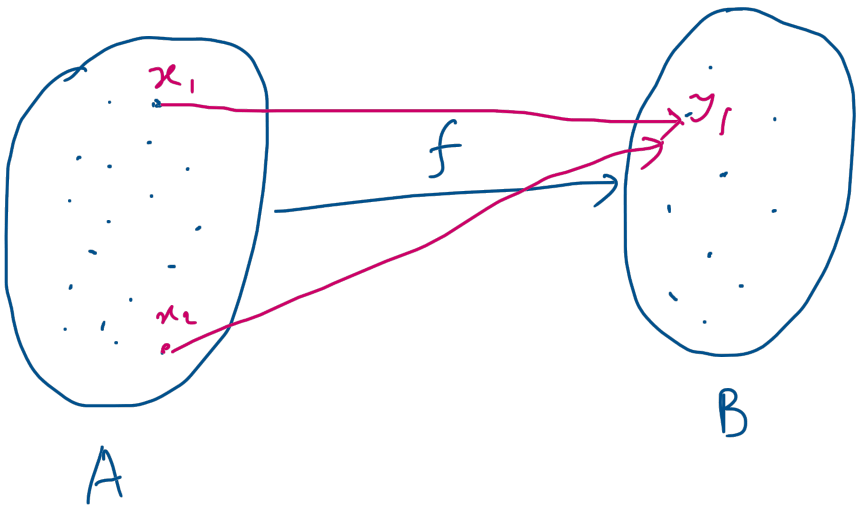
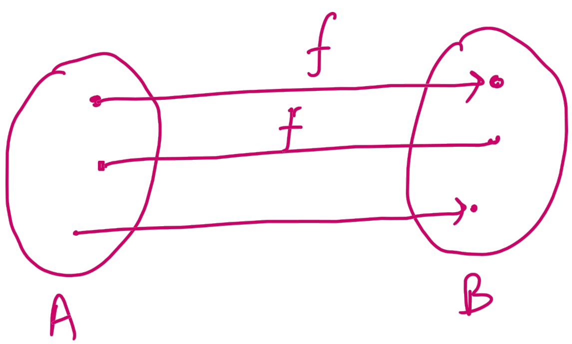
Mathematical Function



function is a rule which maps values of a set (A) with values of another (B) such that the mapping ensures only one output given one input.



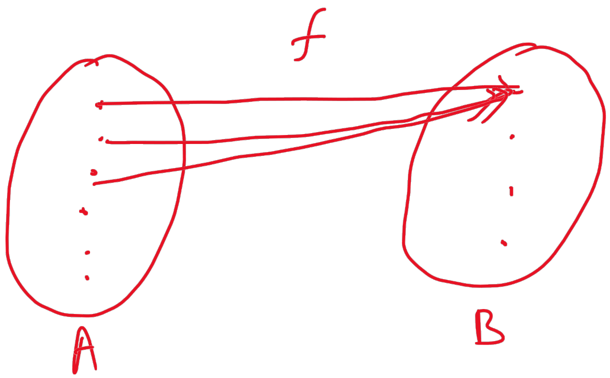
one-to-one mapping

$f : A \rightarrow B$

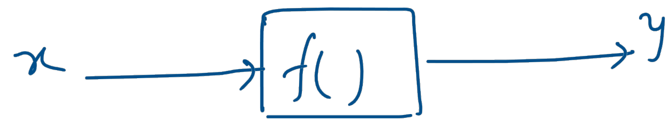
if $x \in A$ & $f(x) \in B$

$A \rightarrow$ Domain

$B \rightarrow$ Co-domain / Range

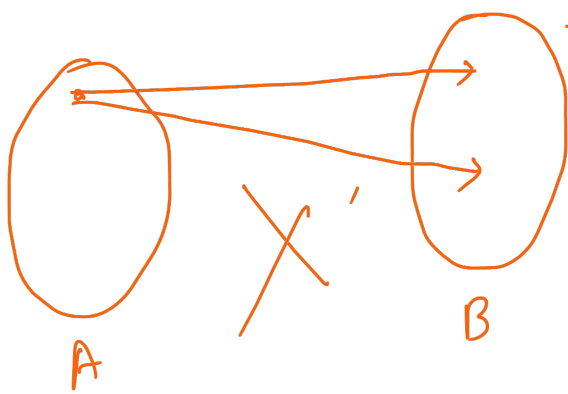


$x \in A$ $\qquad\qquad$ $y = f(x) \in B$

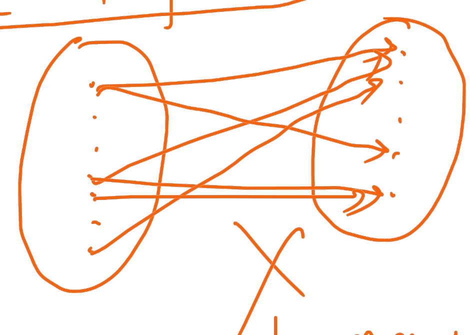


many to one mapping

These are not functions



one to many mapping



many to many mapping

$$y = f(x) = \underline{2x+3}$$

$x_1, x_2 \longrightarrow y \quad \boxed{(x_1 \neq x_2)}$

$y = 2x_1 + 3$ or $y = 2x_2 + 3$

$2x_1 + 3 = 2x_2 + 3$

$\Rightarrow 2x_1 = 2x_2$

$\Rightarrow x_1 = x_2$.

$$y = 3x^2$$

$x = 2, \quad x = -2$

$\downarrow \qquad \downarrow$

$y = 12 \qquad y = 12$

$$y = |x| + 3$$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$x = +5 \rightarrow y = 8$

$x = -5 \rightarrow y = 8$

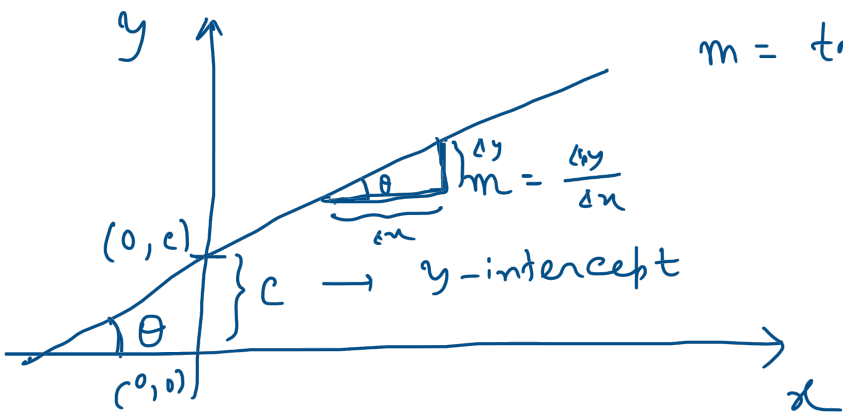
$f(x) = y = mx + c$

$x \in R \qquad y \in R \qquad f : R \rightarrow R$

Linear function

$$y = f(x) = \underline{m}x + \underline{c}$$

m and c are constants

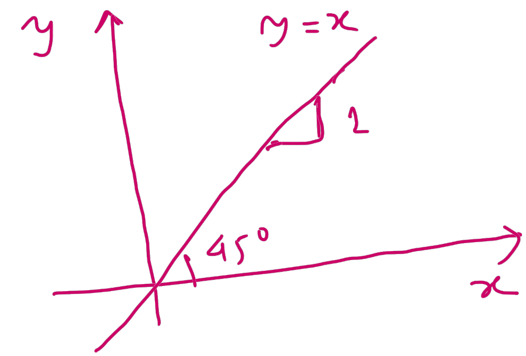


$\theta \rightarrow$ inclination

$m = \tan(\theta) \rightarrow$ slope

$\boxed{y = ax + b}$ → This represents a straight line in 2-D

$y = x$

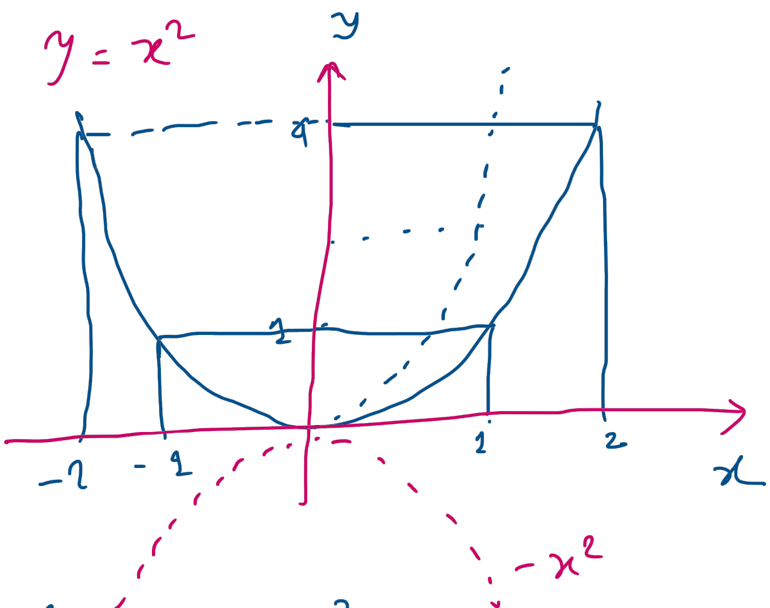


$y = 5$

Quadratic function

$$y = ax^2 + bx + c \quad (a \neq 0)$$

$y = x^2$

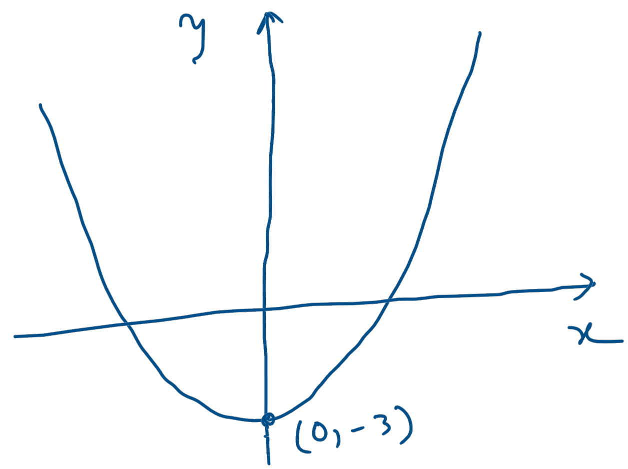


$f(x) = y = x^2$

$x \in R$ $\quad$ $y \in R^+$

$f : R \to R^+$

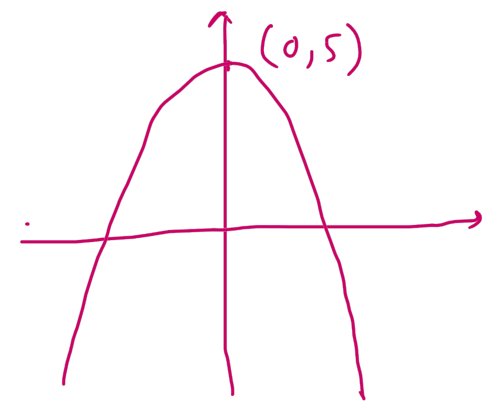
$y = 2x^2 - 3$



$x \in R$, $y \in [-3, \infty)$

$f : R \to [-3, \infty)$

$y = -3x^2 + 5$



$x \in R$

$y \in (-\infty, +5]$

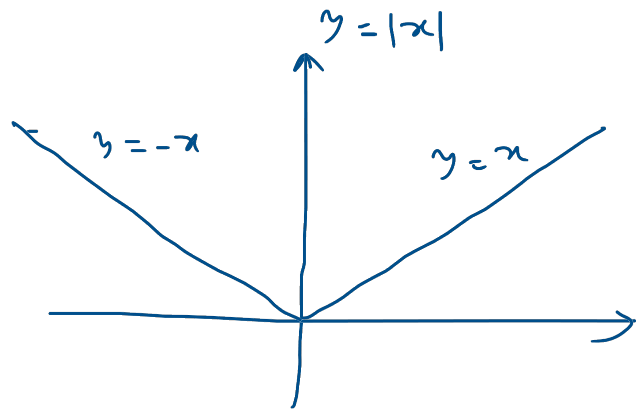
$f : R \to (-\infty, +5]$

$y = |x| \longrightarrow$ mod function

$x > 0$, $y = x$

$x = 0$, $y = 0$

$x < 0$, $y = -x$



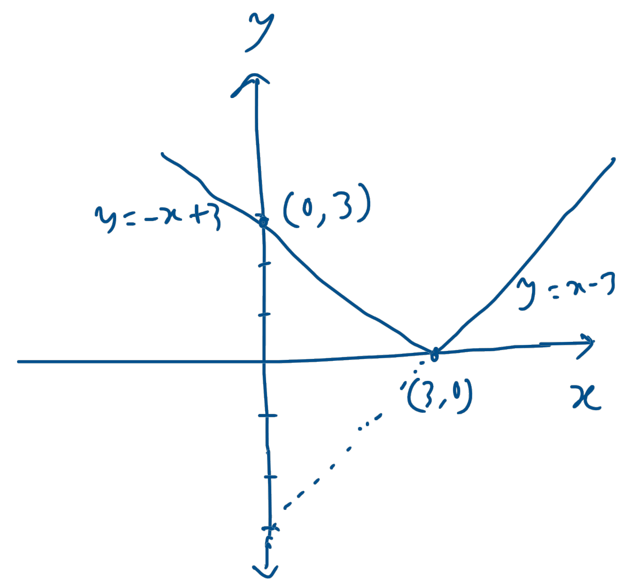
$y = |x - 3|$

$x - 3 = z$

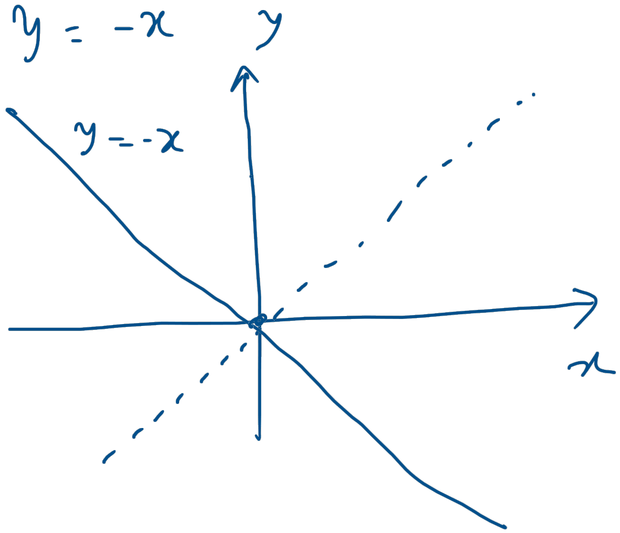
$y = |z|$

$$y = \begin{cases} z & z > 0 \\ -z & z < 0 \end{cases}$$

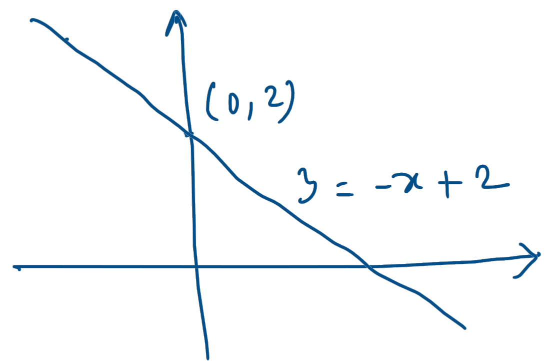
$$y = \begin{cases} x - 3 & ; \ x > 3 \\ -x + 3 & ; \ x < 3 \end{cases}$$



$y = -x$



$y = -x + 2$



$y = x^2$

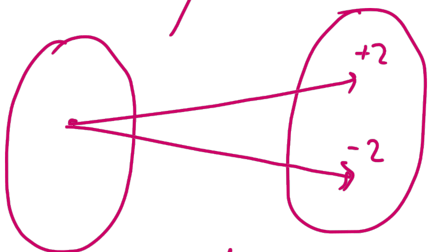
$\Rightarrow x = \sqrt{y}$?

$y = +4$

$x = \sqrt{y}$

$x = -\sqrt{y}$

$x = \pm\sqrt{y}$



not a function

$y = 2x + 3$

$\Rightarrow x = \dfrac{y-3}{2}$

$y = x^2$

$R^+ \quad R$

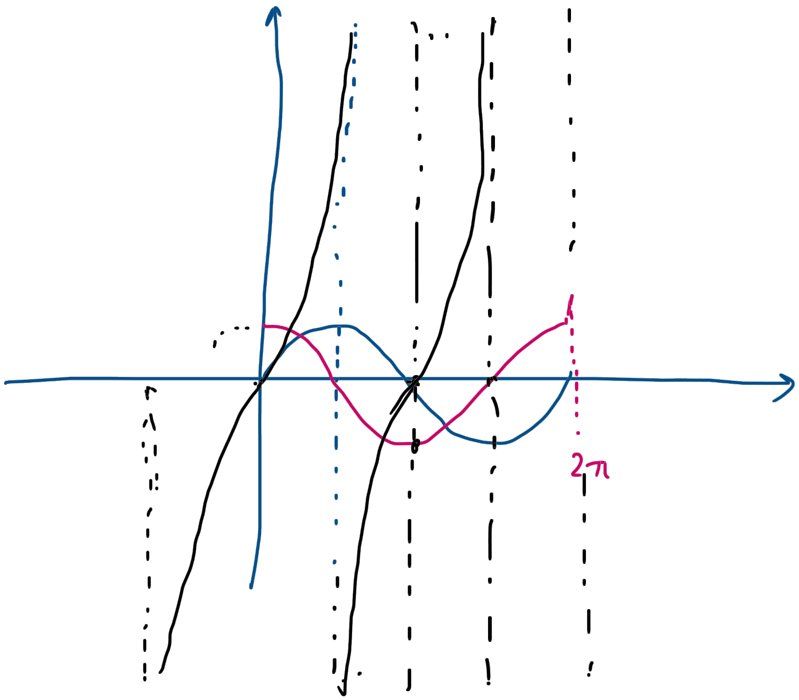
$y = \sin(x)$

$y = \cos(x)$

$y = \tan(x)$

$y = e^x$ (exponentiation) $\quad e = 2.718 \ldots$

$y = \log_e(x)$



$$\boxed{y = a^x}$$

if $a = e \rightarrow \boxed{y = e^x}$

$2 \times 2 \times 2 = 2^3$

$2 \times 2 \times 2 \times 2 \times 2 = 2^{\boxed{5}}$

$a^{\boxed{x}} \quad x \in R$

$a^{0.5}$

$$x^a \times x^b = x^{(a+b)}$$

$$2^3 \times 2^2 = 2^5$$

$$x^{m/n} = \left(x^m\right)^{1/n}$$

$$= \sqrt[n]{x^m}$$

$$\frac{x^a}{x^b} = x^{(a-b)}$$

Say $b = a$ $\therefore$ $\underbrace{\frac{x^a}{x^a}}_{1} = x^{(a-a)} = x^0 \Rightarrow \boxed{x^0 = 1}$

$$\left(x^m\right)^n = x^{mn} = \left(x^n\right)^m$$

$$x^a \cdot x^a = x^{(2a)}$$

$$= \left(x^2\right)^a$$

$$x^{1/2} \times x^{1/2} = x$$

$$\Rightarrow x^{1/2} = \sqrt{x}$$

$$x^{-b} = x^{(0-b)} = \frac{x^0}{x^b} = \frac{1}{x^b}$$

Logarithm (log)

$\underline{a}^{x} = \underline{y}$

$\Rightarrow x = \log_a(y)$

$2^x = 32$

$\Rightarrow x = ?$

$x = \log_2 32$

$(3 \cdot 5)^x = 29 \cdot 32$

$(-3)^{-0 \cdot 5}$

$\log_{\underline{a}}(\underline{x})$ $\hookrightarrow$ base

$a \neq 0$

$a \neq 1$

$a > 0$

$x > 0$

$1^x = 1$

$2^x = -32$

$\Rightarrow x = ?$

$\log_{10}(a) \rightarrow$ base-10 log

$e = 2.718 \ldots\ldots$

$\log_e(a) \rightarrow$ base - e log (natural) $(\ln)$

$\log_2(a) \rightarrow$ base-2 log

$\ln 2$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\ldots$$

$$a^{b_1} \times a^{b_2} \times a^{b_3} = a^{(b_1 + b_2 + b_3)}$$

$$\log_a \left(a^{b_1} \cdot a^{b_2} \cdot a^{b_3}\right) = \log_a \left\{a^{(b_1 + b_2 + b_3)}\right\} = \boxed{b_1 + b_2 + b_3}$$

$\rightarrow \quad x = b^y \Rightarrow y = \log_b x$

$\Rightarrow \boxed{x = b^{\log_b x}}$

$5 = 3^{\log_3 5} = e^{\log_e 5} = 10^{\log_{10} 5}$

$\rightarrow \quad x = b^{\log_b x}, \quad y = b^{\log_b y}, \quad z = b^{\log_b z}$

$\Rightarrow xyz = b^{\log_b x} \times b^{\log_b y} \times b^{\log_b z} = b^{(\log_b x + \log_b y + \log_b z)}$

$\Rightarrow \log_b(\underline{xyz}) = \log_b x + \log_b y + \log_b z$

log transforms multiplication into addition.

$p_1, p_2, p_3, \ldots, p_n$

$\underline{p_1 \times p_2 \times p_3 \times \cdots \times p_n}$

$\underline{-[\log(p_1) + \log(p_2) + \cdots + \log(p_n)]}$

$= -$

$\log_{10}(\underline{1000000}) = (10^6) = 6$

$\log_{10}(\underline{10^{-6}}) = -6$

$0 < \quad < 1$

$0 \cdot \underline{1} \times 0 \cdot 2 = 0 \cdot 02$

$0 \cdot 1 \times 0 \cdot 05 \times 0 \cdot 008 \times \cdots = \underline{\qquad}$

$a^0 = 1$

$\Rightarrow \log_a 1 = 0$

$\log(x) = y$

$\text{antilog}(y) = x$

$\log_{10}(\underline{10^{-32}}) = -32$

$0 < \quad < 1$

$10^9, \ldots, 1$

$\boxed{0 - 10}$

$\log_2 (32 \times 64) = 5 + 6 = 11$

$\Rightarrow 32 \times 64 = 2^{11} = 2048$

$\rule{6cm}{0.4pt} \times \rule{6cm}{0.4pt}$

$\Rightarrow$

$$\underbrace{(x \cdot x \cdot x \cdot \cdots \cdot x)}_{n\text{-times}} = x^n$$

$\Rightarrow \log (x \cdot x \cdot x \cdot \cdots \cdots x) = \log (x^n)$

$\Rightarrow \underbrace{\log x + \log x + \cdots + \log x}_{n\text{-times}} = \log (x^n)$

$\Rightarrow n \log x = \log (x^n)$

$\log 27^{32}$

$= 32 \log 27$

$\log (x^a \cdot y^b \cdot z^c)$

$= \log (x^a) + \log (x^b) + \log (z^c)$

$= a \log x + b \log y + c \log z$

Base transformation

$$\log_a x = \log_b x \times \log_a b \qquad (a \& b \neq 1 \ \& \ a \& b > 0)$$

$\boxed{b^y = x}$ $\Rightarrow \log_a (b^y) = \log_a x$

$\Rightarrow y = \log_b x$ $\Rightarrow y(\log_a b) = \log_a x$

$\Rightarrow \boxed{\log_b x \cdot \log_a b = \log_a x}$

$\log(xyz) = \log x + \log y + \log z$

$\log(x/y) = \log x - \log y$

$$\boxed{\log_b x = \frac{\log_a x}{\log_a b}}$$

$\log_3 81 = \log_9 81 \times \log_3 9$

$\Rightarrow \log_9 81 = \dfrac{\log_3 81}{\log_3 9}$

Sum-notation

$$x_1, x_2, x_3, \ldots, x_n$$

$$x_1 + x_2 + x_3 + \ldots + x_n = \sum_{i=1}^{n} x_i$$

Product notation

$$x_1 \cdot x_2 \cdot x_3 \cdot \ldots \cdot x_n$$

$$\prod_{i=1}^{n} x_i = x_1 \cdot x_2 \cdot x_3 \cdots x_n$$

$$\log_b\left(\prod_{i=1}^{n} x_i\right) = \sum_{i=1}^{n} \log_b(x_i)$$

Differential Calculus

Differentiation of function of one variable

$$y = f(x) = 3x^2$$

$$\Delta x = h$$

$$f(x+h) = 3(x+h)^2$$

$$\therefore \Delta y = f(x+h) - f(x)$$

$$= 3(x+h)^2 - 3x^2$$

$$= 3\{(x+h)^2 - x^2\} = 3(\cancel{x^2} + 2xh + h^2 - \cancel{x^2})$$

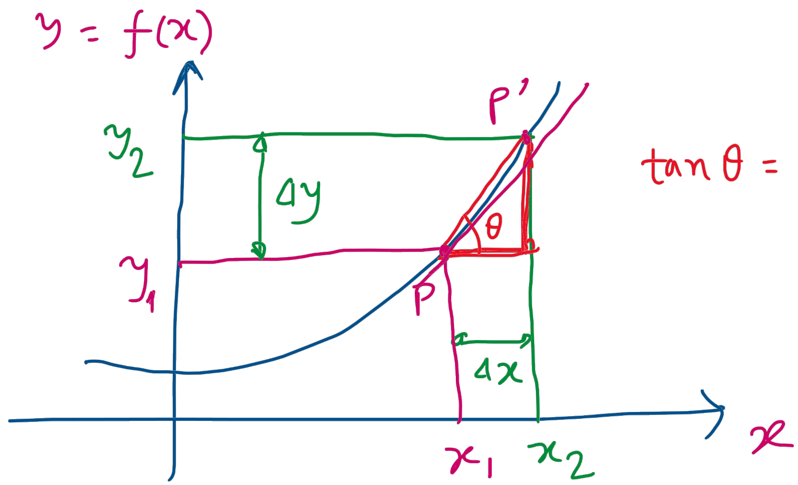
$$= 3(2x+h)h$$

$$\Rightarrow \frac{\Delta y}{\Delta x} = 3(2x+h) = 3(2x + \Delta x)$$

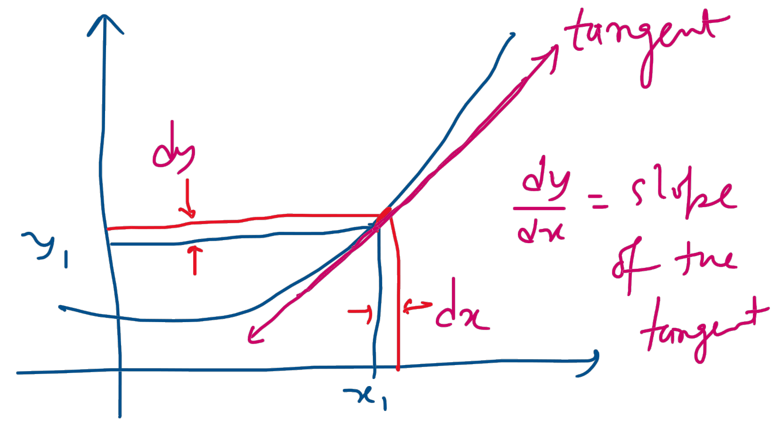
$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

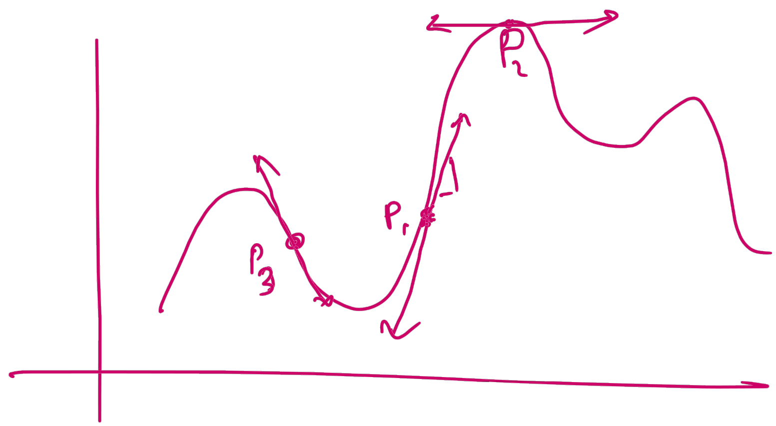
$$\frac{\Delta y}{\Delta x} = 3(2x + \cancel{\Delta x}^{\,0})$$

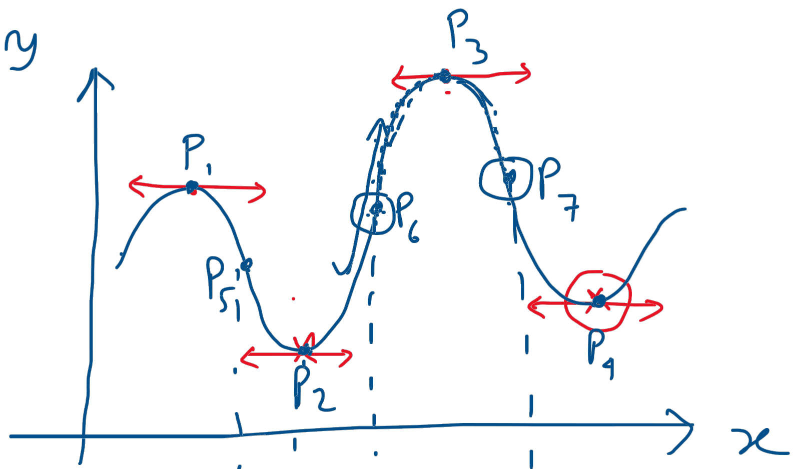
$$\boxed{\frac{dy}{dx} = 6x}$$



$$\tan\theta = \frac{\Delta y}{\Delta x}$$



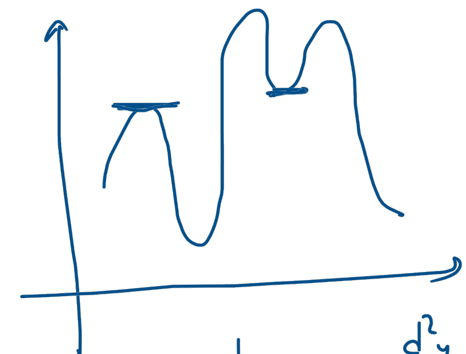




$$\left.\frac{dy}{dx}\right|_{x=P_1,\,P_2,\,P_3,\,P_4} = 0$$

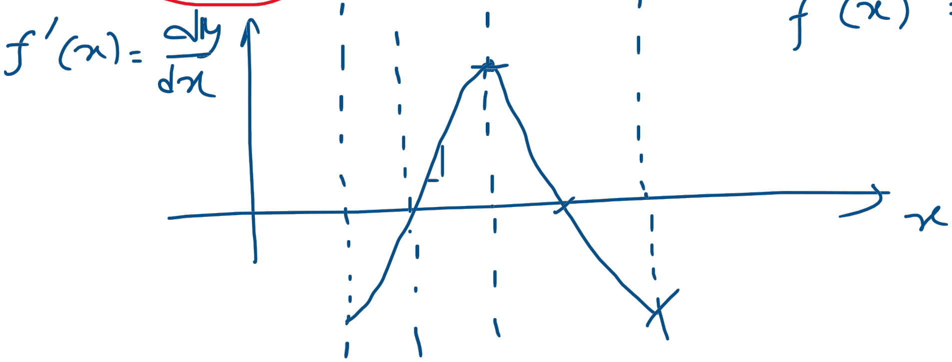
$$\left.\frac{dy}{dx}\right|_{x=P_5,\,P_7} < 0$$

$$\left.\frac{dy}{dx}\right|_{x=P_6} > 0$$



$\frac{dy}{dx} = 0$ → points to maxima / minima

$$f''(x) = \frac{d^2y}{dx^2} < 0$$



$$f''(x) = \frac{d^2y}{dx^2} > 0$$

maxima : $\frac{dy}{dx} = 0$, $\frac{d^2y}{dx^2} < 0$

minima : $\frac{dy}{dx} = 0$, $\frac{d^2y}{dx^2} > 0$

- Formula of differentiation
- Examples of maxima / minima
- Chain rule of differentiation (composite function)
- Scipy . module for numerical diff.
- Integration
- Geometrical interpretation of integration
- Scipy integration.