DNV·GL

CLASS GUIDELINE

DNVGL-CG-0036

Edition July 2019

Calculation of gear rating for marine transmissions

DNV GL AS

FOREWORD

DNV GL class guidelines contain methods, technical requirements, principles and acceptance criteria related to classed objects as referred to from the rules.

Any comments may be sent by e-mail to *rules@dnvgl.com*

CHANGES – CURRENT

This document supersedes the December 2015 edition of DNVGL-CG-0036.

Changes in this document are highlighted in red colour. However, if the changes involve a whole chapter, section or subsection, normally only the title will be in red colour.

Changes July 2019

Topic	*Reference*	*Description*
Clarification	Sec.2 [8]	Added reference to DNVGL-RU-SHIP Pt.2 and DNVGL-CP-0247 for high grade (clean steel forging).
Correction in formula Z_N	Sec.2 [9]	Corrected error in formula for life factor Z_N.
Clarification	Sec.3 [7]	- for high grade added reference to material rules and DNVGL-CP-0247 - for approved process added reference to DNVGL-CP-0247

Editorial corrections

In addition to the above stated changes, editorial corrections may have been made.

CONTENTS

SECTION 1 BASIC PRINCIPLES AND GENERAL INFLUENCE FACTORS

1 Scope and basic principles

The gear rating procedures given in this class guideline are mainly based on the ISO6336 Part 1 to 5 (cylindrical gears), and partly on ISO 10300 Part 1 to 3 (bevel gears) and ISO *Technical Reports on Scuffing and Fatigue Damage Accumulation*, but especially applied for marine purposes, such as marine propulsion and important auxiliaries onboard ships and mobile offshore units.

The calculation procedures cover gear rating as limited by contact stresses (pitting, spalling or case crushing), tooth root stresses (fatigue breakage or overload breakage), and scuffing resistance. Even though no calculation procedures for other damages such as wear, grey staining (micropitting), etc. are given, such damages may limit the gear rating.

The class guideline applies to enclosed parallel shaft gears, epicyclic gears and bevel gears (with intersecting axis). However, open gear trains may be considered with regard to tooth strength, i.e. part 1 and 3 may apply. Even pinion-rack tooth strength may be considered, but since such gear trains often are designed with non-involute pinions, the calculation procedure of pinion-racks is described in Appendix C.

Steel is the only material considered.

The methods applied throughout this document are only valid for a transverse contact ratio $1 < \varepsilon_\alpha < 2$. If $\varepsilon_\alpha > 2$, either special considerations shall be made, or suggested simplification may be used.

All influence factors are defined regarding their physical interpretation. Some of the influence factors are determined by the gear geometry or have been established by conventions. These factors shall be calculated in accordance with the equations provided. Other factors are approximations, which are clearly stated in the text by terms as «may be calculated as». These approximations are substitutes for exact evaluations where such are lacking or too extensive for practical purposes, or factors based on experience. In principle, any suitable method may replace these approximations.

Bevel gears are calculated on basis of virtual (equivalent) cylindrical gears using the geometry of the midsection. The virtual (helical) cylindrical gear shall be calculated by using all the factors as a real cylindrical gear with some exceptions. These exceptions are mentioned in connection with the applicable factors. Wherever a factor or calculation procedure has no reference to either cylindrical gears or bevel gears, it is generally valid, i.e. combined for both cylindrical and bevel.

In order to minimise the volume of this class guideline such combinations are widely used, and everywhere it is necessary to distinguish, it is clearly pointed out by local headings such as:

— cylindrical gears
— bevel gears

The permissible contact stresses, tooth root stresses and scuffing load capacity depend on the safety factors as required in the respective rule sections.

Terms as endurance limit and static strength are used throughout this class guideline.

Endurance limit shall be understood as the fatigue strength in the range of cycles beyond the lower knee of the σ–N curves, regardless if it is constant or drops with higher number of cycles.

Static strength shall be understood as the fatigue strength in the range of cycles less than at the upper knee of the σ–N curves.

For gears that are subjected to a limited number of cycles at different load levels, a cumulative fatigue calculation applies. Information on this is given in Appendix A.

When the term infinite life is used, it means number of cycles in the range 10^8 to 10^{10}.

2 Symbols, nomenclature and units

The main symbols as influence factors (K, Z, Y and X with indices) etc. are presented in their respective headings. Symbols which are not explained in their respective sections are as follows:

a	=	centre distance (mm).
b	=	facewidth (mm).
d	=	reference diameter (mm).
d_a	=	tip diameter (mm).
d_b	=	base diameter (mm).
d_w	=	working pitch diameter (mm).
h_a	=	addendum (mm).
h_{a0}	=	addendum of tool ref. to m_n.
h_{fp}	=	addendum of basic rack ref. to m_n (= h_{a0}).
h_{Fe}	=	bending moment arm (mm) for tooth root stresses for application of load at the outer point of single tooth pair contact.
h_{Fa}	=	bending moment arm (mm) for tooth root stresses for application of load at tooth tip.
HB	=	Brinell hardness.
HV	=	Vickers hardness.
HRC	=	Rockwell C hardness
m_n	=	normal module.
n	=	rev. per minute.
N_L	=	number of load cycles.
q_s	=	notch parameter.
R_a	=	average roughness value (μm).
R_y	=	peak to valley roughness (μm).
R_z	=	mean peak to valley roughness (μm).
s_{an}	=	tooth top land thickness (mm).
s_{at}	=	transverse top land thickness (mm).
s_{Fn}	=	tooth root chord (mm) in the critical section.
s_{pr}	=	protuberance value of tool minus grinding stock, equal residual undercut of basic rack, ref. to m_n.
T	=	torque (Nm).
u	=	gear ratio (per stage).
v	=	linear speed (m/s) at reference diameter.
x	=	addendum modification coefficient.
z	=	number of teeth.
z_n	=	virtual number of spur teeth.
α_n	=	normal pressure angle at ref. cylinder.

α_t = transverse pressure angle at ref. cylinder.

α_a = transverse pressure angle at tip cylinder.

α_{wt} = transverse pressure angle at pitch cylinder.

β = helix angle at ref. cylinder.

β_b = helix angle at base cylinder.

β_a = helix angle at tip cylinder.

ε_α = transverse contact ratio.

ε_β = overlap ratio.

ε_γ = total contact ratio.

ρ_{a0} = tip radius of tool ref. to m_n.

ρ_{fp} = root radius of basic rack ref. to m_n (= ρ_{a0}).

ρ_C = effective radius (mm) of curvature at pitch point.

ρ_F = root fillet radius (mm) in the critical section.

σ_B = ultimate tensile strength (N/mm^2).

σ_y = yield strength resp. 0.2% proof stress (N/mm^2).

Index 1 refers to the pinion, 2 to the wheel.
Index n refers to normal section or virtual spur gear of a helical gear.
Index w refers to pitch point.
Index v refers to the virtual (equivalent) helical cylindrical gear.
Index m refers to the midsection of the bevel gear.
Special additional symbols for bevel gears are as follows:

Σ	=	angle between intersection axis.
ϑ_K	=	angle modification (Klingelnberg)
m_0	=	tool module (Klingelnberg)
δ	=	pitch cone angle.
x_{sm}	=	tooth thickness modification coefficient (midface).
R	=	pitch cone distance (mm).

3 Geometrical definitions

For internal gearing z_2, a; d_{a2}, d_{w2}, d_2 and d_{b2} are negative, x_2 is positive if d_{a2} is increased, i.e. the numeric value is decreased.

The pinion has the smaller number of teeth, i.e.

$$|u| = \frac{|z_2|}{z_1} \geq 1$$

For calculation of surface durability b is the common facewidth on pitch diameter.

For tooth strength calculations b_1 or b_2 are facewidths at the respective tooth roots. If b_1 or b_2 differ much from b above, they are not to be taken more than 1 module on either side of b.

Cylindrical gears

$\tan \alpha_t$	$= \tan \alpha_n / \cos \beta$
$\tan \beta_b$	$= \tan \beta \cos \alpha_t$
$\tan \beta_a$	$= \tan \beta\, d_a / d$
$\cos \alpha_a$	$= d_b/d_a$
d	$= z\, m_n / \cos \beta$
m_t	$= m_n / \cos \beta$
d_b	$= d \cos \alpha_t = d_w \cos \alpha_{wt}$
a	$= 0.5\, (d_{w1} + d_{w2})$
d_{w1}/d_{w2}	$= z_2 / z_1$
$\text{inv}\, \alpha$	$= \tan \alpha - \alpha$ (radians)
$\text{inv}\, \alpha_{wt}$	$= \text{inv}\, \alpha_t + 2 \tan \alpha_n (x_1 + x_2)/(z_1 + z_2)$
z_n	$= z / (\cos^2 \beta_b \cos \beta)$

$$\varepsilon_\alpha = \frac{\xi_{fw1} + \xi_{aw1}}{T_1}$$

where ξ_{fw1} shall be taken as the smaller of:

— $\xi_{fw1} = \tan \alpha_{wt}$

— $\xi_{fw1} = \tan\alpha_{wt} - \tan \operatorname{acos} \dfrac{d_{b1}}{d_{soi1}}$

— $\xi_{fw1} = \left(\tan \operatorname{acos} \dfrac{d_{b2}}{d_{a2}} - \tan\alpha_{wt} \right) \dfrac{z_2}{z_1}$

and

$\xi_{aw1} = \xi_{fw2} \dfrac{z_1}{z_2}$, where ξ_{fw2} is calculated as ξ_{fw1}

substituting the values for the wheel by the values for the pinion and vice versa.

$$T_1 = \frac{2\pi}{z_1}$$

$$d_{soi1} = 2 \cdot \left[\left(\frac{d}{2} - m_n \left(h_{fp} - x_1 - \rho_{fp} + \rho_{fp} \cdot \sin\alpha_n \right) \right)^2 + \left(\frac{m_n (h_{fp} - x_1 - \rho_{fp} + \rho_{fp} \cdot \sin\alpha_n)}{\tan\alpha_t} \right)^2 \right]^{\frac{1}{2}}$$

$$\varepsilon_\beta = \frac{b \sin\beta}{\pi m_n}$$

(for double helix, b shall be taken as the width of one helix).

$\varepsilon_y \quad = \quad \varepsilon_\alpha + \varepsilon_\beta$

$\rho_C \quad = \quad \dfrac{a \;\; u \sin\alpha_{wt}}{\cos\beta_b \, (1+u)^2}$

$v \quad = \quad \dfrac{\pi}{60} n_1 d_1 10^{-3}$

$p_{bt} \quad = \quad \dfrac{\pi m_n \cos \alpha_t}{\cos \beta}$

$$s_{at} = d_a \left[\frac{\frac{\pi}{2} + 2\,x \tan\alpha_n}{z} + \mathrm{inv}\,\alpha_t - \mathrm{inv}\alpha_a \right]$$

$$s_{an} = s_{at} \cos\beta_a$$

4 Bevel gear conversion formulae and specific formulae

Conversion of bevel gears to virtual equivalent helical cylindrical gears is based on the bevel gear midsection. The conversion formulae are:

Number of teeth:

$$z_{v1.2} = z_{1,2} / \cos\delta_{1,2}$$
$$(\delta_1 + \delta_2 = \Sigma)$$

$$u_v = \frac{z_{v2}}{z_{v1}}$$

Gear ratio:

$$\tan\alpha_{vt} = \tan\alpha_n / \cos\beta_m$$

$$\tan\beta_{bm} = \tan\beta_m \cos\alpha_{vt}$$

$$p_{btm} = \frac{\pi\, m_{nm} \cos\alpha_{vt}}{\cos\beta_m}$$

Base pitch:

$$d_{v1.2} = \frac{d_{m1,2}}{\cos\delta_{1,2}}$$

Reference, pitch, diameters:

Centre distance:

$$a_v = 0.5\,(d_{v1} + d_{v2})$$

Tip diameters:

$$d_{va\,1.2} = d_{v\,1,2} + 2\,h_{am\,1,2}$$

Addenda:

– for gears with constant addenda (Klingelnberg):

$$h_{am\,1,2} = m_{mn}\,(1 + x_{m\,1,2})$$

– for gears with variable addenda (Gleason):

$$h_{am\,1,2} = h_{a\,1,2} - b/2 \tan(\delta_{a\,1,2} - \delta_{1,2})$$

(when h_a is addendum at outer end and δ_a is the outer cone angle).

$$x_{m1,2} = \frac{h_{am1,2} - h_{am2,1}}{2\,m_{mn}}$$

Addendum modification coefficients:

Base circle:

$d_{vb\ 1,2} = d_{v\ 1,2} \cos \alpha_{vt}$

Transverse contact ratio:*)

$$\varepsilon_\alpha = \frac{0.5\sqrt{d_{va1}^2 - d_{vb1}^2} + 0.5\sqrt{d_{va2}^2 - d_{vb2}^2} - a_v \sin \alpha_{vt}}{p_{btm}}$$

Overlap ratio*) (theoretical value for bevel gears with no crowning, but used as approximations in the calculation procedures):

$$\varepsilon_\beta = \frac{b \sin \beta_m}{\pi\, m_{nm}}$$

Total contact ratio:*)

$$\varepsilon_\gamma = \sqrt{\varepsilon_\alpha^2 + \varepsilon_\beta^2}$$

*) Note that index «v» is left out in order to combine formulae for cylindrical and bevel gears.

$$v_{mt} = \frac{\pi}{60} n_1\, d_{m1}\, 10^{-3}$$

Tangential speed at midsection:

$$\rho_{vc} = \frac{a_v\, u_v \sin \alpha_{vt}}{\cos \beta_{bm} (1 + u_v)^2}$$

Effective radius of curvature (normal section):

$$l_b = \frac{b\,\varepsilon_\alpha}{\cos \beta_{bm}} \frac{\sqrt{\varepsilon_\gamma^2 - ((2 - \varepsilon_\alpha)(1 - \varepsilon_\beta))^2}}{\varepsilon_\gamma^2} \qquad \text{if } \varepsilon_\beta < 1$$

Length of line of contact:

$$l_b = \frac{b\,\varepsilon_\alpha}{\varepsilon_\gamma \cos\beta_{bm}} \qquad \text{if } \varepsilon_\beta \geq 1$$

5 Nominal tangential load, F_t, F_{bt}, F_{mt} and F_{mbt}

The nominal tangential load (tangential to the reference cylinder with diameter d and perpendicular to an axial plane) is calculated from the nominal (rated) torque T transmitted by the gear set.

Cylindrical gears

$$F_t = \frac{2000\,T}{d} \qquad F_{bt} = \frac{F_t}{\cos\alpha_t}$$

Bevel gears

$$F_{mt} = \frac{2000\,T}{d_m} \qquad F_{mbt} = \frac{F_{mt}}{\cos\alpha_{vt}}$$

6 Application factors, K_A and K_{AP}

The application factor K_A accounts for dynamic overloads from sources external to the gearing.

It is distinguished between the influence of repetitive cyclic torques K_A ([6.1]) and the influence of temporary occasional peak torques K_{AP} ([6.2]).

Calculations are always to be made with K_A. In certain cases additional calculations with K_{AP} may be necessary.

For gears with a defined load spectrum the calculation with a K_A may be replaced by a fatigue damage calculation as given in Appendix A.

6.1 K_A

For gears designed for long or infinite life at nominal rated torque, K_A is defined as the ratio between the maximum repetitive cyclic torque applied to the gear set and nominal rated torque.

This definition is suitable for main propulsion gears and most of the auxiliary gears.

K_A can be determined by measurements or system analysis, or may be ruled by conventions (ice classes). (For the purpose of a preliminary (but not binding) calculation before K_A is determined, it is advised to apply either the max. values mentioned below or values known from similar plants.)

a) For main propulsion gears K_A can be taken from the (mandatory) torsional vibration analysis, thereby considering all permissible driving conditions.[*)]

   Unless specially agreed, the rules do not allow K_A in excess of 1.50 for diesel propulsion.[*)] With turbine or electric propulsion K_A would normally not exceed 1.2. However, special attention should be given to thrusters that are arranged in such a way that heavy vessel movements and/or manoeuvring can cause severe load fluctuations. This means e.g. thrusters positioned far from the rolling axis of vessels that could be susceptible to rolling. If leading to propeller air suction, the conditions may be even worse.

The above mentioned movements or manoeuvring will result in increased propeller excitation. If the thruster is driven by a diesel engine, the engine mean torque is limited to 100%. However, thrusters driven by electric motors can suffer temporary mean torque much above 100% unless a suitable load control system (limiting available e-motor torque) is provided.

b) For main propulsion gears with ice class notation special regulations apply for load spectrum and peak load, see DNVGL-CG-0041 *Ice strengthening of propulsion machinery*.

c) For a *power take off* (PTO) branch from a main propulsion gear with ice class, ice shocks result in negative torques. It is assumed that the PTO branch is unloaded when the ice shock load occurs.

The influence of these reverse shock loads may be taken into account as follows:

The negative torque (reversed load), expressed by means of an application factor based on rated forward load (T or F_t), is $K_{Areverse} = K_{A\ ice} - 1$ (the minus 1 because no mean torque assumed). K_{Aice} to be calculated as in the ice class rules. This $K_{Areverse}$ should be used for back flank considerations such as pitting and scuffing.

The influence on tooth bending strength (forward direction) may be simplified by using the factor

$Y_M = 1 - 0.3 \cdot K_{Areverse} / K_A$.

d) For diesel driven auxiliaries K_A can be taken from the torsional vibration analysis, if available. For units where no vibration analysis is required (< 200 kW) or available, it is advised to apply K_A as the upper allowable value 1.50.*)

e) For turbine or electro driven auxiliaries the same as for c) applies, however the practical upper value is 1.2.

*) For diesel driven gears, more information on K_A for misfiring and normal driving is given in Appendix B.

6.2 K_{AP}

The peak overload factor K_{AP} is defined as the ratio between the temporary occasional peak overload torque and the nominal rated torque.

For plants where high temporary occasional peak torques can occur (i.e. in excess of the above mentioned K_A), the gearing (if nitrited) has to be checked with regard to static strength. Unless otherwise specified the same safety factors as for infinite life apply.

The scuffing safety shall be specially considered, whereby the K_A applies in connection with the bulk temperature, and the K_{AP} applies for the flash temperature calculation and should replace K_A in the formulae in Sec.4 [3.1], Sec.4 [3.2] and Sec.4 [4.1].

K_{AP} can be evaluated from the torsional impact vibration calculation (as required by the rules).

If the overloads have a duration corresponding to several revolutions of the shafts, the scuffing safety has to be considered on basis of this overload, both with respect to bulk and flash temperature.

For plants *without* additional ice class notation, K_{AP} should normally not exceed 1.5.

6.3 Frequent overloads

For plants where high overloads or shock loads occur regularly, the influence of this shall be considered by means of cumulative fatigue, (see Appendix A).

7 Load sharing factor, K_γ

The load sharing factor K_γ accounts for the misdistribution of load in multiple-path transmissions (dual tandem, epicyclical, double helix etc.). K_γ is defined as the ratio between the max. load through an actual path and the evenly shared load.

7.1 General method

K_γ mainly depends on accuracy and flexibility of the branches (e.g. quill shaft, planet support, external forces etc.), and should be considered on basis of measurements or of relevant analysis as e.g.:

$$K_\gamma = \frac{\delta + f}{\delta}$$

δ = total compliance of a branch under full load (assuming even load share) referred to gear mesh.

f = $\sqrt{f_1^2 + f_2^2 + f_3^2 + - - - -}$
where f_1, f_2 etc. are the main individual errors that may contribute to a misdistribution between the branches. E.g. tooth pitch errors, planet carrier pitch errors, bearing clearance influences etc. Compensating effects should also be considered.

For double helical gears:

An external axial force F_{ext} applied from sources outside the actual gearing (e.g. thrust via or from a tooth coupling) will cause a misdistribution of forces between the two helices. Expressed by a load sharing factor the

$$K_\gamma = 1 \pm \frac{F_{ext}}{F_t \cdot \tan\beta}$$

If the direction of F_{ext} is known, the calculation should be carried out separately for each helix, and with the tangential force corrected with the pertinent K_γ. If the direction of F_{ext} is unknown, both combinations shall be calculated, and the higher σ_H or σ_F to be used.

7.2 Simplified method

If no relevant analysis is available the following may apply:

For epicyclic gears:

$$K_\gamma = 1 + 0.25\sqrt{n_{p1} - 3}$$

where n_{pl} = number of planets (≥ 3).

For multistage gears with locked paths and gear stages separated by quill shafts (see figure below):

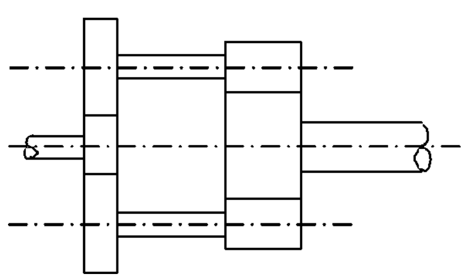


Figure 1 Locked paths gear

$$K_\gamma = 1 + (0.2/\phi)$$

where φ = quill shaft twist (degrees) under full load.

8 Dynamic Factor, K_v

The dynamic factor K_v accounts for the internally generated dynamic loads.

K_v is defined as the ratio between the maximum load that dynamically acts on the tooth flanks and the maximum externally applied load F_t K_A K_γ.

In the following 2 different methods ([8.1] and [8.2]) are described. In case of controversy between the methods, the next following is decisive, i.e. the methods are listed with increasing priority.

It is important to observe the limitations for the method in [8.1]. In particular the influence of lateral stiffness of shafts is often underestimated and resonances occur at considerably lower speed than determined in [8.1].

However, for low speed gears with $v \cdot z_1 < 300$ calculations may be omitted and the dynamic factor simplified to $K_v = 1.05$.

8.1 Single resonance method

For a single stage gear K_v may be determined on basis of the relative proximity (or resonance ratio) N between actual speed n_1 and the lowest resonance speed n_{E1}.

$$N = \frac{n_1}{n_{E1}}$$

Note that for epicyclic gears n is the relative speed, i.e. the speed that multiplied with z gives the mesh frequency.

8.1.1 Determination of critical speed

It is not advised to apply this method for multimesh gears for $N > 0.85$, as the influence of higher modes has to be considered, see [8.2]. In case of significant lateral shaft flexibility (e.g. overhung mounted bevel gears), the influence of coupled bending and torsional vibrations between pinion and wheel should be considered if $N \geq 0.75$, see [8.1].

$$n_{E1} = \frac{30 \cdot 10^3}{\pi z_1} \sqrt{\frac{c_\gamma}{m_{red}}}$$

where:

c_γ is the actual mesh stiffness per unit facewidth, see [11].

For gears with inactive ends of the facewidth, as e.g. due to high crowning or end relief such as often applied for bevel gears, the use of c_γ in connection with determination natural frequencies may need correction. c_γ is defined as stiffness per unit facewidth, but when used in connection with the total mesh stiffness, it is not as simple as $c_\gamma \cdot b$, as only a part of the facewidth is active. Such corrections are given in [11].

m_{red} is the reduced mass of the gear pair, per unit facewidth and referred to the plane of contact.

For a single gear stage where no significant inertias are closely connected to neither pinion nor wheel, m_{red} is calculated as:

$$m_{red} = \frac{m_1 \, m_2}{m_1 + m_2}$$

The individual masses per unit facewidth are calculated as

$$m_{1,2} = \frac{I_{1,2}}{b\,(d_{b1,2}/2)^2}$$

where I is the polar moment of inertia (kgm^2).

The inertia of bevel gears may be approximated as discs with diameter equal the midface pitch diameter and width equal to b. However, if the shape of the pinion or wheel body differs much from this idealised cylinder, the inertia should be corrected accordingly.

For all kind of gears, if a significant inertia (e.g. a clutch) is very rigidly connected to the pinion or wheel, it should be added to that particular inertia (pinion or wheel). If there is a shaft piece between these inertias, the torsional shaft stiffness alters the system into a 3-mass (or more) system. This can be calculated as in [8.2], but also simplified as a 2-mass system calculated with only pinion and wheel masses.

8.1.2 Factors used for determination of Kv

Non-dimensional gear accuracy dependent parameters:

$$B_p = \frac{c'\left(f_{pt} - y_p\right)}{F_t K_A K_\gamma / b}$$

$$B_f = \frac{c'\left(F_\alpha - y_f\right)}{F_t K_A K_\gamma / b}$$

Non-dimensional tip relief parameter:

$$B_k = \left| 1 - \frac{C_a \cdot c'}{F_t \cdot K_A \cdot K_\gamma / b} \right|$$

For gears of quality grade (ISO 1328) Q = 7 or coarser, $B_k = 1$.

For gears with Q ≤ 6 and excessive tip relief, B_k is limited to max. 1.

For gears (all quality grades) with tip relief of more than $2 \cdot C_{eff}$ (see Sec.4 [3.2]) the reduction of ε_α has to be considered (see Sec.4 [4.3]).

Where:

f_{pt}	=	the single pitch deviation (ISO 1328), max. of pinion or wheel
F_α	=	the total profile form deviation (ISO 1328), max. of pinion or wheel (Note: F_α is p.t. not available for bevel gears, thus use $F_\alpha = f_{pt}$)
y_p and y_f	=	the respective running-in allowances and may be calculated similarly to y_α in [12], i.e. the value of f_{pt} is replaced by F_α for y_f.
c´	=	the single tooth stiffness, see [11]

C_a	=	the amount of tip relief, see Sec.4 [3.3]. In case of different tip relief on pinion and wheel, the value that results in the greater value of B_k shall be used. If C_a is zero by design, the value of running-in tip relief C_{ay} (see [12]) may be used in the above formula.

8.1.3 K_v in the subcritical range:

Cylindrical gears	$N \leq 0.85$
Bevel gears	$N \leq 0.75$

$K_v = 1 + N\ K$

$K = C_{v1}\ B_p + C_{v2}\ B_f + C_{v3}\ B_k$

C_{v1} accounts for the pitch error influence

$C_{v1} = 0.32$

C_{v2} accounts for profile error influence

$C_{v2} = 0.34$ for $\varepsilon_\gamma \leq 2$

$$C_{v2} = \frac{0.57}{\varepsilon_\gamma - 0.3} \quad \text{for } \varepsilon_\gamma > 2$$

C_{v3} accounts for the cyclic mesh stiffness variation

$C_{v3} = 0.23$ for $\varepsilon_\gamma \leq 2$

$$C_{v3} = \frac{0.096}{\varepsilon_\gamma - 1.56} \quad \text{for } \varepsilon_\gamma > 2$$

8.1.4 K_v in the main resonance range:

Cylindrical gears	$0.85 < N \leq 1.15$
Bevel gears	$0.75 < N \leq 1.25$

Running in this range should preferably be avoided, and is only allowed for high precision gears.

$K_v = 1 + C_{v1}\ B_p + C_{v2}\ B_f + C_{v4}\ B_k$

C_{v4} accounts for the resonance condition with the cyclic mesh stiffness variation.

$C_{v4} = 0.90$ for $\varepsilon_\gamma \leq 2$

$$C_{v4} = \frac{0.57 - 0.05\,\varepsilon_\gamma}{\varepsilon_\gamma - 1.44} \quad \text{for } \varepsilon_\gamma > 2$$

8.1.5 K_v in the supercritical range:

Cylindrical gears	$N \geq 1.5$

Bevel gears	$N \geq 1.5$

Special care should be taken as to influence of higher vibration modes, and/or influence of coupled bending (i.e. lateral shaft vibrations) and torsional vibrations between pinion and wheel. These influences are not covered by the following approach.

$$K_v = C_{v5} B_p + C_{v6} B_f + C_{v7}$$

C_{v5} accounts for the pitch error influence.

$C_{v5} = 0.47$

C_{v6} accounts for the profile error influence.

$C_{v6} = 0.47$	for $\varepsilon_\gamma \leq 2$
$C_{v6} = \dfrac{0.12}{\varepsilon_\gamma - 1.74}$	for $\varepsilon_\gamma > 2$

C_{v7} relates the maximum externally applied tooth loading to the maximum tooth loading of ideal, accurate gears operating in the supercritical speed sector, when the circumferential vibration becomes very soft.

$C_{v7} = 0.75$	for $\varepsilon_\gamma \leq 1.5$
$C_{v7} = 0.125 \sin\left(\pi\left(\epsilon_\gamma - 2\right)\right) + 0.875$	for $1.5 < \varepsilon_\gamma \leq 2.5$
$C_{v7} = 1.0$	for $\varepsilon_\gamma > 2.5$

8.1.6 K_v in the intermediate range:

Cylindrical gears	$1.15 < N < 1.5$
Bevel gears	$1.25 < N < 1.5$

Comments raised in [8.1.4] and [8.1.5] should be observed.

K_v is determined by linear interpolation between K_v for N = 1.15 respectively 1.25 and N = 1.5 as

Cylindrical gears

$$K_v = K_{v(N=1.5)} + \left(\frac{1.5 - N}{0.35}\right) \cdot \left[K_{v(N=1.15)} - K_{v(N=1.5)}\right]$$

Bevel gears

$$K_v = K_{v(N=1.5)} + \left(\frac{1.5 - N}{0.25}\right) \cdot \left[K_{v(N=1.25)} - K_{v(N=1.5)}\right]$$

8.2 Multi-resonance method

For high speed gear (v > 40 m/s), for multimesh medium speed gears, for gears with significant lateral shaft flexibility etc. it is advised to determine K_v on basis of relevant dynamic analysis.

Incorporating lateral shaft compliance requires transformation of even a simple pinion-wheel system into a lumped multi-mass system. It is advised to incorporate all relevant inertias and torsional shaft stiffnesses into an equivalent (to pinion speed) system. Thereby the mesh stiffness appears as an equivalent torsional stiffness:

$$c_\gamma \, b \, (d_{b1}/2)^2 \quad \text{(Nm/rad)}$$

The natural frequencies are found by solving the set of differential equations (one equation per inertia). Note that for a gear put on a laterally flexible shaft, the coupling bending-torsionals is arranged by introducing the gear mass and the lateral stiffness with its relation to the torsional displacement and torque in that shaft.

Only the natural frequency (ies) having high relative displacement and relative torque through the actual pinion-wheel flexible element, need(s) to be considered as critical frequency (ies).

K_v may be determined by means of the method mentioned in [8.1] thereby using N as the least favourable ratio (in case of more than one pinion-wheel dominated natural frequency). I.e. the N-ratio that results in the highest K_v has to be considered.

The level of the dynamic factor may also be determined on basis of simulation technique using numeric time integration with relevant tooth stiffness variation and pitch/profile errors.

9 Face load factors, $K_{H\beta}$ and $K_{F\beta}$

The face load factors, $K_{H\beta}$ for contact stresses and for scuffing, $K_{F\beta}$ for tooth root stresses, account for non-uniform load distribution across the facewidth.

$K_{H\beta}$ is defined as the ratio between the maximum load per unit facewidth and the mean load per unit facewidth.

$K_{F\beta}$ is defined as the ratio between the maximum tooth root stress per unit facewidth and the mean tooth root stress per unit facewidth. The mean tooth root stress relates to the considered facewidth b_1 respectively b_2.

Note that facewidth in this context is the design facewidth b, even if the ends are unloaded as often applies to e.g. bevel gears.

The plane of contact is considered.

9.1 Relations between $K_{H\beta}$ and $K_{F\beta}$

$$K_{F\beta} = K_{H\beta}^{\left(\frac{1}{1 + h/b + (h/b)^2}\right)}$$

where h/b is the ratio tooth height/facewidth. The maximum of h_1/b_1, and h_2/b_2 shall be used, but not higher than 1/3. For double helical gears, use only the facewidth of one helix.

If the tooth root facewidth (b_1 or b_2) is considerably wider than b, the value of $K_{F\beta(1or2)}$ shall be specially considered as it may even exceed $K_{H\beta}$.

E.g. in pinion-rack lifting systems for jack up rigs, where $b = b_2 \approx m_n$ and $b_1 \approx 3\ m_n$, the typical $K_{H\beta} \approx K_{F\beta 2} \approx 1$ and $K_{F\beta 1} \approx 1.3$.

9.2 Measurement of face load factors

Primarily,

$K_{F\beta}$ may be determined by a number of strain gauges distributed over the facewidth. Such strain gauges must be put in exactly the same position relative to the root fillet. Relations in [9.1] apply for conversion to $K_{H\beta}$.

Secondarily,

$K_{H\beta}$ may be evaluated by observed contact patterns on various defined load levels. It is imperative that the various test loads are well defined. Usually, it is also necessary to evaluate the elastic deflections. Some teeth at each 90 degrees shall be painted with a suitable lacquer. Always consider the poorest of the contact patterns.

After having run the gear for a suitable time at test load 1 (the lowest), observe the contact pattern with respect to extension over the facewidth. Evaluate that $K_{H\beta}$ by means of the methods mentioned in this section. Proceed in the same way for the next higher test load etc., until there is a full face contact pattern. From these data, the initial mesh misalignment (i.e. without elastic deflections) can be found by extrapolation, and then also the $K_{H\beta}$ at design load can be found by calculation and extrapolation. See example.

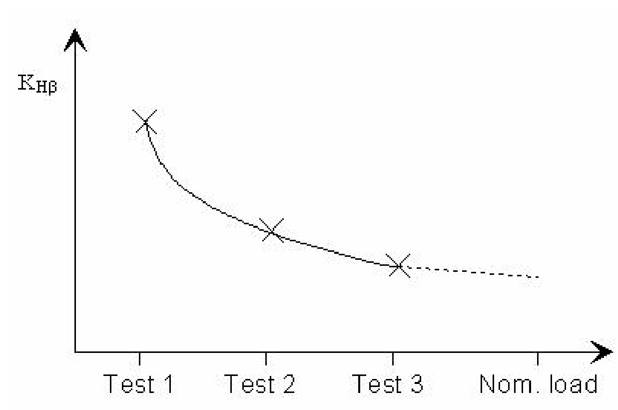


Figure 2 Example of experimental determination of K_{Hb}

It must be considered that inaccurate gears may accumulate a larger observed contact pattern than the actual single mesh to mesh contact patterns. This is particularly important for lapped bevel gears. Ground or hard metal hobbed bevel gears are assumed to present an accumulated contact pattern that is practically equal the actual single mesh to mesh contact patterns. As a rough guidance the (observed) accumulated contact pattern of lapped bevel gears may be reduced by 10% in order to assess the single mesh to mesh contact pattern which is used in [9.9].

9.3 Theoretical determination of $K_{H\beta}$

The methods described in [9.3] to [9.8] may be used for cylindrical gears. The principles may to some extent also be used for bevel gears, but a more practical approach is given in [9.9].

General: For gears where the tooth contact pattern cannot be verified during assembly or under load, all assumptions shall be well on the safe side.

$K_{H\beta}$ shall be determined in the plane of contact.

The influence parameters considered in this method are:

— mean mesh stiffness c_γ (see [11]) (if necessary, also variable stiffness over b)
— mean unit load $F_m/b = F_{bt}\ K_A\ K_\gamma\ K_\nu/b$ (for double helical gears, see [7] for use of K_γ)
— misalignment f_{sh} due to elastic deflections of shafts and gear bodies (both pinion and wheel)
— misalignment f_{defl} due to elastic deflections of and working positions in bearings
— misalignment f_{be} due to bearing clearance tolerances
— misalignment f_{ma} due to manufacturing tolerances
— helix modifications as crowning, end relief, helix correction
— running in amount y_β (see [12]).

In practice several other parameters such as centrifugal expansion, thermal expansion, housing deflection, etc. contribute to $K_{H\beta}$. However, these parameters are not taken into account unless in special cases when being considered as particularly important.

When all or most of the a.m. parameters shall be considered, the most practical way to determine $K_{H\beta}$ is by means of a graphical approach, described in [9.3.1].

If c_γ can be considered constant over the facewidth, and no helix modifications apply, $K_{H\beta}$ can be determined analytically as described in [9.3.2].

9.3.1 Graphical method

The graphical method utilises the superposition principle, and is as follows:

— Calculate the mean mesh deflection δ_M as a function of F_m /b and c_γ, see [11].
— Ddraw a base line with length b, and draw up a rectangular with height δ_M. (The area δ_M b is proportional to the transmitted force).
— Calculate the elastic deflection f_{sh} in the plane of contact. Balance this deflection curve around a zero line, so that the areas above and below this zero line are equal.

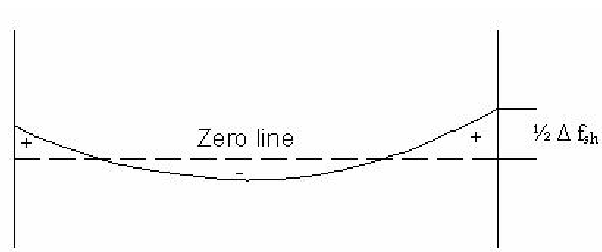


Figure 3 f_{sh} balanced around zero line

— Superimpose these ordinates of the f_{sh} curve to the previous load distribution curve. (The area under this new load distribution curve is still δ_M b.).
— Calculate the bearing deflections and/or working positions in the bearings and evaluate the influence f_{defl} in the plane of contact. This is a straight line and is balanced around a zero line as indicated in Figure 5, but with one distinct direction. Superimpose these ordinates to the previous load distribution curve.
— The amount of crowning, end relief or helix correction (defined in the plane of contact) shall be balanced around a zero line similarly to f_{sh}.

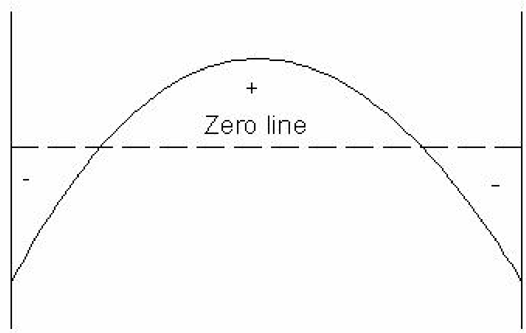


Figure 4 Crowning C_c balanced around zero line

— Superimpose these ordinates to the previous load distribution curve. In case of high crowning etc. as e.g. often applied to bevel gears, the new load distribution curve may cross the base line (the real zero line). The result is areas with negative load that is not real, as the load in those areas should be zero. Thus corrective actions must be made, but for practical reasons it may be postponed to after next operation.
— The amount of initial mesh misalignment, $f_{ma} + f_{be}$ (defined in the plane of contact), shall be balanced around a zero line. If the direction of $f_{ma} + f_{be}$ is known (due to initial contact check), or if the direction of f_{be} is known due to design (e.g. overhang bevel pinion), this should be taken into account. If direction unknown, the influence of $f_{ma} + f_{be}$ in both directions as well as equal zero, should be considered.

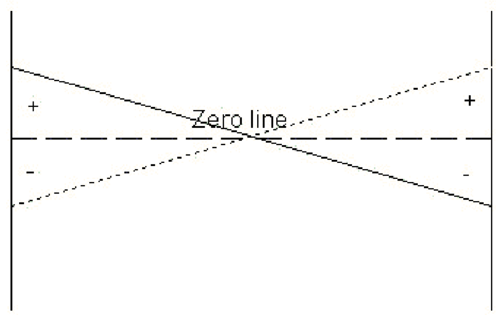


Figure 5 f_{ma}+f_{be} in both directions, balanced around zero line.

Superimpose these ordinates to the previous load distribution curve. This results in up to 3 different curves, of which the one with the highest peak shall be chosen for further evaluation.

— If the chosen load distribution curve crosses the base line (i.e. mathematically negative load), the curve shall be corrected by adding the negative areas and dividing this with the active facewidth. The (constant) ordinates of this rectangular correction area shall be subtracted from the positive part of the load distribution curve. It is advisable to check that the area covered under this new load distribution curve is still equal δ_M b.
— If c_γ cannot be considered as constant over b, then correct the ordinates of the load distribution curve with the local (on various positions over the facewidth) ratio between local mesh stiffness and average mesh stiffness c_γ (average over the active facewidth only). Note that the result shall be a curve that covers the same area δ_M b as before.
— The influence of running in y_β shall be determined as in [12] whereby the value for $F_{\beta x}$ shall be taken as twice the distance between the peak of the load distribution curve and δ_M.
— Determine

$$K_{H\beta} = \frac{peak\ of\ curve - y_\beta}{\delta_M}$$

9.3.2 Simplified analytical method for cylindrical gears

The analytical approach is similar to [9.3.1] but has a more limited application as c_γ is assumed constant over the facewidth and no helix modification applies.

— Calculate the elastic deflection f_{sh} in the plane of contact. Balance this deflection curve around a zero line, so that the area above and below this zero line are equal, see Figure 3. The max. positive ordinate is ½Δf_{sh}.

— Calculate the initial mesh alignment as

$$F_{\beta x} = \left|\Delta f_{sh} \pm f_{ma} \pm f_{be} \pm f_{defl}\right|$$

The negative signs may only be used if this is justified and/or verified by a contact pattern test. Otherwise, always use positive signs. If a negative sign is justified, the value of $F_{\beta x}$ is not to be taken less than the largest of each of these elements.

— Calculate the effective mesh misalignment as $F_{\beta y} = F_{\beta x} - y_\beta$ (y_β see [12])

— Determine

$$K_{H\beta} = 1 + \frac{c_\gamma\, F_{\beta\gamma}\, b}{2\, F_m} \quad \text{for } K_{H\beta} \leq 2$$

or

$$K_{H\beta} = \sqrt{\frac{2\ c_\gamma\, F_{\beta\gamma}\, b}{F_m}} \quad \text{for } K_{H\beta} > 2$$

where c_γ as used here is the effective mesh stiffness, see [11].

9.4 Determination of f_{sh}

f_{sh} is the mesh misalignment due to elastic deflections. Usually it is sufficient to consider the combined mesh deflection of the pinion body and shaft and the wheel shaft. The calculation shall be made in the plane of contact (of the considered gear mesh), and to consider all forces (incl. axial) acting on the shafts. Forces from other meshes can be parted into components parallel respectively vertical to the considered plane of contact. Forces vertical to this plane of contact have no influence on f_{sh}.

It is advised to use following diameters for toothed elements:

$d + 2 \cdot m_n$	for bending and shear deflection
$d + 2\ m_n\ (x - h_{a0} + 0.2)$	for torsional deflection

Usually, f_{sh} is calculated on basis of an evenly distributed load. If the analysis of $K_{H\beta}$ shows a considerable maldistribution in term of hard end contact, or if it is known by other reasons that there exists a hard end contact, the load should be correspondingly distributed when calculating f_{sh}. In fact, the whole $K_{H\beta}$ procedure can be used iteratively. 2 to 3 iterations will be enough, even for almost triangular load distributions.

9.5 Determination of f_{defl}

f_{defl} is the mesh misalignment in the plane of contact due to bearing deflections and working positions (housing deflection may be included if determined).

First the journal working positions in the bearings shall be determined. The influence of external moments and forces must be considered. This is of special importance for twin pinion single output gears with all 3 shafts in one plane.

For rolling bearings f_{defl} is further determined on basis of the elastic deflection of the bearings. An elastic bearing deflection depends on the bearing load and size and number of rolling elements. Note that the bearing clearance tolerances are not included here.

For fluid film bearings f_{defl} is further determined on basis of the lift and angular shift of the shafts due to lubrication oil film thickness. Note that f_{be} takes into account the influence of the bearing clearance tolerance.

When working positions, bearing deflections and oil film lift are combined for all bearings, the angular misalignment as projected into the plane of the contact shall be determined. f_{defl} is this angular misalignment (radians) times the face-width.

9.6 Determination of f_{be}

f_{be} is the mesh misalignment in the plane of contact due to tolerances in bearing clearances. In principle f_{be} and f_{defl} could be combined. But as f_{defl} can be determined by analysis and has a distinct direction, and f_{be} is dependent on tolerances and in most cases has no distinct direction (i.e. ± tolerance), it is practicable to separate these two influences.

Due to different bearing clearance tolerances in both pinion and wheel shafts the two shaft axis will have an angular misalignment in the plane of contact that is superimposed to the working positions determined in [9.5]. f_{be} is the facewidth times this angular misalignment. Note that f_{be} may have a distinct direction or be given as a ± tolerance, or a combination of both. For combination of ± tolerance it is adviced to use

$$f_{be} = \pm\sqrt{f_{be_1}^{2} + f_{be_2}^{2} + \ldots\ldots}$$

f_{be} is particularly important for overhang designs, for gears with widely different kinds of bearings on each side, and when the bearings have wide tolerances on clearances. In general it shall be possible to replace standard bearings without causing the real load distribution to exceed the design premises. For slow speed gears with journal bearings, the expected wear should also be considered.

9.7 Determination of f_{ma}

f_{ma} is the mesh misalignment due to manufacturing tolerances (helix slope deviation) of pinion $f_{H\beta1}$, wheel $f_{H\beta2}$ and housing bore.

For gear without specifically approved requirements to assembly control, the value of f_{ma} shall be determined as

$$f_{ma} = \sqrt{f_{H\beta1}^{2} + f_{H\beta2}^{2}}$$

For gears with specially approved assembly control, the value of f_{ma} will depend on those specific requirements.

9.8 Comments to various gear types

For double helical gears, $K_{H\beta}$ shall be determined for both helices. Usually an even load share between the helices can be assumed. If not, the calculation shall be made as described in [7.1].

For planetary gears the free floating sun pinion suffers only twist, no bending. It must be noted that the total twist is the sum of the twist due to each mesh. If the value of $K_\gamma \neq 1$, this must be taken into account when calculating the total sun pinion twist (i.e. twist calculated with the force per mesh without K_γ, and multiplied with the number of planets).

When planets are mounted on spherical bearings, the mesh misalignments sun-planet respectively planet-annulus will be balanced. I.e. the misalignment will be the average between the two theoretical individual misalignments. The faceload distribution on the flanks of the planets can take full advantage of this. However, as the sun and annulus mesh with several planets with possibly different lead errors, the sun and annulus cannot obtain the above mentioned advantage to the full extent.

9.9 Determination of $K_{H\beta}$ for bevel gears

If a theoretical approach similar to [9.3] to [9.8] is not documented, the following may be used.

$$K_{H\beta} = 1.85 \cdot \left(1.85 - \frac{b_{eff}}{b}\right) \cdot K_{test}$$

b_{eff} / b represents the relative active facewidth (regarding lapped gears, see [9.2] last part).

Higher values than $b_{eff} / b = 0.90$ are normally not to be used in the formula.

For dual directional gears it may be difficult to obtain a high b_{eff} / b in both directions. In that case the smaller b_{eff} / b shall be used.

K_{test} represents the influence of the bearing arrangement, shaft stiffness, bearing stiffness, housing stiffness etc. on the faceload distribution and the verification thereof. Expected variations in length- and height-wise tooth profile is also accounted for to some extent.

a) $K_{test} = 1$ For ground or hard metal hobbed gears with the specified contact pattern verified at full rating or at full torque slow turning at a condition representative for the thermal expansion at normal operation.

   It also applies when the bearing arrangement/support has insignificant elastic deflections and thermal axial expansion. However, each initial mesh contact must be verified to be within acceptance criteria that are calibrated against a type test at full load. Reproduction of the gear tooth length- and height-wise profile must also be verified. This can be made through 3D measurements or by initial contact movements caused by defined axial offsets of the pinion (tolerances to be agreed upon).

b) $K_{test} = 1 + 0.4 \cdot (b_{eff}/b - 0.6)$ For designs with possible influence of thermal expansion in the axial direction of the pinion. The initial mesh contact verified with low load or spin test where the acceptance criteria are calibrated against a type test at full load.

c) $K_{test} = 1.2$ if mesh is only checked by toolmaker's blue or by spin test contact. For gears in this category $b_{eff.}/b > 0.85$ is not to be used in the calculation.

10 Transversal load distribution factors, $K_{H\alpha}$ and $K_{F\alpha}$

The transverse load distribution factors, $K_{H\alpha}$ for contact stresses and for scuffing, $K_{F\alpha}$ for tooth root stresses account for the effects of pitch and profile errors on the transversal load distribution between 2 or more pairs of teeth in mesh.

The following relations may be used:

Cylindrical gears

$$K_{F\alpha} = K_{H\alpha} = \frac{\varepsilon_\gamma}{2}\left(0.9 + 0.4\frac{c_\gamma\left(f_{pt} - y_\alpha\right)b}{F_{tH}}\right)$$

valid for $\varepsilon_\gamma \leq 2$

$$K_{F\alpha} = K_{H\alpha} = 0.9 + 0.4\sqrt{\frac{2\left(\varepsilon_\gamma - 1\right)}{\varepsilon_\gamma}}\frac{c_\gamma\left(f_{pt} - y_\alpha\right)b}{F_{tH}}$$

valid for $\varepsilon_\gamma > 2$

where:

F_{tH}	=	$F_t\ K_A\ K_\gamma\ K_v\ K_{H\beta}$
c_γ	=	see [11]
γ_α	=	see [12]
f_{pt}	=	maximum single pitch deviation (µm) of pinion or wheel, or maximum total profile form deviation F_α of pinion or wheel if this is larger than the maximum single pitch deviation.

Guidance note:

In case of adequate equivalent tip relief adapted to the load, half of the above mentioned f_{pt} can be introduced. A tip relief is considered adequate when the average of C_{a1} and C_{a2} is within ±40% of the value of C_{eff} in Sec.4 [3.2]:

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

Limitations of $K_{H\alpha}$ and $K_{F\alpha}$:

If the calculated values for

$K_{F\alpha} = K_{H\alpha} < 1$, use $K_{F\alpha} = K_{H\alpha} = 1.0$

If the calculated value of $K_{H\alpha} > \dfrac{\varepsilon_\gamma}{\varepsilon_\alpha Z_\varepsilon^2}$ use $K_{H\alpha} = \dfrac{\varepsilon_\gamma}{\varepsilon_\alpha Z_\varepsilon^2}$

If the calculated value of $K_{F\alpha} > \dfrac{\varepsilon_\gamma}{\varepsilon_\alpha Y_\varepsilon}$ use $K_{F\alpha} = \dfrac{\varepsilon_\gamma}{\varepsilon_\alpha Y_\varepsilon}$

where:

$$Y_\varepsilon = 0.25 + \frac{0.75}{\varepsilon_{\alpha n}}$$ (for $\varepsilon_{\alpha n}$ see Sec.3 [3.1].c)

Bevel gears

For ground or hard metal hobbed gears, $K_{F\alpha} = K_{H\alpha} = 1$

For lapped gears, $K_{F\alpha} = K_{H\alpha} = 1.1$

11 Tooth stiffness constants, c´ and c_γ

The tooth stiffness is defined as the load which is necessary to deform one or several meshing gear teeth having 1 mm facewidth by an amount of 1 μm, in the plane of contact.

c´ is the maximum stiffness of a single pair of teeth.

c_γ is the mean value of the mesh stiffness in a transverse plane (brief term: mesh stiffness).

Both valid for high unit load. (Unit load = $F_t \cdot K_A \cdot K_\gamma / b$).

Cylindrical gears

The real stiffness is a combination of the progressive Hertzian contact stiffness and the linear tooth bending stiffnesses. For high unit loads the Hertzian stiffness has little importance and can be disregarded. This approach is on the safe side for determination of $K_{H\beta}$ and $K_{H\alpha}$. However, for moderate or low loads K_v may be underestimated due to determination of a too high resonance speed.

The linear approach is described in A.

An optional approach for inclusion of the non-linear stiffness is described in B.

11.1 The linear approach

$$c' = \frac{0.8\cos\beta}{q} C_R C_B$$

and

$$c_\gamma = c'(0.75\varepsilon_\alpha + 0.25)$$

where:

$$C_B = \left[1 + 0.5\left(1.2 - \frac{h_{a01} + h_{a02}}{2}\right)\right]\left[1 - 0.02(20 - \alpha_n)\right]$$

$$q = 0.04723 + \frac{0.15551}{z_{n1}} + \frac{0.25791}{z_{n2}} - 0.00635\,x_1 - \frac{0.11654\,x_1}{z_{n1}} - 0.00193\,x_2 - \frac{0.24188\,x_2}{z_{n2}} + 0.00529\,x_1^2 + 0.00182\,x_2^2$$

(for internal gears, use z_{n2} equal infinite and $x_2 = 0$).

$h_{a0} = h_{fp}$ for all practical purposes.

C_R considers the increased flexibility of the wheel teeth if the wheel is not a solid disc, and may be calculated as:

$$C_R = 1 + \frac{\ln(b_s/b)}{5\,e^{(s_R/5\,m_n)}}$$

where:

b_s	=	thickness of a central web
s_R	=	average thickness of rim (net value from tooth root to inside of rim).

The formula is valid for $b_s / b \geq 0.2$ and $s_R/m_n \geq 1$. Outside this range of validity and if the web is not centrally positioned, C_R has to be specially considered.

Note: C_R is the ratio between the average mesh stiffness over the facewidth and the mesh stiffness of a gear pair of solid discs. The local mesh stiffness in way of the web corresponds to the mesh stiffness with $C_R = 1$. The local mesh stiffness where there is no web support will be less than calculated with C_R above. Thus, e.g. a centrally positioned web will have an effect corresponding to a longitudinal crowning of the teeth. See also [9.3.1] regarding $K_{H\beta}$.

11.2 The non-linear approach

In the following an example is given on how to consider the non-linearity.

The relation between unit load F/b as a function of mesh deflection δ is assumed to be a progressive curve up to 500 N/mm and from there on a straight line. This straight line when extended to the baseline is assumed to intersect at 10µm.

The relation between unit load F/b as a function of mesh deflection δ is assumed to be a progressive curve up to 500 N/mm and from there on a straight line. This straight line when extended to the baseline is assumed to intersect at 10µm.

With these assumptions the unit force F/b as a function of mesh deflection δ can be expressed as:

$$\frac{F}{b} = K(\delta - 10) \quad \text{for} \quad \frac{F}{b} > 500$$

$$\frac{F}{b} = K\left(\delta - 10\sqrt{\frac{F/b}{500}}\right) \quad \text{for} \quad \frac{F}{b} < 500$$

with $\frac{F}{b} = \frac{F_t}{b} \cdot K_A \cdot K_\gamma$ etc. (N/mm), i.e. unit load incorporating the relevant factors as:

$K_A \cdot K_\gamma$ for determination of K_v.
$K_A \cdot K_\gamma \cdot K_v$ for determination of $K_{H\beta}$.
$K_A \cdot K_\gamma \cdot K_v \cdot K_{H\beta}$ for determination of $K_{H\alpha}$.

δ = mesh deflection (μm)
K = applicable stiffness (c' or c_γ)

Use of stiffnesses for K_V, $K_{H\beta}$ and $K_{H\alpha}$
For calculation of K_v and $K_{H\alpha}$ the stiffness is calculated as follows:

When F/b < 500, the stiffness is determined as $\frac{\Delta F/b}{\Delta\delta}$

where the increment is chosen as e.g. Δ F/b = 10 and thus

$$\Delta\delta = \frac{F/b + 10}{K} + 10\sqrt{\frac{F/b + 10}{500}}$$

When F / b > 500, the stiffness is c' or c_γ.

For calculation of $K_{H\beta}$ the mesh deflection δ is used directly, or an equivalent stiffness determined as $\frac{F}{b \cdot \delta}$.

Bevel gears
In lack of more detailed relationship between stiffness and geometry the following may be used.

$$c' = 13\frac{b_{eff}}{0.85b} \qquad c_\gamma = 16\frac{b_{eff}}{0.85b}$$

b_{eff} not to be used in excess of 0.85 b in these formulae.
Bevel gears with heightwise and lengthwise crowning have progressive mesh stiffness. The values mentioned above are only valid for high loads. They should not be used for determination of C_{eff} (see Sec.4 [3.2]) or $K_{H\beta}$ (see [9.3.1]).

12 Running-in allowances

The running-in allowances account for the influence of running-in wear on the various error elements.
y_α respectively y_β are the running-in amounts which reduce the influence of pitch and profile errors, respectively influence of localised faceload.
C_{ay} is defined as the running-in amount that compensates for lack of tip relief.
The following relations may be used:
For not surface hardened steel

$$y_\alpha = \frac{160}{\sigma_{Hlim}} f_{pt}$$

$$y_\beta = \frac{320}{\sigma_{H\,lim}} f_{\beta x}$$

with the following upper limits:

V	≤ 5 m/s	5-10 m/s	> 10 m/s
$y_{\alpha\ max}$	none	$\frac{12800}{\sigma_{Hlim}}$	$\frac{6400}{\sigma_{Hlim}}$
$y_{\beta\ max}$	none	$\frac{25600}{\sigma_{Hlim}}$	$\frac{12800}{\sigma_{Hlim}}$

For surface hardened steel

y_α = 0.075 f_{pt} but not more than 3 for any speed

y_β = 0.15 $F_{\beta x}$ but not more than 6 for any speed

For all kinds of steel

$$C_{ay} = \frac{1}{18}\left(\frac{\sigma_{H\,lim}}{97} - 18.45\right)^2 + 1.5$$

When pinion and wheel material differ, the following applies:

- Use the larger of f_{pt1} - $y_{\alpha 1}$ and f_{pt2} - $y_{\alpha 2}$ to replace f_{pt} - y_α in the calculation of $K_{H\alpha}$ see [10] and K_v see [8].
- Use $y_\beta = \frac{1}{2}\left(y_{\beta 1} + y_{\beta 2}\right)$ in the calculation of $K_{H\beta}$ see [9].
- Use $C_a = \frac{1}{2}\left(C_{ay\,1} + C_{ay\,2}\right)$ in the calculation of K_v see [8].
- Use $C_{a1} = C_{a2} = \frac{1}{2}\left(C_{ay1} + C_{ay\,2}\right)$ in the scuffing calculation see Sec.4 if no design tip relief is foreseen.

SECTION 2 CALCULATION OF SURFACE DURABILITY

1 Scope and general remarks

Part 2 includes the calculations of flank surface durability as limited by pitting, spalling, case crushing and subsurface yielding. Endurance and time limited flank surface fatigue is calculated by means of [2] to [12]. In a way also tooth fractures starting from the flank due to subsurface fatigue is included through the criteria in [13].

Pitting itself is not considered as a critical damage for slow speed gears. However, pits can create a severe notch effect that may result in tooth breakage. This is particularly important for surface hardened teeth, but also for high strength through hardened teeth. For high-speed gears, pitting is not permitted.

Spalling and case crushing are considered similar to pitting, but may have a more severe effect on tooth breakage due to the larger material breakouts, initiated below the surface. Subsurface fatigue is considered in [13].

For jacking gears (self-elevating offshore units) or similar slow speed gears designed for very limited life, the max. static (or very slow running) surface load for surface hardened flanks is limited by the subsurface yield strength.

For case hardened gears operating with relatively thin lubrication oil films, grey staining (micropitting) may be the limiting criterion for the gear rating. Specific calculation methods for this purpose are not given here, but are under consideration for future revisions. Thus depending on experience with similar gear designs, limitations on surface durability rating other than those according to [2] to [13] may be applied.

2 Basic equations

Calculation of surface durability (pitting) for spur gears is based on the contact stress at the inner point of single pair contact or the contact at the pitch point, whichever is greater.

Calculation of surface durability for helical gears is based on the contact stress at the pitch point.

For helical gears with $0 < \varepsilon_\beta < 1$, a linear interpolation between the above mentioned applies.

Calculation of surface durability for spiral bevel gears is based on the contact stress at the midpoint of the zone of contact.

Alternatively for bevel gears the contact stress may be calculated with the program “BECAL”. In that case, K_A and K_v shall be included in the applied tooth force, but not $K_{H\beta}$ and $K_{H\alpha}$. The calculated (real) Hertzian stresses shall be multiplied with Z_K in order to be comparable with the permissible contact stresses.

The contact stresses calculated with the method in part 2 are based on the Hertzian theory, but do not always represent the real Hertzian stresses.

The corresponding permissible contact stresses σ_{HP} shall be calculated for both pinion and wheel.

2.1 Contact stress

Cylindrical gears

$$\sigma_H = Z_{B,D} Z_H Z_E Z_\varepsilon Z_\beta \sqrt{\frac{F_t(u+1)}{d_1 bu} K_A K_\gamma K_v K_{H\beta} K_{H\alpha}}$$

where:

$Z_{B,D}$ = Zone factor for inner point of single pair contact for pinion resp. wheel (see [3.2]).

Z_H = Zone factor for pitch point (see [3.1]).

Z_E = Elasticity factor (see [4]).

Z_ε = Contact ratio factor (see [5]).

Z_β = Helix angle factor (see [6]).

F_t, K_A , K_γ , K_v , $K_{H\beta}$, $K_{H\alpha}$, see Sec.1 [5] to Sec.1 [10].

d_1, b, u, see Sec.1 [2] to Sec.1 [5].

Bevel gears

$$\sigma_H = 1.05 \cdot Z_M Z_E Z_K \sqrt{\frac{F_{mt}(u_v + 1)}{d_{v1} b u_v} K_A K_\gamma K_v K_{H\beta} K_{H\alpha}}$$

where:

1.05 is a correlation factor to reach real Hertzian stresses (when $Z_K = 1$)

Z_E, K_A etc. see above.

Z_M mid-zone factor, see [3.3].
Z_K bevel gear factor, see [7].

F_{mt}, d_{v1}, u_v, see Sec.1 [2] – Sec.1 [5].

It is assumed that the heightwise crowning is chosen so as to result in the maximum contact stresses at or near the mid-point of the flanks.

2.2 Permissible contact stress

$$\sigma_{HP} = \frac{\sigma_{Hlim} Z_N}{S_H} Z_L Z_v Z_R Z_W Z_X$$

where:

$\sigma_{H\,lim}$ = Endurance limit for contact stresses (see [8]).

Z_N = Life factor for contact stresses (see [9]).

S_H = Required safety factor according to the rules.

Z_L,Z_v,Z_R = Oil film influence factors (see [10]).

Z_W = Work hardening factor (see [11]).

Z_X = Size factor (see [12]).

3 Zone factors Z_H, $Z_{B/D}$ and Z_M

3.1 Zone factor Z_H

The zone factor, Z_H, accounts for the influence on contact stresses of the tooth flank curvature at the pitch point and converts the tangential force at the reference cylinder to the normal force at the pitch cylinder.

$$Z_H = \sqrt{\frac{2\cos\beta_b \cos\alpha_{wt}}{\cos^2\alpha_t \sin\alpha_{wt}}}$$

3.2 Zone factors $Z_{B,D}$

The zone factors, $Z_{B,D}$, account for the influence on contact stresses of the tooth flank curvature at the inner point of single pair contact in relation to Z_H. Index B refers to pinion D to wheel.

For $\varepsilon_\beta \geq 1$, $Z_{B,D} = 1$

For internal gears, $Z_D = 1$

For $\varepsilon_\beta = 0$ (spur gears)

$$Z_B = \frac{\tan\alpha_{wt}}{\sqrt{\left[\sqrt{\left(\frac{d_{a1}}{d_{b1}}\right)^2 - 1} - \frac{2\pi}{z_1}\right]\left[\sqrt{\left(\frac{d_{a2}}{d_{b2}}\right)^2 - 1} - (\varepsilon_\alpha - 1)\frac{2\pi}{z_2}\right]}}$$

$$Z_D = \frac{\tan\alpha_{wt}}{\sqrt{\left[\sqrt{\left(\frac{d_{a2}}{d_{b2}}\right)^2 - 1} - \frac{2\pi}{z_2}\right]\left[\sqrt{\left(\frac{d_{a1}}{d_{b1}}\right)^2 - 1} - (\varepsilon_\alpha - 1)\frac{2\pi}{z_1}\right]}}$$

If $Z_B < 1$, use $Z_B = 1$

If $Z_D < 1$, use $Z_D = 1$

For $0 < \varepsilon_\beta < 1$

$Z_{B,D} = Z_{B,D}$ (for spur gears) – ε_β ($Z_{B,D}$ (for spur gears) – 1)

3.3 Zone factor Z_M

The mid-zone factor Z_M accounts for the influence of the contact stress at the mid point of the flank and applies to spiral bevel gears.

$$Z_M = \sqrt{\frac{2\cos\beta_{bm}\tan\alpha_{vt} d_{v1} d_{v2}}{\left(\sqrt{d_{va1}^2 - d_{vb1}^2} - \varepsilon_\alpha p_{btm}\right)\left(\sqrt{d_{va2}^2 - d_{vb2}^2} - \varepsilon_\alpha p_{btm}\right)}}$$

This factor is the product of Z_H and Z_{M-B} in ISO 10300 with the condition that the heightwise crowning is sufficient to move the peak load towards the midpoint.

3.4 Inner contact point

For cylindrical or bevel gears with very low number of teeth the inner contact point (A) may be close to the base circle. In order to avoid a wear edge near A, it is required to have suitable tip relief on the wheel.

4 Elasticity factor, Z_E

The elasticity factor, Z_E, accounts for the influence of the material properties as modulus of elasticity and Poisson's ratio on the contact stresses.

For steel against steel Z_E = 189.8

5 Contact ratio factor, Z_ε

The contact ratio factor Z_ε accounts for the influence of the transverse contact ratio ε_α and the overlap ratio ε_β on the contact stresses.

$$Z_\varepsilon = \sqrt{\frac{1}{\varepsilon_\alpha}} \quad \text{for } \varepsilon_\beta \geq 1$$

$$Z_\varepsilon = \sqrt{\frac{4-\varepsilon_\alpha}{3}\left(1-\varepsilon_\beta\right)+\frac{\varepsilon_\beta}{\varepsilon_\alpha}} \quad \text{for } \varepsilon_\beta < 1$$

6 Helix angle factor, Z_β

The helix angle factor, Z_β, accounts for the influence of helix angle (independent of its influence on Z_ε) on the surface durability.

$$Z_\beta = \frac{1}{\sqrt{\cos\beta}}$$

7 Bevel gear factor, Z_K

The bevel gear factor accounts for the difference between the real Hertzian stresses in spiral bevel gears and the contact stresses assumed responsible for surface fatigue (pitting). Z_K adjusts the contact stresses in such a way that the same permissible stresses as for cylindrical gears may apply.

The following may be used: Z_K = 0.80

8 Values of endurance limit, σ_{Hlim} and static strength, σ_{H10^5}, σ_{H10^3}

σ_{Hlim} is the limit of contact stress that may be sustained for $5 \cdot 10^7$ cycles, without the occurrence of progressive pitting.

For most materials $5 \cdot 10^7$ cycles are considered to be the beginning of the endurance strength range or lower knee of the σ-N curve. (See also Life Factor Z_N). However, for nitrided steels $2 \cdot 10^6$ apply.

For this purpose, pitting is defined by

- — for not surface hardened gears: pitted area ≥ 2% of total active flank area.
- — for surface hardened gears: pitted area ≥ 0.5% of total active flank area, or ≥ 4% of one particular tooth flank area.

σ_{H10^5} and σ_{H10^3} and are the contact stresses which the given material can withstand for 10^5 respectively 10^3 cycles without subsurface yielding or flank damages as pitting, spalling or case crushing when adequate case depth applies.

The listed values in Table 1 for σ_{Hlim}, σ_{H10^5} and σ_{H10^3} may only be used for materials subjected to a quality control as the one referred to in the rules. Results of approved fatigue tests may also be used as the basis for establishing these values. The defined survival probability is 99%.

Table 1 Endurance limt and static strenght

Material	σ_{Hlim}	σ_{H10^5}	σ_{H10^3}
Alloyed case hardened steels (surface hardness 58-63 HRC) — of specially approved high grade [1)] — of normal grade	 1650 1500	 2500 2400	 3100 3100
Nitrided steel of approved grade, gas nitrided (surface hardness 700 to 800 HV)	1250	1.3 σ_{Hlim}	1.3 σ_{Hlim}
Alloyed quenched and tempered steel, bath or gas nitrided (surface hardness 500 to 700 HV)	1000	1.3 σ_{Hlim}	1.3 σ_{Hlim}
Alloyed, flame or induction hardened steel (surface hardness 500 to 650 HV)	0.75 HV + 750	1.6 σ_{Hlim}	4.5 HV
Alloyed quenched and tempered steel	1.4 HV + 350	1.6 σ_{Hlim}	4.5 HV
Carbon steel	1.5 HV + 250	1.6 σ_{Hlim}	1.6 σ_{Hlim}
These values refer to forged or hot rolled steel. For cast steel the values for σ_{Hlim} shall be reduced by 15%. [1)] Clean steel forgings, see DNVGL-RU-SHIP Pt.2 Ch.2 Sec.6 [5] and DNVGL-CP-0247 Sec.3 [2.13].			

9 Life factor, Z_N

The life factor, Z_N, takes account of a higher permissible contact stress if only limited life (number of cycles, N_L) is demanded or lower permissible contact stress if very high number of cycles apply.

If this is not documented by approved fatigue tests, the following method may be used:

For all steels except nitrided:

$N_L \geq 5 \cdot 10^7$: $Z_N = 1$ or $Z_N = \left(\frac{5 \cdot 10^7}{N_L}\right)^{0.0157}$

I.e. $Z_N = 0.92$ for 10^{10} cycles.

The $Z_N = 1$ from $5 \cdot 10^7$ on, may only be used when the material cleanliness is of approved high grade (see DNVGL-RU-SHIP Pt.4 Ch.2 Sec.2) and the lubrication is optimised by a specially approved filtering process.

$10^5 < N_L < 5 \cdot 10^7$: $$Z_N = \left(\frac{5 \cdot 10^7}{N_L}\right)^{0.37 \cdot \log Z_{N10^5}}$$

$N_L = 10^5$: $$Z_N = Z_{N10^5} = \frac{\sigma_{H10^5} \cdot Z_{X10^5} \cdot Z_{Wst}}{\sigma_{H\lim} \cdot Z_L \cdot Z_V \cdot Z_R \cdot Z_X \cdot Z_W}$$

$10^3 < N_L < 10^5$: $$Z_N = Z_{N10^5} \cdot \left(\frac{10^5}{N_L}\right)^{0.5 \cdot \log\left(\frac{Z_{N10^3}}{Z_{N10^5}}\right)}$$

$N_L \leq 10^3$ $$Z_N = Z_{N10^3} = \frac{\sigma_{H10^3} \cdot Z_{X10^3} \cdot Z_{Wst}}{\sigma_{H\lim} \cdot Z_L \cdot Z_V \cdot Z_R \cdot Z_X \cdot Z_W}$$

(but not less than Z_{N10^5})

For nitrided steels:

$N_L \geq 2 \cdot 10^6$: $Z_N = 1$ or $Z_N = \left(\frac{2 \cdot 10^6}{N_L}\right)^{0,0098}$

That is, $Z_N = 0.92$ for 10^{10} cycles.

The $Z_N = 1$ from $2 \cdot 10^6$ on, may only be used when the material cleanliness is of approved high grade (see DNVGL-RU-SHIP Pt.4 Ch.2) and the lubrication is optimised by a specially approved filtering process.

$10^5 < N_L < 2 \cdot 10^6$ $$Z_N = \left(\frac{2 \cdot 10^6}{N_L}\right)^{0.7686 \log Z_{N10^5}}$$

$N_L \leq 10^5$ $$Z_N = Z_{N10^5} = \frac{1.3 Z_{Wst} Z_{X10^5}}{Z_L Z_V Z_R Z_W Z_X}$$

Guidance note:

When no index indicating number of cycles is used, the factors are valid for $5 \cdot 10^7$ (respectively $2 \cdot 10^6$ for nitriding) cycles.

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

10 Influence factors on lubrication film, Z_L, Z_V and Z_R

The lubricant factor, Z_L, accounts for the influence of the type of lubricant and its viscosity, the speed factor, Z_V, accounts for the influence of the pitch line velocity and the roughness factor, Z_R, accounts for influence of the surface roughness on the surface endurance capacity.

The following methods may be applied in connection with the endurance limit:

	Surface hardened steels	*Not surface hardened steels*
Z_L	$0.91 + \frac{0.36}{(1.2 + 134/\nu_{40})^2}$	$0.83 + \frac{0.68}{(1.2 + 134/\nu_{40})^2}$
Z_V	$0.93 + \frac{0.14}{\sqrt{0.8 + (32/v)}}$	$0.85 + \frac{0.30}{\sqrt{0.8 + (32/v)}}$
Z_R	$\left(\frac{3}{R_{Zrel}}\right)^{0.08}$	$\left(\frac{3}{R_{Zrel}}\right)^{0.15}$

where:

ν_{40} = Kinematic oil viscosity at 40ºC (mm²/s). For case hardened steels the influence of a high bulk temperature (see 4. Scuffing) should be considered. For example, bulk temperatures in excess of 120ºC for long periods may cause reduced flank surface endurance limits.
For values of $\nu_{40} > 500$, use $\nu_{40} = 500$.

R_{Zrel} = The mean roughness between pinion and wheel (after running in) relative to an equivalent radius of curvature at the pitch point ρ_c = 10mm.

$$R_{Zrel} = 0.5\,(R_{Z1} + R_{Z2})\left(\frac{10}{\rho_c}\right)^{\frac{1}{3}}$$

R_Z = Mean peak to valley roughness (μm) (DIN definition) (roughly $R_Z = 6\,R_a$)

For $N_L \leq 10^5$: $Z_L\ Z_V\ Z_R = 1.0$

11 Work hardening factor, Z_W

The work hardening factor, Z_W, accounts for the increase of surface durability of a soft steel gear when meshing the soft steel gear with a surface hardened or substantially harder gear with a smooth surface.

The following approximation may be used for the endurance limit:

Surface hardened steel against not surface hardened steel:

$$Z_W = \left[1.2 - \frac{HB - 130}{1700}\right]\left[\frac{3}{R_{Zeq}}\right]^{0.15}$$

where:

HB = the Brinell hardness of the soft member For HB > 470, use HB = 470 For HB < 130, use HB = 130

R_{Zeq} = equivalent roughness

$$R_{Zeq} = R_{ZH}\left(\frac{R_{ZH}}{R_{ZS}}\right)^{0.66}\left(\frac{15000}{\nu_{40}\ v\ \rho_c}\right)^{0.33}$$

If $R_{Zeq} > 16$, then use $R_{Zeq} = 16$

If $R_{Zeq} < 1.5$, then use $R_{Zeq} = 1.5$

where:

R_{ZH}	=	surface roughness of the hard member before run in.
R_{ZS}	=	surface roughness of the soft member before run in
ν_{40}	=	see [10].

If values of $Z_W < 1$ are evaluated, $Z_W = 1$ should be used for flank endurance. However, the low value for Z_W may indicate a potential wear problem.

Through hardened pinion against softer wheel:

$$Z_W = 1 + (u-1)\cdot\left(0.00898\frac{HB_1}{HB_2} - 0.00829\right)$$

For $\frac{HB_1}{HB_2} \le 1.2$ use $Z_W = 1$

For $\frac{HB_1}{HB_2} > 1.7$ use $\frac{HB_1}{HB_2} = 1.7$

For $u > 20$, use $u = 20$

For static strength ($< 10^5$ cycles):

Surface hardened against not surface hardened

$Z_{Wst} = 1.05$

Through hardened pinion against softer wheel

$Z_{Wst} = 1$

12 Size factor, Z_X

The size factor accounts for statistics indicating that the stress levels at which fatigue damage occurs decrease with an increase of component size, as a consequence of the influence on subsurface defects combined with small stress gradients, and of the influence of size on material quality.

Z_X may be taken unity provided that subsurface fatigue for surface hardened pinions and wheels is considered, e.g. as in the following subsection [13].

13 Subsurface fatigue

This is only applicable to surface hardened pinions and wheels. The main objective is to have a subsurface safety against fatigue (endurance limit) or deformation (static strength) which is at least as high as the safety S_H required for the surface. The following method may be used as an approximation unless otherwise documented.

The high cycle fatigue (>$3 \cdot 10^6$ cycles) is assumed to mainly depend on the orthogonal shear stresses. Static strength (<10^3 cycles) is assumed to depend mainly on equivalent stresses (von Mises). Both are influenced by residual stresses, but this is only considered roughly and empirically.

The subsurface working stresses at depths inside the peak of the orthogonal shear stresses respectively the equivalent stresses are only dependent on the (real) Hertzian stresses. Surface related conditions as expressed by Z_L, Z_V and Z_R are assumed to have a negligible influence.

The real Hertzian stresses σ_{HR} are determined as:

For helical gears with $\varepsilon_\beta \geq 1$:

$$\sigma_{HR} = \sigma_H$$

For helical gears with $\varepsilon_\beta < 1$ and spur gears:

$$\sigma_{HR} = \sigma_H \cdot \frac{\sqrt{1 - \varepsilon_\beta + \frac{\varepsilon_\beta}{\varepsilon_\alpha}}}{Z_\varepsilon}$$

For bevel gears:

$$\sigma_{HR} = \sigma_H \cdot \frac{1}{Z_K}$$

The necessary hardness HV is given as a function of the net depth t_z (net = after grinding or hard metal hobbing, and perpendicular to the flank).

The coordinates t_z and HV shall be compared with the design specification, such as:

— for flame and induction hardening; t_{HVmin}, HV_{min}
— for nitriding; t_{400min}, HV = 400
— for case hardening; t_{550min}, HV = 550; t_{400min}, HV = 400 and t_{300min}, HV = 300 (the latter only if the core hardness < 300. If the core hardness > 400, the t_{400} shall be replaced by a fictive $t_{400} = 1.6\ t_{550}$).

In addition the specified surface hardness is not to be less than the max necessary hardness (at $t_z = 0.5a_H$). This applies to all hardening methods.

For high cycle fatigue (>$3 \cdot 10^6$ cycles) the following applies:

$$HV = 0.4 \cdot \sigma_{HR} \cdot S_H \cdot \cos\left(\frac{\frac{t_Z}{a_H} - 0.5}{\frac{t_Z}{a_H} + 0.5} \cdot 90° \right)$$

applicable to $\frac{t_Z}{a_H} \geq 0.5$

For $\frac{t_z}{a_H} < 0.5$ the value for $\frac{t_z}{a_H} = 0.5$ applies

$$a_H = 1.2 \cdot \frac{\sigma_{HR} \cdot S_H \cdot \rho_c}{56300}$$

Where a_H is half the hertzian contact width multiplied by an empirical factor of 1.2 that takes into account the possible influence of reduced compressive residual stresses (or even tensile residual stresses) on the local fatigue strength.

If any of the specified hardness depths including the surface hardness is below the curve described by HV = f (t_z), the actual safety factor against subsurface fatigue is determined as follows:

— reduce S_H stepwise in the formula for HV and a_H until all specified hardness depths and surface hardness balance with the corrected curve. The safety factor obtained through this method is the safety against subsurface fatigue.

For static strength (<10^3 cycles) the following applies:

$$HV = 0.19 \cdot \sigma_{HR} \cdot S_H \cdot \cos\left(\frac{\frac{t_z}{a_{Hst}} - 0.6}{\frac{t_z}{a_{Hst}} + 0.7} \cdot 90°\right)$$

applicable to $\frac{t_z}{a_{Hst}} \geq 0.6$

$$a_{Hst} = \frac{\sigma_{HR} \cdot S_H \cdot \rho_c}{56300}$$

In the case of insufficient specified hardness depths, the same procedure for determination of the actual safety factor as above applies.

For limited life fatigue (10^3 < cycles < $3 \cdot 10^6$):

For this purpose it is necessary to extend the correction of safety factors to include also higher values than required. I.e. in the case of more than sufficient hardness and depths, the safety factor in the formulae for both high cycle fatigue and static strength shall be increased until necessary and specified values balance.

The actual safety factor for a given number of cycles N between 10^3 and $3 \cdot 10^6$ is found by linear interpolation in a double logarithmic diagram.

$$\log S_{HN} = \frac{\log S_{H\,3\cdot10^6} - \log S_{H\,10^3}}{3.477} \cdot \log N - \quad 0.8628 \cdot \log S_{H3\cdot10^6} + 1.8628 \cdot \log S_{H10^3}$$

SECTION 3 CALCULATION OF TOOTH STRENGTH

1 Scope and general remarks

This section includes the calculation of tooth root strength as limited by tooth root cracking (surface or subsurface initiated) and yielding.

For rim thickness $s_R \geq 3.5 \cdot m_n$ the strength is calculated by means of [2] to [13]. For cylindrical gears the calculation is based on the assumption that the highest tooth root tensile stress arises by application of the force at the outer point of single tooth pair contact of the virtual spur gears. The method has, however, a few limitations that are mentioned in [6].

For bevel gears the calculation is based on force application at the tooth tip of the virtual cylindrical gear. Subsequently the stress is converted to load application at the mid point of the flank due to the heightwise crowning.

Bevel gears may also be calculated with the program BECAL. In that case, K_A and K_v shall be included in the applied tooth force, but not $K_{F\beta}$ and $K_{F\alpha}$.

In case of a thin annulus or a thin gear rim etc, radial cracking can occur rather than tangential cracking (from root fillet to root fillet). Cracking can also start from the compression fillet rather than the tension fillet. For rim thickness $s_R < 3.5 \cdot m_n$ a special calculation procedure is given in [15] and [16], and a simplified procedure in [14].

A tooth breakage is often the end of the life of a gear transmission. Therefore, a high safety S_F against breakage is required.

It should be noted that this part 3 does not cover fractures caused by:

- oil holes in the tooth root space
- wear steps on the flank
- flank surface distress such as pits, spalls or grey staining.

Especially the latter is known to cause oblique fractures starting from the active flank, predominately in spiral bevel gears, but also sometimes in cylindrical gears.

Specific calculation methods for these purposes are not given here, but are under consideration for future revisions. Thus, depending on experience with similar gear designs, limitations other than those outlined in this section may be applied.

2 Tooth root stresses

The local tooth root stress is defined as the max. principal stress in the tooth root caused by application of the tooth force. That is, the stress ratio R = 0. Other stress ratios such as for e.g. idler gears (R ≈ -1.2), shrunk on gear rims (R > 0), etc. are considered by correcting the permissible stress level.

2.1 Local tooth root stress

The local tooth root stress for pinion and wheel may be assessed by strain gauge measurements or FE calculations or similar. For both measurements and calculations all details shall be agreed in advance.

Normally, the stresses for pinion and wheel are calculated as:

Cylindrical gears:

$$\sigma_F = \frac{F_t}{b\, m_n} Y_F\, Y_S\, Y_\beta\, K_A\, K_\gamma\, K_v\, K_{F\beta}\, K_{F\alpha}$$

where:

Y_F = Tooth form factor (see [3]).
Y_S = Stress correction factor (see [4]).
Y_β = Helix angle factor (see [6]).

F_t, K_A, K_γ, K_v, $K_{F\beta}$, $K_{F\alpha}$, see Sec.1 [5] – Sec.1 [10].

b, see Sec.1 [3].

Bevel gears:

$$\sigma_F = \frac{F_{mt}}{b\, m_{mn}} Y_{Fa}\, Y_{Sa}\, Y_\varepsilon\, K_A\, K_\gamma\, K_v\, K_{F\beta}\, K_{F\alpha}$$

where:

Y_{Fa} = Tooth form factor, see [3].
Y_{Sa} = Stress correction factor, see [4].
Y_ε = Contact ratio factor, see [5].

F_{mt}, K_A, etc., see Sec.1 [5] to Sec.1 [10].

b, see Sec.1 [3].

2.2 Permissible tooth root stress

The permissible local tooth root stress for pinion respectively wheel for a given number of cycles, N, is:

$$\sigma_{FP} = \frac{\sigma_{FE} Y_M Y_N}{S_F} Y_{\delta relT} Y_{RrelT} Y_X Y_C$$

Note that all these factors Y_M etc. are applicable to $3 \cdot 10^6$ cycles when used in this formula for σ_{FP}. The influence of other number of cycles on these factors is covered by the calculation of Y_N.
where:

σ_{FE}	=	Local tooth root bending endurance limit of reference test gear (see [7]).
Y_M	=	Mean stress influence factor which accounts for other loads than constant load direction, e.g. idler gears, temporary change of load direction, prestress due to shrinkage, etc. (see [8]).
Y_N	=	Life factor for tooth root stresses related to reference test gear dimensions (see [9]).
S_F	=	Required safety factor according to the rules.
$Y_{\delta relT}$	=	Relative notch sensitivity factor of the gear to be determined, related to the reference test gear (see [10]).
Y_{RrelT}	=	Relative (root fillet) surface condition factor of the gear to be determined, related to the reference test gear (see [11]).
Y_X	=	Size factor (see [12]).

Y_C = Case depth factor (see [13]).

3 Tooth form factors Y_F, Y_{Fa}

The tooth form factors Y_F and Y_{Fa} take into account the influence of the tooth form on the nominal bending stress.

Y_F applies to load application at the outer point of single tooth pair contact of the virtual spur gear pair and is used for cylindrical gears.

Y_{Fa} applies to load application at the tooth tip and is used for bevel gears.

Both Y_F and Y_{Fa} are based on the distance between the contact points of the 30-tangents at the root fillet of the tooth profile for external gears, respectively 60 tangents for internal gears.

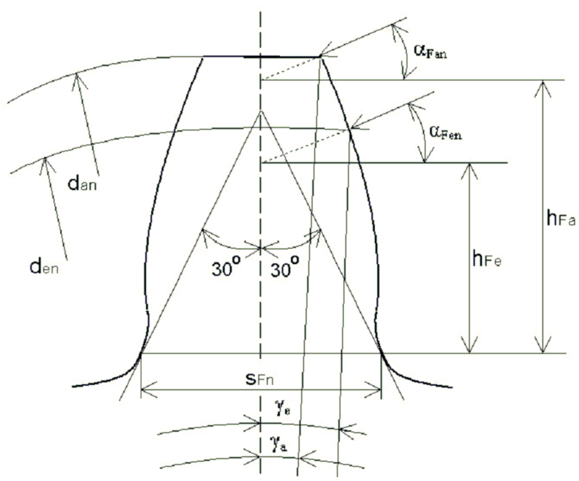


Figure 1 External tooth in normal section

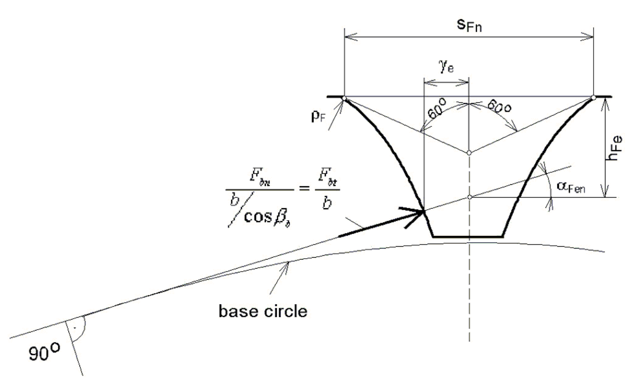


Figure 2 Internal tooth in normal section

Definitions:

$$Y_F = \frac{6 \frac{h_{Fe}}{m_n} \cos \alpha_{Fen}}{\left(\frac{s_{Fn}}{m_n} \right)^2 \cos \alpha_n}$$

$$Y_{Fa} = \frac{6 \frac{h_{Fa}}{m_n} \cos \alpha_{Fan}}{\left(\frac{s_{Fn}}{m_n} \right)^2 \cos \alpha_n}$$

In the case of helical gears, Y_F and Y_{Fa} are determined in the normal section, i.e. for a virtual number of teeth.

Y_{Fa} differs from Y_F by the bending moment arm h_{Fa} and α_{Fan} and can be determined by the same procedure as Y_F with exception of h_{Fe} and α_{Fan}. For h_{Fa} and α_{Fan} all indices $_e$ will change to $_a$ (tip).

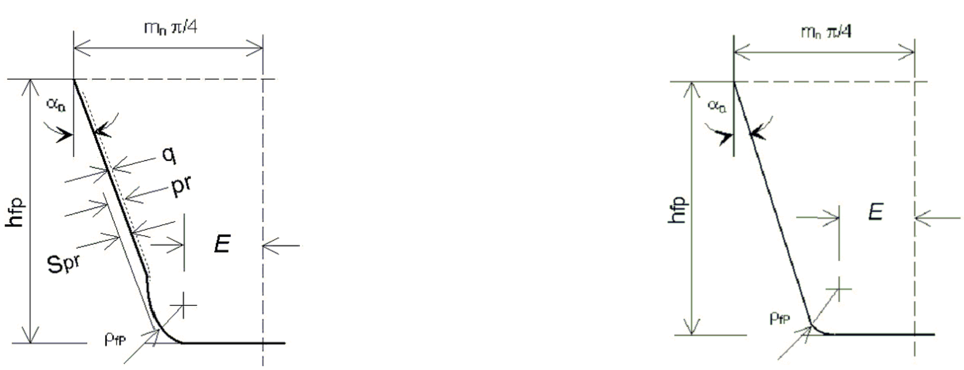
The following formulae apply to cylindrical gears, but may also be used for bevel gears when replacing:

m_n with m_{nm}

z_n with z_{vn}

α_t with α_{vt}

β with β_m



with undercut without undercut

Figure 3 Dimensions and basic rack profile of the teeth (finished profile)

Tool and basic rack data such as h_{fP}, ρ_{fp} and s_{pr} etc. are referred to m_n, i.e. dimensionless.

3.1 Determination of parameters

$$E = \left[\frac{\pi}{4} - h_{fP} \tan \alpha_n - \frac{\rho_{fP}'(1 - \sin \alpha_n) - s_{pr}}{\cos \alpha_n}\right] m_n$$

For external gears $\rho_{fP}' = \rho_{fP}$

For internal gears $\rho_{fP}' = \rho_{fP} + \frac{(x_0 + h_{fP} - \rho_{fP})^{1.95}}{3.156 \cdot 1.036^{z_0}}$

where

z_0 = number of teeth of pinion cutter
x_0 = addendum modification coefficient of pinion cutter
h_{fP} = addendum of pinion cutter
ρ_{fP} = tip radius of pinion cutter.

$$G = \rho_{fp}' - h_{fp} + x$$

$$H = \frac{2}{z_n}\left(\frac{\pi}{2} - \frac{E}{m_n}\right) - \tau$$

with

$\tau = \frac{\pi}{3}$ for external gears

$\tau = \frac{\pi}{6}$ for internal gears

$$\vartheta = 2\frac{G}{z_n} \tan \vartheta - H$$

(to be solved iteratively, suitable start value $\vartheta = \frac{\pi}{6}$

for external gears and $\frac{\pi}{3}$ for internal gears).

a) Tooth root chord s_{Fn}:
For external gears

$$\frac{s_{Fn}}{m_n} = z_n \sin\left(\frac{\pi}{3} - \vartheta\right) + \sqrt{3}\left(\frac{G}{\cos\vartheta} - \rho_{fp}'\right)$$

For bevel gears with a tooth thickness modification:

x_{sm} affects mainly s_{Fn}, but also h_{Fe} and α_{Fen}. The total influence of x_{sm} on Y_{Fa} Y_{sa} can be approximated by only adding 2 x_{sm} to s_{Fn} / m_n.
For internal gears

$$\frac{s_{Fn}}{m_n} = z_n \sin\left(\frac{\pi}{6} - \vartheta\right) + \left(\frac{G}{\cos\vartheta} - \rho_{fP}'\right)$$

b) Root fillet radius ρ_F at 30º tangent:

$$\frac{\rho_F}{m_n} = \rho_{fp}' + \frac{2G^2}{\cos\vartheta\left(z_n \cos^2\vartheta - 2G\right)}$$

c) Determination of bending moment arm h_F:
$d_n = z_n\, m_n$

$$\varepsilon_{\alpha n} = \frac{\varepsilon_\alpha}{\cos^2\beta_b}$$

$d_{an} = d_n + 2\, h_a$

$p_{bn} = \pi\, m_n \cos\alpha_n$

$d_{bn} = d_n \cos\alpha_n$

$$d_{en} = \frac{2z}{|z|}\sqrt{\left(\frac{\sqrt{d_{an}^2 - d_{bn}^2}}{2} - \frac{z}{|z|} p_{bn}\left(\varepsilon_{\alpha n} - 1\right)\right)^2 + \frac{d_{bn}^2}{4}}$$

$$\alpha_{en} = \arccos\frac{d_{bn}}{d_{en}}$$

$$\gamma_e = \frac{1}{z_n}\left(\frac{\pi}{2} + 2\,x\,\tan\alpha_n\right) + \operatorname{inv}\alpha_n - \operatorname{inv}\alpha_{en}$$

$\alpha_{Fen} = \alpha_{en} - \gamma_e$

For external gears

$$\frac{h_{Fe}}{m_n} = \frac{1}{2}\left[\left(\cos\gamma_e - \sin\gamma_e \tan\alpha_{Fen}\right)\frac{d_{en}}{m_n} - z_n \cos\left(\frac{\pi}{3} - \vartheta\right) - \frac{G}{\cos\vartheta} + \rho_{fp}'\right]$$

For internal gears

$$\frac{h_{Fe}}{m_n} = \frac{1}{2}\left[\left(\cos\gamma_e - \sin\gamma_e \cdot \tan\alpha_{Fen}\right)\cdot\frac{d_{en}}{m_n} - z_n \cdot \cos\left(\frac{\pi}{6} - \vartheta\right) - \sqrt{3}\left(\frac{G}{\cos\vartheta} - \rho_{fP}'\right)\right]$$

3.2 Gearing with $\varepsilon_{\alpha n} > 2$

For deep tooth form gearing produced with a verified grade of accuracy of 4 or better, and with applied profile $(2 \le \varepsilon_{\alpha n} \le 2.5)$ modification to obtain a trapezoidal load distribution along the path of contact, the Y_F may be corrected by the factor Y_{DT} as:

$$Y_{DT} = 2.366 - 0.666\varepsilon_{\alpha n} \text{ for } 2.05 \le \varepsilon_{\alpha n} \le 2.50$$

$$Y_{DT} = 1.0 \quad \text{for} \quad \varepsilon_{\alpha n} < 2.05$$

4 Stress correction factors Y_S, Y_{Sa}

The stress correction factors Y_S and Y_{Sa} take into account the conversion of the nominal bending stress to the local tooth root stress. Thereby Y_S and Y_{Sa} cover the stress increasing effect of the notch (fillet) and the fact that not only bending stresses arise at the root. A part of the local stress is independent of the bending moment arm. This part increases the more the decisive point of load application approaches the critical tooth root section.

Therefore, in addition to its dependence on the notch radius, the stress correction is also dependent on the position of the load application, i.e. the size of the bending moment arm.

Y_S applies to the load application at the outer point of single tooth pair contact, Y_{Sa} to the load application at tooth tip.

Y_S can be determined as follows:

$$Y_S = \left(1.2 + 0.13L\right) q_s^{\left(\frac{1}{1.21 + 2.3/L}\right)}$$

$$\text{where: } L = \frac{s_{Fn}}{h_{Fe}} \text{ and} \qquad q_s = \frac{s_{Fn}}{2\rho_F} \text{ (see 3.3)}$$

Y_{Sa} can be calculated by replacing h_{Fe} with h_{Fa} in the above formulae.

Guidance note:

a) Range of validity $1 < q_s < 8$

In case of sharper root radii (i.e. produced with tools having too sharp tip radii), Y_S resp. Y_{Sa} must be specially considered.

b) In case of grinding notches (due to insufficient protuberance of the hob), Y_S resp. Y_{Sa} can rise considerably, and must be multiplied with:

$$\frac{1.3}{1.3 - 0.6\sqrt{\frac{t_g}{\rho_g}}}$$

where:

t_g	depth of the grinding notch
ρ_g	radius of the grinding notch

c) The formulae for Y_S resp. Y_{Sa} are only valid for α_n = 20°. However, the same formulae can be used as a safe approximation for other pressure angles.

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

5 Contact ratio factor Y_ε

The contact ratio factor Y_ε covers the conversion from load application at the tooth tip to the load application at the mid point of the flank (heightwise) for bevel gears.

The following may be used:

$$Y_\varepsilon = 0.625$$

6 Helix angle factor Y_β

The helix angle factor Y_β takes into account the difference between the helical gear and the virtual spur gear in the normal section on which the calculation is based in the first step. In this way it is accounted for that the conditions for tooth root stresses are more favourable because the lines of contact are sloping over the flank.

The following may be used (β input in degrees):

$Y_\beta = 1 - \varepsilon_\beta \, \beta/120$

When $\varepsilon_\beta > 1$, use $\varepsilon_\beta = 1$ and when β > 30°, use β = 30° in the formula.

However, the above equation for Y_β may only be used for gears with β > 25° if adequate tip relief is applied to both pinion and wheel (adequate = at least $0.5 \cdot C_{eff}$, see Sec.4 [3.2]).

7 Values of endurance limit, σ_{FE}

σ_{FE} is the local tooth root stress (max. principal) which the material can endure permanently with 99% survival probability. $3 \cdot 10^6$ load cycles is regarded as the beginning of the endurance limit or the lower knee of the σ – N curve. σ_{FE} is defined as the unidirectional pulsating stress with a minimum stress of zero (disregarding residual stresses due to heat treatment). Other stress conditions such as alternating or prestressed etc. are covered by the conversion factor Y_M.

σ_{FE} can be found by pulsating tests or gear running tests for any material in any condition. If the approval of the gear shall be based on the results of such tests, all details on the testing conditions have to be approved by the Society. Further, the tests may have to be made under the Society's supervision.

If no fatigue tests are available, the listed values in Table 1 for σ_{FE} may be used for materials subjected to a quality control as the one referred to in the rules.

Section 3

Table 1 Endurance limit

Material	σ_{FE}
Alloyed case hardened steels [1)] (fillet surface hardness 58 to 63 HRC)	
— of specially approved high grade[2)]	1050
— of normal grade	
— CrNiMo steels with approved process[3)]	1000
— CrNi and CrNiMo steels generally	920
— MnCr steels generally	850
Nitriding steel of approved grade, quenched, tempered and gas nitrided (surface hardness 700 – 800 HV)	840
Alloyed quenched and tempered steel, bath or gas nitrided (surface hardness 500 to 700 HV)	720
Alloyed quenched and tempered steel, flame or induction hardened [4)] (incl. entire root fillet) (fillet surface hardness 500 to 650 HV)	0.7 HV + 300
Alloyed quenched and tempered steel, flame or induction hardened (excl. entire root fillet) (σ_B = u.t.s. of base material)	0.25 σ_B + 125
Alloyed quenched and tempered steel	0.4 σ_B + 200
Carbon steel	0.25 σ_B + 250

Guidance note:

All numbers given above are valid for separate forgings and for blanks cut from bars forged according to a qualified procedure, see DNVGL-RU-SHIP Pt.4 Ch.2 Sec.3. For rolled steel, the values shall be reduced with 10%. For blanks cut from forged bars, that are not qualified as mentioned above, the values shall be reduced with 20%, For cast steel, reduce with 40%.

1) These values are valid for a root radius

- being unground. If, however, any grinding is made in the root fillet area in such a way that the residual stresses may be affected, σ_{FE} shall be reduced by 20%. (If the grinding also leaves a notch, see [4]).
- with fillet surface hardness 58 to 63 HRC. In case of lower surface hardness than 58 HRC, σ_{FE} shall be reduced with 20·(58 – HRC) where HRC is the detected hardness. (This may lead to a permissible tooth root stress that varies along the facewidth. If so, the actual tooth root stresses may also be considered along facewidth)
- not being shot peened. In case of approved shot peening, σ_{FE} may be increased by 200 for gears where σ_{FE} is reduced by 20% due to root grinding. Otherwise σ_{FE} may be increased by 100 for $m_n \leq 6$ and 100 – 5 (m_n - 6) for $m_n > 6$. However, the possible adverse influence on the flanks regarding grey staining should be considered, and if necessary the flanks should be masked.

2) Clean steel forgings, see DNVGL-RU-SHIP Pt.2 Ch.2 Sec.6 [5] and DNVGL-CP-0247 Sec.3 [2.13].

3) Approved process, see DNVGL-CP-0247 Sec.3 [2.12].

4) The fillet shall not be ground after surface hardening. Regarding possible root grinding, see 1).

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

8 Mean stress influence factor, Y_M

The mean stress influence factor, Y_M, takes into account the influence of other working stress conditions than pure pulsations (R = 0), such as e.g. load reversals, idler gears, planets and shrinkfitted gears.

Y_M (Y_{Mst}) are defined as the ratio between the endurance (or static) strength with a stress ratio R ¹ 0, and the endurance (or static) strength with R = 0.

Y_M and Y_{Mst} apply only to a calculation method that assesses the positive (tensile) stresses and is therefore suitable for comparison between the calculated (positive) working stress σ_F and the permissible stress σ_{FP} calculated with Y_M or Y_{Mst}.

For thin rings (annulus) in epicyclic gears where the “compression” fillet may be decisive, special considerations apply, see [16].

The following method may be used within a stress ratio −1.2 < R < 0.5.

8.1 For idlers, planets and PTO with ice class

$$Y_M \text{ or } Y_{Mst} = \frac{1}{1 - R\dfrac{1-M}{1+M}}$$

where:

R = stress ratio = min. stress divided by max. stress.

For designs with the same force applied on both forward- and back-flank, R may be assumed to – 1.2.

For designs with considerably different forces on forward- and back-flank, such as e.g. a marine propulsion wheel with a power take off pinion, R may be assessed as:

$$-1.2\,\frac{\text{force per unit facewidt of p.t.o.}}{\text{force per unit facewidth of the main branch}}$$

For a power take off (PTO) with ice class, see Sec.1 [6.1] c.

M considers the mean stress influence on the endurance (or static) strength amplitudes.

M is defined as the reduction of the endurance strength amplitude for a certain increase of the mean stress divided by that increase of the mean stress.

Following M values may be used:

	Endurance limit	*Static strength*
Case hardened	0.8 – 0.15 Y_s [1)]	0.7
If shot peened	0.4	0.6
Nitrided	0.3	0.3
Induction or flame hardened	0.4	0.6
Not surface hardened steel	0.3	0.5
Cast steels	0.4	0.6

1) For bevel gears, use Y_s = 2 for determination of M.

The listed M values for the endurance limit are independent of the fillet shape (Y_S), except for case hardening. In principle there is a dependency, but wide variations usually only occur for case hardening, e.g. smooth semicircular fillets versus grinding notches.

8.2 For gears with periodical change of rotational direction

For case hardened gears with full load applied periodically in both directions, such as side thrusters, the same formula for Y_M as for idlers (with R = – 1.2) may be used together with the M values for endurance limit. This simplified approach is valid when the number of changes of direction exceeds 100 and the total number of load cycles exceeds $3 \cdot 10^6$.

For gears of other materials, Y_M will normally be higher than for a pure idler, provided the number of changes of direction is below $3 \cdot 10^6$. A linear interpolation in a diagram with logarithmic number of changes of direction may be used, i.e. from Y_M = 0.9 with one change to Y_M (idler) for $3 \cdot 10^6$ changes. This is applicable to Y_M for endurance limit. For static strength, use Y_M as for idlers.

For gears with occasional full load in reversed direction, such as the main wheel in a reversing gear box, Y_M = 0.9 may be used.

8.3 For gears with shrinkage stresses and unidirectional load

For endurance strength:

$$Y_M = 1 - \frac{2M}{1+M} \frac{\sigma_{fit}}{\sigma_{FE}}$$

σ_{FE} is the endurance limit for R = 0.

For static strength, Y_{Mst} = 1 and σ_{fit} accounted for in [9] b.

σ_{fit} is the shrinkage stress in the fillet (30° tangent) and may be found by multiplying the nominal tangential (hoop) stress with a stress concentration factor:

$$scf_{fit.} = 1.5 - \frac{2\rho_F}{m_n}$$

8.4 For shrink-fitted idlers and planets

When combined conditions apply, such as idlers with shrinkage stresses, the design factor for endurance strength is:

$$Y_M = \frac{1}{1 - R\frac{1-M}{1+M}} - \frac{2M}{(1+M)\cdot(1-R)} \frac{\sigma_{fit}}{\sigma_{FE}}$$

Symbols as above, but note that the stress ratio R in this particular connection should disregard the influence of σ_{fit}, i.e. R normally equal – 1.2.

For static strength:

$$Y_{Mst} = \frac{1}{1 - R\frac{1-M}{1+M}}$$

The effect of σ_{fit} is accounted for in [9] b.

8.5 Additional requirements for peak loads

The total stress range ($\sigma_{max} - \sigma_{min}$) in a tooth root fillet is not to exceed:

$\frac{2.25\sigma_y}{S_F}$ for not surface hardened fillets

$\frac{5HV}{S_F}$ for surface hardened fillets

9 Life factor, Y_N

The life factor, Y_N, takes into account that, in the case of limited life (number of cycles), a higher tooth root stress can be permitted and that lower stresses may apply for very high number of cycles.

Decisive for the strength at limited life is the σ – N – curve of the respective material for given hardening, module, fillet radius, roughness in the tooth root, etc. That is, the factors $Y_{\delta relT}$, Y_{RelT}, Y_X and Y_M have an influence on Y_N.

If no σ – N – curve for the actual material and hardening etc. is available, the following method may be used.

Determination of the σ – N – curve:

a) Calculate the permissible stress σ_{FP} for the beginning of the endurance limit ($3 \cdot 10^6$ cycles), including the influence of all relevant factors as S_F, $Y_{\delta relT}$, Y_{RelT}, Y_X, Y_M and Y_C,i.e.

   $\sigma_{FP} = \sigma_{FE} \cdot Y_M \cdot Y_{\delta relT} \cdot Y_{RelT} \cdot Y_X \cdot Y_C / S_F$

b) Calculate the permissible «static» stress ($\leq 10^3$ load cycles) including the influence of all relevant factors as S_{Fst}, $Y_{\delta relTst}$, Y_{Mst} and Y_{Cst}:

$$\sigma_{FPst} = \frac{1}{S_{Fst}}\left(\sigma_{Fst} \cdot Y_{Mst} \cdot Y_{\delta relTst} \cdot Y_{Cst} - \sigma_{fit}\right)$$

   where σ_{Fst} is the local tooth root stress which the material can resist without cracking (surface hardened materials) or unacceptable deformation (not surface hardened materials) with 99% survival probability.

	σ_{Fst}
Alloyed case hardened steel [1)]	2300
Nitriding steel, quenched, tempered and gas nitrided (surface hardness 700 to 800 HV)	1250
Alloyed quenched and tempered steel, bath or gas nitrided (surface hardness 500 to 700 HV)	1050
Alloyed quenched and tempered steel, flame or induction hardened (fillet surface hardness 500 – 650 HV)	1.8 HV + 800
Steel with not surface hardened fillets, the smaller value of [2)]	$1.8\ \sigma_B$ or $2.25\ \sigma_y$

	σ_{Fst}
1) This is valid for a fillet surface hardness of 58 to 63 HRC. In case of lower fillet surface hardness than 58 HRC, σ_{Fst} shall be reduced with 30·(58 – HRC) where HRC is the actual hardness. Shot peening or grinding notches are not considered to have any significant influence on σ_{Fst}. 2) Actual stresses exceeding the yield point (σ_y or $\sigma_{0.2}$) will alter the residual stresses locally in the "tension" fillet respectively "compression" fillet. This is only to be utilised for gears that are not later loaded with a high number of cycles at lower loads that could cause fatigue in the "compression" fillet.	

c) Calculate Y_N as:

$N_L > 3\cdot10^6$	$Y_N = 1$ or $$Y_N = \left(\frac{3\cdot10^6}{N_L}\right)^{0.01}$$ i.e. $Y_n = 0.92$ for 10^{10} The $Y_N = 1$ from $3\cdot10^6$ on may only be used when special material cleanness applies, see DNVGL-RU-SHIP Pt.4 Ch.2.
$10^3<N_L<3\cdot10^6$	$$Y_N = \left(\frac{3\cdot10^6}{N_L}\right)^{exp}$$ $$exp = 0.2876\ \log\frac{\sigma_{FPst}\text{ for }10^3\text{ cycles}}{\sigma_{FP}\text{ for }3\cdot10^6\text{ cycles}}$$
$N_L < 10^3$	$$Y_N = \frac{\sigma_{FPst}\text{ for }10^3\text{ cycles}}{\sigma_{FP}\text{ for }3\cdot10^6\text{ cycles}}$$ or simply use σ_{FPst} as mentioned in b) directly.

Guidance on number of load cycles N_L for various applications:

— For propulsion purpose, normally $N_L = 10^{10}$ at full load (yachts etc. may have lower values).
— For auxiliary gears driving generators that normally oper-ate with 70 to 90% of rated power, $N_L = 10^8$ with rated power may be applied.

10 Relative notch sensitivity factor, $Y_{\delta relT}$

The dynamic (respectively static) relative notch sensitivity factor, $Y_{\delta relT}$ ($Y_{\delta relTst}$) indicate to which extent the theoretically concentrated stress lies above the endurance limits (respectively static strengths) in the case of fatigue (respectively overload) breakage.

$Y_{\delta relT}$ is a function of the material and the relative stress gradient. It differs for static strength and endurance limit.

The following method may be used:

For endurance limit:

for not surface hardened fillets:

$$Y_{\delta relT} = \frac{1 + \left(0.135 - 1.22 \cdot 10^{-4} \cdot \sigma_{0,2}\right)\sqrt{1 + 2\,q_s}}{1.33 - 3 \cdot 10^{-4} \cdot \sigma_{0.2}}$$

for all surface hardened fillets except nitrided:

$$Y_{\delta relT} = \frac{\left(1 + 0.0245\sqrt{1 + 2\,q_s}\right)}{1.06}$$

for nitrided fillets:

$$Y_{\delta relT} = \frac{\left(1 + 0.142\sqrt{1 + 2\,q_s}\right)}{1.347}$$

For static strength:

for not surface hardened fillets[1)]:

$$Y_{\delta relTst} = \frac{1 + 0.82\left(Y_s - 1\right)\left(300/\sigma_{0.2}\right)^{0.25}}{1 + 0.82\left(300/\sigma_{0.2}\right)^{0.25}}$$

for surface hardened fillets except nitrided:

$$Y_{\delta relTst} = 0.44\ Y_S + 0.12$$

for nitrided fillets:

$$Y_{\delta relTst} = 0.6 + 0.2\ Y_S$$

1) These values are only valid if the local stresses do not exceed the yield point and thereby alter the residual stress level. See also [9] b footnote 2.

11 Relative surface condition factor, Y_{RrelT}

The relative surface condition factor, Y_{RrelT}, takes into account the dependence of the tooth root strength on the surface condition in the tooth root fillet, mainly the dependence on the peak to valley surface roughness.

Y_{RrelT} differs for endurance limit and static strength.

The following method may be used:

For endurance limit:

$$Y_{RrelT} = 1.675 - 0.53\ (R_y + 1)^{0.1}$$

for surface hardened steels and alloyed quenched and tempered steels except nitrided

$$Y_{RrelT} = 5.3 - 4.2\ (R_y + 1)^{0.01}$$

for carbon steels

$$Y_{\delta relT} = 4.3 - 3.26\ (R_y + 1)^{0.005}$$

for nitrided steels

For static strength:

$Y_{RrelTst} = 1$ for all R_y and all materials.

For a fillet without any longitudinal machining trace, $R_y \approx R_z$.

12 Size factor, Y_X

The size factor, Y_X, takes into account the decrease of the strength with increasing size. Y_X differs for endurance limit and static strength.

The following may be used:

For endurance limit:

$Y_X = 1$	for $m_n \leq 5$	generally
$Y_X = 1.03 - 0.006\ m_n$	for $5 < m_n < 30$	for not surface hardened steels
$Y_X = 0.85$	for $m_n \geq 30$	
$Y_X = 1.05 - 0.01\ m_n$	for $5 < m_n < 25$	for surface hardened steels
$Y_X = 0.8$	for $m_n \geq 25$	

For static strength:

$Y_{Xst} = 1$ for all m_n and all materials.

13 Case depth factor, Y_C

The case depth factor, Y_C, takes into account the influence of hardening depth on tooth root strength.

Y_C applies only to surface hardened tooth roots, and is different for endurance limit and static strength.

In case of insufficient hardening depth, fatigue cracks can develop in the transition zone between the hardened layer and the core. For static strength, yielding shall not occur in the transition zone, as this would alter the surface residual stresses and therewith also the fatigue strength.

The major parameters are case depth, stress gradient, permissible surface respectively subsurface stresses, and subsurface residual stresses.

The following simplified method for Y_C may be used.

Y_C consists of a ratio between permissible subsurface stress (incl. influence of expected residual stresses) and permissible surface stress. This ratio is multiplied with a bracket containing the influence of case depth and stress gradient. (The empirical numbers in the bracket are based on a high number of teeth, and are somewhat on the safe side for low number of teeth.)

Y_C and Y_{Cst} may be calculated as given below, but calculated values above 1.0 shall be put equal 1.0.

For endurance limit:

$$Y_C = \frac{\text{const.}}{\sigma_{FE}} \left(1 + \frac{3\,t}{\rho_F + 0.2\,m_n}\right)$$

For static strength:

$$Y_{Cst} = \frac{const.}{\sigma_{Fst}} \left(1 + \frac{3t}{\rho_F + 0.2 m_n} \right)$$

where const. and t are connected as:

Hardening process	*t =*	*endurance limit const =*	*static strength const =*
Case hardening	t_{550}	640	1900
	t_{400}	500	1200
	t_{300}	380	800
Nitriding	t_{400}	500	1200
Induction- or flame hardening	t_{HVmin}	1.1 HV_{min}	2.5 HV_{min}

For symbols, see Sec.2 [13].

In addition to these requirements to minimum case depths for endurance limit, some upper limitations apply to case hardened gears:

The max. depth to 550 HV should not exceed

1) 1/3 of the top land thickness s_{an} unless adequate tip relief is applied (see Sec.1 [10]).
2) 0.25 m_n. If this is exceeded, the following applies additionally in connection with endurance limit:

$$Y_C = 1 - \left(\frac{t_{550\,max}}{m_n} - 0.25 \right)$$

14 Thin rim factor Y_B

Where the rim thickness is not sufficient to provide full support for the tooth root, the location of a bending failure may be through the gear rim, rather than from root fillet to root fillet.

Y_B is *not* a factor used to convert calculated root stresses at the 30° tangent to actual stresses in a thin rim tension fillet. Actually, the compression fillet can be more susceptible to fatigue.

Y_B is a simplified empirical factor used to derate thin rim gears (external as well as internal) when no detailed calculation of stresses in both tension and compression fillets are available.

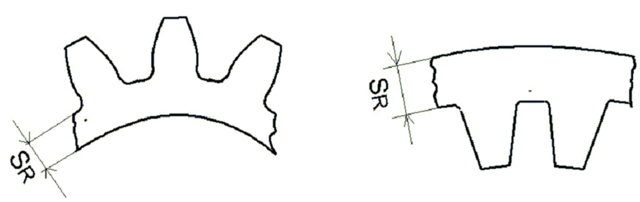


Figure 4 Examples on thin rims

Y_B is applicable in the range 1.75 < s_R/m_n < 3.5.

Y_B = 1.15 · ln (8.324 · m_n/s_R)

(for $s_R/m_n \geq 3.5$, Y_B = 1)

(for $s_R/m_n \leq 1.75$, use 3.15)

σ_F as calculated in [2.1] shall be multiplied with Y_B when s_R/m_n < 3.5. Thus Y_B is used for both high and low cycle fatigue.

Guidance note:

This method is considered to be on the safe side for external gear rims. However, for internal gear rims without any flange or web stiffeners the method may not be on the safe side, and it is advised to check with the method in [15]/[16]

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

15 Stresses in thin rims

For rim thickness s_R < 3.5 m_n the safety against rim cracking has to be checked.

The following method may be used.

15.1 General

The stresses in the standardised 30º tangent section, tension side, are slightly reduced due to decreasing stress correction factor with decreasing relative rim thickness s_R/m_n. On the other hand, during the complete stress cycle of that fillet, a certain amount of compression stresses are also introduced. The complete stress range remains approximately constant. Therefore, the standardised calculation of stresses at the 30º tangent may be retained for thin rims as one of the necessary criteria.

The maximum stress range for thin rims usually occurs at the 60º to 80º tangents, both for «tension» and «compression» side. The following method assumes the 75º tangent to be the decisive. Therefore, in addition to the a.m. criterion applied at the 30º tangent, it is necessary to evaluate the max. and min. stresses at the 75º tangent for both «tension» (loaded flank) fillet and «compression» (back-flank) fillet. For this purpose the whole stress cycle of each fillet should be considered, but usually the following simplification is justified:

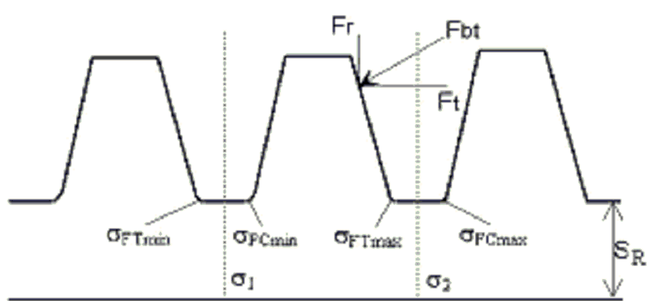


Figure 5 Nomenclature of fillets

Index «T» means «tensile» fillet, «C» means «compression» fillet.

σ_{FTmin} and σ_{FCmax} are determined on basis of the nominal rim stresses times the stress concentration factor, Y_{75}.

σ_{FCmin} and σ_{FTmax} are determined on basis of superposition of nominal rim stresses times Y_{75} plus the tooth bending stresses at 75º tangent.

15.2 Stress concentration factors at the 75º tangents

The nominal rim stress consists of bending stresses due to local bending moments, tangential stresses due to the tangential force F_t, and radial shear stresses due to F_r.

The major influence is given by the bending stresses. The influence of the tangential stresses is minor, and even though its stress concentration factor is slightly higher than for bending, it is considered to be safe enough when the sum of these nominal stresses are combined with the stress concentration factor for bending. The influence of the radial shear stress is neglected.

The stress concentration factor relating nominal rim stresses to local fillet stresses at the 75º tangent may be calculated as:

$$Y_{75} = \frac{3 \cdot \sqrt{\frac{s_R}{m_n}}}{1.85\sqrt{\rho_{75}} + \sqrt{\frac{s_R}{m_n}}\sqrt{\rho_{75}}}$$

where ρ_{75} is the root radius at the 75º tangent ref. to m_n. Usually ρ_{75} is closed to the tool radius ρ_{ao}, and

$\rho_{75} = \rho_{ao}$

is a safe approximation compensating for the a.m. simplifications to the «unsafe» side.

The tooth root stresses of the loaded tooth are decreasing with decreasing relative rim thickness, approximately with the empirical factor

$$Y_{corr} = 1 - \frac{1}{3}\left(\frac{m_n}{s_R}\right)^{1.5}$$

15.3 Nominal rim stresses

The bending moment applied to the rim consists of a part of the tooth tilt moment F_t (h_F + 0.5 s_R) and the bending caused by the radial force F_r.

The sectional modulus (first moment of area) which is used for determination of the nominal bending stresses is not necessarily the same for the 2 a.m. bending moments. If flanges, webs, etc. outside the toothed section contribute to stiffening the rim against various deflections, the influence of these stiffeners should be considered. E.g. an end flange will have an almost negligible influence on the effective sectional modulus for the stresses due to tooth tilt as the deflection caused by the tilt moment is rather small and would not much involve the flange. On the other hand, the radial forces, as for instance from the meshes in an annulus, would cause considerable radial deflections that the flange might restrict to a substantial amount. When considering the stiffening of such flanges or webs on basis of simplified models, it is advised to use an effective rim thickness $s_R' = s_R + 0.2\ m_n$ for the first moment of area of the rim (toothed part) cross section.

For a high number of rim teeth, it may be assumed that the rim bending moments in the fillets adjacent to the loaded tooth are of the same magnitude as right under the applied force. This assumption is reasonable for an annulus, but rather much on the safe side for a hollow pinion.

The influence of F_t on the nominal tangential stress is simplified by half of it for compressive stresses (σ_1) and the other half for tensile stresses (σ_2). Applying these assumptions, the nominal rim stresses adjacent to the loaded tooth are:

$$\sigma_1 = \frac{-0.5 F_t (h_F + 0.5 s_R)}{W_T} - \frac{F_r R\, f(\vartheta)}{W_R} - \frac{F_t}{2A}$$

$$\sigma_2 = \frac{0.5 F_t (h_F + 0.5 s_R)}{W_T} - \frac{F_r R\, f(\vartheta)}{W_R} + \frac{F_t}{2A}$$

where σ_1, σ_2 see Figure 5.

A	=	minimum area of cross section (usually b_R s_R).
W_T	=	the sectional modulus of rim with respect to tooth tilt moment (usually $b_R s_r^2/6$).
W_R	=	the sectional modulus of the rim including the influence of stiffeners as flanges, webs etc. ($W_R \geq W_T$).
b_R	=	the width of the rim.
R	=	the radius of the neutral axis in the rim, i.e. from wheel centre to midpoint of rim.
$f(\vartheta)$	=	a function for bending moment distribution around the rim. For a rim (pinion) with one mesh only, the $f(\vartheta)$ at the position of load application is 0.24. For an annulus with 3 or more meshes, $f(\vartheta)$ at the position of each load application is approx.: 3 planets $f(\vartheta)$ = 0.19 4 planets $f(\vartheta)$ = 0.14 5 planets $f(\vartheta)$ = 0.11 6 planets $f(\vartheta)$ = 0.09

It must be checked if the max. (tensile) stress in the compression fillet really occurs when the fillet is adjacent to the loaded tooth. In principle, the stress variation through a complete rotation should be considered, and the max. value used. The max. value is usually never less than 0. For an annulus, e.g. the tilt moment is zero in the mid position between the planet meshes, whilst the bending moment due to the radial forces is half of that at the mesh but with opposite sign.

If these formulae are applied to idler gears, as e.g. planets, the influence of nominal tangential stresses must be corrected by deleting $F_t/(2\ A)$ for σ_1, and using F_t/A for σ_2. Further, the influence of F_r on the nominal bending stresses is usually negligible due to the planet bearing support.

15.4 Root fillet stresses

Determination of min. and max. stresses in the «tension» fillet:

Minimum stress:

$$\sigma_{FTmin} = K\ \sigma_1 Y_{75}$$

Maximum stress:

$$\sigma_{FTmax} = K\ \sigma_2 Y_{75} + 0.3\,\sigma_F Y_{corr}$$

where:

0.3 is an empirical factor relating the tension stresses (σ_F) at the 30º tangent to the part of the tension stresses at the 75º tangent which add to the rim related stresses. (0.3 also takes into account that full superposition of nominal stresses times stress concentration factors from both «sides of the corner fillet» would result in too high stresses.)

$$K = K_A \cdot K_\gamma \cdot K_v \cdot K_{F\beta} \cdot K_{F\alpha}$$

Determination of min. and max. stresses in the «compression» fillet:

Minimum stress:

$$\sigma_{FCmin} = K\,\sigma_1 Y_{75} - 0.36\,\sigma_F Y_{corr}$$

Maximum stress:

$$\sigma_{FCmax} = K\,\sigma_2 Y_{75}$$

where:

0.36 is an empirical factor relating the tension stresses (σ_F) at the 30º tangent to the part of the compression stresses at the 75º tangent which add to the rim related stresses.

For gears with reversed loads as idler gears and planets there is no distinct «tension» or «compression» fillet. The minimum stress σ_{Fmin} is the minimum of σ_{FTmin} and σ_{FCmin} (usually the latter is decisive). The maximum stress σ_{FTmax} is the maximum of σ_{FTmax} and σ_{FCmax} (usually the former is decisive).

16 Permissible stresses in thin rims

16.1 General

The safety against fatigue fracture respectively overload fracture shall be at least at the same level as for solid gears. The «ordinary» criteria at the 30º tangent apply as given in [1] through [3].

Additionally the following criteria at the 75º tangent may apply.

16.2 For >3·10^6 cycles

The permissible stresses for the «tension» fillets and for the «compression» fillets are determined by means of a relevant fatigue diagram.

If the actual tooth root stress (tensile or compressive) exceeds the yield strength to the material, the induced residual stresses shall be taken into account.

For determination of permissible stresses the following is defined:

R = stress ratio, i.e $\frac{\sigma_{FTmin}}{\sigma_{FTmax}}$ respectively $\frac{\sigma_{FCmin}}{\sigma_{FCmax}}$

Δσ = stress range, i.e. $\sigma_{FTmax} - \sigma_{FTmin}$ resp. $\sigma_{FCmax} - \sigma_{FCmin}$

(For idler gears and planets $\frac{\sigma_{Fmin}}{\sigma_{Fmax}}$ and $\Delta\sigma = \sigma_{Fmax}\ \sigma_{Fmin}$).

The permissible stress range $\Delta\sigma_p$ for the «tension» respectively «compression» fillets can be calculated as:

For R > −1

$$\Delta\sigma_p = \frac{1.3}{1 + 0.3\frac{1+R}{1-R}}\sigma_{FP}$$

For $-\infty < R < -1$

$$\Delta\sigma_p = \frac{1.3}{1+0.15\dfrac{1+R}{1-R}}\sigma_{FP}$$

where:

σ_{FP} = see [2], determined for unidirectional stresses (Y_M = 1).

If the yield strength σ_y is exceeded in either tension or compression, residual stresses are induced. This may be considered by correcting the stress ratio R for the respective fillets (tension or compression).

For example, if $|\sigma_{FCmin}| > \sigma_y$, (i.e. exceeded in compression), the difference $\Delta = |\sigma_{FCmin}| - \sigma_y$ affects the stress ratio as

$$R = \frac{\sigma_{FCmin}+\Delta}{\sigma_{FCmax}+\Delta} = \frac{-\sigma_y}{\sigma_{FCmax}+\Delta}$$

Similarly the stress ratio in the tension fillet may require correction.

If the yield strength is exceeded in tension, $\sigma_{FTmax} > \sigma_y$, the difference $\Delta = \sigma_{FTmax} - \sigma_y$ affects the stress ratio as

$$R = \frac{\sigma_{FTmin}-\Delta}{\sigma_{FTmax}-\Delta} = \frac{\sigma_{FTmin}-\Delta}{\sigma_y}$$

Checking for possible exceeding of the yield strength has to be made with the highest torque and with the K_{AP} if this exceeds K_A.

16.3 For ≤ 10^3 cycles

The permissible stress range $\Delta\sigma_p$ is not to exceed:

For $R > -1$

$$\Delta\sigma_{pst} = \frac{1.5}{1+0.5\dfrac{1+R}{1-R}}\sigma_{FPst}$$

For $-\infty < R < -1$

$$\Delta\sigma_{pst} = \frac{1.5}{1+0.25\dfrac{1+R}{1-R}}\sigma_{FPst}$$

For all values of R, $\Delta\sigma_{pst}$ is limited by:

not surface hardened $\dfrac{2.25\sigma_y}{S_F}$

surface hardened $\dfrac{5\,HV}{S_F}Y_C$

Definition of $\Delta\sigma$ and R, see [16.2], with particular attention to possible correction of R if the yield strength is exceeded.

σ_{FPst} see [2], determined for unidirectional stresses (Y_M = 1) and < 10^3 cycles.

16.4 For 10^3 < cycles < $3 \cdot 10^6$

$\Delta\sigma_p$ shall be determined by linear interpolation a log-log diagram.

$\Delta\sigma_p$ at N_L load cycles is:

$$\Delta\sigma_{p\,N_L} = \left(\frac{3 \cdot 10^6}{N_L}\right)^{exp} \Delta\sigma_{p\,3\cdot 10^6}$$

$$exp = 0.2876 \log \frac{\Delta\sigma_{p\,10^3}}{\Delta\sigma_{p\,3\cdot 10^6}}$$

SECTION 4 CALCULATION OF SCUFFING LOAD CAPACITY

1 Introduction

High surface temperatures due to high loads and sliding velocities can cause lubricant films to break down. This seizure or welding together of areas of tooth surface is termed scuffing.

In contrast to pitting and fatigue breakage which show a distinct incubation period, a single short overloading can lead to a scuffing failure. In the ISO-TR13989 two criteria are mentioned. The method used in this class guideline is based on the principles of the flash temperature criterion.

Guidance note:

Bulk temperature in excess of 120ºC for long periods may have an adverse effect on the surface durability, see Sec.2 [11].

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

2 General criteria

In no point along the path of contact the local contact temperature may exceed the permissible temperature, i.e.:

$$\vartheta_B \leq \frac{\vartheta_S - \vartheta_{oil}}{S_S} + \vartheta_{oil}$$

$$\vartheta_B \leq \vartheta_S - 50$$

where:

ϑ_B	=	max. contact temperature along the path of contact. $\vartheta_B = \vartheta_{MB} + \vartheta_{fla\ max}$
ϑ_{MB}	=	bulk temperature, see [3.4].
$\vartheta_{fla\ max}$	=	max. flash temperature along the path of contact, see [4].
ϑ_S	=	scuffing temperature, see below.
ϑ_{oil}	=	oil temperature before it reaches the mesh (max. applicable for the actual load case to be used, i.e. normally alarm temperature, except for ice classes where a max. expected temperature applies).
S_S	=	required safety factor according to the Rules.

The scuffing temperature may be calculated as:

$$\vartheta_S = 80 + \left(0.857 + 1.12 \cdot \left(\frac{100}{\nu_{40}}\right)^{0.02} \cdot X_{wrelT}\right) FZG^2 X_L$$

where:

X_{wrelT}	=	relative welding factor.

<table>
<tr><td colspan="2"></td><td>X_{wrelT}</td></tr>
<tr><td colspan="2">Through hardened steel</td><td>1.0</td></tr>
<tr><td colspan="2">Phosphated steel</td><td>1.25</td></tr>
<tr><td colspan="2">Copperplated steel</td><td>1.50</td></tr>
<tr><td colspan="2">Nitrided steel</td><td>1.50</td></tr>
<tr><td rowspan="3">Casehardened steel</td><td>Less than 10% retained austenite</td><td>1.15</td></tr>
<tr><td>10 – 20% retained austenite</td><td>1.0</td></tr>
<tr><td>20 – 30% retained austenite</td><td>0.85</td></tr>
<tr><td colspan="2">Austenitic steel</td><td>0.45</td></tr>
</table>

FZG = load stage according to FZG-Test A/8.3/90. (Note: This is the load stage where scuffing occurs. However, due to scatter in test results, calculations shall be made with one load stage less than the specification.)

X_L = lubricant factor.

= 1.0 for mineral oils.

= 0.8 for polyalfaolefins.

= 0.7 for non-water-soluble polyglycols.

= 0.6 for water-soluble polyglycols.

= 1.5 for traction fluids.

= 1.3 for phosphate esters.

ν_{40} = kinematic oil viscosity at 40°C (mm^2/s).

Application of other test methods such as the Ryder, the FZG-Ryder R/46.5/74, and the FZG L-42 Test 141/19.5/110 may be specially considered.

For high speed gears with very short time of contact ϑ_S, may be increased as follows provided use of EP-oils.

Addition to the calculated scuffing temperature ϑ_S:

If $t_c \geq 18$ μs, no addition

If $t_c < 18$ μs, add $18 \cdot X_{wrelT} \cdot (18 - t_c)$

where

t_c = contact time (μs) which is the time needed to cross the Hertzian contact width.

$$t_c = 340 \frac{\sigma_H \cdot u}{n_1 \cdot \cos\beta_b \cdot (1 + u)} \qquad [\mu s]$$

σ_H as calculated in Sec.2 [2.1].

For bevel gears, use u_v in stead of u.

3 Influence factors

3.1 Coefficient of friction

The following coefficient of friction may apply:

$$\mu = 0.048\left(\frac{w_{Bt}}{v_{\Sigma C}\,\rho_{redC}}\right)^{0.2} \eta_{oil}^{-0.05}\, R_a^{0.25}\, X_L$$

where:

w_{Bt} = specific tooth load (N/mm)

$v_{\Sigma C}$ = sum of tangential velocities at pitch point. At pitch line velocities > 50 m/s, the limiting value of $v_{\Sigma C}$ at v = 50 m/s shall be used.

ρ_{redC} = relative radius of curvature (transversal plane) at the pitch point

Cylindrical gears

$$w_{Bt} = \frac{F_{bt}}{b} \cdot K_A \cdot K_\gamma \cdot K_v \cdot K_{H\beta} \cdot K_{H\alpha} \qquad \text{(see 1)}$$

$$v_{\Sigma C} = 2\,v \sin \alpha_{tw}$$

$$\rho_{redC} = \rho_C \cos \beta_b$$

Bevel gears

$$w_{Bt} = \frac{F_{mb\,t}}{b} \cdot K_A \cdot K_\gamma \cdot K_v \cdot K_{H\beta} \cdot K_{H\alpha} \qquad \text{(see 1)}$$

$$v_{\Sigma C} = 2\,v_{mt} \sin \alpha_{vt}$$

$$\rho_{redC} = \rho_{vC} \cos \beta_{bm}$$

η_{oil} = dynamic viscosity (mPa s) at ϑ_{oil}, calculated as

$$\eta_{oil} = \frac{\nu_{oil}\ \rho}{1000}$$

where ρ in kg/m^3 approximated as

$$\rho = \rho_{15} - (\vartheta_{oil} - 15)0.7$$

and ν_{oil} is kinematic viscosity at ϑ_{oil} and may be calculated by means of the following equation:

$$\log\log(\nu_{oil} + 0.8) = \log\log(\nu_{100} + 0.8) + \frac{\log 373 - \log(273 + \vartheta_{oil})}{\log 373 - \log 313} \cdot (\log\log(\nu_{40} + 0.8) - \log\log(\nu_{100} + 0.8))$$

R_a = composite arithmetic mean roughness (micron) of pinion an wheel calculated as

$$R_a = 0.5\,(R_{a\,1} + R_{a\,2})$$

This is defined as the roughness on the new flanks i.e. as manufactured.

X_L = see [4.2]

3.2 Effective tip relief Ceff

C_{eff} is the effective tip relief; that amount of tip relief, which compensates for the elastic deformation of the gear mesh, i.e. zero load at the tooth tip. It is assumed (simplified) to be equal for both pinion and wheel.

Cylindrical gears

For helical:

$$C_{eff} = \frac{F_{bt}\ K_A\ K_\gamma}{b\ c_\gamma} \qquad \text{(see 1)}$$

For spur:

$$C_{eff} = \frac{F_{bt}\ K_A\ K_\gamma}{b\ c'} \qquad \text{(see 1)}$$

Alternatively for spur and helical gears the non-linear approach in Sec.1 [11] may be used (taking $C_{eff.}$ = δ).

Bevel gears

$$C_{eff} = \frac{F_{mbt} K_A}{c_\gamma b} \qquad \text{(see 1)}$$

where:

$$c_\gamma = \frac{44 \cdot \varepsilon_\alpha}{2 + \varepsilon_\gamma}$$

3.3 Tip relief and extension

Cylindrical gears

The extension of the tip relief is not to result in an effective contact ratio $\varepsilon_\alpha < 1$ when the gear is unloaded (exceptions to this may only apply for applications where the gear is not to run at light loads). This means that the unrelieved part of the path of contact shall be minimum p_{bt}. It is further assumed that this unrelieved part is placed centrally on the path of contact.

If root relief applies, it has to be calculated as equivalent tip relief. That is, pinion root relief (at mesh position A) is added to C_{a2}, and wheel root relief (at mesh position E) is added to C_{a1}.

If no design tip relief or root relief on the mating gear is specified (i.e. if C_{a1}+ C_{root2} = 0 and visa versa), use the running in amount, see Sec.1 [12].

Bevel gears

Bevel gears shall have heightwise crowning, i.e. no distinct relieved/unrelieved area. This may be treated as tip and root relief. For calculation purposes the root relief is combined with the tip relief of the mating member into an equivalent tip relief. If no resulting tip relieves are specified, the equivalent tip relives may be calculated, as an approximation, based on the tool crowning $C_{a\ tool}$ (per mille of tool module m_0) as follows:

$$C_{a\,1\,eq} = C_{a\,1} + C_{root\,2}$$

$$C_{a\,2\,eq} = C_{a\,2} + C_{root\,1}$$

where:

$$C_{al} = C_{atool1} \cdot m_0 \cdot \left(1 - \frac{A_1 + 2(m_0 - m_n)}{m_0}\right)^2$$

$$A_1 = m_n(1 + x_1) - 0.5 \cdot \sin\alpha_{vt} \cdot \left(\sqrt{d_{va1}^2 - d_{vb1}^2} - d_{vb1} \cdot \tan\alpha_{vt}\right)$$

$$C_{a2} = C_{atool2} \cdot m_0 \cdot \left(1 - \frac{A_2 + 2(m_0 - m_n)}{m_0}\right)^2$$

$$A_2 = m_n(1 + x_2) - 0.5 \cdot \sin\alpha_{vt} \cdot \left(\sqrt{d_{va2}^2 - d_{vb2}^2} - d_{vb2} \cdot \tan\alpha_{vt}\right)$$

$$C_{root1} = C_{atool1} \cdot m_0 \cdot \left(1 - \frac{A_2}{m_0}\right)^2$$

$$C_{root2} = C_{atool2} \cdot m_0 \cdot \left(1 - \frac{A_1}{m_0}\right)^2$$

If the pressure angles of the cutter blades are modified (and verified) to balance the tip relieves, the following may be assumed:

$$C_{a1eq} = C_{a2eq} = 0.5 \cdot \left(\text{sum of calculated}\, C_{a1eq} \text{and}\, C_{a2eq}\right)$$

Throughout the following C_{a1} and C_{a2} mean the equivalent tip relieves C_{a1eq} and C_{a2eq}.

3.4 Bulk temperature

The bulk temperature may be calculated as:

$$\vartheta_{MB} = \vartheta_{oil} + 0.5\, X_s\, X_{mp} \vartheta_{flaaverage}$$

where:

X_S	=	lubrication factor.
	=	1.2 for spray lubrication.
	=	1.0 for dip lubrication (provided both pinion and wheel are dip lubricated and tip speed < 5 m/s).
	=	1.0 for spray lubrication with additional cooling spray (spray on both pinion and wheel, or spray on pinion and dip lubrication of wheel).
	=	0.2 for meshes fully submerged in oil.
		contact factor,
X_{mp}	=	$X_{mp} = 0.5\left(1 + n_p\right)$
n_p	=	number of mesh contact on the pinion (for small gear ratii the number of wheel meshes should be used if higher).
$\vartheta_{flaaverage}$	=	average of the integrated flash temperature (see [4.3]) along the path of contact.

$$\vartheta_{flaaverage} = \frac{\int_{y=A}^{E} \vartheta_{fla}(\Gamma_y)\, d\Gamma_y}{\Gamma_E - \Gamma_A}$$

For high speed gears (v > 50 m/s) it may be necessary to assess the bulk temperature on the basis of a thermal rating of the entire gear transmission.

4 The flash temperature ϑ_{fla}

4.1 Basic formula

The local flash temperature ϑ_{fla} may be calculated as

$$\vartheta_{fla} = 0.325\, \mu\, X_{corr} \left(w_{Bt}\, X_{\Gamma_y}\right)^{3/4} n_1^{1/2} \frac{\left| \sqrt{\rho_{1y}} - \sqrt{\frac{\rho_{2y}}{u}} \right|}{\rho_{redy}^{1/4}}$$

(For bevel gears, replace u with u_v)

and shall be calculated stepwise along the path of contact from A to E.

where:

μ = coefficient of friction, see [3.1].

correction factor taking empirically into account the increased scuffing risk in the beginning of the approach path, due to mesh starting without any previously built up oil film and possible shuffling away oil before meshing if insufficient tip relief.

X_{corr} = $X_{corr} = 1 + \frac{C_{eff} - C_a}{50} \left| \frac{\Gamma_y}{\varepsilon_\alpha (\Gamma_D - \Gamma_A)} \right|^3$

C_a = tip relief of *driven* member. X_{corr} is only applicable in the approach path and if $C_a < C_{eff}$, otherwise 1.0.

w_{Bt} = unit load, see [3.1].

X_{Γ_y} = load sharing factor, see [4.3].

n_1 = pinion r.p.m.

Γ_y, ρ_{ly} etc. see [4.2].

4.2 Geometrical relations

The various radii of flank curvature (transversal plane) are:

ρ_{1y} = pinion flank radius at mesh point y.

ρ_{2y} = wheel flank radius at mesh point y.

ρ_{redy} = equivalent radius of curvature at mesh point y.

$$\rho_{redy} = \frac{\rho_{1y}\,\rho_{2y}}{\rho_{1y} + \rho_{2y}}$$

Cylindrical gears

$$\rho_{1y} = \frac{1+\Gamma_y}{1+u} a \sin\alpha_{tw}$$

$$\rho_{2y} = \frac{u-\Gamma_y}{1+u} a \sin\alpha_{tw}$$

Note that for internal gears, a and u are negative.

Bevel gears

$$\rho_{1y} = \frac{1+\Gamma_y}{1+u_v} a_v \sin\alpha_{vt}$$

$$\rho_{2y} = \frac{u_v-\Gamma_y}{1+u_v} a_v \sin\alpha_{vt}$$

Γ is the parameter on the path of contact, and y is any point between A end E.
At the respective ends, Γ has the following values:

Root pinion/tip wheel
Cylindrical gears

$$\Gamma_A = -\frac{z_2}{z_1}\left(\frac{\sqrt{(d_{a2}/d_{b2})^2 - 1}}{\tan\alpha_{tw}} - 1\right)$$

Bevel gears

$$\Gamma_A = -u_v\left(\frac{\sqrt{(d_{va2}/d_{vb2})^2 - 1}}{\tan\alpha_{vt}} - 1\right)$$

Tip pinion/root wheel

Cylindrical gears

$$\Gamma_E = \frac{\sqrt{(d_{a1}/d_{b1})^2 - 1}}{\tan \alpha_{tw}} - 1$$

Bevel gears

$$\Gamma_E = \frac{\sqrt{(d_{va1}/d_{vb1})^2 - 1}}{\tan \alpha_{vt}} - 1$$

At inner point of single pair contact

Cylindrical gears

$$\Gamma_B = \Gamma_E - \frac{2\,\pi}{z_1\ \tan \alpha_{tw}}$$

Bevel gears

$$\Gamma_B = \Gamma_E - \frac{2\,\pi}{z_{v1}\ \tan \alpha_{vt}}$$

At outer point of single pair contact

Cylindrical gears

$$\Gamma_D = \Gamma_A + \frac{2\,\pi}{z_1\ \tan \alpha_{tw}}$$

Bevel gears

$$\Gamma_D = \Gamma_A + \frac{2\,\pi}{z_{v1}\ \tan \alpha_{vt}}$$

At pitch point $\Gamma_C = 0$.

The points F and G (only applicable to cylindrical gears) limiting the extension of tip relief (so as to maintain a minimum contact ratio of unity for unloaded gears) are at

$$\Gamma_F = \frac{\Gamma_A + \Gamma_B}{2}$$

$$\Gamma_G = \frac{\Gamma_D + \Gamma_E}{2}$$

4.3 Load sharing factor X_Γ

The load sharing factor X_Γ accounts for the load sharing between the various pairs of teeth in mesh along the path of contact.

X_Γ shall be calculated stepwise from A to E, using the parameter Γ_y.

4.3.1 Cylindrical gears with β = 0 and no tip relief

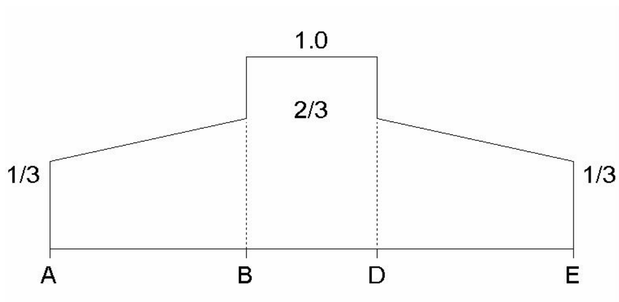


Figure 1

$$X_{\Gamma_y} = \frac{1}{3} + \frac{1}{3}\frac{\Gamma_y - \Gamma_A}{\Gamma_B - \Gamma_A} \qquad \text{for } \Gamma_A \le \Gamma_y < \Gamma_B$$

$$X_{\Gamma_y} = 1 \qquad \text{for } \Gamma_B \le \Gamma_y < \Gamma_D$$

$$X_{\Gamma_y} = \frac{1}{3} + \frac{1}{3}\frac{\Gamma_E - \Gamma_y}{\Gamma_E - \Gamma_D} \text{ for } \Gamma_D < \Gamma_y \le \Gamma_E$$

4.3.2 Cylindrical gears with β = 0 and tip relief

Tip relief on the pinion reduces X_Γ in the range G – E and increases correspondingly X_Γ in the range F – B.

Tip relief on the wheel reduces X_Γ in the range A – F and increases correspondingly X_Γ in the range D – G.

Following remains generally:

$$X_{\Gamma_y} = 1 \qquad \text{for } \Gamma_B \le \Gamma_y \le \Gamma_D$$

$$X_{\Gamma_F} = X_{\Gamma_G} = 1/2$$

In the following it must be distinguished between $C_a < C_{eff}$ respectively $C_a > C_{eff}$. This is shown by an example below where $C_{a1} < C_{eff}$ and $C_{a2} > C_{eff}$.

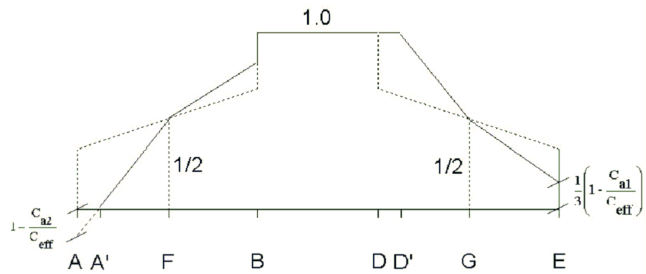


Figure 2

Guidance note:

When C_a > C_{eff} the path of contact is shortened by A – A' respectively E' – E. The single pair contact path is extended into B' respectively D'. If this shift is significant, it is necessary to consider the negative effect on surface durability (B') and bending stresses (B' and D').

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

Range A - F

For $C_{a2} \leq C_{eff}$

$$X_{\Gamma_A} = \frac{1}{3}\left(1 - \frac{C_{a2}}{C_{eff}}\right)$$

$$X_{\Gamma_y} = X_{\Gamma_A} + \frac{\Gamma_y - \Gamma_A}{\Gamma_F - \Gamma_A}\left(\frac{1}{6} + \frac{C_{a2}}{3 \cdot C_{eff}}\right) \text{ for } \Gamma_A \leq \Gamma_y \leq \Gamma_F$$

For $C_{a2} \geq C_{eff}$

$$X_{\Gamma_y} = 0 \qquad \text{for } \Gamma_A \leq \Gamma_y \leq \Gamma_{A'}$$

with $\Gamma_{A'} = \Gamma_A + (\Gamma_F - \Gamma_A)\dfrac{\dfrac{C_{a2}}{C_{eff}} - 1}{\dfrac{C_{a2}}{C_{eff}} - \dfrac{1}{2}}$

$$X_{\Gamma_y} = 1 - \frac{C_{a2}}{C_{eff}} + \frac{\Gamma_y - \Gamma_A}{\Gamma_F - \Gamma_A}\left(\frac{C_{a2}}{C_{eff}} - \frac{1}{2}\right) \text{ for } \Gamma_{A'} \leq \Gamma_y \leq \Gamma_F$$

Range F - B

For $C_{a1} \le C_{eff}$

$$X_{\Gamma_y} = \frac{1}{2} + \frac{\Gamma_y - \Gamma_F}{\Gamma_B - \Gamma_F}\left(\frac{1}{6} + \frac{C_{a1}}{3 \cdot C_{eff}}\right) \qquad \text{for } \Gamma_F \le \Gamma_y \le \Gamma_B$$

For $C_{a1} \ge C_{eff}$

$$X_{\Gamma_y} = \frac{1}{2} + \frac{\Gamma_y - \Gamma_F}{\Gamma_B - \Gamma_F}\left(\frac{C_{a1}}{C_{eff}} - \frac{1}{2}\right) \qquad \text{for } \Gamma_F \le \Gamma_y \le \Gamma_{B'}$$

$$X_{\Gamma_y} = 1 \qquad \text{for } \Gamma_{B'} \le \Gamma_y \le \Gamma_B$$

with $\Gamma_{B'} = \Gamma_F + \dfrac{\Gamma_B - \Gamma_F}{2\dfrac{C_{a1}}{C_{eff}} - 1}$

Range D – G

For $C_{a2} \le C_{eff}$

$$X_{\Gamma_y} = \frac{2}{3} + \frac{C_{a2}}{3\,C_{eff}} - \frac{\Gamma_y - \Gamma_D}{\Gamma_G - \Gamma_D}\left(\frac{1}{6} + \frac{C_{a2}}{3\,C_{eff}}\right) \text{for } \Gamma_D \le \Gamma_y \le \Gamma_G$$

For $C_{a2} \ge C_{eff}$

$$X_{\Gamma_y} = 1 \qquad \text{for } \Gamma_D \le \Gamma_y \le \Gamma_{D'}$$

with $\Gamma_{D'} = \Gamma_D + \left(\Gamma_G - \Gamma_D\right)\dfrac{\dfrac{C_{a2}}{C_{eff}} - 1}{\dfrac{C_{a2}}{C_{eff}} - \dfrac{1}{2}}$

$$X_{\Gamma} = \frac{C_{a2}}{C_{eff}} - \frac{\Gamma_y - \Gamma_D}{\Gamma_G - \Gamma_D}\left(\frac{C_{a2}}{C_{eff}} - \frac{1}{2}\right) \qquad \text{for } \Gamma_{D'} \le \Gamma_y \le \Gamma_G$$

Range G - E

For $C_{a1} \le C_{eff}$

$$X_{\Gamma_E} = \frac{1}{3}\left(1 - \frac{C_{a1}}{C_{eff}}\right)$$

$$X_{\Gamma_y} = \frac{1}{2} - \frac{\Gamma_y - \Gamma_G}{\Gamma_E - \Gamma_G}\left(\frac{1}{6} + \frac{C_{a1}}{3 \cdot C_{eff}}\right) \qquad \text{for } \Gamma_G \le \Gamma_y \le \Gamma_E$$

For $C_{a1} > C_{eff}$

$$X_{\Gamma_y} = \frac{1}{2} - \frac{\Gamma_y - \Gamma_G}{\Gamma_E - \Gamma_G}\left(\frac{C_{a1}}{C_{eff}} - \frac{1}{2}\right) \qquad \text{for } \Gamma_G \le \Gamma_y \le \Gamma_{E'}$$

$$X_{\Gamma_y} = 0 \qquad \text{for } \Gamma_{E'} \le \Gamma_y \le \Gamma_E$$

with $\Gamma_{E'} = \Gamma_G + \dfrac{\Gamma_E - \Gamma_G}{2\dfrac{C_{a1}}{C_{eff}} - 1}$

4.3.3 Gears with β > 0, buttressing

Due to oblique contact lines over the flanks a certain buttressing may occur near A and E.

This applies to both cylindrical and bevel gears with tip relief < C_{eff}. The buttressing X_{butt} is simplified as a linear function within the ranges A – H respectively I – E.

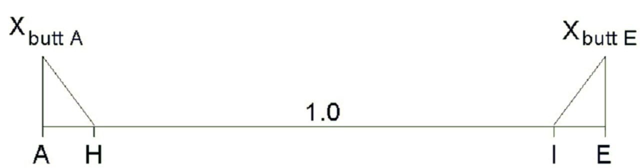


Figure 3

$$X_{butt_{A,E}} = 1.3 \quad \text{when } \varepsilon_\beta \geq 1$$

$$X_{butt_{A,E}} = 1 + 0.3\,\varepsilon_\beta \quad \text{when } \varepsilon_\beta < 1$$

Cylindrical gears

$$\Gamma_H - \Gamma_A = \Gamma_E - \Gamma_I = 0.2 \sin \beta_b$$

Bevel gears

$$\Gamma_H - \Gamma_A = \Gamma_E - \Gamma_I = 0.2 \sin \beta_{bm}$$

4.3.4 Cylindrical gears with $\varepsilon_\gamma \leq 2$ and no tip relief

X_{Γ_y} is obtained by multiplication of X_{Γ_y} in [4.3.1] with X_{butt} in [4.3.3].

4.3.5 Gears with $\varepsilon_\gamma > 2$ and no tip relief

Applicable to both cylindrical and bevel gears.

$$X_{\Gamma_y} = \frac{1}{\varepsilon_\alpha} \qquad \text{for } \Gamma_H \leq \Gamma_y \leq \Gamma_I$$

$$X_{\Gamma_y} = \frac{1}{\varepsilon_\alpha} X_{butt} \qquad \text{for } \Gamma_y < \Gamma_H \text{ and } \Gamma_y > \Gamma_I$$

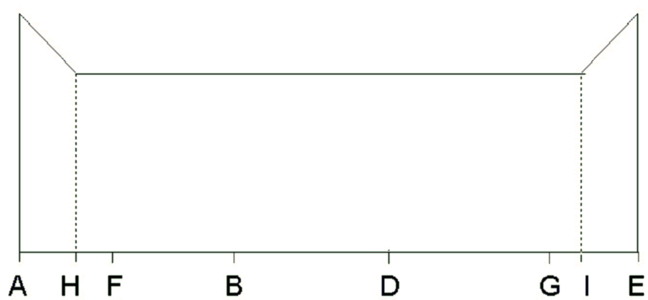


Figure 4

4.3.6 Cylindrical gears with $\varepsilon_\gamma \leq 2$ and tip relief

X_{Γ_y} is obtained by multiplication of X_{Γ_y} in [4.3.2] with X_{butt} in [4.3.3].

4.3.7 Cylindrical gears with $\varepsilon_\gamma > 2$ and tip relief

Tip relief on the pinion (respectively wheel) reduces X_{Γ_y} in the range G – E (respectively A – F) and increases X_{Γ_y} in the range F – G.

X_{Γ_y} is obtained by multiplication of X_{Γ_y} as described below with X_{butt} in [4.3.3].

In the X_Γ example below the influence of tip relief is shown (without the influence of X_{butt}) by means of .

$$C_{a1} > C_{eff} \text{ and } C_{a2} < C_{eff}$$

Tip relief > C_{eff} causes new end points A' respectively E' of the path of contact.

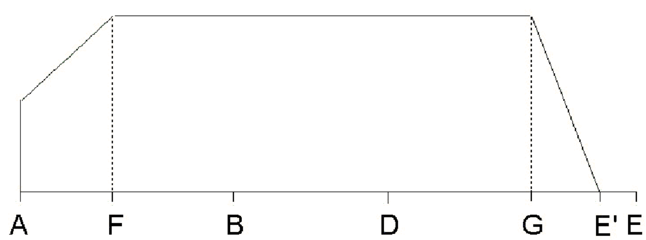


Figure 5

Range A – F

$$X_{\Gamma_y} = \frac{C_{eff} - C_{a2}}{\varepsilon_\alpha \, C_{eff}} + \frac{\Gamma_y - \Gamma_A}{\Gamma_F - \Gamma_A} \, \frac{(\varepsilon_\alpha - 1) C_{a1} + (3\varepsilon_\alpha + 1) C_{a2}}{2\varepsilon_\alpha (\varepsilon_\alpha + 1) C_{eff}}$$

for $\Gamma_A \le \Gamma_y \le \Gamma_F$ if $C_{a2} \le C_{eff}$

and for $\Gamma_{A'} \le \Gamma_y \le \Gamma_F$ if $C_{a2} \ge C_{eff}$

$$X_{\Gamma_y} = 0 \qquad \text{for } \Gamma_A \le \Gamma_y \le \Gamma_{A'} \qquad \text{if } C_{a2} > C_{eff}$$

with $\Gamma_{A'} = \Gamma_A + (\Gamma_F - \Gamma_A) \dfrac{(C_{a2} - C_{eff}) \, 2 (\varepsilon_\alpha + 1)}{(\varepsilon_\alpha - 1) C_{a1} + (3\varepsilon_\alpha + 1) C_{a2}}$

Range F – G

$$X_{\Gamma_y} = \frac{1}{\varepsilon_\alpha} + \frac{(\varepsilon_\alpha - 1)(C_{a1} + C_{a2})}{2\varepsilon_\alpha (\varepsilon_\alpha + 1) C_{eff}} \qquad \text{for } \Gamma_F \le \Gamma_y \le \Gamma_G$$

Range G – E

$$X_{\Gamma_y} = X_{\Gamma_{F-G}} - \frac{\Gamma_y - \Gamma_G}{\Gamma_E - \Gamma_G} \frac{(3\,\varepsilon_\alpha + 1) C_{a1} + (\varepsilon_\alpha - 1) C_{a2}}{2\,\varepsilon_\alpha (\varepsilon_\alpha + 1) C_{eff}}$$

for $\Gamma_G \le \Gamma_y \le \Gamma_E$ if $C_{a1} \le C_{eff}$

and for $\Gamma_G \le \Gamma_y \le \Gamma_{E'}$ if $C_{a1} \ge C_{eff}$

$$X_{\Gamma_y} = 0 \quad \text{for } \Gamma_{E'} \le \Gamma_y \le \Gamma_E \quad \text{if } C_{a1} \ge C_{eff}$$

with $\Gamma_{E'} = \Gamma_E - (\Gamma_E - \Gamma_G) \dfrac{(C_{a1} - C_{eff})\, 2\,(\varepsilon_\alpha + 1)}{(3\,\varepsilon_\alpha + 1) C_{a1} + (\varepsilon_\alpha - 1) C_{a2}}$

4.3.8 Bevel gears with ε_γ more than approx. 1.8 and heightwise crowning

For C_{a1} = C_{a2} = C_{eff} the following applies:

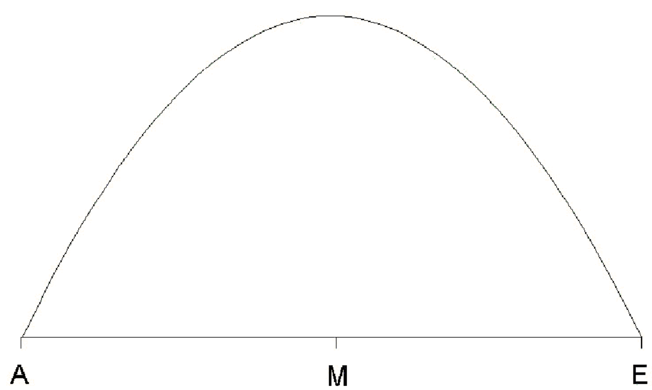


Figure 6

$$\Gamma_M = 0.5(\Gamma_E + \Gamma_A)$$

$$X_{\Gamma_y} = \frac{1.5}{\varepsilon_\alpha} - \frac{6(\Gamma_y - \Gamma_M)^2}{\varepsilon_\alpha^3 (\Gamma_D - \Gamma_A)^2}$$

For tip relief < C_{eff}, X_{Γ_y} is found by linear interpolation between $X_{\Gamma_y(C_a = C_{eff})}$ and $X_{\Gamma_y(C_a = C_0)}$ as in [4.3.5].

The interpolation shall be made stepwise from A to M with the influence of C_{a2} and from M to E with the influence of C_{a1}.(For $C_{a1} \neq C_{a2}$ there is a discontinuity at M.)

For example, with C_{a1} = 0.4 C_{eff} and C_{a2} = 0.55 C_{eff}, then

Range A – M

$$X_{\Gamma_y} = 0.45\, X_{\Gamma_y (C_{a2}=0)} + 0.55\, X_{\Gamma_y (C_{a2}=C_{eff})}$$

Range M – E

$$X_{\Gamma_y} = 0.6\, X_{\Gamma_y (C_{a1}=0)} + 0.4\, X_{\Gamma_y (C_{a1}=C_{eff})}$$

For tip relief > C_{eff} the new end points A' and E' are found as

$$\Gamma_{A'} = \Gamma_A + \frac{\varepsilon_\alpha}{6}\left(\Gamma_D - \Gamma_A\right)\left(\frac{C_{a2}}{C_{eff}} - 1\right)$$

$$\Gamma_{E'} = \Gamma_E + \frac{\varepsilon_\alpha}{6}\left(\Gamma_D - \Gamma_A\right)\left(\frac{C_{a1}}{C_{eff}} - 1\right)$$

Range A – A'

$$X_{\Gamma_y} = 0$$

Range A' – M

$$X_{\Gamma_y} = \frac{1.5}{\varepsilon_\alpha} \frac{3}{4 - \frac{C_{a2}}{C_{eff}}} \left(1 - \frac{\left(\Gamma_y - \Gamma_M\right)^2}{\left(\Gamma_{A'} - \Gamma_M\right)^2}\right)$$

Range M – E'

$$X_{\Gamma_y} = \frac{1.5}{\varepsilon_\alpha} \frac{3}{4 - \frac{C_{a1}}{C_{eff}}} \left(1 - \frac{\left(\Gamma_y - \Gamma_M\right)^2}{\left(\Gamma_{E'} - \Gamma_M\right)^2}\right)$$

Range E' – E

$$X_{\Gamma_y} = 0$$

APPENDIX A FATIGUE DAMAGE ACCUMULATION

The Palmgren-Miner cumulative damage calculation principle is used. The procedure may be applied as follows:

1 Stress spectrum

From the individual torque classes, the torques (T_i) at the peak values of class intervals and the associated number of cycles (N_{Li}) for both pinion and wheel are to be listed from the highest to the lowest torque.

(In case of a cyclic torque variation within the torque classes, it is advised to use the peak torque. If the cyclic variation is such that the same teeth will repeatedly suffer the peak torque, this is a must.)

The stress spectra for tooth roots and flanks (σ_{Fi},σ_{Hi}) with all relevant factors (except K_A) are to be calculated on the basis of the torque spectrum. The load dependent K-factors are to be determined for each torque class.

2 σ–N–curve

The stress versus load cycle curves for tooth roots and flanks (both pinion and wheel) are to be drawn on the basis of *permissible* stresses (i.e. including the demanded minimum safety factors) as determined in 2 respectively 3. If different safety levels for high cycle fatigue and low cycle fatigue are desired, this may be expressed by different demand safety factors applied at the endurance limit respectively at static strength.

3 Damage accumulation

The individual damage ratio D_i at i^{th} stress level is defined as

$$D_i = \frac{N_{Li}}{N_{Fi}}$$

where:

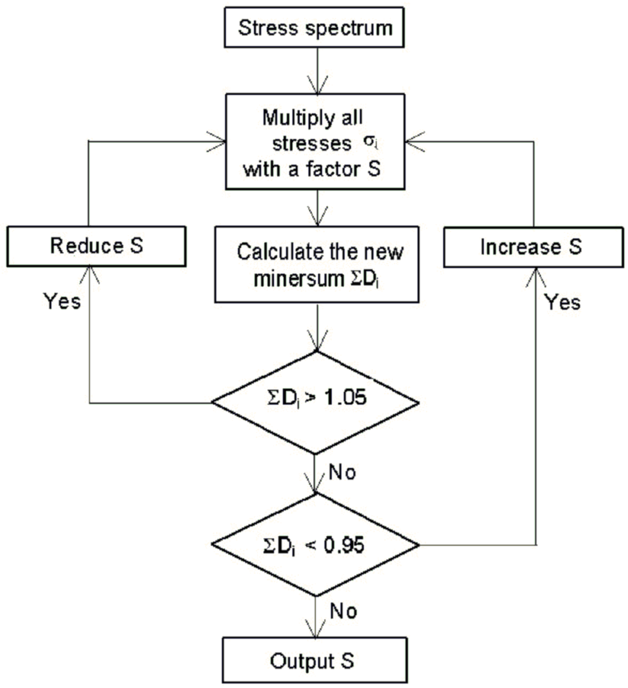
N_{Li}	=	The number of applied cycles at i^{th} stress.
N_{Fi}	=	The number of cycles to failure at i^{th} stress.

Basically stresses σ_i below the permissible stress level for infinite life (if a constant Z_N or Y_N is accepted) do not contribute to the damage sum. However, calculating the actual safety factor S_{act} as described below all the σ_i for which the product S σ_i is bigger than or equal to the permissible stress level for infinite life contribute to the damage sum and thus to the determination of S_{act}. The final value of S is decisive.

(N_{Fi} can be found mathematically by putting the permissible stress σ_{pi} equal the actual stress σ_i, thereby finding the actual life factor. This life factor can be solved with regard to load cycles, i.e. N_{Fi}.)

The damage sum ΣD_i is not to exceed unity.

If $\Sigma D_i \neq 1$, the safety against cumulative fatigue damage is different from the applied demand safety factor. For determination of this theoretical safety factor an iteration procedure is required as described in the following flowchart:



S is correction factor with which the actual safety factor S_{act} can be found.

S_{act} is the demand safety factor (used in determination of the permissible stresses in the σ–N–curve) times the correction factor S.

The full procedure is to be applied for pinion and wheel, tooth roots and flanks.

Guidance note:

If alternating stresses occur in a spectrum of mainly pulsating stresses, the alternating stresses may be replaced by equivalent pulsating stresses, i.e. by means of division with the actual mean stress influence factor Y_M.

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

APPENDIX B APPLICATION FACTORS FOR DIESEL DRIVEN GEARS

For diesel driven gears the application factor K_A depends on torsional vibrations. Both normal operation and misfiring conditions have to be considered.

Normally these two running conditions can be covered by only one calculation.

1 Definitions

$$\text{Normal operation } K_{A\,norm} = \frac{T_0 + T_{v\,norm}}{T_0}$$

where:

T_0	=	rated nominal torque
$T_{v\,norm}$	=	vibratory torque amplitude for normal operation

$$\text{Misfiring operation } K_{A\,misf} = \frac{T + T_{v\,misf}}{T_0}$$

where:

T	=	remaining nominal torque when one cylinder out of action
$T_{v\,misf}$	=	vibratory torque amplitude in misfiring condition. This refers to a *permissible* misfiring condition, i.e. a condition that does not require automatic or immediate corrective actions as speed or pitch reduction.

The normal operation is assumed to last for a very high number of cycles, such as 10^{10}.

The misfiring operation is assumed to last for a limited duration, such as 10^7 cycles.

2 Determination of decisive load

Assuming life factor at 10^{10} cycles as $Y_N = Z_N = 0.92$ which usually is relevant, the calculation may be performed only once with the combination having the highest value of application factor/life factor.

For bending stresses and scuffing, the higher value of

$$\frac{K_{A\,norm}}{0.92} \quad \text{and} \quad \frac{K_{A\,misf}}{0.98}$$

For contact stresses, the higher value of

$$\frac{K_{A\,norm}}{0.92} \quad \text{and} \quad \frac{K_{A\,misf}}{1.13} \quad \left(\text{but } \frac{K_{A\,misf}}{0.97} \text{ for nitrided gears}\right)$$

3 Simplified procedure

Note that this is only a guidance, and is not a binding convention.

$$T = \frac{Z-1}{Z} \cdot T_o \text{ where } Z = \text{number of cylinders}$$

$$T_{V\,norm} = \frac{Z}{24}\left(T_{V\,misf} - T_{V\,ideal}\right) + T_{V\,ideal}$$

where $T_{V\,ideal}$ = vibratory torque with all cylinders perfectly equal.

When using trends from torsional vibration analysis and measurements, the following may be used:

$T_{V\,ideal}$ / T_o is close to zero for engines with few cylinders and using a suitable elastic coupling, and increases with relative coupling stiffness and number of cylinders.

$T_{V\,misf}$ /T_o may be high for engines with few cylinders and decreases with number of cylinders.

This can be indicated as:

$$\frac{T_{V\,ideal}}{T_o} \approx \frac{Z}{200}$$

and

$$\frac{T_{V\,misf}}{T_o} \approx 0.4 - \frac{Z}{80}$$

Inserting this into the formulae for the two application factors, the following guidance can be given:

$K_{A\,misf}$ ≈ 1.12 to 1.18

$K_{A\,norm}$ ≈ 1.10 to 1.15

Since $K_{A\,norm}$ is to be combined with the lower life factors, the decisive load condition will be the normal one, and a K_A of 1.15 will cover most relevant cases, when a suitable elastic coupling is chosen.

APPENDIX C CALCULATION OF PINION-RACK

Pinion-racks used for elevating of mobile offshore units are open gears that are subjected to wear and tear. With normal specifications such as only a few hundred total operation cycles (site to site) the tooth bending stresses for static strength will be decisive for the lay out, however, with exception of surface hardened pinions where case crushing has to be considered.

In the following the use of part 1 and 3 for pinion-racks is shown, including relevant simplifications.

The calculations here only consider the loads due to hoisting / lowering of the legs. In some designs, the gears also take dynamic loads during transit of the vessel. In such cases, the calculations described here must be amended with cumulative fatigue calculation due to the load spectrum during transfer.

1 Pinion tooth root stresses

Since the load spectrum normally is dominated by high torques and few load cycles (in the range up to 1-2000), the static strength is decisive.

The actual stress is calculated as:

$$\sigma_{F1} = \frac{F_t}{b_1 \cdot m_n} \cdot Y_{Fa} \cdot Y_{Sa} \cdot K_{F\beta 1}$$

b_1 is limited to $b_2 + 2 \cdot m_n$.

Y_{Fa} and Y_{Sa} replace Y_F and Y_S because load application at tooth tip has to be assumed for such inaccurate gears.

Pinions often use a non-involute profile in the dedendum part of the flank, e.g. a constant radius equal the radius of curvature at reference circle. For such pinions s_{Fn} and h_{Fa} are to be measured directly on a sectional drawing of the pinion tooth.

Due to high loads and narrow face widths it may be assumed that $K_{F\beta 2} = K_{H\beta} = 1.0$. However, when $b_1 > b_2$, then $K_{F\beta 1} > 1.0$.

If no detailed documentation of $K_{F\beta 1}$ is available, the following may be used:

$K_{F\beta 1} = 1 + 0.15 \cdot (b_1/b_2 - 1)$

The permissible stress (not surface hardened) is calculated as:

$$\sigma_{FP1} = \frac{\sigma_{Fst1}}{S_F} \cdot Y_{\delta relTst}$$

The mean stress influence due to leg lifting may be disregarded.

The actual and permissible stresses should be calculated for the relevant loads as given in the rules.

2 Rack tooth root stresses

The actual stress is calculated as:

$$\sigma_{F2} = \frac{F_t}{b_2 \cdot m_n} \cdot Y_{Fa} \cdot Y_{Sa}$$

See [1] for details.

The permissible stress is calculated as:

$$\sigma_{FP2} = \frac{\sigma_{Fst2}}{S_F} \cdot Y_{\delta relTst}$$

For alloyed steels (Ni, Cr, Mo) with high toughness and ductility the value of $Y_{\delta relTst}$ may be put equal to Y_{Sa}.

3 Surface hardened pinions

For surface hardened pinions the maximum load is not to cause crushing of the hardened layer of the flank.

In principle the calculation described in Sec.2 [13] may be used, but when the theoretical Hertzian stress exceeds the approximately 1.8 times the yield strength of the rack material, plastic deformation will occur. This will limit the peak Hertzian stress but increases the contact width, and thus the penetration of stresses into the depth.

An approximation may be based on an assessment of contact width determined by means of equal areas under the theoretical (elastic range) Hertzian contact and the elastoplastic contact stress (limited to $1.8 \cdot \sigma_y$) with the unknown width.

CHANGES – HISTORIC

December 2015 edition

This is a new document.