Monte Carlo in Movie Production

Ray Tracing and Sampling

Slavomir Kaslev
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- I work at Worldwide FX as Head of R&D

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Studio demo reel

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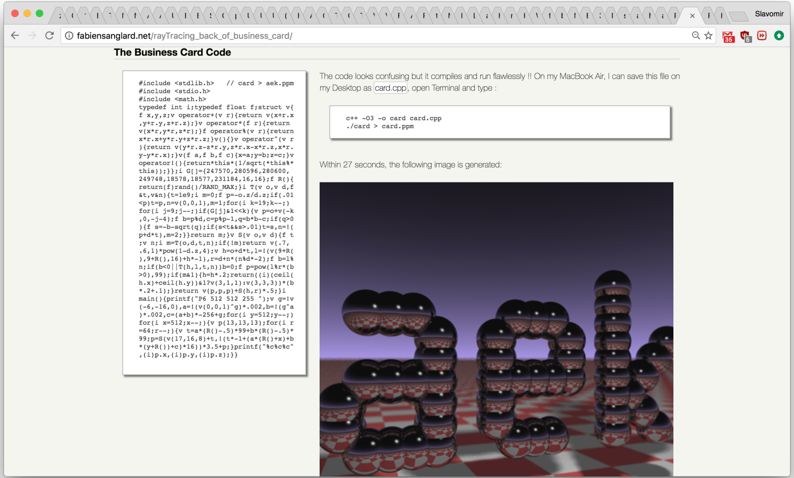
- "Physically Based Rendering: From Theory to Implementation" by Matt Pharr and Greg Humphreys

Bussiness card ray tracer



The Rendering Equation

$$L(p, \omega_o) = L_e(p, \omega_o) + \int_{S^2} f(p, \omega_o, \omega_i) L(t(p, \omega_i), -\omega_i) \left|\cos\theta_i\right| d\omega_i$$

Analytical solution: Operator formulation

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- The light transport operator

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- Power series representation: $L = L_e + \boldsymbol{T} L_e + \boldsymbol{T}^2 L_e + \dots$

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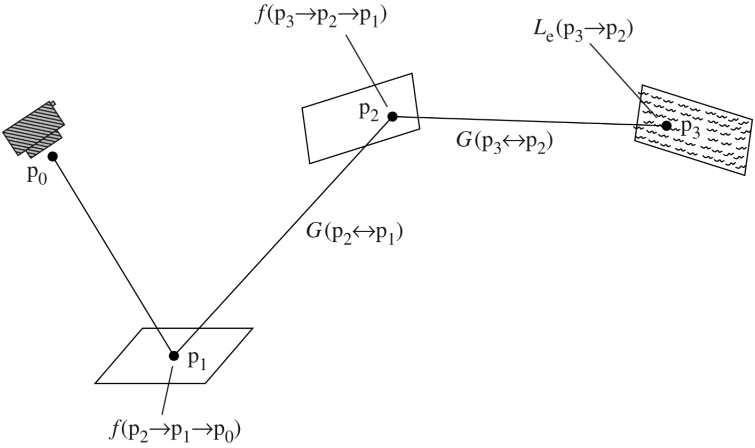
- The solution operator

$$\boldsymbol{S} = (\boldsymbol{I} - \boldsymbol{T})^{-1}$$
$$L = \boldsymbol{S} L_e$$

Numerical solution: Integral over paths

$$L(p' \to p) = L_e(p' \to p) + \int_A f(p'' \to p' \to p) L(p'' \to p') G(p'' \leftrightarrow p') dA(p'')$$

Generating paths



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- Monte Carlo methods were central to the simulations required for the Manhattan Project

Monte Carlo method I

Suppose we want to find the value of I where

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Well by definition of expected value we know that

$$\mathbf{E}[f(X)] = \int_{x \in [0,1]^s} f(x)p(x)dx$$

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$$\begin{aligned}\mathbf{E}[\frac{f(X)}{p(X)}] &= \int_{x\in[0,1]^s} \frac{f(x)}{p(x)} p(x)dx \\ &= \int_{x\in[0,1]^s} f(x)dx = I\end{aligned}$$

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Therefore

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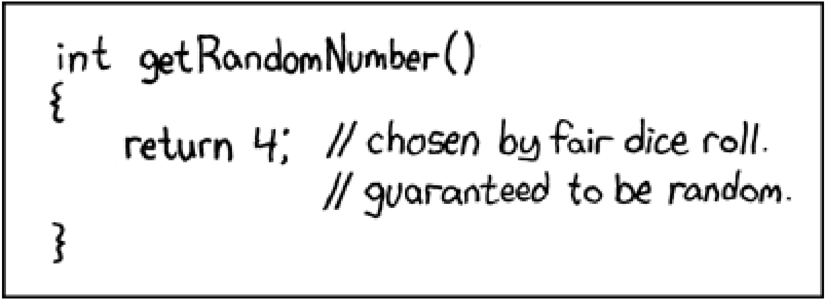
$$\mathbf{E}[\widetilde{I_N}] = I$$

$$\mathbf{V}[\widetilde{I_N}] = \frac{1}{N} \mathbf{V}[\frac{f(X)}{p(X)}]$$

$$\sigma[\widetilde{I_N}] = \frac{1}{\sqrt{N}} \sigma[\frac{f(X)}{p(X)}]$$

Random Numbers: xkcd #221



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- Low discrepancy sequences: van der Corput, Halton, Hammersley, Sobol and others

$$D_N^*(x_1, ..., x_N) \leq C \frac{(\ln N)^s}{N}$$

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- https://github.com/skaslev/pyman

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"All problems in computer graphics can be solved with a matrix inversion." Jim Blinn

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- Sum the series and substitue $\epsilon = 1$

Further reading

- "Physically Based Rendering: From Theory to Implementation" by Matt Pharr and Greg Humphreys
- "Robust Monte Carlo Methods for Light Transport Simulation", Eric Veach, Ph.D. dissertation
- "Light Transport Simulation with Vertex Connection and Merging" by Iliyan Georgiev, Jaroslav Kivnek, Tom Davidovi, Philipp Slusallek
- "Random Number Generation and Quasi-Monte Carlo Methods" by Harald Niederreiter
- "Quantum Mechanics and Path Integrals" by Richard Feynman
- "An Introduction to the Analysis of Algorithms" by Robert Sedgewick and Phillipe Flajolet
- "Mathematical Physics", Carl Bender, https://www.youtube.com/watch?v=LYNOGk3ZjFM
- "fract", source code from this presentation, https://github.com/skaslev/fract

Questions?

Thank you