Logarithm

$2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$

$2^6 = 64$, $2^7 = 128$

$2^x = 32$ then $x = ?$

$2^x = 100$ then $x = ?$

$6 < x < 7$

$\Rightarrow x = \log_2 (32)$

$x = \underbrace{\log_2 100}$

exponent

$\text{base} \rightarrow (b)^x = y$

$\Rightarrow \boxed{x = \underbrace{\log_b (y)}}$

logarithm (log) of y to the base 'b'

Constraints :- $y \neq 0$, $b \neq 1$

$1^{100} = 1$

$1^{1000} = 1$

log 100 to the base 2

$$3^4 \times 3^3 = (3\times3\times3\times3)\times(3\times3\times3) = 3\times3\times3\times3\times3\times3\times3 = 3^7$$

$$b^x \times b^y = b^{x+y}$$

$$b^{x_1} \times b^{x_2} \times b^{x_3} \times \cdots \times b^{x_n} = b^{(x_1+x_2+\cdots+x_n)} = Z$$

$$Z = b^{x_1} \times b^{x_2} \times \cdots \times b^{x_n} = b^{(x_1+x_2\cdots+x_n)}$$

$$\Rightarrow \log_b Z = x_1+x_2+\cdots+x_n$$

$$b^3 \times b^7 = e \Rightarrow \log_b e = 3+7 = 10$$

Rule-1

if $\log_b x = y$ then $x = b^y$ $\quad \log_2 32 = 5 \Rightarrow 2^5 = 32$

$\Rightarrow x = b^y$

$\Rightarrow \boxed{x = b^{(\log_b x)}}$ ✓

Rule-2

$x = b^{\log_b x}$, $y = b^{\log_b y}$, $z = b^{\log_b z}$

$\Rightarrow xyz = b^{\overset{a_1}{(\log_b x)}} \times b^{\overset{a_2}{(\log_b y)}} \times b^{\overset{a_3}{(\log_b z)}}$

$\Rightarrow xyz = b^{a_1} \times b^{a_2} \times b^{a_3}$ $\quad \left(\begin{array}{l} a_1 = \log_b x \\ a_2 = \log_b y \\ a_3 = \log_b z \end{array} \right)$

$\Rightarrow xyz = b^{(a_1+a_2+a_3)}$

$\Rightarrow \log_b(xyz) = a_1+a_2+a_3 \Rightarrow \boxed{\log_b(xyz) = \log_b x + \log_b y + \log_b z}$

$$\underline{x \times y \times z = k}$$

$$\Rightarrow \underbrace{\log_b (x \times y \times z)} = \log_b k \qquad (b \neq 1)$$

$$\Rightarrow \log_b x + \log_b y + \log_b z = \log_b k$$

Log tranforms multiplication to addition.

$$\log_b (x_1 x_2 x_3 \cdots x_n) = \log_b x_1 + \log_b x_2 + \cdots + \log_b x_n$$

Rule - 3

$$\log_b(x_1 x_2 \cdots\cdots x_n) = \underbrace{\log_b x_1 + \log_b x_2 + \cdots + \log_b x_n}_{n \text{ terms.}}$$

$$\log_b(x_1 x_2) = \log_b x_1 + \log_b x_2 \qquad (x_1 = x_2 = x)$$

$$\Rightarrow \log_b(x^2) = \log_b x + \log_b x = 2\log_b x$$

$$\log(x_1 x_2 x_3) = \log_b x_1 + \log_b x_2 + \log_b x_3 \qquad (x_1 = x_2 = x_3 = x)$$

$$\Rightarrow \log(x^3) = \log_b x + \log_b x + \log_b x = 3\log x$$

$$\boxed{\log(x^n) = n\log x}$$

$$\log(x^a y^b z^c) = \log(x^a) + \log(y^b) + \log(z^c)$$
$$= a\log x + b\log y + c\log z$$

Rule-4: $\log_a x = (\log_b x) \times (\log_a b)$ $\quad (a \,\&\, b \neq 1)$

$$\Rightarrow \log_b x = \frac{\log_a x}{\log_a b} \Bigg\} \text{ Base conversion}$$

$\pi = 3.14159 \ldots \ldots$

↳ irrational.

Usually:- We use base-10 Logarithm (common log)

$\log_{10}(x)$

e → Euler's number

$= 2.7 \ldots$

In mathematics we encounter another type of logarithm.

$\log_e(x)$ $\qquad \ln(x)$

↳ $\log_e x$

(natural logarithm)

In computer Science :- Base 2 Logarithm.

$\log_2 x$

$\boxed{x^0 = 1}$ $(x \neq 0)$

$b = 2$

$2^3 = 8$, $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

$2^5 = 32$ $2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

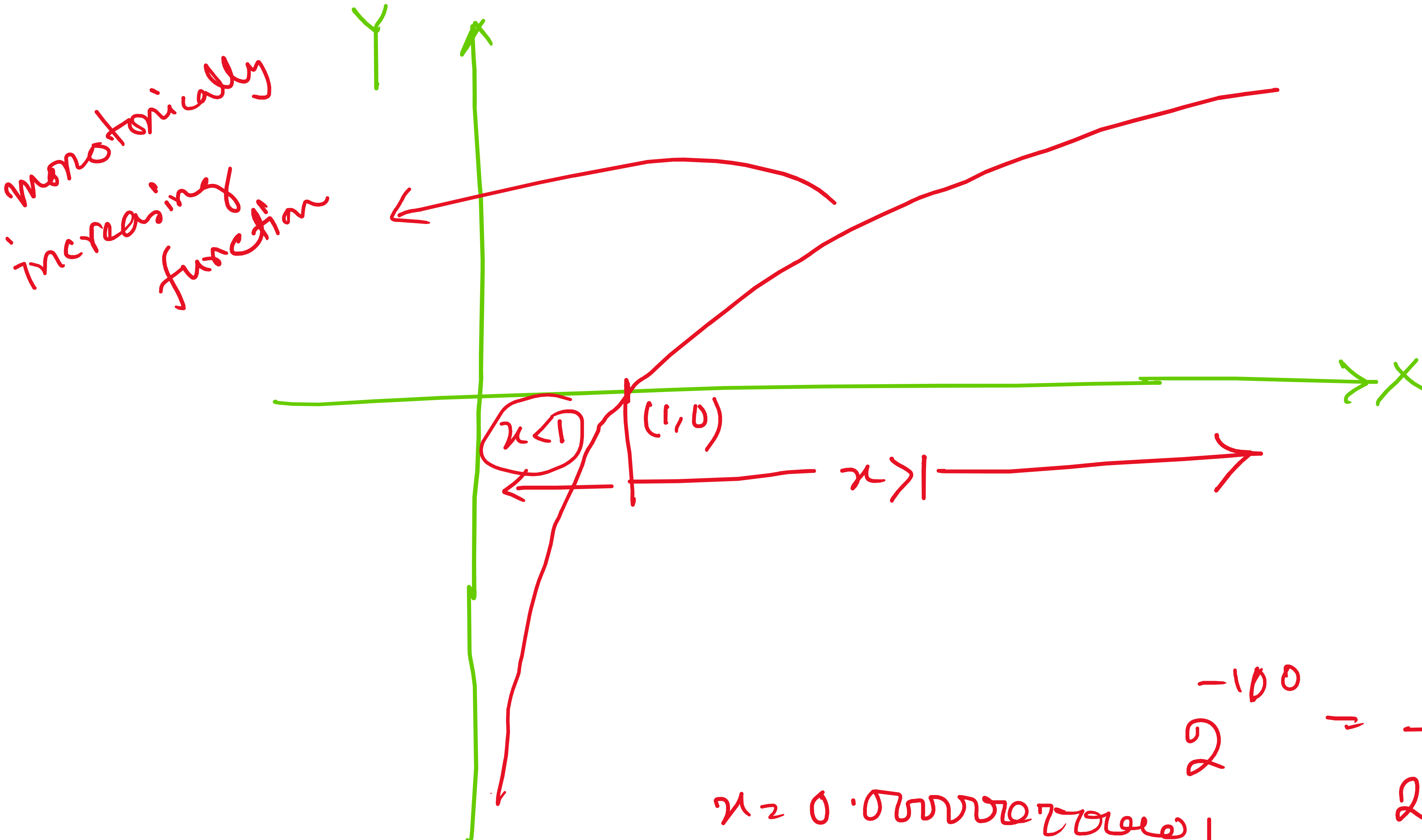
$\log_b(1) = 0$

$\boxed{\log_b(b^{-x}) = -x}$

2^x , $2^{-x} = \frac{1}{2^x}$

$\boxed{b^{-x} = \frac{1}{b^x}}$

$y = \log_b x \quad (b \neq 1, \; b > 1) \quad (x > 0)$



$x = 0.0000000000001$

$x = 1, \quad y = \log_b 1 = 0$

$x > 1, \quad y > 0$

$x < 1, \quad y < 0$

$(2)^x = -\frac{1}{3} = -3$

$2^{1/2} = \sqrt{2}$

$2^{1/3} = \sqrt[3]{2}$

$2^{-100} = \frac{1}{2^{100}}$

$\log_2 (2^{-100}) = -100$

Summation Notation:-

$$x_1 + x_2 + x_3 + \cdots + x_n = \sum_{i=1}^{n} x_i$$

$$\sum_{i=1}^{10} x_i = x_1 + x_2 + x_3 + \cdots + x_{10}$$

$$\sum_{i=1}^{n} x_i^2 = x_1^2 + x_2^2 + \cdots + x_n^2$$

$$\sum_{i=1}^{n} \log_b(x_i) = \log_b x_1 + \log_b x_2 + \log_b x_3 + \cdots + \log_b x_n$$

Product Notation

$$\prod_{i=1}^{n} x_i = x_1 \times x_2 \times x_3 \times \cdots \times x_n$$

$$\prod_{i=1}^{n} 2^{x_i} = 2^{x_1} \times 2^{x_2} \times 2^{x_3} \times \cdots \times 2^{x_n}$$

$$\log_b\left(\prod_{i=1}^{n} x_i\right) = \log_b\left(x_1 \times x_2 \times \cdots \times x_n\right) = \log_b x_1 + \log_b x_2 + \cdots + \log_b x_n$$

$$\Rightarrow \boxed{\log_b\left(\prod_{i=1}^{n} x_i\right) = \sum_{i=1}^{n} \log_b x_i}$$

$$\log_{10}(23718 \times 312446) = \log(23718) + \log(312446)$$

$$(10^c)$$

$$= a + b = c$$

$$\log_2(1024 \times 4096) = \log_2(1024) + \log_2(4096)$$

$$= 10 + 12 = 22$$

$$1024 \times 4096 = 2^{22}$$