Vectors :

Vector is a collection of real numbers (in the form of 1-D array)

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$x_1 \in \mathbb{R}$, $x_2 \in \mathbb{R}$, $x_3 \in \mathbb{R}$, ..., $x_n \in \mathbb{R}$

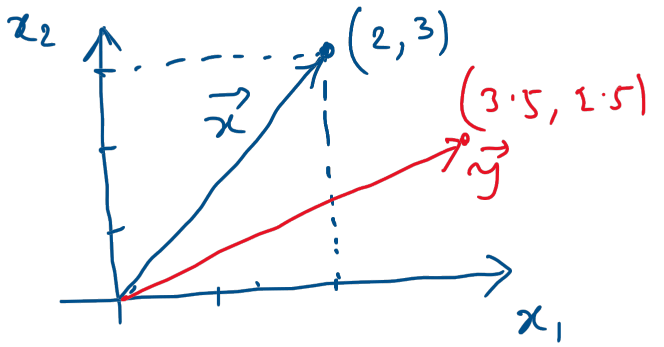
$\vec{x} \in \mathbb{R}^n$

($\mathbb{R}^n \to$ n-dimensional real space).

$$\begin{bmatrix} 3 \\ 4 \\ -2 \end{bmatrix} \to \text{3D. vector}$$

$$\vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \qquad \vec{x} \in \mathbb{R}^2$$

$$\vec{y} = \begin{bmatrix} 3.5 \\ 1.5 \end{bmatrix} \qquad \vec{y} \in \mathbb{R}^2$$



$$\begin{bmatrix} 3 \\ 5 \\ 6 \\ 7 \\ 8 \end{bmatrix} \to \text{5D vector}$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = [x_1, x_2, x_3, \ldots, x_n]^T$$

$T \rightarrow$ transpose

Dimension of vector

$$\dim(\vec{x}) = n$$

$$\vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \qquad \dim(\vec{x}) = 2$$

$$\vec{y} = [0.5, \ 0.7, \ -2.9]^T \qquad \dim(\vec{y}) = 3$$

Vector addition

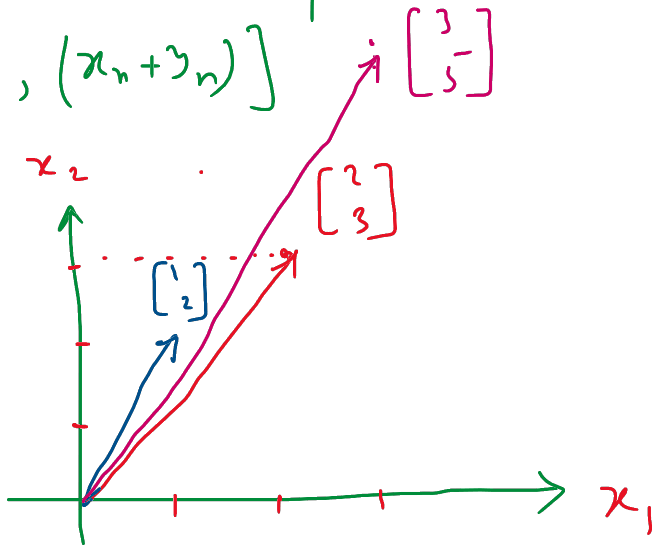
$\vec{x}$, $\vec{y}$ $\qquad$ $\vec{x} + \vec{y}$ (This is only possible when $\dim(\vec{x}) = \dim(\vec{y})$)

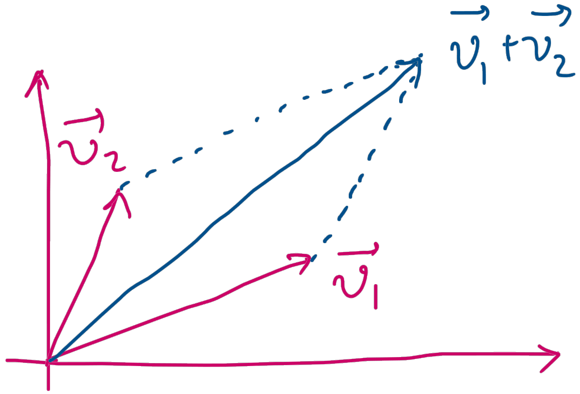
$[2, 3]^T + [3, 4, 5]^T \neq$ not possible

$$\vec{x} = [x_1, x_2, \ldots, x_n]^T \qquad \vec{y} = [y_1, y_2, \ldots, y_n]^T$$

$$\vec{x} + \vec{y} = [(x_1 + y_1), (x_2 + y_2), \ldots, (x_n + y_n)]^T$$

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$



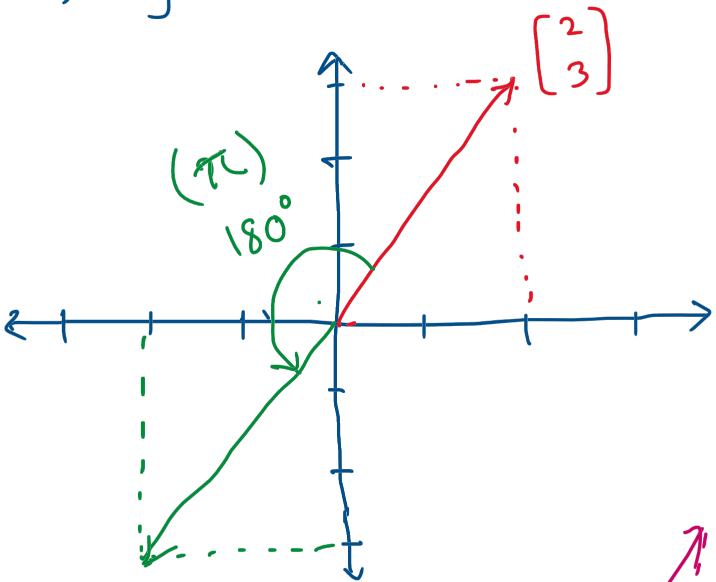


Vector inversion:

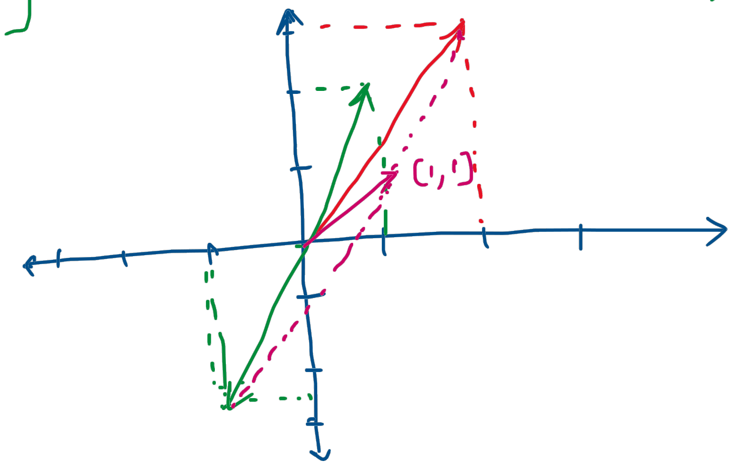
$$\vec{v} = [v_1, v_2, v_3, \ldots, v_n]^T$$

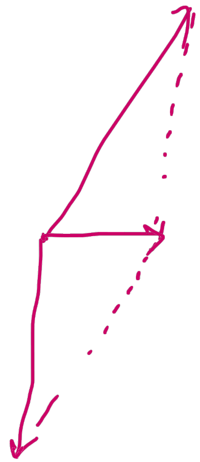
$$\vec{v} = [2, 3]^T \qquad \vec{w} = -\vec{v}$$

$$\Rightarrow \vec{w} = [-2, -3]^T$$

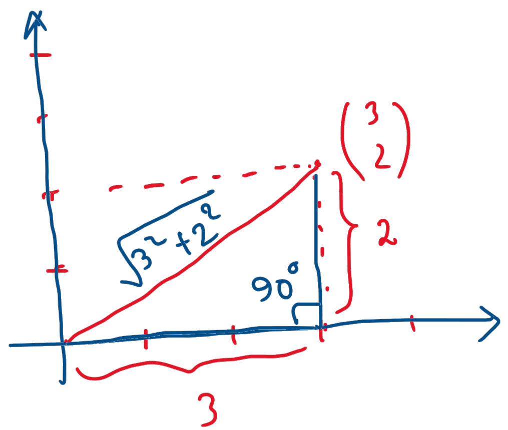


$$\begin{bmatrix}2\\3\end{bmatrix} - \begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}$$





Magnitude of a vector



$$\vec{v} = [v_1, v_2, v_3, \ldots, v_n]^T$$

$\|\vec{v}\|$ = magnitude of vector $\vec{v}$ (L_2 norm)

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2 + \cdots + v_n^2}$$

$$\lambda\vec{v} = [\lambda v_1, \lambda v_2, \lambda v_3, \ldots, \lambda v_n]^T$$

$$\|\lambda\vec{v}\| = \sqrt{\lambda^2 v_1^2 + \lambda^2 v_2^2 + \cdots + \lambda^2 v_n^2} = \sqrt{\lambda^2 (v_1^2 + v_2^2 + \cdots + v_n^2)} = \sqrt{\lambda^2} \cdot \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}$$

$$= \lambda \|\vec{v}\|$$

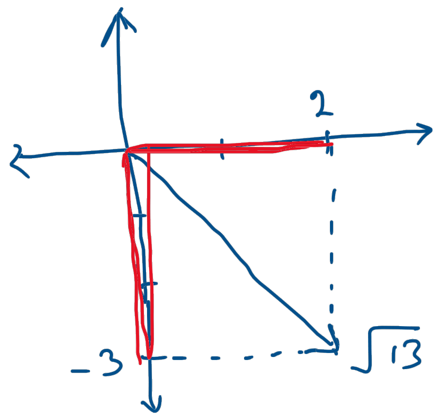
L_1 - norm

$$\vec{v} = [v_1, v_2, \ldots, v_n]^T$$

$$\|\vec{v}\|_1 = L1\text{-norm} = |v_1| + |v_2| + |v_3| + \cdots + |v_n| = \sum_{i=1}^{n} |v_i|$$

$$\vec{v} = \begin{bmatrix} 2 \\ -3 \end{bmatrix} \qquad \|\vec{v}\|_1 = |2| + |-3| = 2 + 3 = \underline{5}$$

$$\|\vec{v}\|_2 = \sqrt{2^2 + 3^2} = \sqrt{13}$$



L_p - norm

$$\|\vec{v}\|_p = \left(v_1^p + v_2^p + v_3^p + \cdots + v_n^p \right)^{1/p}$$

$$v_m = \max(v_1, v_2, \cdots, v_n)$$

$$\frac{L_\infty\text{-norm}}{p \to \infty} = \left[v_m^p \cdot \left\{ \left(\cancel{\frac{v_1}{v_m}}\right)^{p} + \left(\cancel{\frac{v_2}{v_m}}\right)^{p} + \cdots + \left(\cancel{\frac{v_n}{v_m}}\right)^{p} \right\} \right]^{1/p}$$

$(2, 3, 5)$

$$\left(2^p + 3^p + 5^p\right)^{1/p}$$

$$= 5^p \cdot \left[\left(\frac{2}{5}\right)^p + \left(\frac{3}{5}\right)^p + 1^p \right]^{1/p}$$

$$p \to \infty, \quad \left(v_m^p\right)^{1/p} = v_m$$

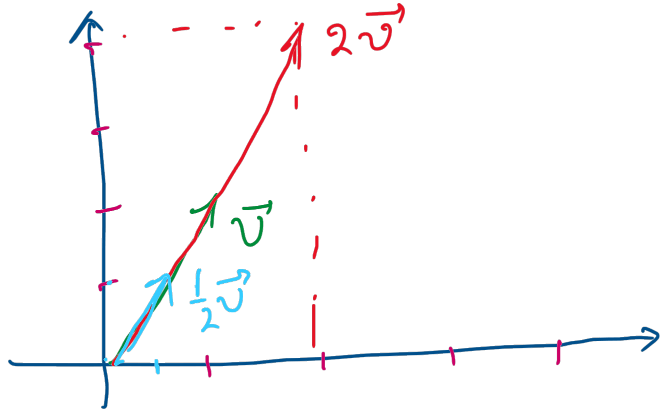
Multiplication :—

1. Multiplying a vector with a scalar

$\vec{w} = \lambda \vec{v}$ $\qquad \lambda \to$ scalar , $\vec{v}$

$$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \qquad \lambda \vec{v} = \begin{bmatrix} \lambda v_1 \\ \lambda v_2 \\ \vdots \\ \lambda v_n \end{bmatrix}$$

$\vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\qquad \lambda = 2$ $\qquad \lambda \vec{v} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$



If we multiply a vector with a -ve number then we flip the vector by 180° & then scale its magnitude.

2. Multiplying a vector with another vector.
(Product of two vectors)

2.1. Inner product (Dot product, Scalar product)

if $\dim(\vec{x}) = \dim(\vec{y})$ $\quad \vec{x} = [x_1, x_2, \dots, x_n]^T, \vec{y} = [y_1, y_2, \dots, y_n]^T$

$$\vec{x} \cdot \vec{y} = \vec{x}^T \vec{y} = x_1 y_1 + x_2 y_2 + x_3 y_3 + \dots + x_n y_n = \sum_{i=1}^{n} x_i y_i$$

$$\begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 & -5 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix} = 10 + 6 + (-5) = 11$$

$$\vec{v} = [v_1, v_2, \dots, v_n]^T$$

$$\begin{bmatrix} 2 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} = 1 + (-1) = 0$$

$$\vec{v}^T \vec{v} = \vec{v} \cdot \vec{v} = v_1^2 + v_2^2 + v_3^2 + \dots + v_n^2 = \|\vec{v}\|^2$$

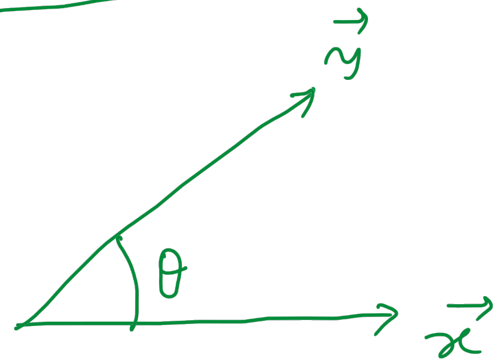
$$\vec{v} = \begin{bmatrix} 2 \\ -3 \end{bmatrix} \qquad \vec{v} \cdot \vec{v} = 2^2 + (-3)^2 = 4 + 9 = 13 = \|\vec{v}\|^2$$

$$\Rightarrow \|\vec{v}\| = \sqrt{\vec{v}^T \vec{v}}$$

Unit vector :- $\|\vec{u}\| = 1$ $\therefore$ $\vec{u}^T\vec{u} = 1$

$$\vec{x} = \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix} \qquad \|\vec{x}\| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$$

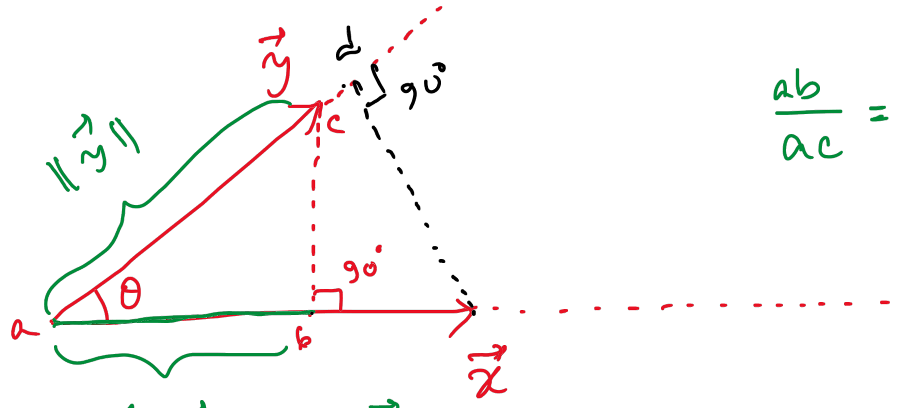
Geometrical interpretation of inner product



$$\vec{x} \cdot \vec{y} = \vec{x}^T\vec{y} = \|\vec{x}\| \cdot \|\vec{y}\| \cos\theta$$

$$\vec{y} \cdot \vec{x} = \vec{x} \cdot \vec{y} \qquad \therefore \boxed{\vec{y}^T\vec{x} = \vec{x}^T\vec{y}}$$

Commutative property



projection of $\vec{y}$ along $\vec{x}$

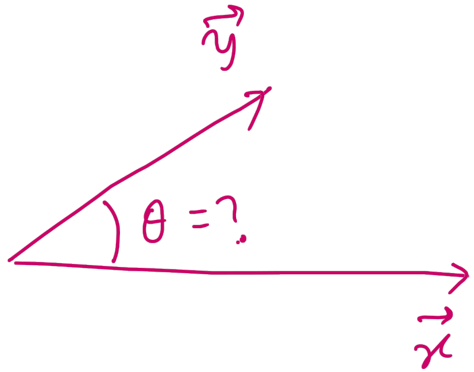
$$\frac{ab}{ac} = \cos\theta \qquad ad = \frac{\vec{x}^T\vec{y}}{\|\vec{y}\|}$$

$$ab = (ac) \cdot \cos\theta$$

$$\Rightarrow ab = \|\vec{y}\| \cos\theta$$

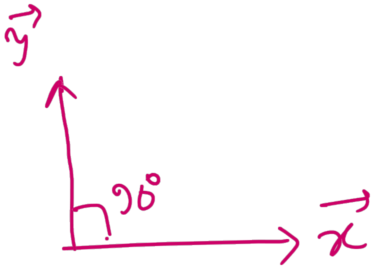
$$\Rightarrow ab\,\|\vec{x}\| = \|\vec{x}\|\,\|\vec{y}\| \cos\theta = \vec{x}^T\vec{y}$$

$$\Rightarrow ab = \vec{x}^T\vec{y} / \|\vec{x}\|$$



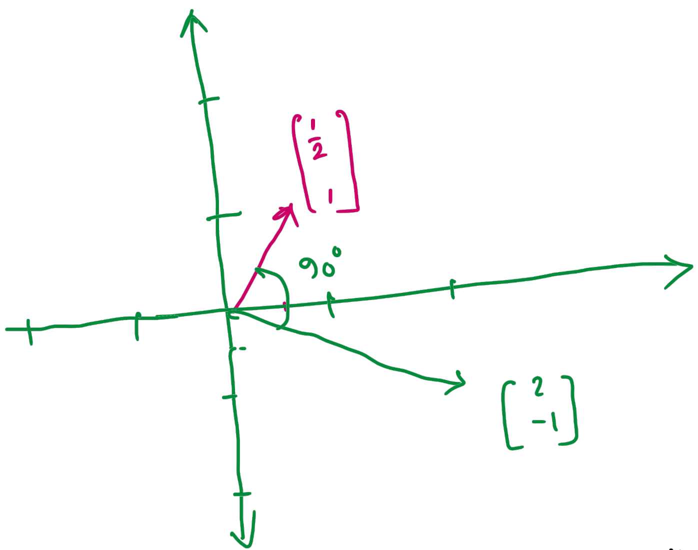
$$\vec{x}^T \vec{y} = \|\vec{x}\| \cdot \|\vec{y}\| \cos\theta$$

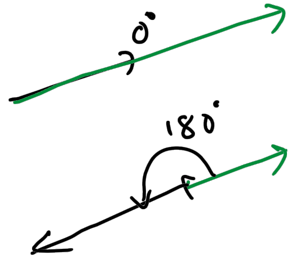
$$\Rightarrow \cos\theta = \frac{\vec{x}^T \vec{y}}{\|\vec{x}\| \, \|\vec{y}\|} \Rightarrow \theta = \cos^{-1}\left(\frac{\vec{x}^T \vec{y}}{\|\vec{x}\| \, \|\vec{y}\|} \right)$$



$$\vec{x}^T \vec{y} = 0$$

$$\begin{bmatrix} 2 \\ -1 \end{bmatrix} \qquad \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}$$





$$\vec{v}_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

find the angle between $\vec{v}_1$ & $\vec{v}_2$

$$\vec{v}_1 \cdot \vec{v}_2 = 1$$

$$\|\vec{v}_1\| = \sqrt{29}$$

$$\|\vec{v}_2\| = \sqrt{10}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{29} \cdot \sqrt{10}} \right)$$