Vector Algebra:

Vectors: Vector is a collection of real numbers (in a form of 1-D array)

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_n \end{bmatrix}$$

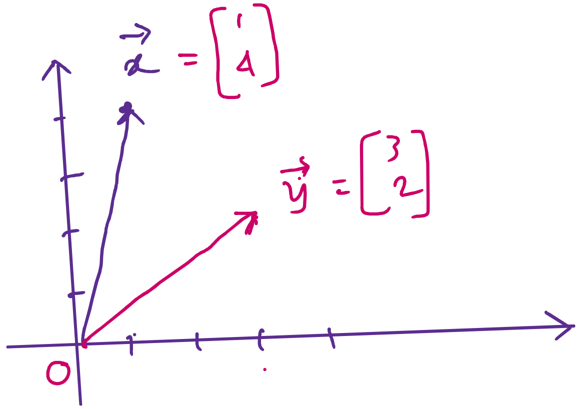
$x_1 \in \mathbb{R}$, $x_2 \in \mathbb{R}$, $x_3 \in \mathbb{R}, \dots,$ $x_n \in \mathbb{R}$

$\vec{x} \in \mathbb{R}^n$ ($\mathbb{R}^n \rightarrow$ n-dimensional real space)

$\begin{bmatrix} 3 \\ 4 \end{bmatrix} \rightarrow$ 2D vector

$\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \rightarrow$ 3D vector

$\vec{x} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ $\quad \vec{y} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ $\quad \vec{x} \in \mathbb{R}^2$, $\vec{y} \in \mathbb{R}^2$



Usually we write vectors as column vector

$\vec{x} = [x_1, x_2, \dots, x_n]^T$ $\quad T \rightarrow$ transpose dimension of a vector $\Rightarrow$ how many elements are there.

$[2, 3]^T \rightarrow \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\dim(\vec{x}) = n \rightarrow \vec{x}$ is a n-dimensional vector.

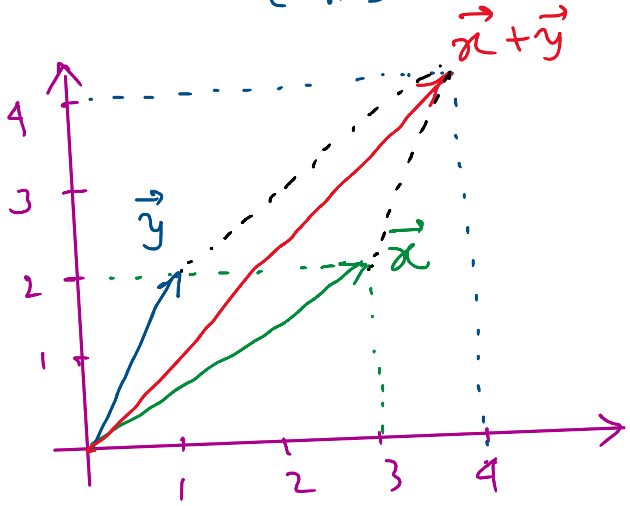
$\vec{y} = [0.3, 0.5, -0.7]^T \quad \therefore \dim(\vec{y}) = 3$

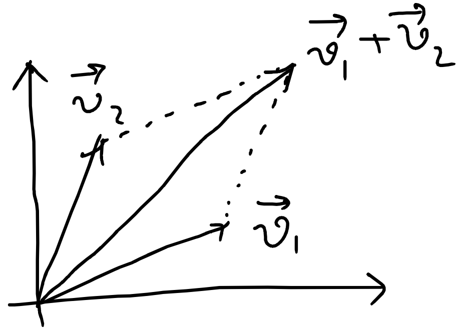
Vector Addition : $\vec{x} + \vec{y} = \vec{z}$ (only possible when $\dim(\vec{x}) = \dim(\vec{y})$)

$\vec{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad \vec{y} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad \vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad \therefore \vec{x} + \vec{y} = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \\ \vdots \\ x_n + y_n \end{bmatrix}$

$\Rightarrow \vec{x} + \vec{y} = \begin{bmatrix} 3+1 \\ 2+2 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$



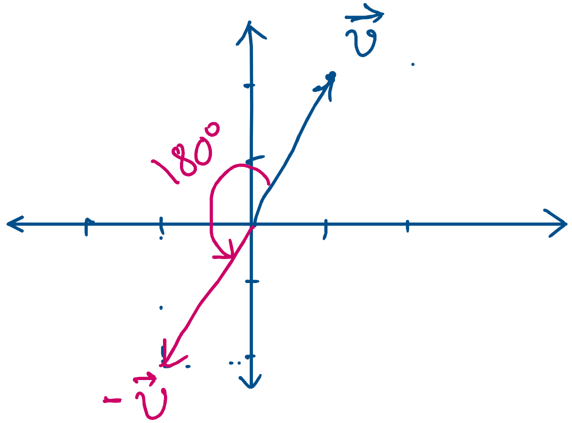


Negative of a vector :

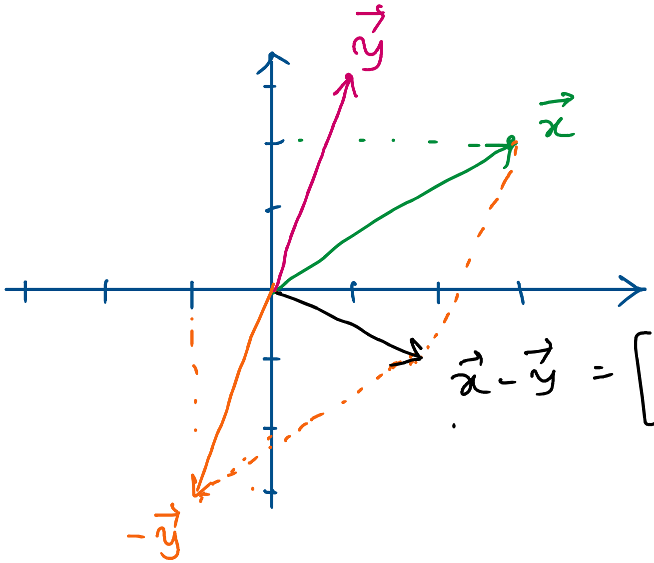
$$\vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \Rightarrow -\vec{v} = \vec{\omega} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

$$\vec{x} = [x_1 , x_2 , \dots , x_n]^T$$

$$-\vec{x} = [-x_1 , -x_2 , \dots , -x_n]^T$$



$$\vec{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \qquad \vec{y} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \qquad \vec{x} - \vec{y} = \begin{bmatrix} 3-1 \\ 2-3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$



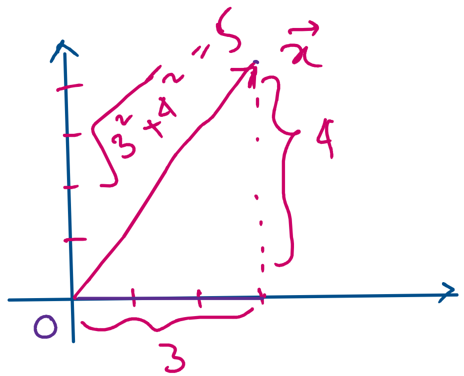
$$\vec{x} - \vec{y} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Magnitude of a Vector :

$$\vec{v} = [v_1, v_2, \dots, v_n]^T$$

$\|\vec{v}\| \rightarrow$ magnitude of $\vec{v}$ $\qquad \|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2} = \sqrt{\sum_{i=1}^{n} v_i^2}$

$\vec{x} = [3, 4]^T \qquad \|\vec{x}\| = ? = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$



$\|\vec{v}\| \longrightarrow$ L_2 norm (euclidean) of a vector $= \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}$

Other types of norms

L_1 norm : $\|\vec{v}\|_1 = \sum_{i=1}^{n} |v_i| = |v_1| + |v_2| + \dots + |v_n|$

$\vec{y} = \begin{bmatrix} 3 \\ -4 \end{bmatrix} \qquad \|\vec{y}\|_1 = 3 + |-4| = 7 \quad , \quad \|\vec{y}\|_2 = \sqrt{3^2 + (-4)^2} = 5$

L_p - norm $\Rightarrow \|\vec{v}\|_p = \left(\sum_{i=1}^{n} v_i^p \right)^{1/p} \qquad \therefore L_3$ - norm $= \left(v_1^3 + v_2^3 + \dots + v_n^3 \right)^{1/3}$