Integral Calculus (Integration)

$$\frac{d}{dx}[F(x)] = f(x)$$

$$\int f(x)\,dx = F(x) + \underline{c}$$

↳ constant of integration.

$$F(x) \xrightarrow{\frac{d}{dx}} f(x) \xrightarrow{\int (\ )dx} F(x) + c$$

$f(x) = 3x^2$ $\qquad \int (3x^2)\,dx = x^3 + c$

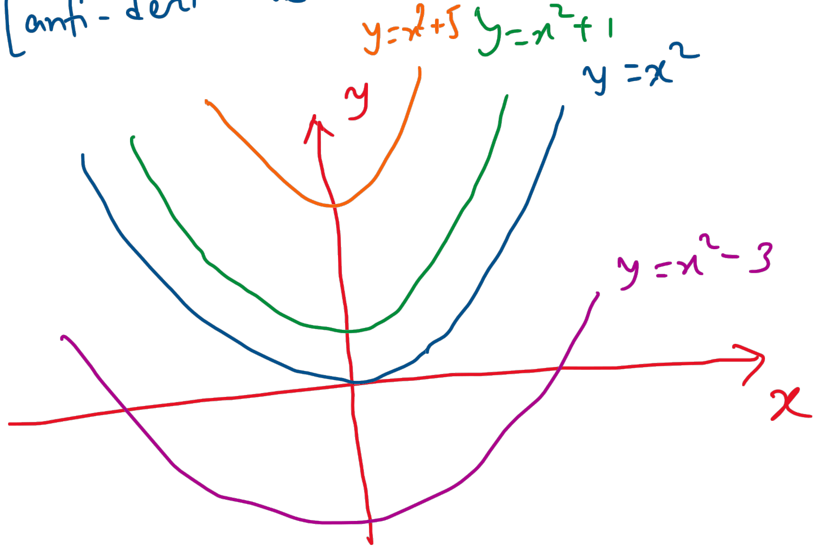
$f(x) = 2x$ $\qquad \int (2x)\,dx = \boxed{x^2 + c}$

Integration

indefinite integral $\left(\int f(x)\,dx\right)$

Definite integral $\left(\int_a^b f(x)\,dx\right)$

[anti-deri've



Definite Integral

Suppose, $\int f(x)\,dx = \underline{F(x)} + C$

$$\int_a^b f(x)\,dx = F(b) - F(a) \rightarrow \text{value (Real)}$$

$$\int_2^4 2x\,dx = ?$$

$$\int 2x\,dx = \underline{x^2} + C \qquad F(x) = x^2$$

$$= F(4) - F(2)$$

$$= 4^2 - 2^2$$

$$= 16 - 4$$

$$= 12$$

$$\int_0^{\pi/2} (\cos x)\,dx = \sin x \Big]_0^{\pi/2}$$

$$\int (\cos x)\,dx = \underline{\sin x} + C$$

$$= \sin(\pi/2) - \sin(0)$$

$$= 1 - 0 = 1$$

Rules of integration

1. $$\int f(x)\, dx = F(x) + c$$

then $$\int k f(x)\, dx = k F(x) + c$$

2. $$\int [a f(x) + b g(x)]\, dx = a \int f(x)\, dx + b \int g(x)\, dx$$

3. $$\frac{d}{dx}\left(x^{n+1}\right) = (n+1)\, x^n \quad \therefore \int (n+1)\, x^n\, dx = x^{n+1} + c'$$

$$\Rightarrow (n+1) \int x^n\, dx = x^{n+1} + c'$$

$$\Rightarrow \boxed{\int x^n\, dx = \frac{x^{n+1}}{n+1} + C} \qquad \boxed{n \neq -1}$$

$$\int x^{3/2}\, dx = \frac{x^{3/2+1}}{\frac{3}{2}+1} + c$$

$$= \frac{2}{5} x^{5/2} + c$$

4. $\int \frac{1}{x}\,dx = \int x^{(-1)}\,dx$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$ $\quad \therefore \int \frac{1}{x}\,dx = \ln(x) + C$ $\qquad \ln \to \log_e$

5. $\int \sin(x)\,dx$

$= -\cos(x) + C$

$\frac{d}{dx}[\cos(x)] = -\sin(x)$

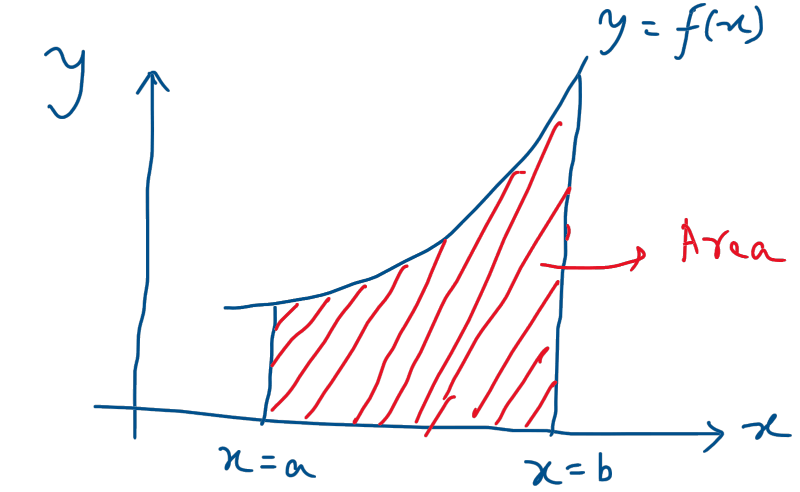
$\Rightarrow \frac{d}{dx}[-\cos(x)] = \sin(x)$

6. $\int \cos(x)\,dx = \sin x + C$

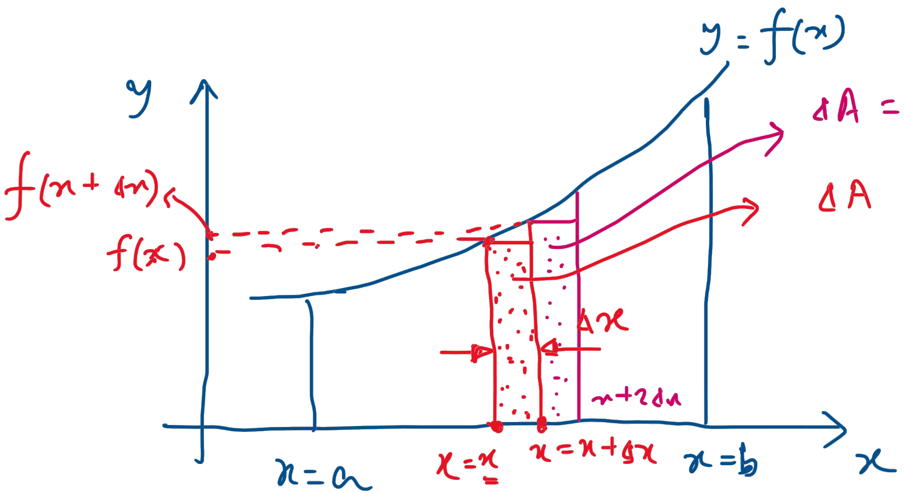
$\int (x^2 + 3\sin x)\,dx = \frac{x^3}{3} - 3\cos x + C$

$= \int x^2\,dx + 3\int \sin x\,dx$

Geometrical interpretation of definite integral :—



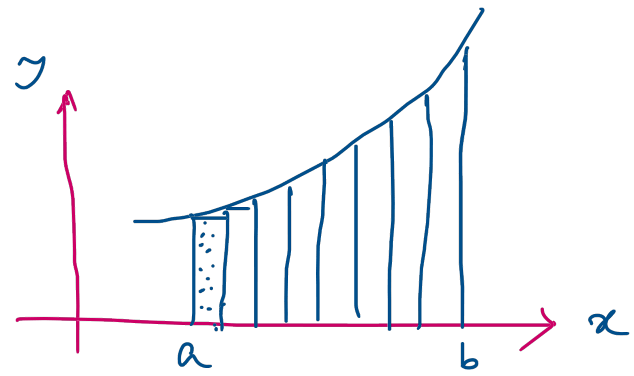
Area under the curve $f(x)$ between $x=a$ to $x=b$



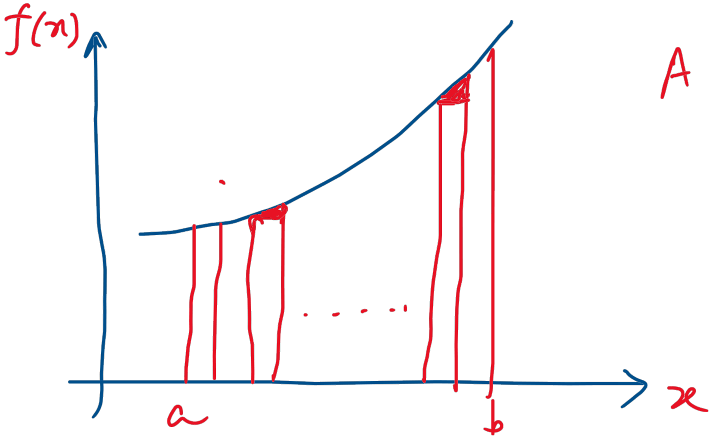
$\Delta A = f(x+\Delta x)\cdot\Delta x$

$\Delta A = f(x)\,\Delta x$

$$n = \frac{b-a}{\Delta x}$$

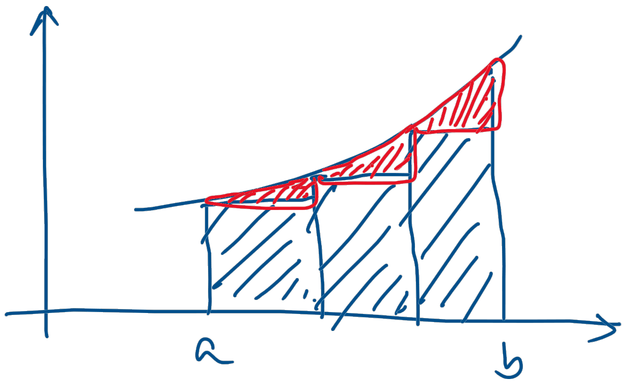


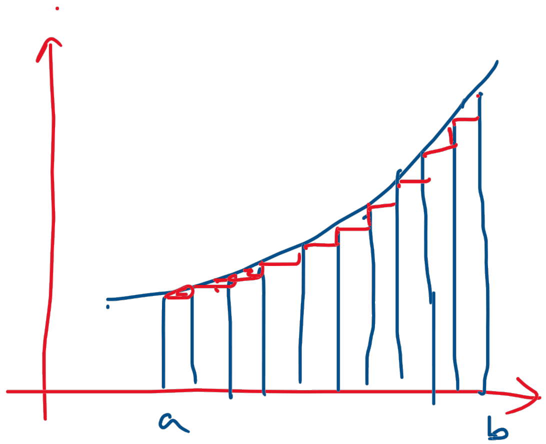
$f(a)\cdot\Delta x + f(a+\Delta x)\cdot\Delta x$
$+\cdots\cdots + f\big(a+(n-1)\Delta x\big)\cdot\Delta x$



$$A \simeq \sum_{x=a}^{b} f(x)\,\Delta x$$

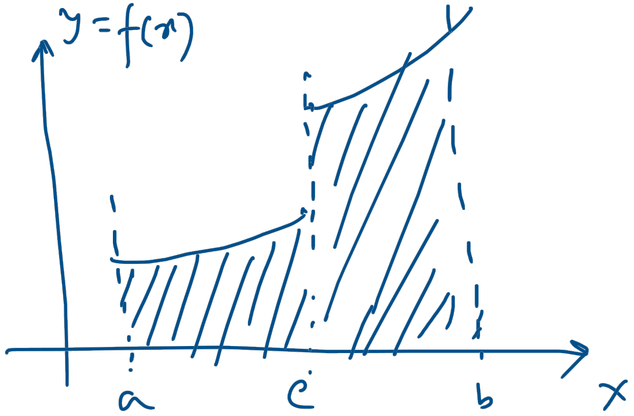
if Δx is vanishingly small



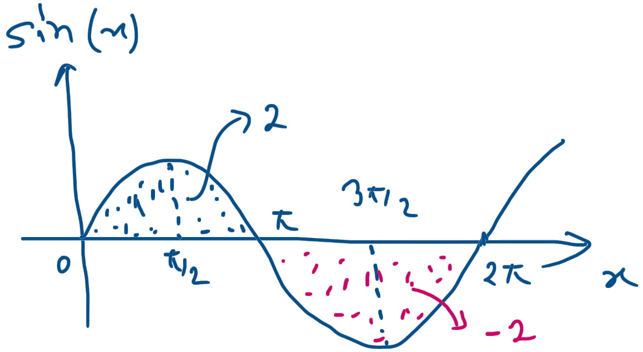


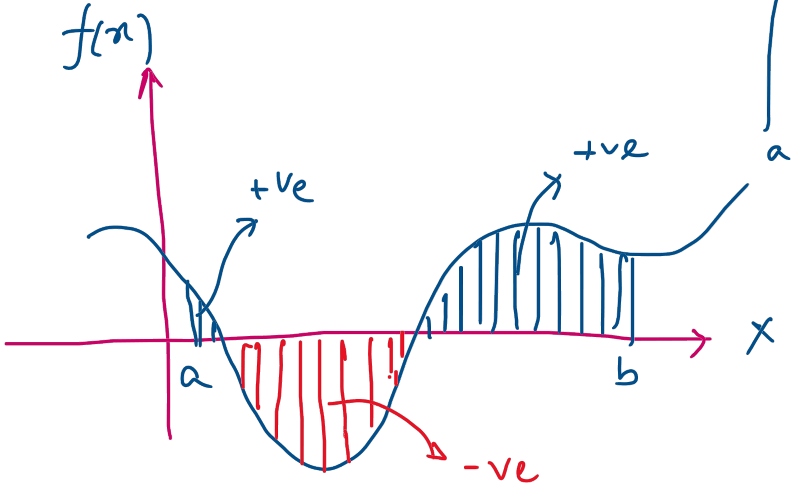
$$\boxed{A = \int_a^b f(x)\,dx}$$

Definite integral represents area under the curve from the lower bound (a) to the upper bound (b)



$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$





$$\int_a^b f(x)\,dx$$

$$\int_0^{\pi} \sin(x)\,dx = -\cos x\Big]_0^{\pi} = -\left(\cos\pi - \cos 0\right)$$

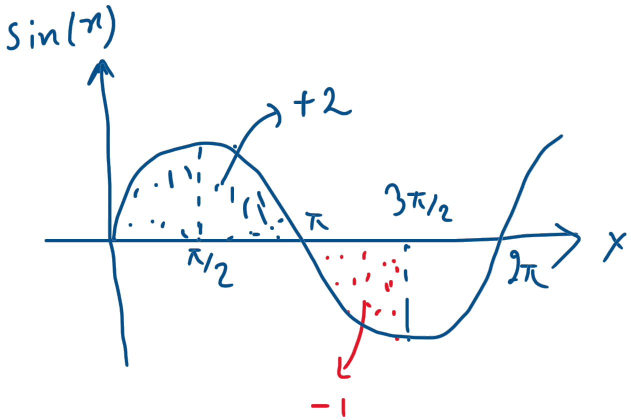
$$= -(-1-1) = -(-2) = 2$$

$$\int_{\pi}^{2\pi} \sin(x)\,dx = -\cos x\Big]_{\pi}^{2\pi} = -\left[\cos(2\pi) - \cos(\pi)\right]$$

$$= -\left[1-(-1)\right]$$

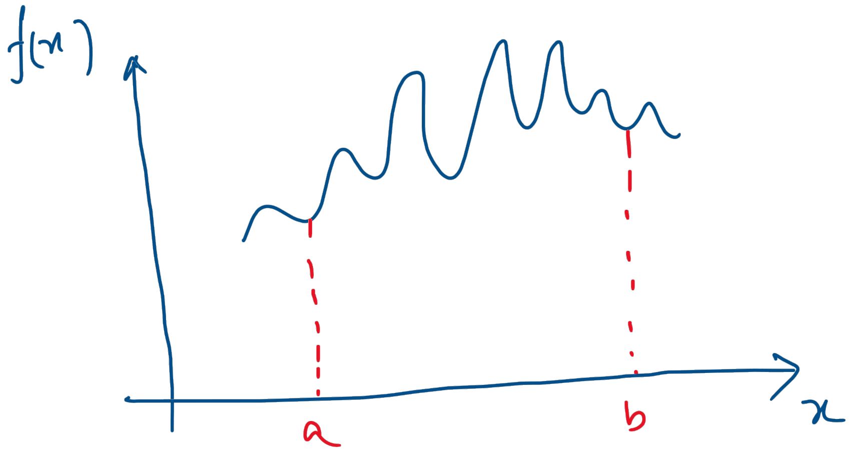
$$= -2$$

$$\int_0^{2\pi} \sin x\,dx = \int_0^{\pi} \sin x\,dx + \int_{\pi}^{2\pi} \sin x\,dx = 2-2 = 0$$



$$\int_0^{3\pi/2} \sin(x)\,dx = -\cos x \Big]_0^{3\pi/2} = -\left[\cos\left(3\pi/2\right) - \cos(0)\right]$$

$$= -\left[0 - 1\right] = +1$$



$\int f(x)\,dx \longrightarrow$ indefinite integral is impossible to compute

$\int_a^b f(x)\,dx \longrightarrow$ this is possible numerically

[numerical integration]

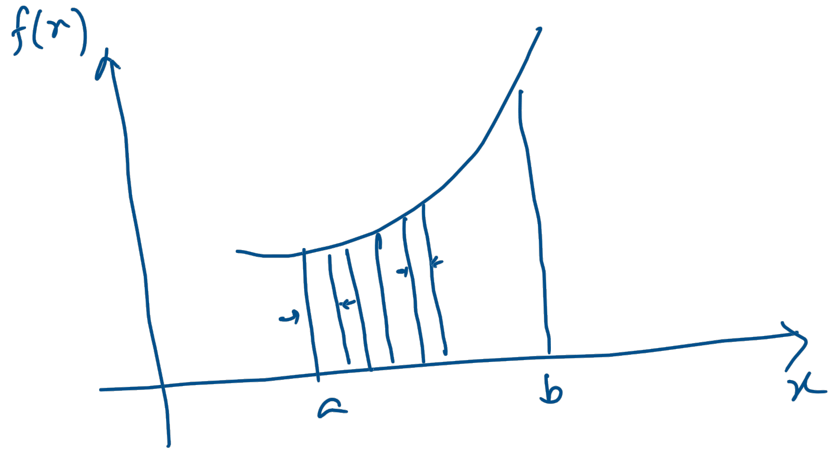
$f(x) = x^2$

$\int f(x)\,dx = \frac{x^3}{3}$

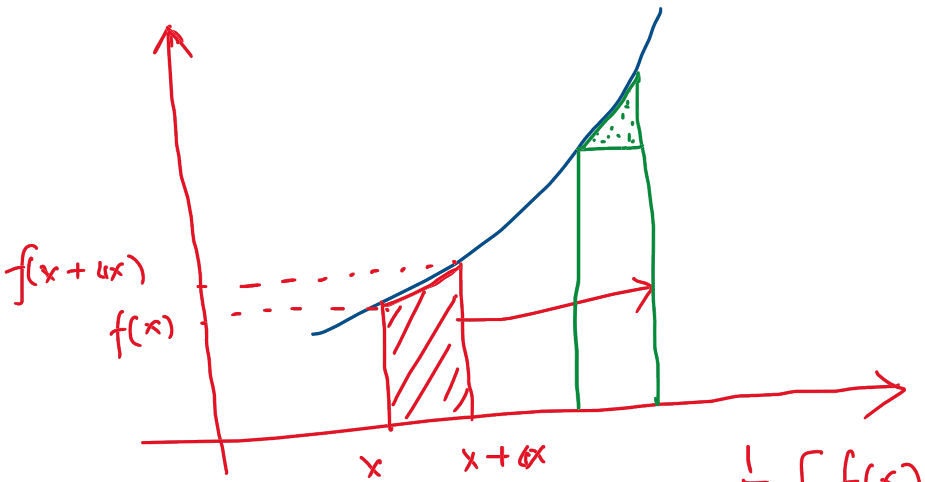
$\int_2^5 f(x)\,dx = \left.\frac{x^3}{3}\right]_2^5 = \frac{1}{3}(5^3 - 2^3)$

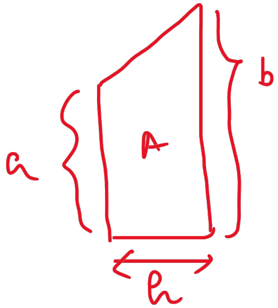
$= \frac{1}{3}(125 - 8)$

$= \frac{1}{3} \times 117 = \boxed{39}$



$\frac{b-a}{n} = \Delta x$





$$A = \frac{1}{2}(a+b)\cdot h$$

$$\underbrace{\frac{1}{2}\left[f(x)+f(x+\Delta x)\right]\Delta x}_{\text{trapezoidal rule of integration.}}$$