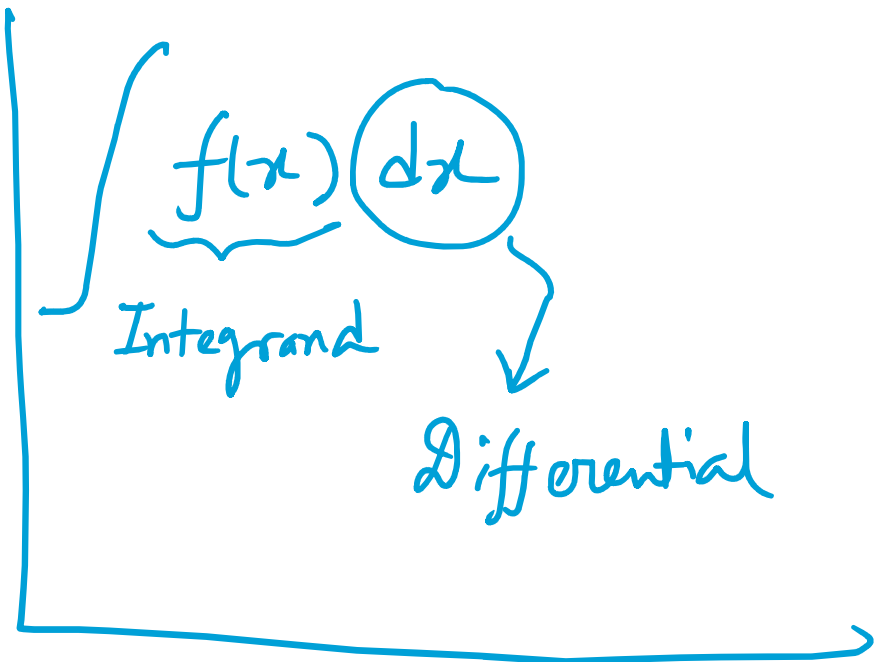
$$\int x^0 dx = x + C$$

$$\Rightarrow \int 1 \cdot dx = x + C$$

$$\frac{d}{dx}(\log_e x) = \frac{1}{x} \quad \therefore \int \frac{1}{x} dx = \log_e x + C$$

$$\frac{d}{dx}(\sin x) = \cos x \quad \Rightarrow \int \cos x dx = \sin x + C$$

$$\frac{d}{dx}(\cos x) = -\sin x \quad \Rightarrow \int \sin x dx = -\cos x + C$$



$$\begin{aligned} \int (3x^2 + \sin x) dx &= 3 \int x^2 dx + \int \sin x dx \\ &= 3 \cdot \left(\frac{x^3}{3} \right) + (-\cos x) + C \\ &= x^3 - \cos x + C \end{aligned}$$

$$\frac{dy}{dx} = f(x) \rightarrow \text{Differential Equation.} \quad \left/ \quad \begin{array}{l} F(b) + c - (F(a) + c) \\ = F(b) - F(a) + \cancel{c} - \cancel{c} \end{array} \right.$$

$$y = \int f(x) dx + C$$

Definite Integration :- Suppose $\int f(x) dx = \underline{F}(x) + C$

$$\int_a^b f(x) dx = F(b) - F(a)$$

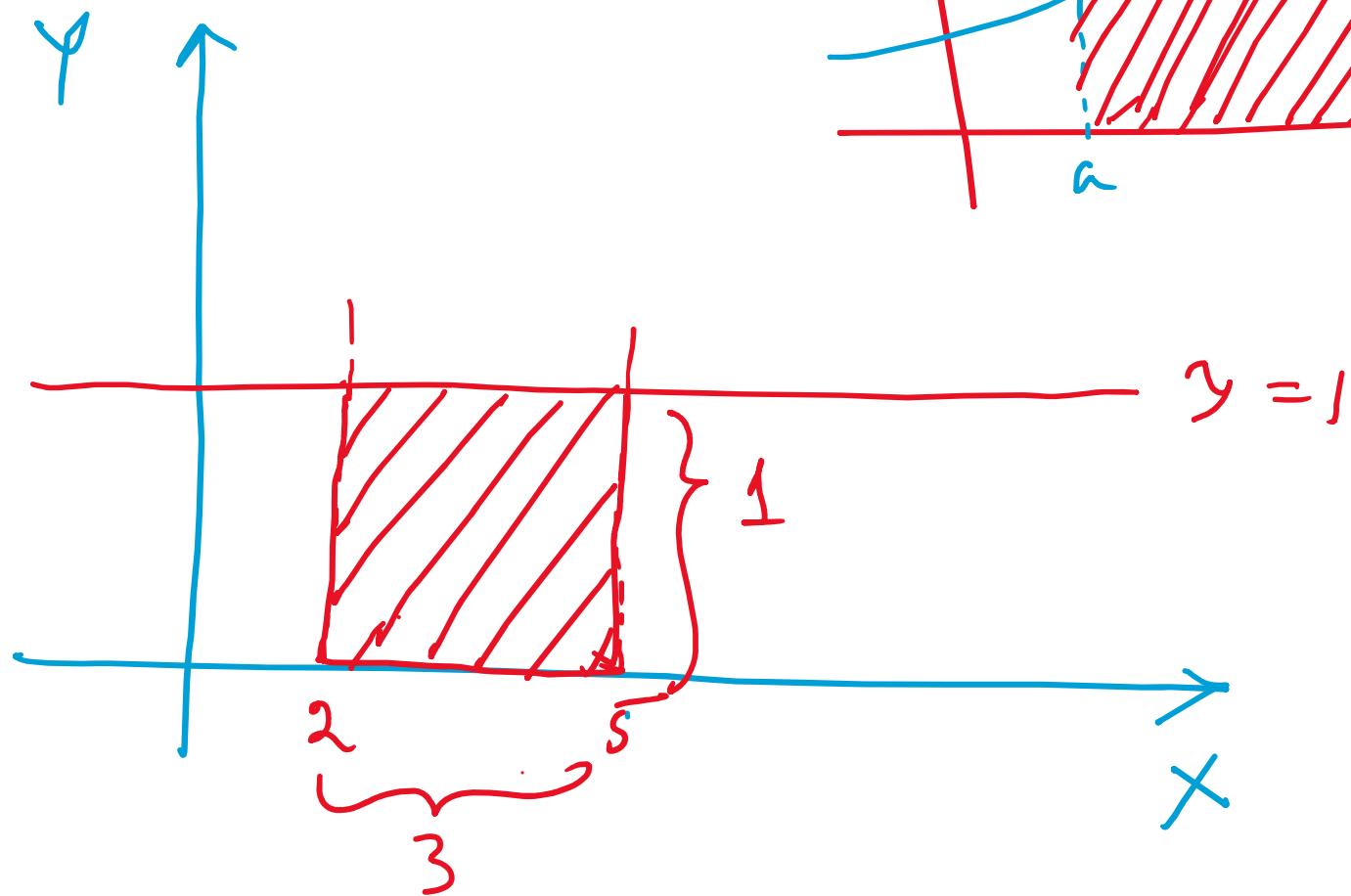
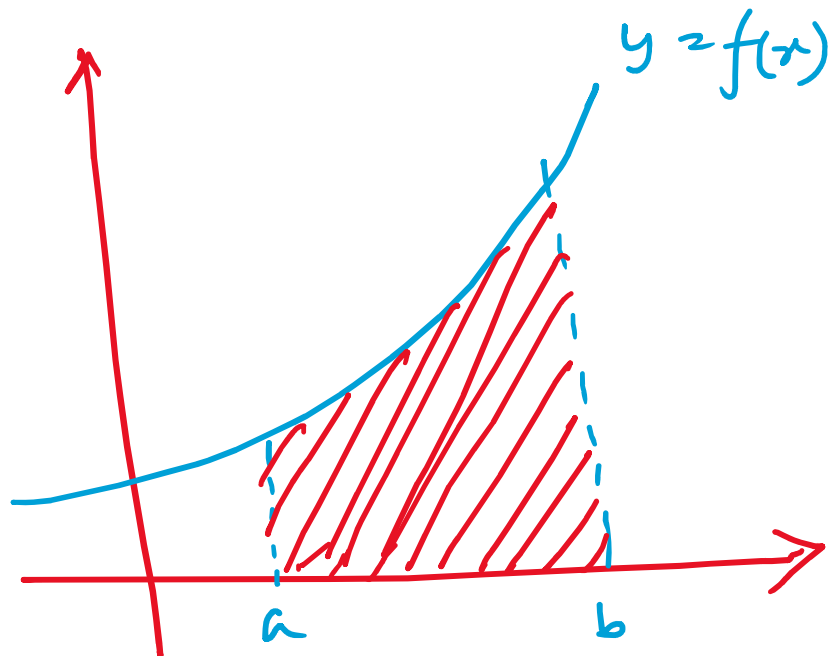
$$\begin{aligned} \int_0^2 3x^2 dx &= [x^3]_0^2 \\ &= 2^3 - 0^3 \\ &= 8 \end{aligned}$$

$$\begin{aligned} \int_0^\pi \sin x dx &= [-\cos x]_0^\pi = -[\cos x]_0^\pi = -(\cos \pi - \cos 0) \\ &= -(-1 - 1) = 2 \end{aligned}$$

Geometrical Interpretation of Definite Integral:-

$$\sum_{a}^b \Rightarrow \int \text{Sum}$$

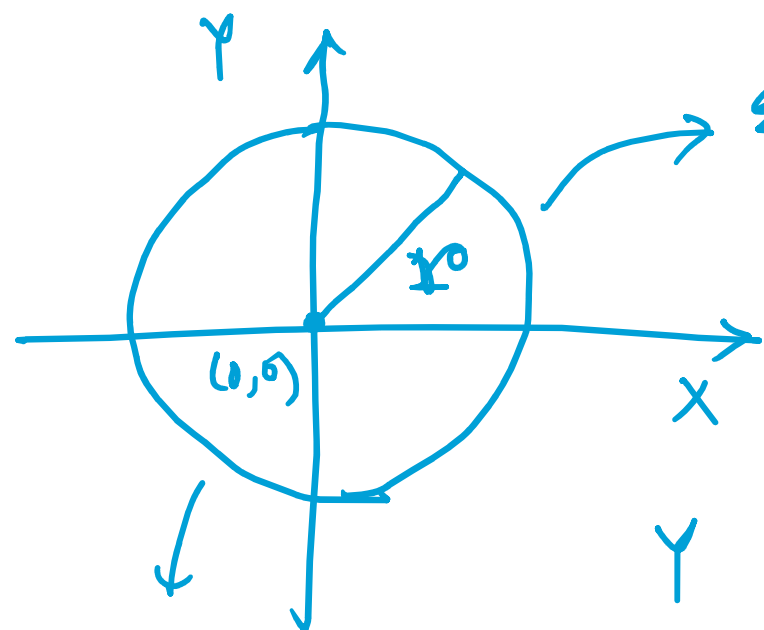
$$\int_a^b f(x) dx$$



$$\int_2^5 1 \cdot dx = [x]_2^5 = 5 - 2 = \underline{3}$$

$$y = 1$$

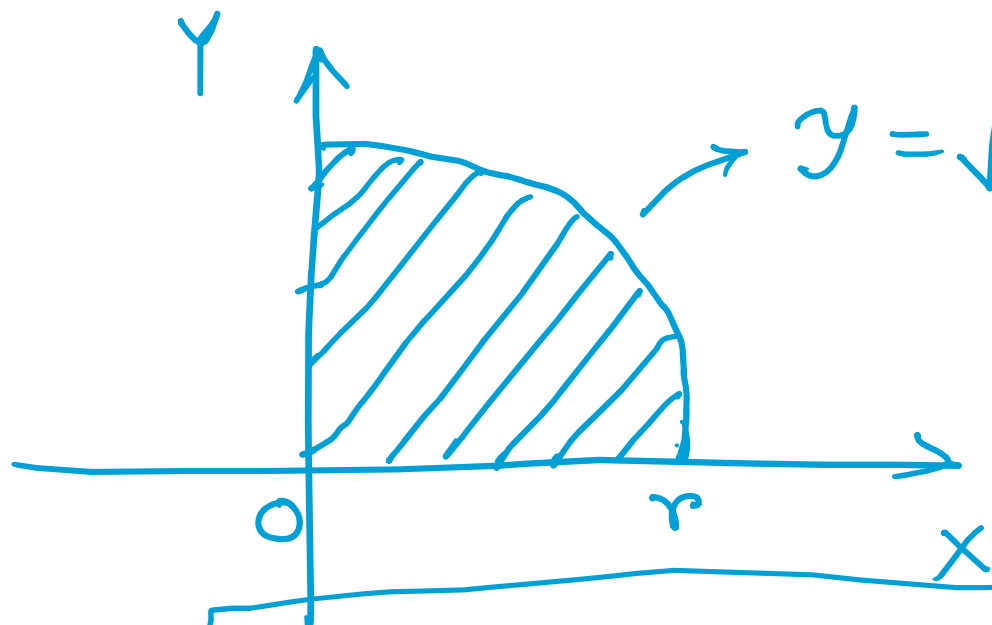
$\int_a^b f(x) dx \rightarrow$ It is the measure of the area under the curve $y = f(x)$ between $x = a$ & $x = b$.



Equation of ~~unit radius~~ circle centred at $(0,0)$ and radius 'r'

$$x^2 + y^2 = r^2$$

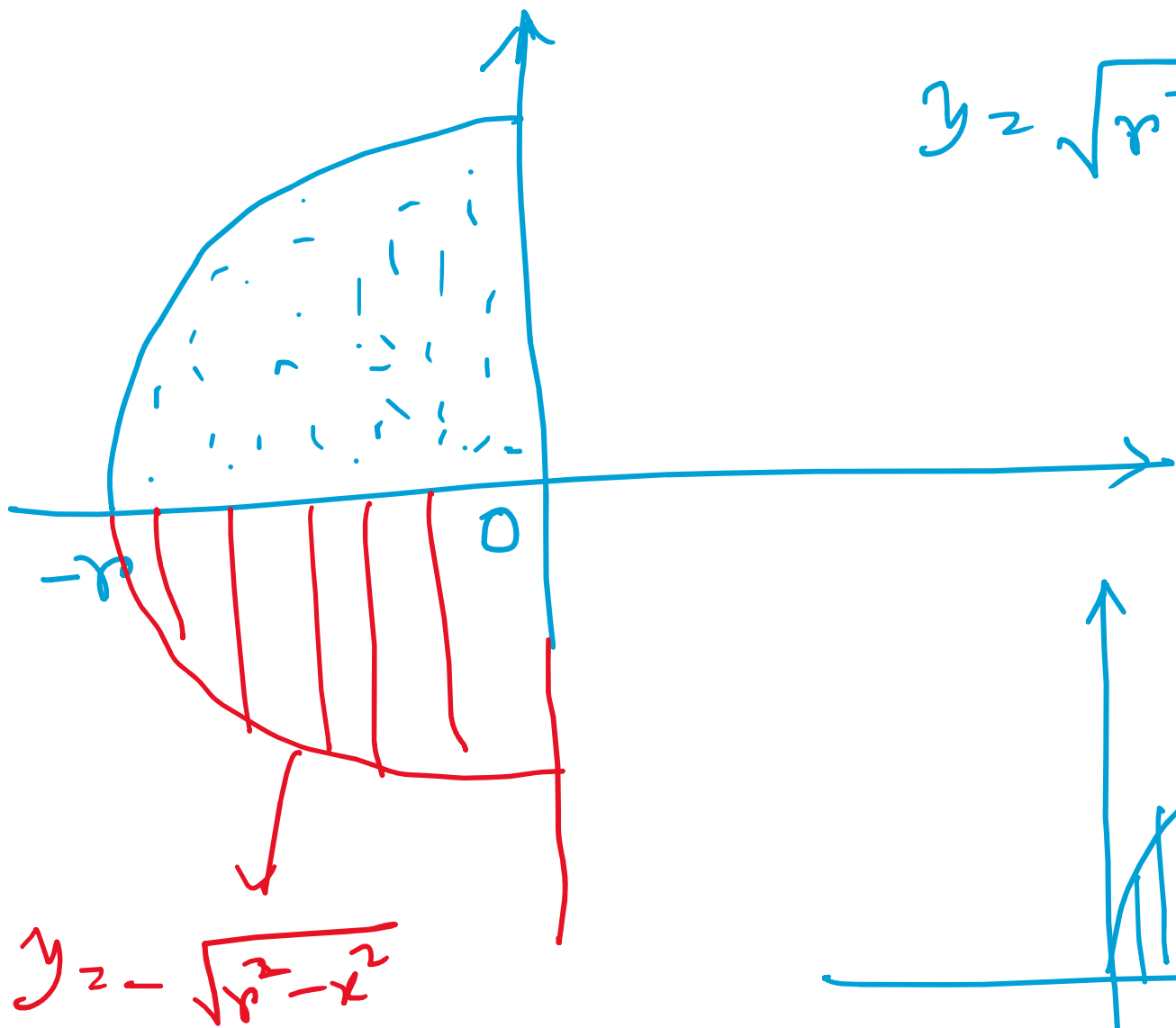
$$\pi r^2$$



$$y = \sqrt{r^2 - x^2}$$

$$\int_0^r \sqrt{r^2 - x^2} \, dx = \frac{\pi}{4} r^2$$

$$\int_0^a \sqrt{a^2 - x^2} \, dx = \frac{\pi}{4} a^2$$



$$y = \sqrt{r^2 - x^2}$$

$$\int_{-r}^0 \sqrt{r^2 - x^2} dx = \frac{\pi}{4} r^2$$

$$\int_2^5 (-1) dx = \underline{-3}$$

