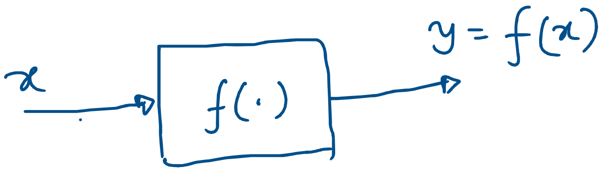
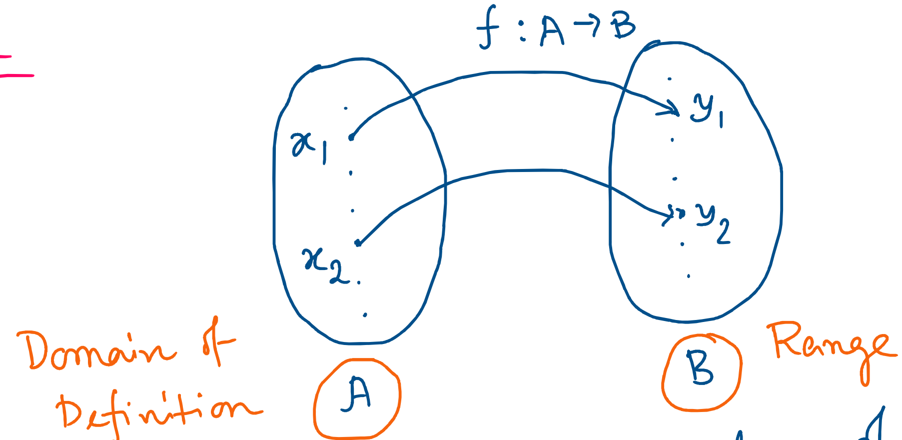
Mathematics for Data Science

1) Functions (Σ, Π notation)
2) Linear function, quadratic function, exponential function, logarithmic function
3) Calculus of Single variable
   - Differentiation
   - Geometrical interpretation of differentiation
   - Rules of differentiation
4) Composite function
5) Chain rule of differentiation
6) Maxima and Minima.

Function of a single variable

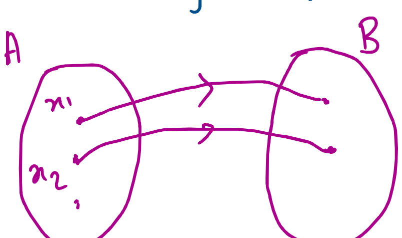




A mathematical function is a rule that maps values of a set (A) with values of another set (B) such that only one output is obtained given one input.

$y_1 = f(x_1)$ & $y_2 = f(x_2)$

if $x_1 = x_2$ then $y_1 = y_2$



$f(x_1) = y_1$
$f(x_2) = y_2$

$g(f(x)) = x$

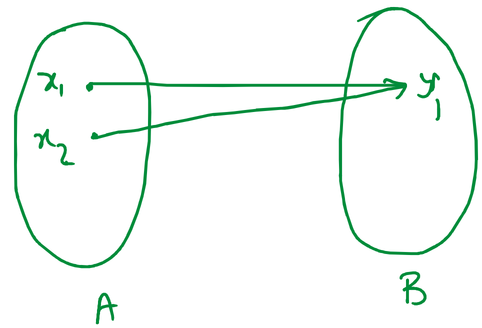
g is the inverse of 'f'

One-to-one mapping (Bijection)

g exist only when f is a bijection

$y = x^2$

$(-2) \to 4$
$(+2) \to 4$

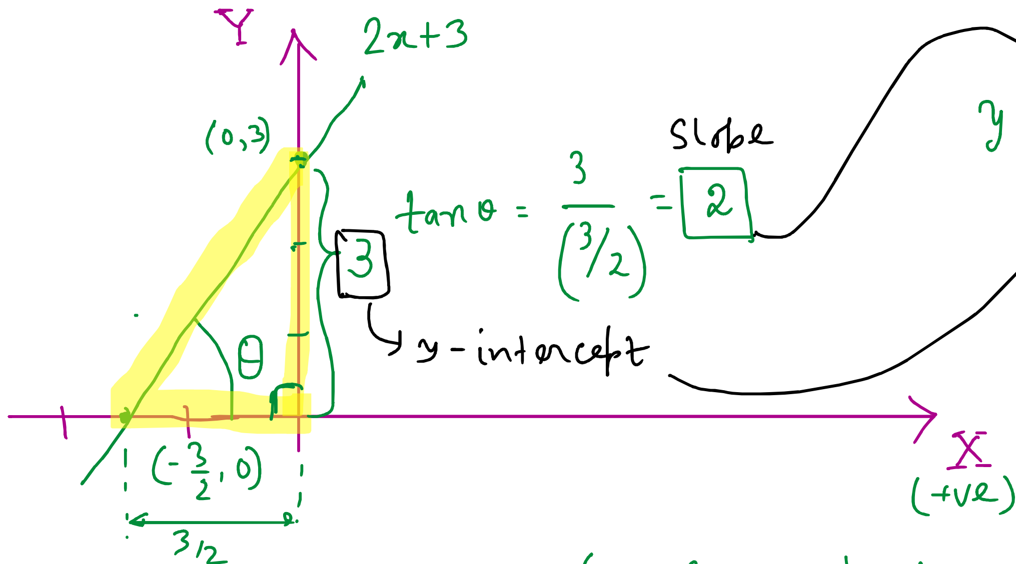


many to one mapping

Linear Function

$y = \underline{m}x + \underline{c} \Rightarrow \boxed{ax + by + c = 0} \Rightarrow y = \left(-\frac{a}{b}\right)x + \left(-\frac{c}{b}\right)$

Equation of st. line



$\tan\theta = \frac{3}{(3/2)} = \boxed{2}$ Slope

$y = 2x + 3$

x	0	$-3/2$
y	3	0

$x = 1, \; y = 5$

$x = 2, \; y = 7$

$x = 3, \; y = 9$

$\theta \rightarrow$ inclination (angle made by the St. line with +ve direction of x-axis)

$(\tan\theta) \rightarrow$ slope

$y = mx + c$

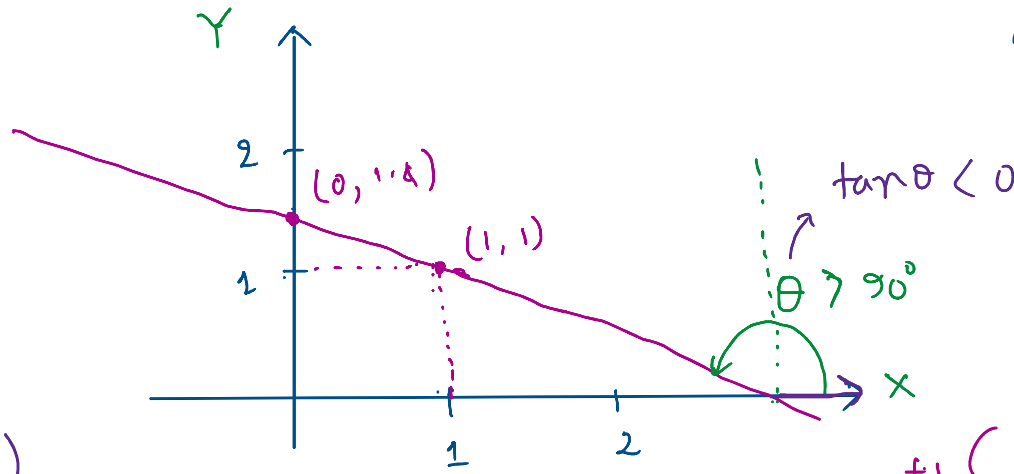
m: Slope, c: y-intercept

$2x + 5y - 7 = 0$

Slope = ? $\left(-\frac{2}{5}\right)$ y-intercept = ? $\left(\frac{7}{5}\right)$

$\Rightarrow 5y = -2x + 7$

$\Rightarrow y = \left(-\frac{2}{5}\right)x + \left(\frac{7}{5}\right)$



$$2x + 5y - 7 = 0 \Rightarrow y = \left(-\frac{2}{5}\right)x + \frac{7}{5}$$

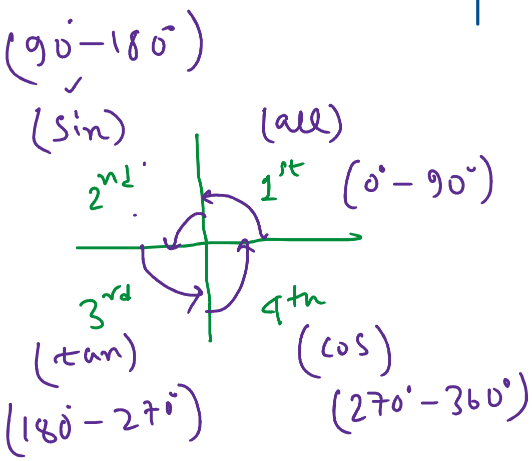
x	0	1
y	$7/5$	1

$= 1.4$

$x = 1 \Rightarrow y = 1$ $\quad -\frac{2}{5}$

$+1 \quad x = 2 \Rightarrow y = \frac{3}{5}$ $\quad -\frac{2}{5}$

$+1 \quad x = 3 \Rightarrow y = \frac{1}{5}$



Key takeaway

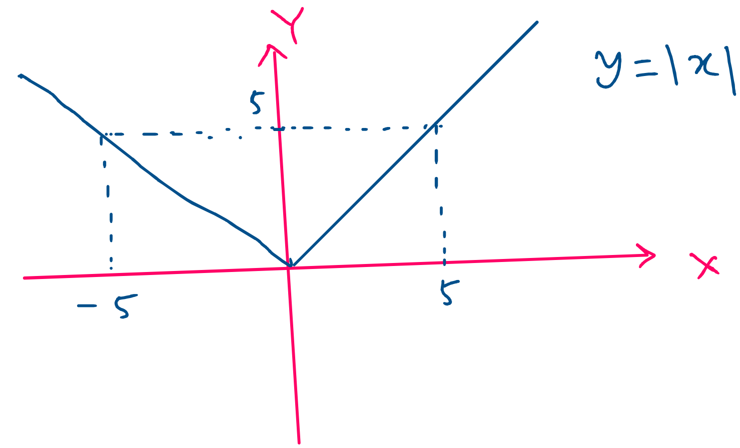
→ $\boxed{y = mx + c}$ → m : slope & c : y-intercept

→ $ax + by + c = 0$

Domain : $\mathbb{R}$, Range : $\mathbb{R}$ $\qquad$ $\mathbb{R}$ → set of real numbers.

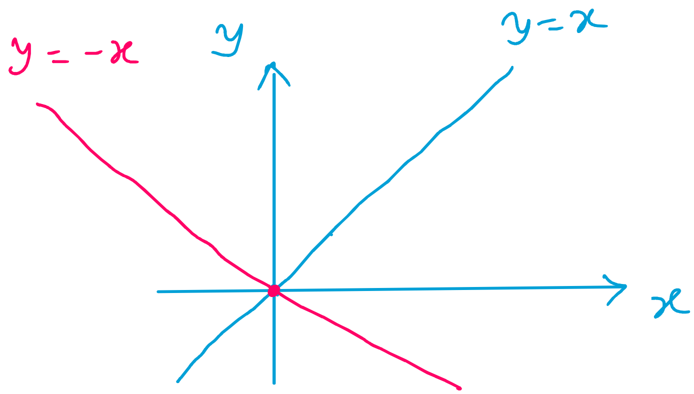
Absolute value function

$$y = |x| = \begin{cases} x & ; \ x \geq 0 \\ -x & ; \ x < 0 \end{cases}$$



$x = +4$, $y = 4$

$x = -3$, $y = 3$



Domain : $\mathbb{R}$

Range : $\mathbb{R}^+$: Set of +ve real numbers.

$y = x$

x	0	1	2
y	0	1	2

$y = -x$

x	0	-1	1
y	0	1	-1

quick challenge :- $y = 2|x-3| + 5$

1) Plot the graph of y vs x
2) find the domain
3) find the range

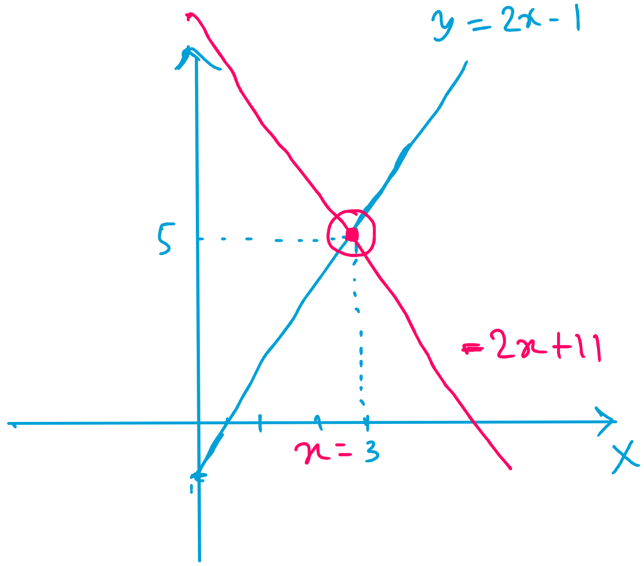
$|x-3|$

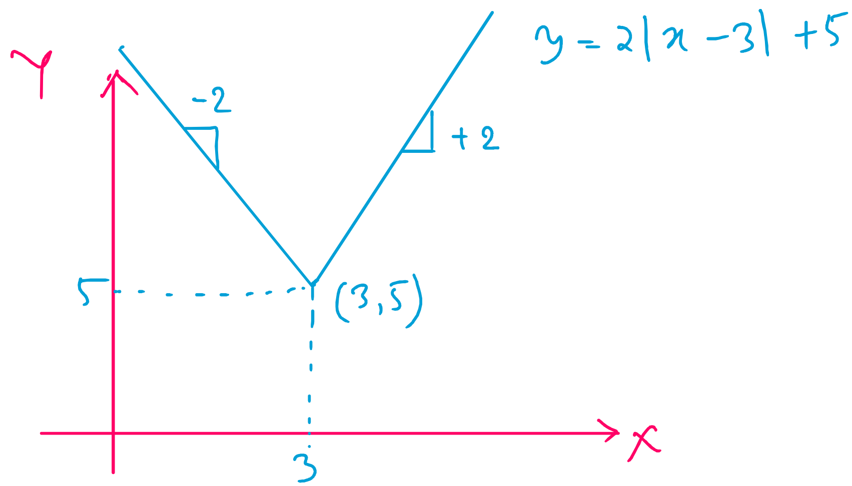
$x=0 : |x-3| = 3$

$x=6 : |x-3| = 3$

$$|x-3| = \begin{cases} x-3 \ ; & x-3 \geq 0 \Rightarrow x \geq 3 \\ -x+3 \ , & x-3 < 0 \Rightarrow x < 3 \end{cases}$$

$$y = \begin{cases} 2(x-3)+5 \ ; & x \geq 3 \\ 2(-x+3)+5 \ ; & x < 3 \end{cases} \Rightarrow y = \begin{cases} 2x-1 \ ; & x \geq 3 \\ -2x+11 \ ; & x < 3 \end{cases}$$





Domain : $(-\infty, +\infty)$ or $\mathbb{R}$

Range : $[5, +\infty)$

$[2, 3] \rightarrow$ closed interval $2 \leq x \leq 3$

$(2, 3) \rightarrow$ open interval $2 < x < 3$

$(2, 3] \rightarrow 2 < x \leq 3$ $\quad$ $[2, 3) \rightarrow 2 \leq x < 3$

Polynomial functions

$$y = a_0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n \rightarrow \text{Polynomial of degree} - n$$

$y = a_0 + a_1 x \rightarrow$ degree $- 1$

$y = a_0 + a_1 x + a_2 x^2 \rightarrow$ quadratic polynomial $\rightarrow$ degree $= 2$

$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \rightarrow$ cubic polynomial $\rightarrow$ degree $= 3$

exponential function

$y = c\, a^{(bx)}$

$\boxed{y = 2^x}$

$\boxed{y = 5.3^{-2x}}$

$\boxed{y = e^x}$

$e \rightarrow$ euler's number $\approx 2.7 \ldots\ldots$

logarithmic function

$$y = a \log_{b}(kx + c)$$

what is logarithm?

$$8 = 2^{(x)} \Rightarrow x = ?$$

$$\Rightarrow \log_2(8) = \log_2(2^x) = x$$

$$\log_a(a^x) = x \Rightarrow \boxed{\log_e}(e^x) = x$$

natural logarithm (base-e) ln

$$\ln(x) = \log_e x$$

$$\Rightarrow \underline{16} = 2^{x_1} \Rightarrow x_1 = \log_2 16 = 4$$

$$\Rightarrow \underline{16} = 4^{x_2} \Rightarrow x_2 = \log_4 16 = \underline{2}$$

properties of log

1) $\log(x^a) \Rightarrow a\log(x)$

2) $\log(x \cdot y) \Rightarrow \log(x) + \log(y)$

$$\Rightarrow \log\left(\prod_{i=1}^{n} x_i\right) = \sum_{i=1}^{n} \log(x_i)$$

$$\prod_{i=1}^{n} x_i = x_1 \cdot x_2 \cdot x_3 \cdots\cdots x_n$$

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + \cdots\cdots + x_n$$

$y = \log_{\underline{a}} x \qquad x > 0$

✓ which values can 'a' (base) take

✓ which values can '$\underline{x}$' take (domain) $\Rightarrow x > 0$ $(0, \infty)$

which values can 'y' take (range)

$\Rightarrow \boxed{x = \underline{a}^{y}} \qquad a \neq 0, \quad a \neq 1$

$2^x = 32 \Rightarrow x = \log_2 32 = 5$

$a = (-2)$? $a \not< 0$

$2^x = -32 \Rightarrow x$ is impossible

$(-2)^3 = -8$

$\Rightarrow 3 = \log_{(\cancel{-2})}(\cancel{-8})$ is it a right expression?

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$2^x = \frac{1}{16} \Rightarrow x = -4$

$2^x = 1 \Rightarrow x = 0$

$\log_2\left(\frac{1}{16}\right) = -4$

$y = \log_a(x) \Rightarrow a > 0$ & $a \neq 1$

$\Rightarrow x > 0$;

$\Rightarrow y \in (-\infty, \infty)$ or $\mathbb{R}$

$0 < x < \infty \Rightarrow \frac{0 < x < 1}{\log(x) < 0}, \quad \frac{x = 1,}{\log(x) = 0} \quad \frac{x > 1}{\log(x) > 0}$

10 small numbers, each of them $\approx \boxed{10^{-5}}$

multiply these 10 numbers, the result will be $\approx 10^{-50} \longrightarrow 0$

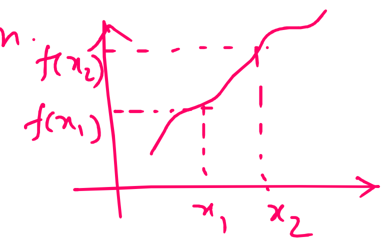
$\underset{10^{-5}}{} p_1, p_2, p_3, \ldots, p_{10} \Rightarrow \prod_{i=1}^{10} p_i \approx 10^{-50} \longrightarrow$ problem

$\Rightarrow \underset{-5}{\underset{\downarrow}{\log(p_1)}}, \underset{-2.5}{\underset{\downarrow}{\log(p_2)}}, \underset{-3}{\underset{\downarrow}{\ldots}}, \Rightarrow \sum_{i=1}^{10} \log(p_i) \approx (-50) \to$ no-problem

Logarithms are widely used in finance, probability calculation, machine learning.

Say x_1, x_2 s.t. $x_2 > x_1$ if $f(x_2) \geq f(x_1)$ $\forall x_1, x_2 \in$ Domain of 'f'

then $f(x)$ is called monotonically increasing function.



if $x_2 > x_1 \Rightarrow f(x_2) \leq f(x_1) \to$ monotonically decreasing.

Composite Functions :

$f\circ g(x) \neq g\circ f(x)$

$$f(x) = x^2 \quad , \quad g(x) = e^x$$

$\log(\_\_)$ >0

$$\Rightarrow g\circ f(x) = g(f(x)) = g(x^2) = e^{x^2}$$

$$\Rightarrow f\circ g(x) = f(g(x)) = f(e^x) = (e^x)^2 = e^{2x}$$

$$f(x) = \sin(3x) \qquad g(x) = \log(2x+3)$$

$$\therefore f\circ g(x) = f(\log(2x+3)) = \sin(3\log(2x+3))$$

$$g\circ f(x) = g(\sin 3x) = \log(\underbrace{\underbrace{2\sin(3x)}_{[-2,2]}+3}_{[1,5]})$$

$\Rightarrow$ Domain : $\mathbb{R}$

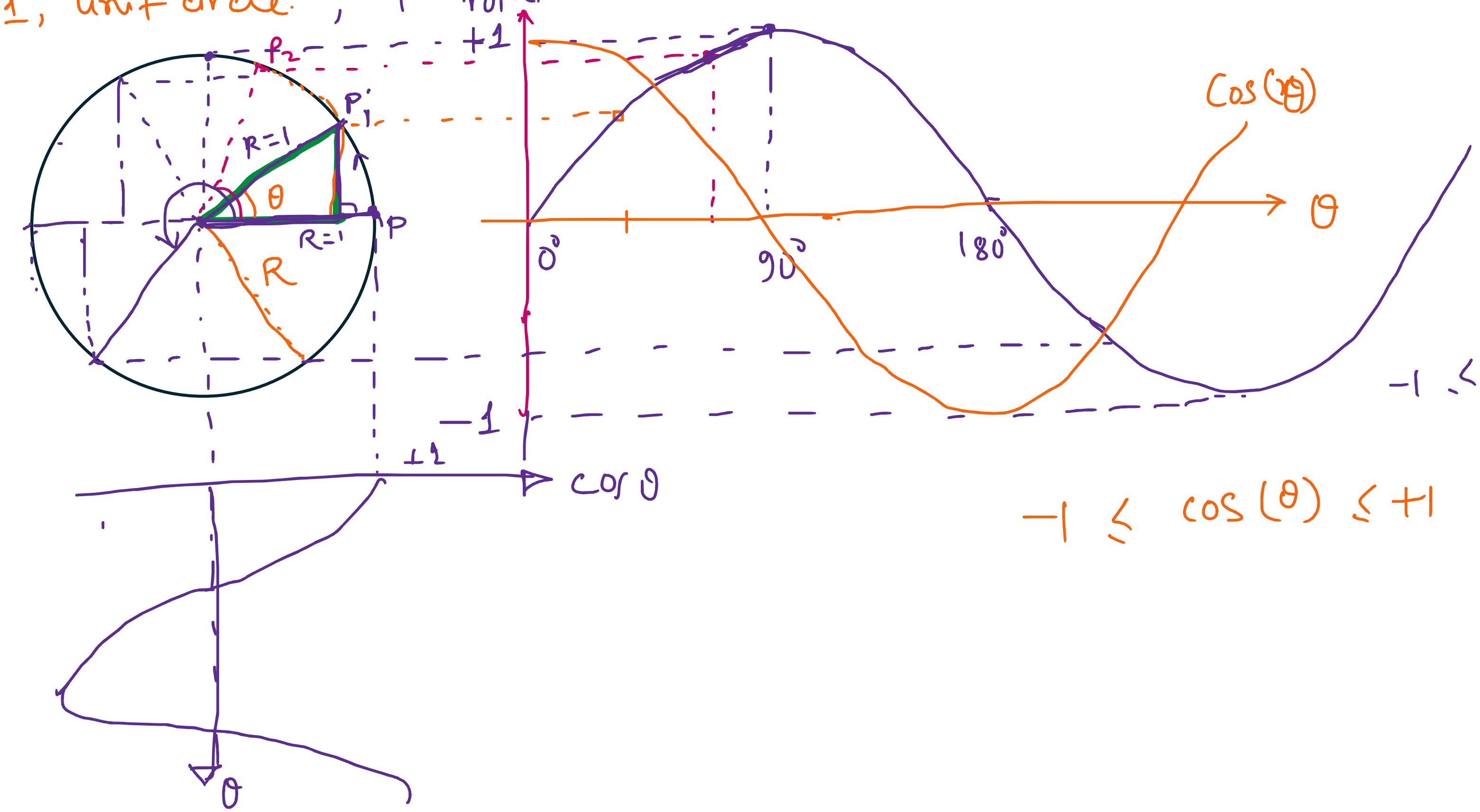
$(-\infty, +\infty)$

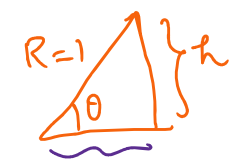
$\rightarrow$ Range : $[0, \log 5]$

Trigonometric Functions

$\sin(x)$, $\cos(x)$, $\tan(x)$, $\operatorname{cosec}(x)$, $\sec(x)$, $\cot(x)$

$R=1$, unit circle , P rotates with uniform angular velocity in anti-clockwise direction





$\frac{b}{R} = \cos\theta$

$\frac{h}{R} = \sin\theta$

$-1 \leq \sin(x) \leq +1$

$-1 \leq \cos(\theta) \leq +1$