

CLASS GUIDELINE

DNVGL-CG-0128

Edition January 2018

Buckling

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FOREWORD

DNV GL class guidelines contain methods, technical requirements, principles and acceptance criteria related to classed objects as referred to from the rules.

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CHANGES - CURRENT

This document supersedes the October 2015 edition of DNVGL-CG-0128

Changes in this document are highlighted in red colour. However, if the changes involve a whole chapter, section or sub-section, normally only the title will be in red colour.

Changes January 2018 entering into force as from date of publication.

Topic	Reference	Description
Improvement of buckling guideline	App.D [2.1.4]	Definition of transverse stress for stiffener buckling application has been added in line with common practice.
	Sec.3 [5.6]	The parameter c has been corrected and the definition of end constraint factor, f_end, has been clarified.
	Sec.3 Table 3	The definition of equivalent plate geometry for plate buckling assessment in way of unstiffened openings has been revised and collected in a new Note 2 to Table 3 . For plate buckling case 1, the missing value for buckling factor K_x for stress ratio between -1 and 0 has been added.
	App.E	A new appendix has been made to describe in detail the hull girder ultimate capacity by multi-step method. This appendix is based on CSR Pt.1 Ch.5 App.2.

Editorial corrections

In addition to the above stated changes, editorial corrections may have been made.

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SECTION 1 INTRODUCTION

1 Objective

The present guideline gives methods and principles applicable for the assessment of buckling and ultimate strength limits (ULS) of load carrying members as used in steel ship hulls or similar plated constructions.

The class guidelines application is based on relevant rules for classification of ships.

The purpose of buckling and ultimate strength criteria are to ensure robust design and controlled behaviour of structures subjected to loads as defined in the rules.

2 Buckling methods

Several sets of buckling and ultimate strength methodologies are given

- Closed form method (CFM) ([Sec.3](#))
- Flat stiffened and unstiffened plates; semi-analytical/numerical method PULS ([Sec.4](#))
- Special shell structures ([Sec.3 \[6\]](#)).

Guidance note:

The closed form method (CFM) is based on semi-empirical formulations. The criteria are given as closed cell formulas (CCF).

The PULS code is based on semi-analytical formulations applying a direct approach for assessing the buckling and ultimate strength limits. A direct approach means here that the (equilibrium) equations describing the physical problem are established and a numerical strategy is used for solving them.

The closed form method (CFM) CSR-H and PULS code are partly supplementary. Both codes cover uni-axially stiffened and unstiffened plate, while the Closed Form Method (CFM) also cover pillars, beams, cross-ties, web plates with cut-outs and curved shells. PULS covers orthogonally stiffened plates, irregular stiffened plates and corrugated panels.

The present guideline covers mainly ship shaped structures. Other structural configurations like frames and special shells are also partly covered according to [Table 1](#).

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

Table 1 Structural configuration and buckling method

STRUCTURAL CONFIGURATION	METHOD REF	
	CFM	PULS element/other
PLATES		
Unstiffened panels (e.g. between stiffeners)	Sec.3 [2.2]	U3
Stiffened panels/bottom/deck/ship side/BHD	Sec.3 [2.1]/Sec.3 [2.3]	S3
Girder web plating/floors/stools	Sec.3 [2.4]	U3/S3/T1
Girder web plating with cut-outs	Sec.3 [2.4.2]	NA
Diaphragms/cofferdams	Sec.3 [2.2]	U3/S3
Corrugated bulkheads/panels	Sec.3 [3]	K3/U3
SPS sandwich plating		0154
Composites FRP/GRP plating		PULS Excel FRP
STRUTS, PILLARS AND CROSS TIES	Sec.3 [5]	
BARS AND FRAME STRUCTURES	Sec.3 [5]	

STRUCTURAL CONFIGURATION	METHOD REF	
SHELLS		
Ship bilge/curved panels	Sec.3 [2.2.6]	
Spherical shells/tanks/cylindrical tank end-closures	Sec.3 [6]	

A brief description of main features behind the ultimate strength principles in relation to more design and strength assessments of ship structures is given in [Sec.2](#).

A description of methods and principles using non-linear FEM codes (NLFM) for assessing capacity limits and permanent sets of structures is given in [Sec.6](#).

A general description of hull girder capacity models are given in [Sec.5](#).

3 Assumptions – limitations

The buckling models and general descriptions of methods etc. in the present guideline assume the standard quasi-static formulations, i.e. dynamic load effects are not considered influencing the ultimate capacity. This is a conservative assumption and applies to most normal environmental design loads such as waves, wind and current.

For ship hull components the ultimate load capacity are to be measured against an extreme load representative for the most probable load over the defined rule lifetime of the vessel (most probable over 25 years in North-Atlantic, probability of 10^{-8}).

4 Buckling checks flow chart – link to rules

The rule [DNVGL-RU-SHIP Pt.3 Ch.8 Buckling](#) and [DNVGL-RU-SHIP Pt.3 Ch.5 Hull Girder strength](#) and their link to the present CG are illustrated in flow chart, [Figure 1](#), with a short description on what type of buckling criteria and cases the different rule sections covers.

Buckling strength criteria at four levels are to be complied with ([Figure 1](#)) in addition the stress independent slenderness requirements ([RU SHIP Pt.3 Ch.8 Sec.2](#))

- 1) Prescriptive buckling requirements ([DNVGL-RU-SHIP Pt.3 Ch.8 Sec.3](#)): Local strength of plates and stiffeners subjected to hull girder stresses, i.e. local “panel by panel” checks across hull section.

Guidance note:

The longitudinal strength check applies nominal stresses and is based on nominal hull girder neutral axis.

Net scantlings: The cross-section properties/nominal stress is based on 50% t subtracted and the buckling capacity is based on net scantlings (i.e. 100% t subtracted)

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

- 2) Hull Girder Strength ([DNVGL-RU-SHIP Pt.3 Ch.8](#)).

Sec.1 Hull Girder Yielding Strength

Guidance note:

Global stress check due to vertical and horizontal bending, torsion and shear.

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

Sec.2 Hull Girder Ultimate Strength (HGUS)

Guidance note:

Ultimate moment capacity ($M_d < M_u$) check based summing ultimate capacity all individual elements covering stress redistributions in hull section due to local element buckling and shift in hull section neutral axis. (Single and Multi-step/Smith M_u models).

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

- 3) Partial ship structural analysis ([DNVGL-RU-SHIP Pt.3 Ch.8 Sec.4; 0127 Finite element analysis](#));

Cargo hold analyses by linear FEM. Local buckling checks of stiffeners and plates "panel by panel" subjected to rule hull girder global loads, local cargo loads and lateral sea pressure

Guidance note:

Net scantlings: The buckling capacity is based on net scantlings (i.e. 100% t subtracted)

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

- 4) Global strength analysis ([DNVGL-RU-SHIP Pt.3 Ch.8 Sec.4; 0127 Finite element analysis](#));

Full ship global strength analyses by linear FEM. Local buckling checks of stiffeners and plates "panel by panel" subjected to rule global loads, local cargo loads and sea pressures

Guidance note:

Net scantlings: The buckling capacity is based on net scantlings (i.e. 100% t subtracted)

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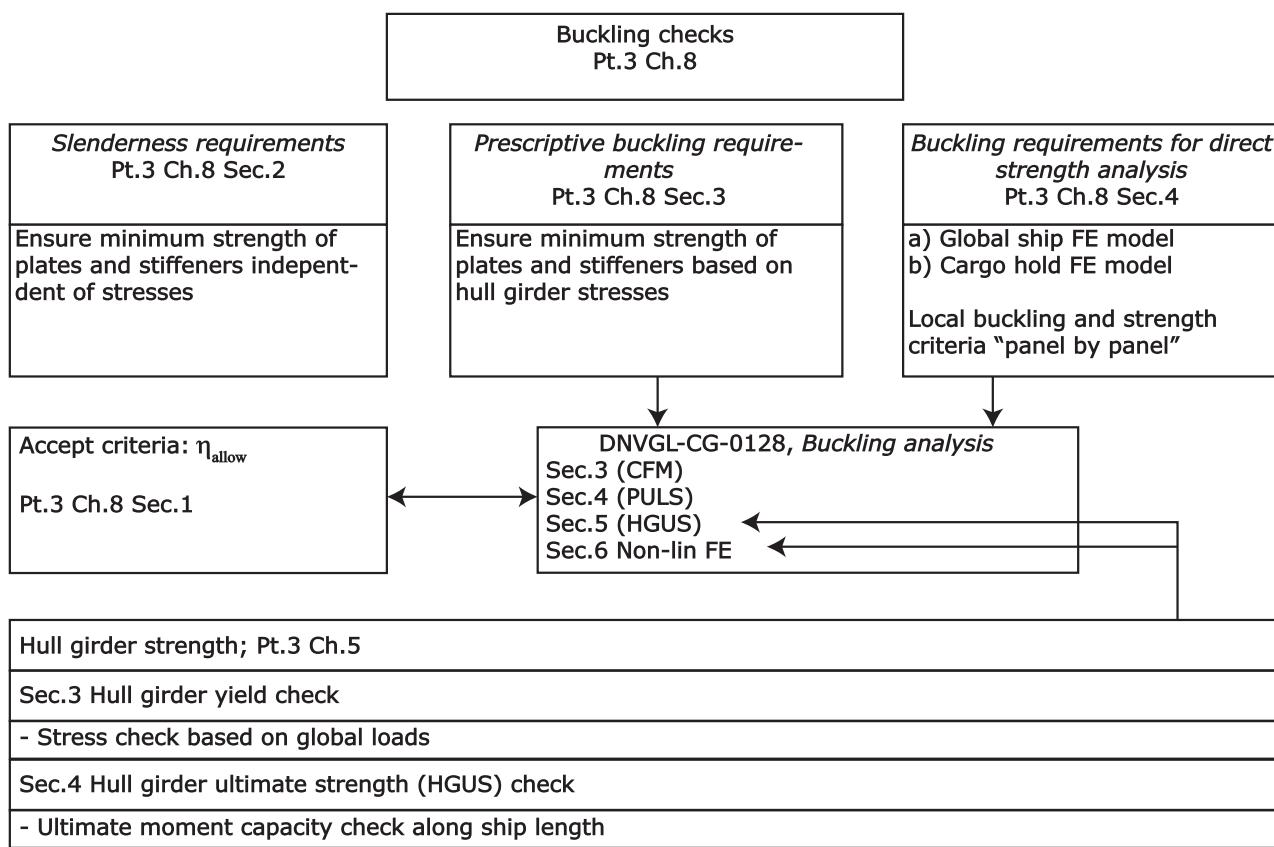


Figure 1 Rule sections on buckling and their links to present class guideline

SECTION 2 ULTIMATE STRENGTH - PRINCIPLES

1 Objectives

The purpose of applying ultimate limit state (ULS) principles in design is to ensure the ship to behave in a controlled manner when subjected to rule design loads and that no structural collapse will occur.

The objective is also to ensure that localized plastic collapse will not take place during the extreme events even though such may not threaten the overall safety and not lead to total collapse of the ship.

2 General principles

2.1 Ultimate strength – plastic buckling

Hull stresses exceeding the ultimate capacity limit of individual structural elements is not accepted as it may lead to major (significant) damages in the form of localized plastic buckles/permanent sets (Figure 1).



Figure 1 Illustrations of local structural collapse/plastic buckling/permanent sets/damages in ship structures

Guidance note:

Marginal and rather localized plastic straining (e.g. plate surface yielding, hot spot straining etc.) will occur at extreme loads and are usually accepted.

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

The ultimate capacity limit used meant to represent characteristic lower bound strength, i.e. it is assumed that the probability of exceedance is in the range of 90% or higher.

Guidance note:

The lower bound strength reflects uncertainties in main parameters such as buckling model approximations, imperfection sensitivity, out-of-flatness as well as material and welding/residual stress characteristics.

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

2.2 Elastic buckling and postbuckling

Elastic buckling limits of structures may be considered a critical limit states depending on the type of structure.

In general elastic buckling of local plate elements in ship structures is not critical due to their redundancy characteristics. Thus large (elastic) deflections are accepted as long as it is ensured that the structure behaves in a controlled manner and its functional and operational requirements are not jeopardized.

Guidance note:

Elastic buckling of plates which are properly supported around the edges display positive post buckling characteristics and loads beyond the eigenvalue can be carried. Thus elastic plate buckling is not a failure mode as such and is acceptable as long as it is ensured the load shedding and stress-redistributions are coped with.

Plates compressed beyond the elastic buckling limit and into the post-buckling range will lose membrane stiffness and the structure will be more flexible than assumed using standard linear methods (reduced "E" modules and Poisson ratio/anisotropic stiffness).

The reduced flexibility in highly compressed areas will lead to load shedding at both a local and overall level.

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

For shell structures of e.g. spherical or cylindrical shapes (LNG tanks), the classical elastic buckling load (eigenvalue) is an upper load limit the shell can carry. That is different from the buckling behaviour of flat plates and implies that elastic buckling is not accepted as it represents the maximum load limit slender structures can carry.

Guidance note:

Elastic buckling of shells will normally show a degree of unstable behaviour with a negative post-buckling characteristic. This means that in particular slender non-perfect shells will display imperfection sensitivity and buckle elastically at a significantly lower load than corresponding to the eigenvalue.

An example of such behaviour for a toro-spherical shell tank end-closure (LNG tank) is shown in [App.B](#).

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

Elastic buckling is a state at which the structure loses its stability and large elastic deflections will start developing rapidly. It is normally associated with the minimum eigenvalue of the perfect structure, i.e. often referred to as classical buckling.

The classical elastic buckling limit may be stable, unstable or neutral with associated load bearing and imperfection sensitivity characteristics as illustrated schematically in [Figure 2](#).

Guidance note:

A stable elastic buckling limit is characterized by a positive post-buckling region, i.e. the structure can carry significantly higher loads than the eigenvalue though at large deflections (e.g. flat plates).

An unstable elastic buckling limit is characterized by a negative post-buckling region, i.e. the load bearing capacity drops below the eigenvalue and deflections grows violently (e.g. cylindrical and spherical shells).

A neutral elastic buckling limit is characterized by a neutral post-buckling region, i.e. the load bearing capacity is neither increasing nor dropping, but stays at the eigenvalue and the deflections grows at an infinitely rate (e.g. pillars, columns and beam-columns).

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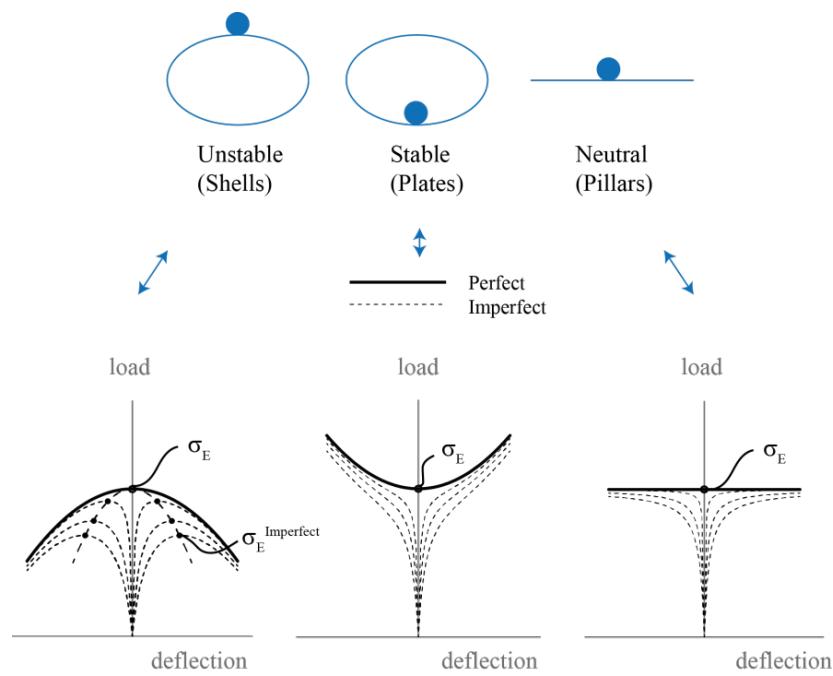


Figure 2 Elastic Stability categorization and load-bearing characteristics, schematically

In relation to buckling it is convenient to define a slenderness parameter λ ;

$$\lambda = \sqrt{\frac{\text{yield}}{\text{elastic buckling}}}$$

A classification of slenderness ranges are:

Slender structures $\lambda > 1.4$

Moderate slender structures $0.6 < \lambda < 1.4$

Stocky structures $0.6 > \lambda$

Guidance note:

The slenderness parameter in the present lambda format is useful as a reference parameter for all type of structures. It gives a measure of the failure being dominant by buckling effects (slender structures $\lambda > 1.4$) or by material yield effects (stocky structures $\lambda < 0.6$).

The slenderness limit $\lambda = 1.4$ is somewhat arbitrary selected but it correspond to the limit used in the well known empirical Johnson-Ostenfeld approach for buckling capacity assessment, beyond which the buckling capacity is set equal to the elastic buckling limit.

For illustration a schematic figure showing the buckling capacity as a function of the slenderness ratio λ is given below (Figure 3). Several design curves are included covering elastic buckling limit (eigenvalue), Johnson-Ostenfeld formula, ultimate capacity limit of plates and squash material yield.

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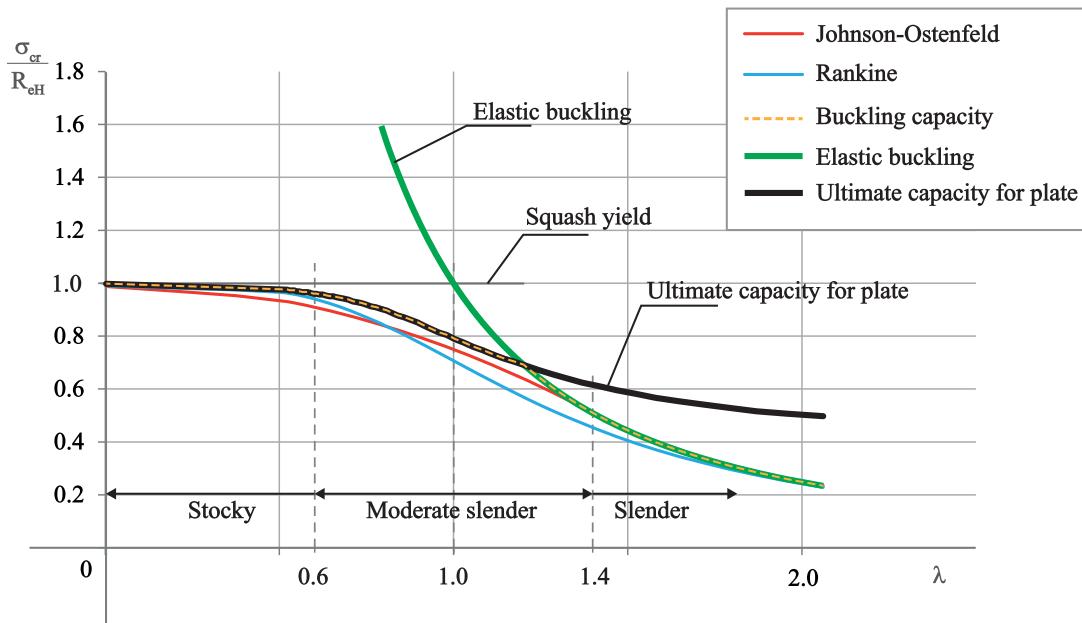


Figure 3 Buckling design curves used for structure as function of slenderness, schematically

2.3 Utilization factor definitions

The actual utilization factor is defined as the ratio between the applied loads and the corresponding buckling limit. The buckling limit is a general term to be understood as the ultimate strength limit unless otherwise specified. It follows that the most general definition is

$$\eta = \frac{\text{load}}{\text{ultimate capacity}}$$

In general a structure is subjected to a set of independent loads, e.g. say three for schematic illustration in 3D load space (P_1, P_2, P_3) e.g. (axial, transverse, shear). A subscript 0 indicates a reference load state which typically will be the rule design loads.

$$\begin{bmatrix} \sigma_{10} \\ \sigma_{20} \\ \sigma_{30} \end{bmatrix}, \begin{bmatrix} P_{10} \\ P_{20} \\ P_{30} \end{bmatrix}; \text{ Reference load state, design Rule loads}$$

A detailed load history is generally not known and the normal approach will be to assume a proportional load history, i.e. all loads are scaled from zero, through the reference state (0) and up to the point where the ultimate capacity (collapse) is identified. The collapse state is given the notation

$$\begin{bmatrix} \sigma_{1u} \\ \sigma_{2u} \\ \sigma_{3u} \end{bmatrix}, \begin{bmatrix} P_{1u} \\ P_{2u} \\ P_{3u} \end{bmatrix}; \text{ Ultimate capacity state, collapse limit for design loads}$$

It follows then that the actual utilization factor in load space is based on the square root measure, i.e.

$$\eta = \frac{\sqrt{\sigma_{10}^2 + \sigma_{20}^2 + \sigma_{30}^2}}{\sqrt{\sigma_{1u}^2 + \sigma_{2u}^2 + \sigma_{3u}^2}} = \frac{1}{\lambda_u} = \frac{W_{act}}{W_u} = \frac{L_0}{L_u} = \frac{1}{\gamma_c}$$

In [Figure 4](#) the present utilization factor concept is illustrated in the 3D (and 2D) load space.

Guidance note:

This definition of utilization factor gives a consistent measure of how far from the collapse limit the structure actually operates. E.g. a utilization factor of 0.67 means that 67% of the capacity is used, while a factor of 1.23 means that it is exceeded by 23%. Increasing the loads by x% gives accordingly x% higher utilization factor etc.

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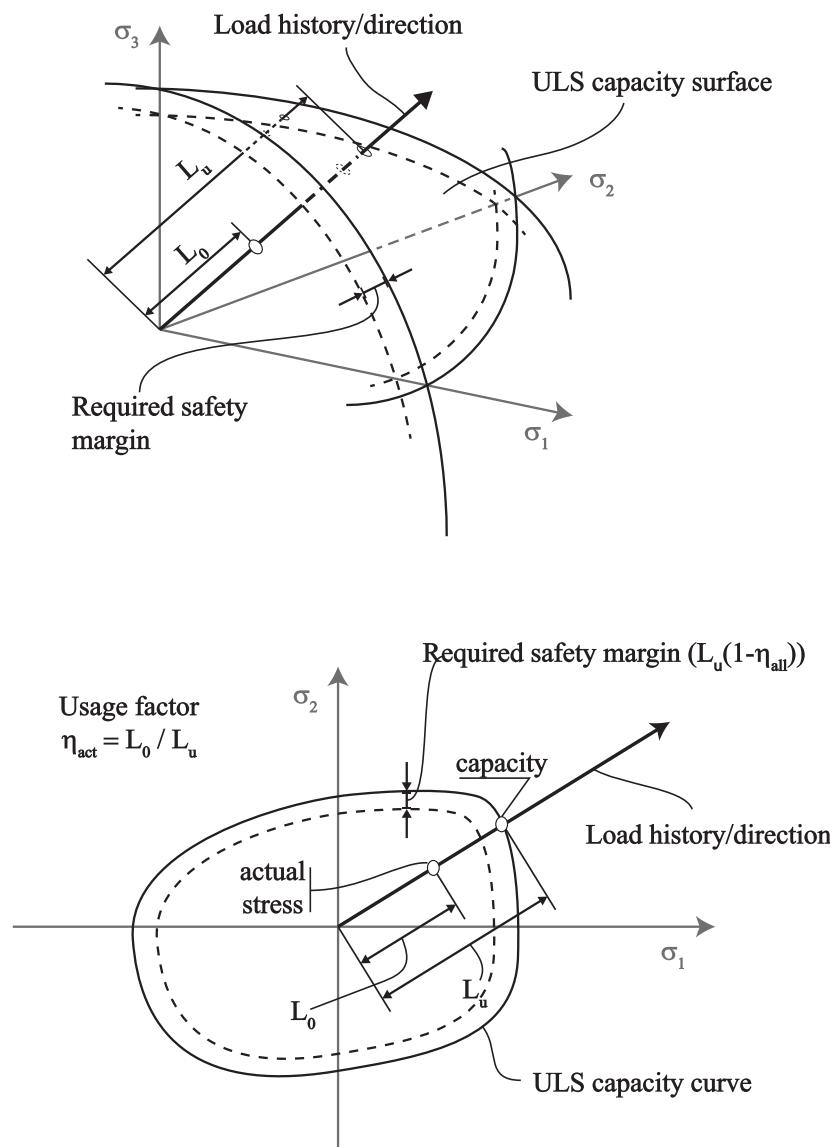


Figure 4 Definition of utilization factors in load space schematically; proportional loading and ULS capacity surface

2.4 Acceptance levels – safety formats

The safety format in a rule context is written generally as

$$\eta \leq \eta_{all}$$

Here η is actual utilization factor defined in [2.3] and η_{all} is the allowable level which is given in the rules, DNVGL-RU-SHIP Pt.3 Ch.8 Sec.1 Table 1. The η_{allow} value includes safety factors as relevant for ship hull structures.

For other structures the acceptance level will be based on a case by case evaluation by the Society.

SECTION 3 CLOSED FORM METHOD (CFM) - BUCKLING CAPACITY

Symbols

For symbols not defined in this section, refer to [DNVGL-RU-SHIP Pt.3 Ch.1 Sec.4](#).

A_s	= net sectional area of the stiffener without attached plating, in mm ²
a	= length of the longer side of the plate panel, in mm
b	= length of the shorter side of the plate panel, in mm
b_{eff}	= effective width of the attached plating of a stiffener, in mm, as defined in [2.3.5]
b_{eff1}	= effective width of the attached plating of a stiffener, in mm, without the shear lag effect taken as: <ul style="list-style-type: none"> — for $\sigma_x > 0$ for prescriptive assessment:
	$b_{eff1} = \frac{C_{x1}b_1 + C_{x2}b_2}{2}$
	for FE analysis:
	$b_{eff1} = C_x b$
	— for $\sigma_x \leq 0$
	$b_{eff1} = b$
b_f	= breadth of the stiffener flange, in mm.
b_1, b_2	= width of plate panel on each side of the considered stiffener, in mm.
C_{x1}, C_{x2}	= reduction factor defined in Table 3 calculated for the EPP1 and EPP2 on each side of the considered stiffener according to case 1.
d	= length of the side parallel to the axis of the cylinder corresponding to the curved plate panel as shown in Table 4 , in mm.
e_f	= distance from attached plating to centre of flange, in mm, as shown in Figure 1 to be taken as: <ul style="list-style-type: none"> $e_f = h_w$ for flat bar profile $e_f = h_w - 0.5 t_f$ for bulb profile $e_f = h_w + 0.5 t_f$ for angle and Tee profiles
F_{long}	= coefficient defined in [2.2.4]
F_{tran}	= coefficient defined in [2.2.5]
h_w	= depth of stiffener web, in mm, as shown in Figure 1
ℓ	= span, in mm, of stiffener equal to spacing between primary supporting members or span of side frame equal to the distance between the hopper tank and top wing tank as defined in DNVGL-RU-SHIP Pt.2 Ch.1 Sec.2 Figure 2
R	= radius of curved plate panel, in mm
R_{eH_P}	= specified minimum yield stress of the plate in N/mm ²
R_{eH_S}	= specified minimum yield stress of the stiffener in N/mm ²
S	= partial safety factor to be taken as: <ul style="list-style-type: none"> — $S = 1.1$ for structures which are exposed to local concentrated loads (e.g. container loads on hatch covers, foundations). — $S = 1.15$ for the following members of General dry cargo ship, Multi-purpose dry cargo ship, Ore carrier or Bulk carrier(without CSR), with freeboard length L_{LL} of not less than 150 m and carrying solid bulk cargoes having a density 1.0 t/m³ and above: stiffeners located on the hatchway coamings, the sloping plate of the topside and hopper

tanks if any, the inner bottom, the inner side if any, the side shell of single side skin construction if any and the top and bottom stools of transverse bulkheads if any.

— $S = 1.0$ for all other cases.

t_p = net thickness of plate panel, in mm

t_w = net stiffener web thickness, in mm

t_f = net flange thickness, in mm

x -axis = local axis of a rectangular buckling panel parallel to its long edge

y -axis = local axis of a rectangular buckling panel perpendicular to its long edge

α = aspect ratio of the plate panel, defined in [Table 3](#) to be taken as:

$$\alpha = \frac{a}{b}$$

β = coefficient taken as:

$$\beta = \frac{1 - \psi}{\alpha}$$

ω = coefficient taken as:

$$\omega = \min(3; \alpha)$$

σ = stress applied on the edge along x axis of the buckling panel, in N/mm²

σ_y = stress applied on the edge along y axis of the buckling panel, in N/mm²

σ_1 = maximum stress, in N/mm²

σ_2 = minimum stress, in N/mm²

σ_E = elastic buckling reference stress, in N/mm² to be taken as:

— for the application of plate limit state according to [\[2.2.1\]](#):

$$\sigma_E = \frac{\pi^2 E}{12(1 - \nu^2)} \left(\frac{t_p}{b} \right)^2$$

— for the application of curved plate panels according to [\[2.2.6\]](#):

$$\sigma_E = \frac{\pi^2 E}{12(1 - \nu^2)} \left(\frac{t_p}{d} \right)^2$$

τ = applied shear stress, in N/mm²

τ_c = buckling strength in shear, in N/mm², as defined in [\[2.2.3\]](#)

ψ = edge stress ratio to be taken as:

$$\psi = \frac{\sigma_2}{\sigma_1}$$

γ = stress multiplier factor acting on loads. When the factor is such that the loads reach the interaction formulae, $\gamma = \gamma_c$.

γ_c = stress multiplier factor at failure.

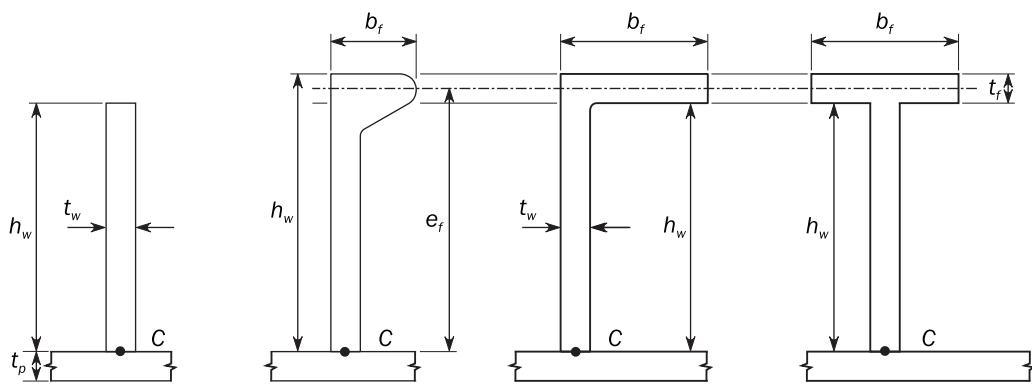


Figure 1 Stiffener cross sections

1 General

1.1 Scope

1.1.1 This section contains the methods for determination of the buckling capacity of plate panels, stiffeners, primary supporting members, struts, pillars, cross ties and corrugated bulkheads.

1.1.2 For the application of this section, the stresses σ_x , σ_y and τ applied on the structural members are defined in:

- DNVGL-RU-SHIP Pt.3 Ch.8 Sec.3 for prescriptive requirements
- DNVGL-RU-SHIP Pt.3 Ch.8 Sec.4 for FE analysis requirements

1.1.3 Ultimate buckling capacity

The ultimate buckling capacity is calculated by applying the actual stress combination and then increasing or decreasing the stresses proportionally until the interaction formulae defined in [2.1.1], [2.2.1], and [2.3.4] are equal to 1.0.

1.1.4 Buckling utilisation factor

The buckling utilisation factor of the structural member is equal to the highest utilisation factor obtained for the different buckling modes.

1.1.5 Lateral pressure

The lateral pressure is to be considered as constant in the buckling strength assessment.

2 Plate and stiffeners

2.1 Overall stiffened panel capacity

2.1.1 The elastic stiffened panel limit state is based on the following interaction formula:

$$\frac{P_z}{c_f} = 1$$

where c_f and P_z are defined in [2.3.4].

2.2 Plate capacity

2.2.1 Plate limit state

Utilization factor for plate buckling is defined as:

$$\eta = \frac{1}{\gamma_c}$$

The plate limit state is based on the following interaction formulae:

$$\begin{aligned} \left(\frac{\gamma_{c1}\sigma_x S}{\sigma_{cx}'} \right)^{e_0} - B \left(\frac{\gamma_{c1}\sigma_x S}{\sigma_{cx}'} \right)^{\frac{e_0}{2}} \left(\frac{\gamma_{c1}\sigma_y S}{\sigma_{cy}'} \right)^{\frac{e_0}{2}} + \left(\frac{\gamma_{c1}\sigma_y S}{\sigma_{cy}'} \right)^{e_0} + \left(\frac{\gamma_{c1}|\tau|S}{\tau_c'} \right)^{e_0} &= 1 \\ \left(\frac{\gamma_{c2}\sigma_x S}{\sigma_{cx}'} \right)^{\frac{2}{0.25}} \beta_p + \left(\frac{\gamma_{c2}|\tau|S}{\tau_c'} \right)^{\frac{2}{0.25}} \beta_p &= 1 \text{ for } \sigma_x \geq 0 \\ \left(\frac{\gamma_{c3}\sigma_y S}{\sigma_{cy}'} \right)^{\frac{2}{0.25}} \beta_p + \left(\frac{\gamma_{c3}|\tau|S}{\tau_c'} \right)^{\frac{2}{0.25}} \beta_p &= 1 \text{ for } \sigma_y \geq 0 \\ \frac{\gamma_{c4}|\tau|S}{\tau_c'} &= 1 \end{aligned}$$

with

$$\gamma_c = \min(\gamma_{c1}, \gamma_{c2}, \gamma_{c3}, \gamma_{c4})$$

where:

σ_x, σ_y	= applied normal stress to the plate panel, in N/mm ² , to be taken as defined in [2.2.7]
τ	= applied shear stress to the plate panel, in N/mm ²
σ_{cx}'	= ultimate buckling stress, in N/mm ² , in direction parallel to the longer edge of the buckling panel as defined in [2.2.3]
σ_{cy}'	= ultimate buckling stress, in N/mm ² , in direction parallel to the shorter edge of the buckling panel as defined in [2.2.3]
τ_c'	= ultimate buckling shear stresses, in N/mm ² , as defined in [2.2.3]
$\gamma_{c1}, \gamma_{c2}, \gamma_{c3}, \gamma_{c4}$	= stress multiplier factors at failure for each of the above different limit states. γ_{c1} and γ_{c2} are only to be considered when $\sigma_x \geq 0$ and $\sigma_y \geq 0$ respectively
B	= coefficient given in Table 1
e_0	= coefficient given in Table 1
β_p	= plate slenderness parameter not to be taken less than: $\beta_p = \frac{b}{t_p} \sqrt{\frac{R_{eH} - P}{E}} \text{ or } 1.0, \text{ whichever is greater.}$

Table 1 Definition of coefficients B and e_0

Applied stress	B	e_0
$\sigma_x \geq 0$ and $\sigma_y \geq 0$	$0.7 - 0.3\beta_p/\alpha^2$	$2/\beta_p^{0.25}$
$\sigma_x < 0$ or $\sigma_y < 0$	1.0	2.0

When a plate under combined axial and shear stresses has different effective plate breadths b , see note 2 in [Table 3](#), calculation of the exponent e_0 (see [Table 1](#)) is to be performed using a plate slenderness parameter β_p based on the larger effective plate breadth b .

2.2.2 Reference degree of slenderness

The reference degree of slenderness is to be taken as:

$$\lambda = \sqrt{\frac{R_{eH_P}}{K\sigma_E}}$$

where:

K = buckling factor, as defined in [Table 3](#) and [Table 4](#).

2.2.3 Ultimate buckling stresses

The ultimate buckling stresses of plate panels, in N/mm², are to be taken as:

$$\sigma_{cx}' = C_x R_{eH_P}$$

$$\sigma_{cy}' = C_y R_{eH_P}$$

The ultimate buckling stress of plate panels subject to shear, in N/mm², is to be taken as:

$$\tau_c' = C_\tau \frac{R_{eH_P}}{\sqrt{3}}$$

where:

C_x, C_y, C_τ = reduction factors, as defined in [Table 3](#).

— for the 1st Equation of [\[2.2.1\]](#), when $\sigma_x < 0$ or $\sigma_y < 0$, the reduction factors are to be taken as:

$$C_x = C_y = C_\tau = 1$$

— for the other cases:

— For SP-A and UP-A, C_y is calculated according to [Table 3](#) by using

$$c_1 = \left(1 - \frac{1}{\alpha}\right) \geq 0$$

— For SP-A and UP-A evaluated with the acceptance criteria AC-III in accident condition, C_y is calculated according to [Table 3](#) by using

$$c_1 = 0$$

— For SP-B and UP-B, C_y is calculated according to [Table 3](#) by using

$$c_1 = 1$$

- For vertically stiffened longitudinal plating with the shorter edges supported by longitudinal plating/decks and located within $0.7z$, from horizontal neutral axis, where z , is the distance from horizontal neutral axis to equivalent deck line or baseline respectively as defined in DNVGL-RU-SHIP Pt.3 Ch.5 Sec.2 [1.6]: C_y is calculated according to [Table 3](#) by using

$$c_1 = \left(1 - \frac{1}{\alpha}\right) \geq 0$$

- For corrugation of corrugated bulkheads, C_y is calculated according to [Table 3](#) by using

$$c_1 = \left(1 - \frac{1}{\alpha}\right) \geq 0$$

The boundary conditions for plates are to be considered as simply supported, see cases 1, 2 and 15 of [Table 3](#). If the boundary conditions differ significantly from simple support, a more appropriate boundary condition can be applied according to the different cases of [Table 3](#) subject to the agreement of the Society.

2.2.4 Correction factor F_{long}

The correction factor F_{long} depending on the edge stiffener types on the longer side of the buckling panel is defined in [Table 2](#). An average value of F_{long} is to be used for plate panels having different edge stiffeners. For stiffener types other than those mentioned in [Table 2](#), the value of c is to be agreed by the Society. In such a case, value of c higher than those mentioned in [Table 2](#) can be used, provided it is verified by buckling strength check of panel using non-linear FE analysis and deemed appropriate by the Society.

Table 2 Correction factor F_{long}

Structural element types		F_{long}	c
Unstiffened Panel		1.0	N/A
Stiffened Panel	Stiffener not fixed at both ends	1.0	N/A
	Flat bar ¹⁾	$F_{long} = c + 1 \text{ for } \frac{t_w}{t_p} > 1$	0.10
	Bulb profile		0.30
	Angle profile	$F_{long} = c\left(\frac{t_w}{t_p}\right)^3 + 1 \text{ for } \frac{t_w}{t_p} \leq 1$	0.40
	T profile		0.30
	Girder of high rigidity (e.g. bottom transverse)	1.4	N/A
Stiffener fixed at both ends	U type profile fitted on hatch cover ²⁾	<ul style="list-style-type: none"> — Plate on which the U type profile is fitted <ul style="list-style-type: none"> — For $b_2 < b_1$: $F_{long} = 1$ — For $b_2 \geq b_1$: $F_{long} = \left(1.55 - 0.55 \frac{b_1}{b_2} \right) \left[1 + c\left(\frac{t_w}{t_p}\right)^3 \right]$ <ul style="list-style-type: none"> — Other plate of the U type profile: $F_{long} = 1$ 	0.2

1) t_w is the net web thickness, in mm, without the correction defined in [2.3.2].

2) b_1 and b_2 are defined in Symbols.

2.2.5 Correction factor F_{tran}

The correction factor F_{tran} is to be taken as:

- For vertically stiffened longitudinal plating with the shorter edges supported by longitudinal plating/decks and located within $0.7z$, from horizontal neutral axis, where z , is the distance from horizontal neutral axis to equivalent deck line or baseline respectively as defined in DNVGL-RU-SHIP Pt.3 Ch.5 Sec.2 [1.6]:
 - $F_{tran} = 1.25$ when the two adjacent frames are supported by one tripping bracket fitted in way of the adjacent plate panels.
 - $F_{tran} = 1.33$ when the two adjacent frames are supported by two tripping brackets each fitted in way of the adjacent plate panels.
 - $F_{tran} = 1.15$ elsewhere.
- For other cases: $F_{tran} = 1$

2.2.6 Curved plate panels

This requirement for curved plate limit state is applicable when $R/t_p \leq 2500$. Otherwise, the requirement for plate limit state given in [2.2.1] is applicable.

Utilization factor for curved plate buckling is defined as:

$$\eta = \frac{1}{\gamma_c}$$

The curved plate limit state is based on the following interaction formula:

$$\left(\frac{\gamma_c \sigma_{ax} S}{C_{ax} R_{eH_P}} \right)^{1.25} - 0.5 \left(\frac{\gamma_c \sigma_{ax} S}{C_{ax} R_{eH_P}} \right) \left(\frac{\gamma_c \sigma_{tg} S}{C_{tg} R_{eH_P}} \right) + \left(\frac{\gamma_c \sigma_{tg} S}{C_{tg} R_{eH_P}} \right)^{1.25} + \left(\frac{\gamma_c \tau \sqrt{3} S}{C_\tau R_{eH_P}} \right)^2 = 1.0$$

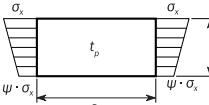
where:

- σ_{ax} = applied axial stress to the cylinder corresponding to the curved plate panel, in N/mm²
In case of tensile axial stresses, $\sigma_{ax} = 0$.
- σ_{tg} = applied tangential stress to the cylinder corresponding to the curved plate panel, in N/mm²
In case of tensile tangential stresses, $\sigma_{tg} = 0$.
- C_{ax}, C_{tg}, C_τ = buckling reduction factor of the curved plate panel, as defined in [Table 4](#).

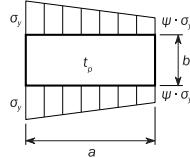
Each term of the above condition shall not exceed 1.0.

The stress multiplier factor, γ_c , of the curved plate panel need not be taken less than the stress multiplier factor, γ_c , for the expanded plate panel according to [\[2.2.1\]](#).

Table 3 Buckling factor and reduction factor for plane plate panels

Case	Stress ratio ψ	Aspect ratio α	Buckling factor K	Reduction factor C
1 	$\psi \leq 0$	$1 \leq \alpha \leq \psi$	$K_x = F_{long} \frac{8.4}{\psi + 1.1}$	<ul style="list-style-type: none"> When $\sigma_x \leq 0$: $C_x = 1$ When $\sigma_x > 0$: $C_x = 1 \text{ for } \lambda \leq \lambda_c$ $C_x = c \left(\frac{1}{\lambda} - \frac{0.22}{\lambda^2} \right) \text{ for } \lambda > \lambda_c$ <p>where: $c = (1.25 - 0.12\psi) \leq 1.25$ $\lambda_c = \frac{c}{2} \left(1 + \sqrt{1 - \frac{0.88}{c}} \right)$</p>
	$0 < \psi \leq 1$		$K_x = F_{long} [7.63 - \psi(6.26 - 10\psi)]$	
	$\psi > 1$		$K_x = F_{long} [5.975(1 - \psi)^2]$	

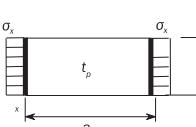
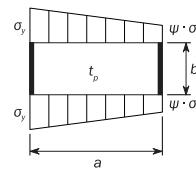
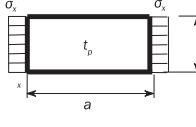
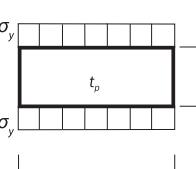
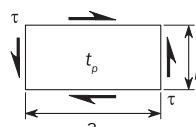
Case	Stress ratio ψ	Aspect ratio α	Buckling factor K	Reduction factor C
	$1 \geq \psi \geq 0$		$K_y = F_{tran} \frac{2 \left(1 + \frac{1}{\alpha^2}\right)^2}{1 + \psi + \frac{(1 - \psi)}{100} \left(\frac{2.4}{\alpha^2} + 6.9f_1\right)}$	<ul style="list-style-type: none"> When $\sigma_y \leq 0$: $C_y = 1$ When $\sigma_y > 0$: $C_y = c \left(\frac{1}{\lambda} - \frac{R + F^2(H - R)}{\lambda^2} \right)$ <p>where: $c = (1.25 - 0.12\psi) \leq 1.25$ $R = \lambda(1 - \lambda/c)$ for $\lambda < \lambda_c$ $R = 0.22$ for $\lambda \geq \lambda_c$ $\lambda_c = 0.5c (1 + \sqrt{1 - 0.88/c})$ $F = \left[1 - \left(\frac{K}{0.91} - 1\right)/\lambda_p^2\right]c_1 \geq 0$ $\lambda_p^2 = \lambda^2 - 0.5$ for $1 \leq \lambda_p^2 \leq 3$ c_1 as defined in [2.2.3]</p> $H = \lambda - \frac{2\lambda}{c(T + \sqrt{T^2 - 4})} \geq R$ $T = \lambda + \frac{14}{15\lambda} + \frac{1}{3}$
2			$K_y = \frac{200F_{tran}(1 + \beta^2)^2}{(1 - f_3)(100 + 2.4\beta^2 + 6.9f_1 + 23f_2)}$	
	$0 > \psi \geq 1 - ((4a)/3)$	$\alpha > 6(1 - \psi)$	$f_1 = 0.6 \left(\frac{1}{\beta} + 14\beta \right)$ but no greater than $14.5 - 0.35\beta^2$ $f_2 = f_3 = 0$	
	$3(1 - \psi) \leq \alpha \leq 6(1 - \psi)$		$f_1 = \frac{1}{\beta} - 1$ $f_2 = f_3 = 0$	

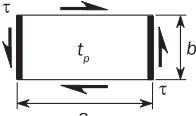
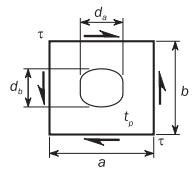
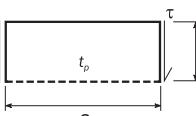
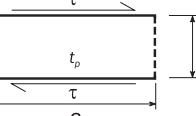


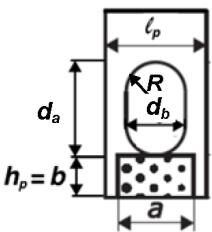
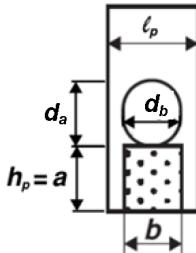
Case	Stress ratio ψ	Aspect ratio α	Buckling factor K	Reduction factor C
		$1.5(1-\psi) \leq \alpha \leq 3(1-\psi)$	$f_1 = \frac{1}{\beta} - (2 - \omega\beta)^4 - 9(\omega\beta - 1)\left(\frac{2}{3} - \beta\right)$ $f_2 = f_3 = 0$	
		$1-\psi \leq \alpha \leq 1.5(1-\psi)$	<ul style="list-style-type: none"> – For $\alpha > 1.5$: $f_1 = 2\left(\frac{1}{\beta} - 16\left(1 - \frac{\omega}{3}\right)^4\right)\left(\frac{1}{\beta} - 1\right)$ $f_2 = 3\beta - 2$ $f_3 = 0$ <ul style="list-style-type: none"> – For $\alpha \leq 1.5$: $f_1 = 2\left(\frac{1.5}{1-\psi} - 1\right)\left(\frac{1}{\beta} - 1\right)$ $f_2 = \frac{\psi(1 - 16f_4^2)}{1 - \alpha}$ $f_3 = 0$ $f_4 = [1.5 - \min(1.5; \alpha)]^2$	
		$0.75(1-\psi) \leq \alpha \leq (1-\psi)$	$f_1 = 0$ $f_2 = 1 + 2.31(\beta - 1) - 48\left(\frac{4}{3} - \beta\right)f_4^2$ $f_3 = 3f_4\left(\beta - 1\right)\left(\frac{f_4}{1.81} - \frac{\alpha - 1}{1.31}\right)$ $f_4 = [1.5 - \min(1.5; \alpha)]^2$	

Case	Stress ratio ψ	Aspect ratio α	Buckling factor K	Reduction factor C
	$\psi < 1 - ((4\alpha)/3)$		$K_y = 5.972 F_{tran} \frac{\beta^2}{1 - f_3}$ <p style="text-align: center;">where:</p> $f_3 = f_5 \left(\frac{f_5}{1.81} + \frac{1 + 3\psi}{5.24} \right)$ $f_5 = \frac{9}{16} [1 + \max(-1; \psi)]^2$	
3	$\psi \approx 0$	2)		
	$\psi \approx 1$		$K_x = \frac{4 \left(0.425 + \frac{1}{\alpha^2} \right)}{3\psi + 1}$	
4	$\psi \approx 0$	2)		$C_x = 1 \text{ for } \lambda \leq 0.7$ $C_x = \frac{1}{\lambda^2 + 0.51} \text{ for } \lambda > 0.7$
	$\psi \approx 1$		$K_x = \left(0.425 + \frac{1}{\alpha^2} \right) \frac{3 - \psi}{2}$	
5		$\alpha \geq 1.64$	$K_x = 1.28$	
		$\alpha < 1.64$	$K_x = \frac{1}{\alpha^2} + 0.56 + 0.13\alpha^2$	

Case	Stress ratio ψ	Aspect ratio α	Buckling factor K	Reduction factor C
6	$\psi \geq 0$	$\alpha \leq 1$	3) $K_y = \frac{4(0.425 + \alpha^2)}{(3\psi + 1)\alpha^2}$	
	$\psi > 0$	$\alpha > 1$	3) $K_y = 4(0.425 + \alpha^2)(1 + \psi) \frac{1}{\alpha^2} - 5\psi(1 - 3.42\psi) \frac{1}{\alpha^2}$	
7	$\psi \geq 1$	$\alpha \leq 1$		$C_x = 1 \text{ for } \lambda \leq 0.7$ $C_x = \frac{1}{\lambda^2 + 0.51} \text{ for } \lambda > 0.7$
			$K_y = (0.425 + \alpha^2) \frac{(3 - \psi)}{2\alpha^2}$	
8		-	$K_y = 1 + \frac{0.56}{\alpha^2} + \frac{0.13}{\alpha^4}$	
9		-	$K_x = 6.97$	$C_x = 1 \text{ for } \lambda \leq 0.83$ $C_x = 1.13 \left(\frac{1}{\lambda} - \frac{0.22}{\lambda^2} \right)$ for $\lambda > 0.83$
10		-	$K_y = 4 + \frac{2.07}{\alpha^2} + \frac{0.67}{\alpha^4}$	$C_y = 1 \text{ for } \lambda \leq 0.83$ $C_y = 1.13 \left(\frac{1}{\lambda} - \frac{0.22}{\lambda^2} \right)$ for $\lambda > 0.83$

Case	Stress ratio ψ	Aspect ratio α	Buckling factor K	Reduction factor C
11 	-	$\alpha \geq 4$	$K_x = 4$	$C_x = 1 \text{ for } \lambda \leq 0.83$ $C_x = 1.13 \left(\frac{1}{\lambda} - \frac{0.22}{\lambda^2} \right)$ for $\lambda > 0.83$
		$\alpha < 4$	$K_x = 4 + 2.74 \left(\frac{4-\alpha}{3} \right)^4$	
12 	-		$K_y = K_y$ determined as per case 2	<ul style="list-style-type: none"> - For $\alpha < 2$: $C_y = C_{y2}$ - For $\alpha \geq 2$: $C_y = \left(1.06 + \frac{1}{10\alpha} \right) C_{y2}$ <p>where:</p> <p>$C_{y2} = C_y$ determined as per case 2</p>
13 	-	$\alpha \geq 4$	$K_x = 6.97$	$C_x = 1 \text{ for } \lambda \leq 0.83$ $C_x = 1.13 \left(\frac{1}{\lambda} - \frac{0.22}{\lambda^2} \right)$ for $\lambda > 0.83$
		$\alpha < 4$	$K_x = 6.97 + 3.1 \left(\frac{4-\alpha}{3} \right)^4$	
14 	-		$K_y = \frac{6.97}{\alpha^2} + \frac{3.1}{\alpha^2} \left(\frac{4 - \frac{1}{\alpha}}{3} \right)^4$	$C_x = 1 \text{ for } \lambda \leq 0.83$ $C_x = 1.13 \left(\frac{1}{\lambda} - \frac{0.22}{\lambda^2} \right)$ for $\lambda > 0.83$
15 	-		$K_\tau = \sqrt{3} \left(5.34 + \frac{4}{\alpha^2} \right)$	$C_\tau = 1 \text{ for } \lambda \leq 0.84$ $C_\tau = \frac{0.84}{\lambda} \text{ for } \lambda > 0.84$

Case	Stress ratio ψ	Aspect ratio α	Buckling factor K	Reduction factor C
16		-	$K_\tau = \sqrt{3} \left[5.34 + \max\left(\frac{4}{\alpha^2}; \frac{7.15}{\alpha^{2.5}}\right) \right]$	
17		-	$K_\tau = K_{\tau \text{ case 11}} r$ <p> $K_{\tau \text{ case 11}}$ = K_τ according to case 11 r = opening reduction factor taken as: $r = \left(1 - \frac{d_a}{a}\right) \left(1 - \frac{d_b}{b}\right)$ with $\frac{d_a}{a} \leq 0.7 \text{ and } \frac{d_b}{b} \leq 0.7$ </p>	
18		-	4) $K_\tau = 3^{0.5} \left(0.6 + \frac{4}{\alpha^2} \right)$	$C_\tau = 1 \text{ for } \lambda \leq 0.84$ $C_\tau = \frac{0.84}{\lambda} \text{ for } \lambda > 0.84$
19		-	5) $K_\tau = 8$	
Edge boundary conditions:				
----- Plate edge free. _____ Plate edge simply supported. _____ Plate edge clamped.				
Note 1: Cases listed are general cases. Each stress component (σ_x, σ_y) is to be understood in local coordinates.				

Case	Stress ratio ψ	Aspect ratio α	Buckling factor K	Reduction factor C
Note 2: Unsupported edge of plane plate panels in way of openings				
If the length of a regular opening is longer than half the width of the plate panel ($d_a > 0.5 \ell_p$, $l_0 > 0.5 l_p$, the length of the unsupported edge of a plane plate panel in way of a regular opening may be shortened for the buckling assessment as follows:				
 <p>Case 3 ($l_\sigma = a$) and Case 18 ($l_t = a$)</p>  <p>Case 6 ($l_\sigma = b$) and Case 19 ($l_t = b$)</p>				
d_b = width, in mm, of the opening d_a = height, in mm, of the opening ℓ_p = width, in mm, of plate panel h_p = height, in mm, of plate panel R = radius, in mm of opening corrected radius, in mm, of opening to be taken as: R_c = $R_c = \max\left(d_b - \frac{4}{3}R; \frac{2}{9}\ell_p\right) \quad (1)$				
For case 3 and 6, normal stresses:				
Corrected length of free edge, in mm, to be taken as:				
$\ell_\sigma = \min\left(\ell_p; 100 \frac{R_c}{\ell_p} t_p \sqrt{\frac{2h_p}{d_a} + 1}\right)$				
If $\ell_\sigma \geq h_p$ --> Case 3 is to be applied with $a = \ell_\sigma$, $b = h_p$ and σ_x as normal stress				
If $\ell_\sigma < h_p$ --> Case 6 is to be applied with $a = h_p$, $b = \ell_\sigma$ and σ_y as normal stress				
For small openings, when the reduction factor for case 3, C_x , or the reduction factor for case 6, C_y , exceeds the reduction factor for the plate without opening, C , for case 2 is applicable.				

For cases 18 and 19, shear stresses:

Corrected length of free edge, in mm, to be taken as:

$$\ell_\tau = \min\left(\ell_p; 2.1R_c \sqrt{\frac{2h_p}{d_a} + 1}\right)$$

If $\ell_\tau \geq h_p$ --> Case 18 is to be applied with $a = \ell_\tau$, $b = h_p$ and τ as shear stress

If $\ell_\tau < h_p$ --> Case 19 is to be applied with $a = h_p$, $b = \ell_\tau$ and τ as shear stress

For small openings, the reduction factor C_τ for Case 17 is applicable.

Table 4 Buckling factor and reduction factor for curved plate panel with $R_0/t_p \leq 2500$

Case	Aspect ratio	Buckling factor K	Reduction factor C
1	$\frac{d}{R_0} \leq 0.5\sqrt{\frac{R_0}{t_p}}$	$K_{ax} = 1 + \frac{2}{3} \frac{d^2}{R_0 t_p}$	For general application: $C_{ax} = 1$ for $\lambda \leq 0.25$ $C_{ax} = 1.233 - 0.933\lambda$ for $0.25 < \lambda \leq 1$ $C_{ax} = \frac{0.3}{\lambda^3}$ for $1 < \lambda \leq 1.5$ $C_{ax} = \frac{0.2}{\lambda^2}$ for $\lambda > 1.5$
	$\frac{d}{R_0} > 0.5\sqrt{\frac{R_0}{t_p}}$	$K_{ax} = 0.267 \frac{d^2}{R_0 t_p} \left[3 - \frac{d}{R_0} \sqrt{\frac{t_p}{R_0}} \right] \geq 0.4 \frac{d^2}{R_0 t_p}$	For curved single fields, e.g. bilge strake, which are bounded by plane panels as shown in DNVGL-RU-SHIP Pt.3 Ch.6 Sec.4 Figure 1 : $C_{ax} = \frac{0.65}{\lambda^2} \leq 1.0$

Case	Aspect ratio	Buckling factor K	Reduction factor C
2a	$\frac{d}{R_0} \leq 1.63\sqrt{\frac{R_0}{t_p}}$	$K_{tg} = \frac{d}{\sqrt{R_0 t_p}} + 3 \frac{(R_0 t_p)^{0.175}}{d^{0.35}}$	For general application: $C_{tg} = 1$ for $\lambda \leq 0.4$ $C_{tg} = 1.274 - 0.686\lambda$ for $0.4 < \lambda \leq 1.2$ $C_{tg} = \frac{0.65}{\lambda^2}$ for $\lambda > 1.2$ For curved single fields, e.g. bilge strake, which are bounded by plane panels as shown in DNVGL-RU-SHIP Pt.3 Ch.6 Sec.4 Figure 1:
2b	$\frac{d}{R_0} > 1.63\sqrt{\frac{R_0}{t_p}}$	$K_{tg} = 0.3 \frac{d^2}{R_0^2} + 2.25 \left(\frac{R_0}{dt_p} \right)^2$	$C_{tg} = \frac{0.8}{\lambda^2} \leq 1.0$
			$p_e = \text{external pressure in } [\text{N/mm}^2]$
3	$\frac{d}{R_0} \leq \sqrt{\frac{R_0}{t_p}}$	$K_{tg} = \frac{0.6d}{\sqrt{R_0 t_p}} + \frac{\sqrt{R_0 t_p}}{d} - 0.3 \frac{R_0 t_p}{d^2}$	As in load case 2a.
	$\frac{d}{R_0} > \sqrt{\frac{R_0}{t_p}}$	$K_{tg} = 0.3 \frac{d^2}{R_0^2} + 0.291 \left(\frac{R_0}{dt_p} \right)^2$	
4	$\frac{d}{R_0} \leq 8.7\sqrt{\frac{R_0}{t_p}}$	$K_\tau = \sqrt{3} \sqrt{28.3 + \frac{0.67d^3}{R_0^{1.5} t_p^{1.5}}}$	$C_\tau = 1$ for $\lambda \leq 0.4$ $C_\tau = 1.274 - 0.686\lambda$ for $0.4 < \lambda \leq 1.2$ $C_\tau = \frac{0.65}{\lambda^2}$ for $\lambda > 1.2$
	$\frac{d}{R_0} > 8.7\sqrt{\frac{R_0}{t_p}}$	$K_\tau = \sqrt{3} \frac{0.28d^2}{R_0 \sqrt{R_0 t_p}}$	
Explanations for boundary conditions:			
<ul style="list-style-type: none"> - - - - Plate edge free. — Plate edge simply supported. — Plate edge clamped. 			

2.2.7 Applied normal stress to plate panel

The normal stress, σ_x and σ_y , in N/mm², to be applied for the plate panel capacity calculation as given in [2.2.1] shall be taken as follows:

- For FE analysis, the reference stresses as defined in [DNVGL-RU-SHIP Pt.3 Ch.8 Sec.4](#).
- For prescriptive assessment, the axial or transverse compressive stresses calculated according to [DNVGL-RU-SHIP Pt.3 Ch.8 Sec.3 \[2.2.1\]](#), at load calculation points of the considered elementary plate panel, as defined in [DNVGL-RUSHIP Pt.3 Ch.3 Sec.7 \[2\]](#).
- For grillage analysis where the stresses are obtained based on beam theory, the stresses shall be taken as:

- If $\sigma_{xb} \leq 0$:

$$\sigma_x = \frac{\sigma_{xb} + v \cdot \sigma_{yb}}{1 - v^2} \quad \text{and} \quad \sigma_y = \sigma_{yb}$$

- If $\sigma_{yb} \leq 0$:

$$\sigma_x = \sigma_{xb} \quad \text{and} \quad \sigma_y = \frac{\sigma_{yb} + v \cdot \sigma_{xb}}{1 - v^2}$$

- In other cases:

$$\sigma_x = \frac{\sigma_{xb} + v \cdot \sigma_{yb}}{1 - v^2}$$

$$\sigma_y = \frac{\sigma_{yb} + v \cdot \sigma_{xb}}{1 - v^2}$$

where:

σ_{xb}, σ_{yb} = stress, in N/mm², from grillage beam analysis respectively along x or y axis of the attached buckling panel.

The shear stress τ , in N/mm², to be applied for the plate panel capacity calculation as given in [\[2.2.1\]](#) shall be taken as follows:

- For FE analysis, the reference shear stresses as defined in [DNVGL-RU-SHIP Pt.3 Ch.8 Sec.4 \[2.4\]](#).
- For prescriptive assessment, the shear stresses calculated according to [DNVGL-RU-SHIP Pt.3 Ch.8 Sec.4 \[2.2.1\]](#), at load calculation points of the considered elementary plate panel, as defined in [DNVGL-RUSHIP Pt.3 Ch.3 Sec.7 \[2\]](#).
- For grillage beam analysis, $\tau = 0$ in the attached buckling panel.

2.3 Stiffeners

2.3.1 Buckling modes

The following buckling modes shall be checked:

- Stiffener induced failure (SI).
- Associated plate induced failure (PI).

2.3.2 Web thickness of flat bar

For accounting the decrease of the stiffness due to local lateral deformation, the effective web thickness of flat bar stiffener, in mm, is to be used in [2.3.4] for the calculation of the net sectional area, A_s , the net section modulus, Z , and the moment of inertia, I , of the stiffener and is taken as:

$$t_{w_red} = t_w \left[1 - \frac{2\pi^2}{3} \left(\frac{h_w}{s} \right)^2 \left(1 - \frac{b_{eff1}}{s} \right) \right]$$

2.3.3 Idealisation of bulb profile

Bulb profiles shall be considered as equivalent angle profiles, as defined in DNVGL-RU-SHIP Pt.3 Ch.3 Sec.7 [1.4.1].

2.3.4 Ultimate buckling capacity

When $\sigma_a + \sigma_b + \sigma_w > 0$, the ultimate buckling capacity for stiffeners is to be checked according to the following interaction formula:

$$\frac{\gamma_c \sigma_a + \sigma_b + \sigma_w}{R_{eH}} S = 1$$

where:

σ_a = effective axial stress, in N/mm², at mid span of the stiffener, acting on the stiffener with its attached plating:

$$\sigma_a = \sigma_x \frac{s t_p + A_s}{b_{eff1} t_p + A_s}$$

σ_x = nominal axial stress, in N/mm², acting on the stiffener with its attached plating.

— For FE analysis, σ_x is the FE corrected stress as defined in [2.3.6] in the attached plating in the direction of the stiffener axis.

— For prescriptive assessment, σ_x is the axial stress calculated according to DNVGL-RU-SHIP Pt.3 Ch.8 Sec.3 [2.2.1] at load calculation point of the stiffener, as defined in DNVGL-RU-SHIP Pt.3 Ch.3 Sec.7 [3].

— For grillage beam analysis, σ_x is the stress acting along the x-axis of the attached buckling panel.

R_{eH} = specified minimum yield stress of the material, in N/mm²:

$$R_{eH} = R_{eH_S} \quad \text{for stiffener induced failure (SI).}$$

$$R_{eH} = R_{eH_P} \quad \text{for plate induced failure (PI).}$$

σ_b = bending stress in the stiffener, in N/mm²:

$$\sigma_b = \frac{M_0 + M_1}{1000Z}$$

- Z = net section modulus of stiffener, in cm^3 , including effective width of plating according to [2.3.5], to be taken as:
 - The section modulus calculated at the top of stiffener flange for stiffener induced failure (SI).
 - The section modulus calculated at the attached plating for plate induced failure (PI).
- C_{PI} = plate induced failure pressure coefficient:
 $C_{PI} = 1$ if the lateral pressure is applied on the side opposite to the stiffener.
 $C_{PI} = -1$ if the lateral pressure is applied on the same side as the stiffener.
- C_{SI} = stiffener induced failure pressure coefficient:
 $C_{SI} = -1$ if the lateral pressure is applied on the side opposite to the stiffener.
 $C_{SI} = 1$ if the lateral pressure is applied on the same side as the stiffener.
- M_1 = bending moment, in Nmm, due to the lateral load P :
- $$M_1 = C_i \frac{|P| s \ell^2}{24 \cdot 10^3} \quad \text{for continuous stiffener}$$
- $$M_1 = C_i \frac{|P| s \ell^2}{8 \cdot 10^3} \quad \text{for sniped stiffener}$$
- P = lateral load, in kN/m^2 .
 - for FE analysis, P is the average pressure as defined in DNVGL-RU-SHIP Pt.3 Ch.8 Sec.4 [2.5.2] in the attached plating.
 - for prescriptive assessment, P is the pressure calculated at load calculation point of the stiffener, as defined in DNVGL-RU-SHIP Pt.3 Ch.3 Sec.7 [3].
- C_i = pressure coefficient:
 $C_i = C_{SI}$ for stiffener induced failure (SI).
 $C_i = C_{PI}$ for plate induced failure (PI).
- M_0 = bending moment, in Nmm, due to the lateral deformation w of stiffener:
- $$M_0 = F_E \left(\frac{P_z w}{c_f - P_z} \right) \quad \text{with } c_f - P_z > 0$$
- F_E = ideal elastic buckling force of the stiffener, in N:

$$\left(\frac{\pi}{\ell} \right)^2 EI 10^4$$
 The lateral buckling strength of non-continuous stiffeners with significant restraint against end rotation may be specially considered.
- I = moment of inertia, in cm^4 , of the stiffener including effective width of attached plating according to [2.3.5]. I is to comply with the following requirement:

$$I \geq \frac{st_p^3}{12 \cdot 10^4}$$
- t_p = net thickness of plate, in mm, to be taken as
 - for prescriptive requirements: the mean thickness of the two attached plating panels,
 - for FE analysis: the thickness of the considered elementary plate panel (EPP) on one side of the stiffener.
- P_z = nominal lateral load, in N/mm^2 , acting on the stiffener due to stresses, σ_x, σ_y and τ , in the attached plating in way of the stiffener mid span:

$$P_z = \frac{t_p}{s} \left[\sigma_{xl} \left(\frac{\pi s}{\ell} \right)^2 + 2c\gamma\sigma_y + \sqrt{2}\tau_1 \right]$$

$$\sigma_{xl} = \gamma\sigma_x \left(1 + \frac{A_s}{st_p} \right) \text{ but not less than 0}$$

$$\tau_1 = \gamma \left| \tau \right| - t_p \sqrt{R_{eH,p} E \left(\frac{m_1}{a^2} + \frac{m_2}{b^2} \right)} \text{ but not less than 0}$$

σ_y = stress applied on the edge along y axis of the buckling panel, in N/mm², but not less than 0.

- for FE analysis, σ_y is the FE corrected stress as defined in [2.3.6] in the attached plating in the direction perpendicular to the stiffener axis
- for prescriptive assessment, σ_y is the maximum compressive stress calculated according to DNVGL-RU-SHIP Pt.3 Ch.8 Sec.3 [2.2.1], at load calculation points of the stiffener attached plating, as defined in DNVGL-RU-SHIP Pt.3 Ch3 Sec.7 [2]
- For grillage beam analysis, σ_y is the stress acting along the y -axis of the attached buckling panel

τ = applied shear stress, in N/mm².

- for FE analysis, τ is the reference shear stress as defined in DNVGL-RU-SHIP Pt.3 Ch.8, Sec4 [2.4.2] in the attached plating
- for prescriptive assessment, τ is the shear stress calculated according to DNVGL-RU-SHIP Pt.3 Ch.8 Sec.3 [2.2.1] at load calculation point of the stiffener attached plating, as defined in DNVGL-RU-SHIP Pt.3 Ch.3 Sec.7 [2]
- for grillage beam analysis, $\tau = 0$ in the attached buckling panel

m_1, m_2 = coefficients taken equal to:

$$m_1 = 1.47, m_2 = 0.49 \quad \text{for } \alpha \geq 2$$

$$m_1 = 1.96, m_2 = 0.37 \quad \text{for } \alpha < 2$$

c = factor taking into account the stresses in the attached plating acting perpendicular to the stiffener's axis:

$$c = 0.5(1 + \psi) \quad \text{for } 0 \leq \psi \leq 1$$

$$c = \frac{1}{2(1 - \psi)} \quad \text{for } \psi < 0$$

ψ = edge stress ratio for case 2 according to Table 3

w = deformation of stiffener, in mm:

$$w = w_0 + w_1$$

w_0 = assumed imperfection, in mm, to be taken as:

$$w_0 = \frac{\ell}{1000} \quad \text{in general}$$

$w_0 = -w_{na}$ for stiffeners sniped at both ends considering stiffener induced failure (SI)

$w_0 = w_{na}$ for stiffeners sniped at both ends considering plate induced failure (PI)

w_{na} = distance from the mid-point of attached plating to the neutral axis of the stiffener calculated with the effective width of the attached plating according to [2.3.5]

w_1 = deformation of stiffener, in mm, at mid-point of stiffener span due to lateral load P . In case of uniformly distributed load, w_1 is to be taken as:

$$w_1 = C_i \frac{|P| s \ell^4}{384 \cdot 10^7 EI} \quad \text{in general}$$

$$w_1 = C_i \frac{5 |P| s \ell^4}{384 \cdot 10^7 EI} \quad \text{for stiffeners sniped at both ends}$$

c_f = elastic support provided by the stiffener, in N/mm²:

$$c_f = F_E \left(\frac{\pi}{\ell} \right)^2 (1 + c_p)$$

$$c_p = \frac{1}{1 + \frac{0.91}{c_{xa}} \left(\frac{12 \cdot I_{10}}{st_p^3} - 1 \right)}$$

c_{xa} = coefficient to be taken as:

$$c_{xa} = \left(\frac{\ell}{2s} + \frac{2s}{\ell} \right)^2 \quad \text{for } l \geq 2s$$

$$c_{xa} = \left(1 + \left(\frac{\ell}{2s} \right)^2 \right)^2 \quad \text{for } l < 2s$$

σ_w = stress due to torsional deformation, in N/mm², to be taken as:

$$\sigma_w = E y_w \left(\frac{t_f}{2} + h_w \right) \Phi_0 \left(\frac{\pi}{l} \right)^2 \left(\frac{1}{1 - \frac{0.4 R_e H_S}{\sigma_{ET}}} - 1 \right) \quad \text{for stiffener induced failure (SI).}$$

$$\sigma_w = 0 \quad \text{for plate induced failure (PI).}$$

y_w = distance, in mm, from centroid of stiffener cross section to the free edge of stiffener flange, to be taken as:

$$y_w = \frac{t_w}{2} \quad \text{for flat bar.}$$

$$y_w = b_f - \frac{h_w t_w^2 + t_f b_f^2}{2 A_s} \quad \text{for angle and bulb profiles.}$$

$$y_w = \frac{b_f}{2} \quad \text{for T profile.}$$

Φ_0 = coefficient taken as:

$$\Phi_0 = \frac{\ell}{h_w} 10^{-3}$$

σ_{ET} = reference stress for torsional buckling, in N/mm²:

$$\sigma_{ET} = \frac{E}{I_p} \left(\frac{\varepsilon \pi^2 I_\omega 10^2}{\ell^2} + 0.385 I_T \right)$$

I_p = net polar moment of inertia of the stiffener, in cm⁴, about point C as shown in [Figure 1](#), as defined in [Table 5](#)

I_T = net St. Venant's moment of inertia of the stiffener, in cm⁴, as defined in [Table 5](#)

I_w = net sectional moment of inertia of the stiffener, in cm^6 , about point C as shown in Figure 1, as defined in Table 5.

ε = degree of fixation.

$$\varepsilon = 1 + \frac{\left(\frac{\ell}{\pi}\right)^2 10^{-3}}{\sqrt{I_w \left(\frac{0.75s}{t_p^3} + \frac{e_f - 0.5t_f}{t_w^3} \right)}}$$

A_w = net web area, in mm^2 .

A_f = net flange area, in mm^2 .

Table 5 Moments of inertia

	Flat bars ¹⁾	Bulb, angle and T profiles
I_P	$\frac{h_w^3 t_w}{3 \cdot 10^4}$	$\left(\frac{A_w (e_f - 0.5t_f)^2}{3} + A_f e_f^2 \right) 10^{-4}$
I_T	$\frac{h_w t_w^3}{3 \cdot 10^4} \left(1 - 0.63 \frac{t_w}{h_w} \right)$	$\frac{(e_f - 0.5t_f)t_w^3}{3 \cdot 10^4} \left(1 - 0.63 \frac{t_w}{e_f - 0.5t_f} \right) + \frac{b_f t_f^3}{3 \cdot 10^4} \left(1 - 0.63 \frac{t_f}{b_f} \right)$
I_ω	$\frac{h_w^3 t_w^3}{36 \cdot 10^6}$	<p>for bulb and angle profiles.</p> $\frac{A_f e_f^2 b_f^2}{12 \cdot 10^6} \left(\frac{A_f + 2.6A_w}{A_f + A_w} \right)$ <p>for T profiles.</p> $\frac{b_f^3 t_f e_f^2}{12 \cdot 10^6}$

1) t_w is the net web thickness, in mm, not t_{w_red} as defined in [2.3.2].

2.3.5 Effective width of attached plating

The effective width of attached plating of stiffeners, b_{eff} , in mm, is to be taken as:

— For $\sigma_x > 0$:

— For FE analysis,

$$b_{eff} = \min(C_x b; \chi_s s)$$

— For prescriptive assessment,

$$b_{eff} = \min\left(\frac{C_{x1} b_1 + C_{x2} b_2}{2}; \chi_s s\right)$$

— For $\sigma_x \leq 0$:

$$b_{eff} = \chi_s s$$

where:

χ_s = effective width coefficient to be taken as:

$$\chi_s = \min \left[\frac{\frac{1.12}{1.75}}{1 + \left(\frac{\ell_{eff}}{s} \right)^{1.6}}; 1.0 \right] \quad \text{for} \quad \frac{\ell_{eff}}{s} \geq 1$$

$$\chi_s = 0.407 \frac{\ell_{eff}}{s} \quad \text{for} \quad \frac{\ell_{eff}}{s} < 1$$

ℓ_{eff} = effective length of the stiffener, in mm, taken as:

$$\ell_{eff} = \frac{l}{\sqrt{3}} \quad \text{for stiffener fixed at both ends}$$

$$\ell_{eff} = 0.75l \quad \text{for stiffener simply supported at one end and fixed at the other}$$

$$\ell_{eff} = l \quad \text{for stiffener simply supported at both ends}$$

2.3.6 FE corrected stresses for stiffener capacity

When the reference stresses σ_x and σ_y obtained by FE analysis according to DNVGL-RU-SHIP Pt.3 Ch.8 Sec.4 [2.4] are both compressive, they shall be corrected according to the following formulae:

— If $\sigma_x < v\sigma_y$:

$$\sigma_{xcor} = 0$$

$$\sigma_{ycor} = \sigma_y$$

— If $\sigma_y < v\sigma_x$:

$$\sigma_{xcor} = \sigma_x$$

$$\sigma_{ycor} = 0$$

— In the other cases:

$$\sigma_{xcor} = \sigma_x - v\sigma_y$$

$$\sigma_{ycor} = \sigma_y - v\sigma_x$$

2.4 Primary supporting members

2.4.1 Web plate in way of openings

The web plate of primary supporting members with openings is to be assessed for buckling based on the combined axial compressive and shear stresses.

The web plate adjacent to the opening on both sides is to be considered as individual unstiffened plate panels as shown in Table 6.

The interaction formulae of [2.2.1] are to be used with:

$$\sigma_x = \sigma_{av}$$

$$\sigma_y = 0$$

$$\tau = \tau_{av}$$

where:

σ_{av} = weighted average compressive stress, in N/mm², in the area of web plate being considered, i.e. P1, P2 or P3 as shown in [Table 6](#).

For the application of the [Table 6](#), the weighted average shear stress is to be taken as:

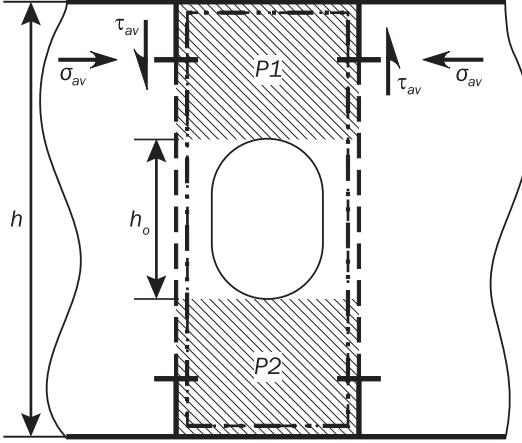
— Opening modelled in primary supporting members:

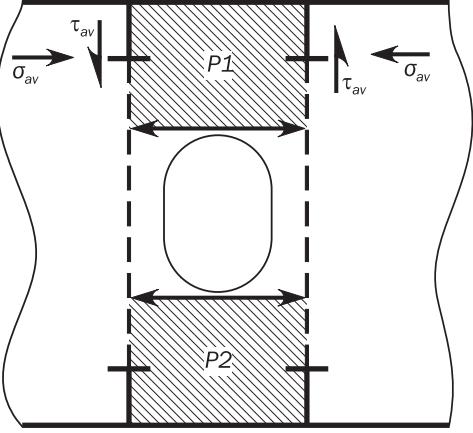
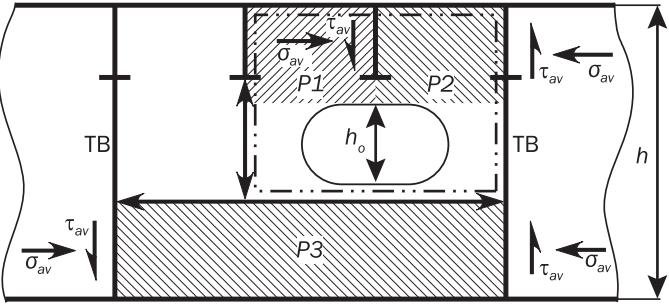
τ_{av} = weighted average shear stress, in N/mm², in the area of web plate being considered, i.e. P1, P2 or P3 as shown in [Table 6](#).

— Opening not modelled in primary supporting members:

τ_{av} = weighted average shear stress, in N/mm², given in [Table 6](#).

Table 6 Reduction factors

Configuration	C_x, C_y	C_T	
		Opening modelled in PSM	Opening not modelled in PSM
(a) Without edge reinforcements:	<p>Separate reduction factors are to be applied to areas P1 and P2 using case 3 or case 6 in Table 3, with edge stress ratio: $\psi = 1.0$</p> 	<p>Separate reduction factors are to be applied to areas P1 and P2 using case 18 or case 19 in Table 3 with: $\tau_{av} = \tau_{av}(\text{web}) h / (h - h_0)$</p>	<ul style="list-style-type: none"> When case 17 of Table 3 is applicable: A common reduction factor is to be applied to areas P1 and P2 using case 137 in Table 3 for area marked with: $\tau_{av} = \tau_{av}(\text{web})$ When case 17 of Table 3 is not applicable: Separate reduction factors are to be applied to areas P1 and P2 using case 18 or case 19 in Table 3 with: $\tau_{av} = \tau_{av}(\text{web}) h / (h - h_0)$

Configuration	C_x, C_y	C_T	
		Opening modelled in PSM	Opening not modelled in PSM
(b) With edge reinforcements:	<p>Separate reduction factors are to be applied for areas P_1 and P_2 using C_x for case 1 or C_y for case 2 in Table 3 with stress ratio: $\psi = 1.0$</p> 	Separate reduction factors are to be applied to areas P_1 and P_2 using case 15 in Table 3 with: $\tau_{av} = \tau_{av}(\text{web}) h / (h - h_0)$	Separate reduction factors are to be applied for areas P_1 and P_2 using case 15 in Table 3 with: $\tau_{av} = \tau_{av}(\text{web}) h / (h - h_0)$
(c) Example of hole in web:		<p>Panels P_1 and P_2 are to be evaluated in accordance with (a). Panel P_3 is to be evaluated in accordance with (b).</p>	

Note 1: Web panels to be considered for buckling in way of openings are shown shaded and numbered P_1 , P_2 , etc.

Where:

h = height, in m, of the web of the primary supporting member in way of the opening

h_0 = height, in m, of the opening measured in the depth of the web

$\tau_{av}(\text{web})$ = weighted average shear stress, in N/mm^2 , over the web height h of the primary supporting member.

2.4.2 The equivalent plate panel of web plate of primary supporting members crossed by perpendicular stiffeners shall be idealised as shown in [Figure 2](#).

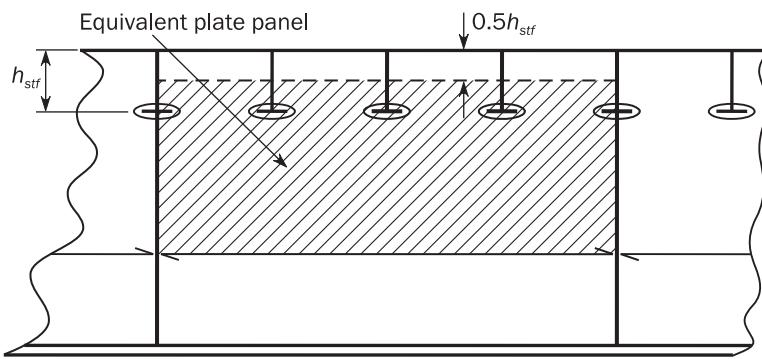


Figure 2 Web plate idealisation

3 Corrugated bulkhead

The buckling utilisation factor of flange and web of corrugation of corrugated bulkheads is based on the combination of in plane stresses and shear stress.

The buckling stresses to be used in the interaction formulation in [2.2.1] are to be determined based on case 1, 2 and 15, together with the following coefficients:

- $\alpha = 2$
- $\psi_x = \psi_y = 1$

4 Struts, pillars and cross ties

4.1 Buckling utilisation factor

The buckling utilisation factor, η , for axially compressed struts, pillars and cross ties is to be taken as:

$$\eta = \frac{\sigma_{av}}{\sigma_{cr}}$$

where:

- σ_{av} = average axial compressive stress in the member, in N/mm².
 σ_{cr} = minimum critical buckling stress, in N/mm², taken as:

$$\sigma_{cr} = \sigma_E \quad \text{for} \quad \sigma_E \leq 0.5R_{eH}$$

$$\sigma_{cr} = \left(1 - \frac{R_{eH}S}{4\sigma_E}\right)R_{eH} \quad \text{for} \quad \sigma_E > 0.5R_{eH}$$

- σ_E = minimum elastic compressive buckling stress, in N/mm², according to [4.2] to [4.4]
 R_{eH} = specified minimum yield stress of the considered member, in N/mm². For built up members, the lowest specified minimum yield stress is to be used.

The correction of panel breadth is applicable also for other slot configurations provided that the web or collar plate is attached to at least one side of the passing stiffener.

4.2 Elastic column buckling stress

The elastic compressive column buckling stress, σ_{EC} , in N/mm² of members subject to axial compression is to be taken as:

$$\sigma_{EC} = \pi^2 E f_{end} \frac{I}{A \ell_{pill}^2} 10^{-4}$$

where:

- I = net moment of inertia about the weakest axis of the cross section, in cm⁴.
- A = net cross sectional area of the member, in cm².
- ℓ_{pill} = length of the member, in m, taken as:
 - for pillar and strut: unsupported length of the member
 - for cross tie:
 - i) in centre tank: distance between the flanges of longitudinal stiffeners on the starboard and port longitudinal bulkheads to which the cross tie's horizontal stringer is attached
 - ii) in wing tank: distance between the flanges of longitudinal stiffeners on the longitudinal bulkhead to which the cross tie's horizontal stringer is attached, and the inner hull plating
- f_{end} = end constraint factor for column buckling. Values of f_{end} is given in [Table 7](#):

Guidance note:

The end constraint factor, f_{end} , may in other buckling literatures be represented as an effective length factor, k , to the pillar length as following: $\ell_e = k \ell_{pill}$, and gives the following relation between f_{end} and k factors:

$$f_{end} = \frac{1}{k^2}$$

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

Table 7 End constrain factors.

Buckled shape of member					
f_{end}	4.0	2.0	1.0	1.0	0.25
k	0.5	0.7	1.0	1.0	2.0

For cross ties a f_{end} factor of 2.0 to be used.

A pillar end may be considered fixed when brackets of adequate size are fitted. Such brackets are to be supported by structural members with greater bending stiffness than the pillar.

4.3 Elastic torsional buckling stress

The elastic torsional buckling stress, σ_{ET} , in N/mm², with respect to axial compression of members is to be taken as:

$$\sigma_{ET} = \frac{GI_{sv}}{I_{pol}} + \frac{\pi^2 f_{end} E c_{warp}}{I_{pol} \ell_{pill}^2} 10^{-4}$$

where:

- I_{sv} = net St. Venant's moment of inertia, in cm⁴, see [Table 8](#) for examples of cross sections
- I_{pol} = net polar moment of inertia about the shear centre of cross section, in cm⁴:

$$I_y + I_z + A(y_0^2 + z_0^2)$$
- c_{warp} = net sectorial moment of inertia, in cm⁶, see [Table 8](#) for examples of cross sections
- ℓ_{pill} = length of the member, in m, as defined in [\[4.2\]](#)
- y_0 = transverse position of shear centre relative to the cross sectional centroid, in cm, see [Table 8](#) for examples of cross sections
- z_0 = vertical position of shear centre relative to the cross sectional centroid, in cm, see [Table 8](#) for examples of cross sections
- A = net cross sectional area, in cm², as defined in [\[4.2\]](#)
- I_y = net moment of inertia about y axis, in cm⁴
- I_z = net moment of inertia about z axis, in cm⁴
- f_{end} = end constraint factor for torsional buckling, see [\[4.2\]](#) or actual values may be used.

4.4 Elastic torsional/column buckling stress

For cross sections where the centroid and the shear centre do not coincide, the interaction between the torsional and column buckling mode is to be examined. The elastic torsional/column buckling stress, σ_{ETF} , with respect to axial compression is to be taken as:

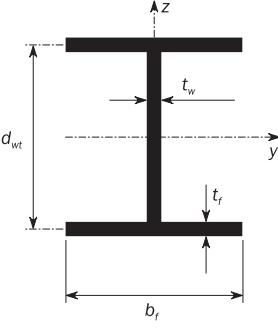
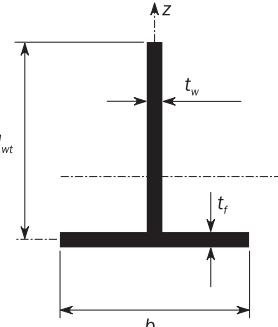
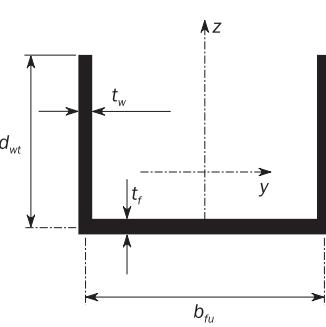
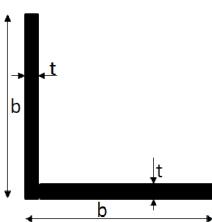
$$\sigma_{ETF} = \frac{1}{2\zeta} \left[(\sigma_{EC} + \sigma_{ET}) - \sqrt{(\sigma_{EC} + \sigma_{ET})^2 - 4\zeta\sigma_{EC}\sigma_{ET}} \right]$$

where:

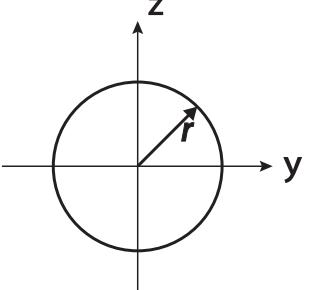
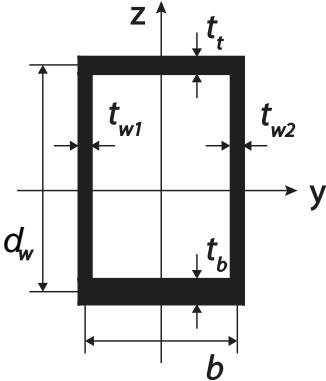
- ζ = coefficient taken as:

$$\zeta = 1 - \frac{(y_0^2 + z_0^2)A}{I_{pol}}$$
- y_0 = transverse position of shear centre relative to the cross sectional centroid, in cm, as defined in [\[4.3\]](#)
- z_0 = vertical position of shear centre relative to the cross sectional centroid, in cm, as defined in [\[4.3\]](#)
- A = net cross sectional area, in cm², as defined in [\[4.2\]](#)
- I_{pol} = net polar moment of inertia about the shear centre of cross section, in cm⁴ as defined in [\[4.3\]](#)
- σ_{EC} = elastic column compressive buckling stress, as defined in [\[4.2\]](#)
- σ_{ET} = elastic torsional buckling stress, as defined in [\[4.3\]](#).

Table 8 Cross sectional properties

	$I_{sv} = \frac{1}{3}(2b_f t_f^3 + d_{wt} t_w^3)10^{-4}$	cm^4
	$c_{warp} = \frac{d_{wt}^2 b_f^3 t_f}{24} 10^{-6}$	cm^6
	$I_{sv} = \frac{1}{3}(b_f t_f^3 + d_{wt} t_w^3)10^{-4}$	cm^4
	$y_0 = 0$	cm
	$z_0 = -\frac{0.5 d_{wt}^2 t_w}{d_{wt} t_w + b_f t_f} 10^{-1}$	cm
	$c_{warp} = \frac{b_f^3 t_f^3 + 4d_{wt}^3 t_w^3}{144} 10^{-6}$	cm^6
	$I_{sv} - n50 = \frac{1}{3}(b_f u t_f^3 + 2d_{wt} t_w^3)10^{-4}$	cm^4
	$y_0 = 0$	cm
	$z_0 = -\frac{d_{wt}^2 t_w 10^{-1}}{2d_{wt} t_w + b_f t_f} - \frac{0.5 d_{wt}^2 t_w 10^{-1}}{d_{wt} t_w + \frac{b_f t_f}{6}}$	cm
	$c_{warp} = \frac{b_f^2 d_{wt}^3 t_w (3d_{wt} t_w + 2b_f t_f)}{12(6d_{wt} t_w + b_f t_f)} 10^{-6}$	cm^6
	$I_{sv} = \frac{2bt^3}{3} 10^{-4}$	cm^4
	$c_{warp} = 0$	cm^6

	$I_{sv} = \frac{1}{3}(2bt_f^3 + ht_w^3)10^{-4}$	cm ⁴
	$c_{warp} = \frac{h_w^2 b^3 t_f (bt_f + h_w t_w)}{12(2bt_f + h_w t_w)} 10^{-6}$	cm ⁶
	$I_{sv} = \frac{1}{3}(b_f1 t_{f1}^3 + 2b_f2 t_{f2}^3 + b_f3 t_{f3}^3 + d_{wt} t_w^3)10^{-4}$	cm ⁴
	$y_0 = 0$	cm
	$z_0 = z_s - \frac{(b_f3 d_{wt} t_{f3} + 0.5 d_{wt}^2 t_w)}{d_{wt} t_w + b_f1 t_{f1} + 2b_f2 t_{f2} + b_f3 t_{f3}} 10^{-1}$	cm
	$c_{warp} = \left(I_{f1} z_s^2 + \frac{I_{f2} b_{f1}^2}{200} + I_{f3} \left(\frac{d_{wt}}{10} - z_s \right)^2 \right)$	cm ⁶
	$I_{f1} = \left(\frac{(b_f1 - t_{f2})^3 t_{f1}}{12} + \frac{b_f2 t_{f2} b_{f1}^2}{2} \right) 10^{-4}$	cm ⁴
	$I_{f2} = \frac{b_{f2}^3 t_{f2}}{12} 10^{-4}$	cm ⁴
	$I_{f3} = \frac{b_{f3}^3 t_{f3}}{12} 10^{-4}$	cm ⁴
	$z_s = \frac{I_{f3} d_{wt}}{I_{f1} + I_{f3}} 10^{-1}$	cm
 $t \ll r$	$I_{sv} = 2\pi r^3 t 10^{-4}$	cm ⁴
	$c_{warp} = 0$	cm ⁶

	$I_{sv} = \frac{1}{2}r^4 10^{-4}$	cm ⁴
	$c_{warp} = 0$	cm ⁶
	$I_{sv} = \frac{4d_w^2 b^2}{\frac{d_w}{t_{w1}} + \frac{d_w}{t_{w2}} + \frac{b}{t_t} + \frac{b}{t_b}} 10^{-4}$	cm ⁴
	$c_{warp} = 0$	cm ⁶

Note 1: All dimensions are in mm.
Note 2: Cross sectional properties are given for typical cross sections. Properties for other cross sections are to be determined by direct calculation.

5 Bars and framework

5.1 Introduction

This chapter treats the buckling of bars and frames. Depending on the loading condition, a bar may be referred to as follows:

- Column* bar subject to pure compression
- Beam* bar subject to pure bending
- Beam-column* bar subject to simultaneous bending and compression.

Buckling modes for bars are categorized as follows (see [Figure 3](#)):

- Column buckling* bending about the axis of least resistance
- Torsional buckling* twisting without bending
- Column-torsional buckling* simultaneous twisting and bending
- Lateral-torsional buckling of beams* simultaneous twisting and bending

Local buckling

local buckling of the plated part of the cross-section (plate-buckling, shell-buckling)

Column buckling may be the critical mode of a slender column with doubly symmetrical cross-section, not susceptible to, or fixed against twisting.

Torsional buckling may be the critical mode of open thin-walled short columns, where the shear centre and the centroid *coincide* (doubly-symmetrical I-shapes, anti-symmetrical Z-shapes, cruciforms etc.).

Column-torsional buckling may be the critical mode of columns where the shear centre and the centroid *not coincide*, and thus are torsionally weak (thin-walled open sections).

Lateral-torsional buckling may be the critical mode when a beam is subjected to bending of its strong axis and not braced against bending of the weak axis.

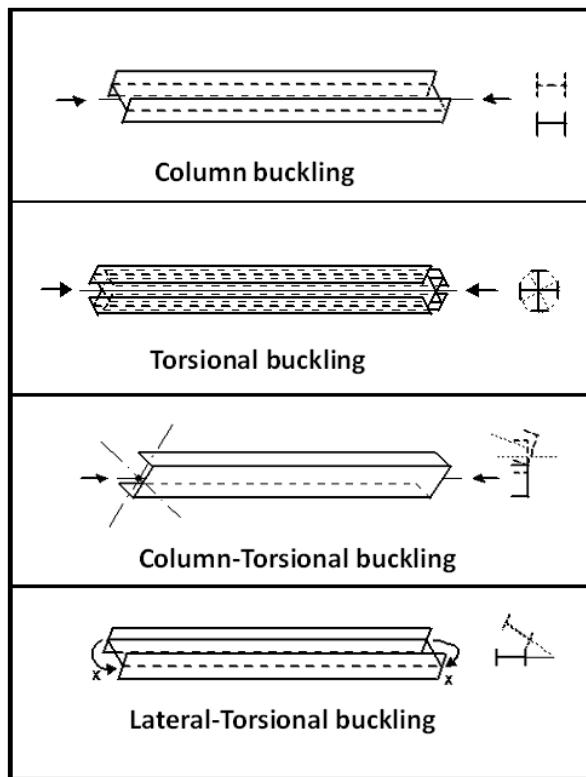


Figure 3 Buckling modes of columns and beams

The buckling mode which corresponds to the lowest buckling load is referred to as the critical buckling mode.

It is assumed that the cross-section of a member under consideration has minimum one axis of symmetry (Z axis). Members with arbitrary cross-sections are subject to special considerations.

The following symbols are used without a specific definition in the text where they appear:

- | | |
|-----|--|
| A | = net cross-sectional area, in cm^2 |
| E | = Young's modulus, in N/mm^2 |

G	= shear modulus ($G = E/2(1 + \nu)$), in N/mm ²
I	= net moment of inertia, in cm ⁴
ℓ_{pill}	= length of member, in m
R_{eH}	= minimum yield stress of the considered member, in N/mm ² . For built up members, the lowest specified minimum yield stress is to be used
ν	= Poisson's ratio
σ_E	= minimum elastic compressive buckling stress, in N/mm ² , according to [4.2] to [4.4]
σ_{cr}	= characteristic buckling resistance, in N/mm ²
σ_{EF}	= elastic torsional buckling stress, in N/mm ²
σ_{EFT}	= elastic column-torsional buckling stress, in N/mm ²
I_p	= net polar moment of inertia about the shear centre, in cm ⁴
z_0	= distance from centroid to shear centre along the z-axis.

5.2 Buckling utilisation factor

The allowable buckling utilisation factor, η , for members subjected compression is defined in DNVGL-RU-SHIP Pt.3 Ch.8 Sec.1 [3.4] and is given by the following:

$$\eta = \frac{\sigma_{av}}{\sigma_{cr}}$$

where:

σ_{av}	= average axial compressive stress in the member, in N/mm ²
σ_{cr}	= minimum critical buckling stress, in N/mm ² .

5.3 Characteristic buckling resistance

The characteristic buckling resistance of a compression member, σ_{cr} , in N/mm², is determined by use of the reduced slenderness, λ .

The reduced slenderness, λ , is defined by:

$$\lambda = \sqrt{\frac{R_{eH}}{\sigma_E}}$$

where:

σ_E	= elastic buckling stress in N/mm ² for the actual buckling mode.
------------	--

A compression member is defined as "stocky" if $\lambda < 0.2$.

A member is consider as "compact" if the reduced slenderness, λ , with respect to local buckling of any part of the member is less than:

- 0.7 for plane parts of the cross section
- 0.5 for curved parts of the cross section.

Local buckling of plated structures are given in [2.2].

A compression member which is defined as both “stocky” and “compact” is not susceptible to buckling.

5.4 Non-dimensional buckling curves

Non-dimensional buckling curves are given in [Figure 4](#). σ_{cr} may be obtained by:

If $\lambda \leq \lambda_0$

$$\sigma_{cr} = R_{eH}$$

If $\lambda > \lambda_0$

$$\sigma_{cr} = \left[\frac{1 + \mu + \lambda^2 - \sqrt{(1 + \mu + \lambda^2)^2 - 4\lambda^2}}{2\lambda^2} \right] \cdot R_{eH}$$

where:

$$\mu = \alpha (\lambda - \lambda_0)$$

The coefficients α and λ_0 are given in [Table 9](#).

Table 9 Numerical values of λ_0 and α

Curve	λ_0	α
a	0.2	0.20
b	0.2	0.35
c	0.2	0.5
e	0.6	0.35

[Table 10](#) defines column curves for commonly used structural sections. Curve e applies to lateral-torsional buckling of beams, [Figure 4](#).

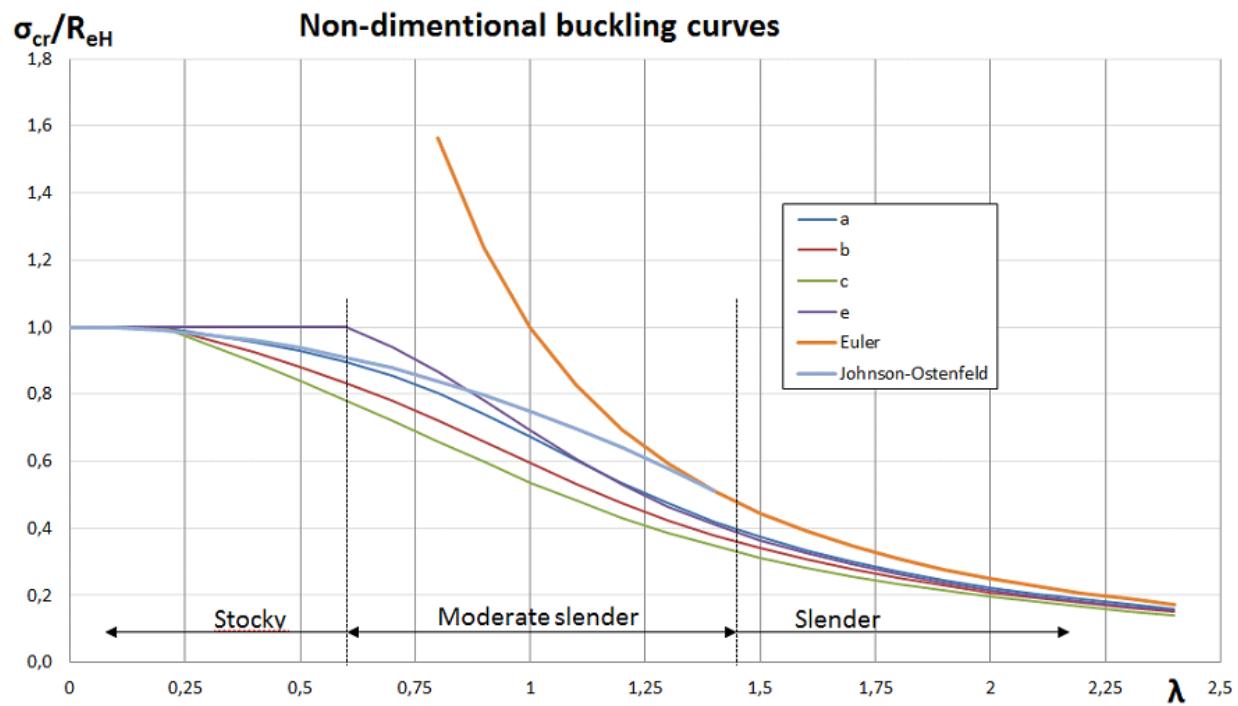


Figure 4 Non-dimensional buckling curves

Table 10 Buckling curves for different cross section

Shape of section	Buckling about axis	Column curve
<p>Rolled tubes Welded tubes (hot finished)</p>	y-y z-z	a
	y-y z-z	
<p>y-y: b,t; z-z: h,d Welded box sections</p>	y-y z-z	b
	y-y z-z	
<p>I and H rolled section</p>	y-y z-z	a b
	y-y z-z	
<p>I and H welded sections</p>	y-y z-z	b c
	y-y z-z	
<p>I and H sections with welded flange cover plates $t=t_{\max}$</p>	y-y z-z	b a
	y-y z-z	
<p>Box sections, stress relieved by heat treatment</p>	y-y z-z	a
	y-y z-z	
<p>I and H sections, stress relieved by heat treatment</p>	y-y z-z	a b
	y-y z-z	
<p>T and L sections y-y: d z-z: t</p>	y-y z-z	c
	y-y z-z	
Channels	y-y z-z	c

5.5 Members exposed to pure axial compression (columns)

For members exposed to axial loads only and not susceptible to local buckling, the elastic buckling stress, σ_E , for column buckling, torsional buckling and torsional/column buckling as given in [4.2], [4.3] and [4.4] shall be considered.

5.6 Members exposed to pure bending (beam)

A beam which is subjected to bending about its strong axis (y -axis) and not restrained against buckling about the weak axis (z -axis) may fail into lateral-torsional buckling mode, see Figure 3. The failure mode is governing when the compression stress reaches a critical value, σ_{bcr} .

The utilization factor for members subjected to pure bending is defined by:

$$\eta = \frac{\sigma_b}{\sigma_{bcr}}$$

where:

- η = allowable utilisation factor given in DNVGL-RU-SHIP Pt.3 Ch.8 Sec.1 [3.4]
- σ_b = effective axial stress due to bending.

σ_{bcr} is represented by curve in Table 9, and reduced slenderness for lateral-torsional buckling, λ_v , is defined as following:

$$\lambda_v = \sqrt{\frac{R_{eH}}{\sigma_{EV}}}$$

where:

- σ_{EV} = elastic lateral-torsional buckling stress:

$$\sigma_{EV} = f_{end} \frac{\pi^2 E I_z c}{Z_{yc} \ell_{pill}^2} 10^{-4}$$

where:

- ℓ_{pill} = unsupported length of the member as given in [4], in m
- Z_{yc} = net section modulus with respect to compression flange, in cm^3
- I_z = net moment of inertia about the weak axis, in cm^4
- c = parameter depending on geometric proportions, bending moment distribution and position of load with respect to the neutral axis
- f_{end} = end constraint factor with respect to lateral-torsional buckling (tripping).

For a beam with constant bending moment (bending moments applied at the ends):

$$c = \left[\frac{C_{warp}}{I_z} + \frac{I_{sv}}{f_{end} I_z} \frac{\ell_{pill}^2 10^4}{2\pi^2 (1+\nu)} \right]^{0.5}$$

where:

$$\begin{aligned} C_{warp} &= \text{warping constant, in cm}^6 \\ I_{sv} &= \text{net St. Venant torsional constant, in cm}^4. \end{aligned}$$

The parameters C_{warp} and I_{sv} are given in [Table 8](#) for commonly used cross sections.

For double-symmetrical H- and I-shape sections where $\sigma_{EV} \sim \sigma_E$, $I_{sv} = 0$ and $Z_{yc} = Ah_w/2$, σ_{EV} may be taken as:

$$\sigma_{EV} = f_{end} \frac{\pi^2 EI_{zc} h_w}{Z_{yc} \ell_{pill}^2} 10^{-4}$$

where:

$$\begin{aligned} I_{zc} &= \text{net moment of inertia, in cm}^4, \text{ of the compression flange (for double-symmetrical H- and I-shape sections } I_{zc} = I_z/2) \\ h_w &= \text{web height, in mm.} \end{aligned}$$

Lateral-torsional buckling need not be considered if:

$$\lambda_v < 0.6, \text{ or if } \ell_{pill} < \ell_{eo}$$

where:

$$\begin{aligned} \ell_{eo} &= 0.55 b_f \sqrt{\frac{A_c h_w E}{10 Z_{yc} R_{eH}}} \\ \ell_{pill} &= \text{unsupported length of member, in m} \\ A_c &= \text{net cross sectional area of compression flange, in cm}^2 \\ b_f &= \text{width of compression flange, in m.} \end{aligned}$$

5.7 Members exposed to axial load and bending (beam-columns)

The utilization factor for members subjected to both compression and bending may be taken as:

$$\eta = \frac{\sigma_a}{\sigma_{acr}} + \frac{\sigma_b}{\left(1 - \frac{\sigma_a}{\sigma_E}\right) \sigma_{bcr}}$$

where:

$$\begin{aligned} \sigma_a &= \text{axial stress, in N/mm}^2, \text{ due to compression.} \\ \sigma_b &= \text{effective axial stress, in N/mm}^2, \text{ due to bending} \\ &\quad \text{Bending about weak (z-axis) or strong axis (y-axis) see options for } \sigma_{bcr}. \\ &\quad \text{For compression members which are braced against joint translation, } \sigma_b \text{ is the maximum bending stress within the middle third of the length of the member, see } \text{Figure 5.} \\ \sigma_{acr} &= \text{characteristic buckling stress, in N/mm}^2, \text{ for axial compression as defined in [5.5]} \\ \sigma_E &= \text{minimum elastic buckling stress, in N/mm}^2, \text{ about weak axis (z-axis).} \end{aligned}$$

σ_{bcr} = characteristic buckling stress, in N/mm², for pure bending as defined in [5.6]. If bending about weak axis (z-axis) then $\sigma_{bcr} = R_{eH}$

For double symmetrical H- and I-shape and rectangular box sections which are subjected to simultaneous axial compression and bending about both axes, the utilization factor may be taken as:

$$\eta = \frac{\sigma_a}{\sigma_{acr}} + \frac{\sigma_{by}}{\left(1 - \frac{\sigma_a}{\sigma_E}\right)\sigma_{bcr}} + \frac{\sigma_{bz}}{\left(1 - \frac{\sigma_a}{\sigma_E}\right)R_{eH}}$$

where:

σ_{by} = effective axial stress, in N/mm², due to bending about strong axis (y-axis)
 σ_{bz} = effective axial stress, in N/mm², due to bending about weak axis (z-axis).

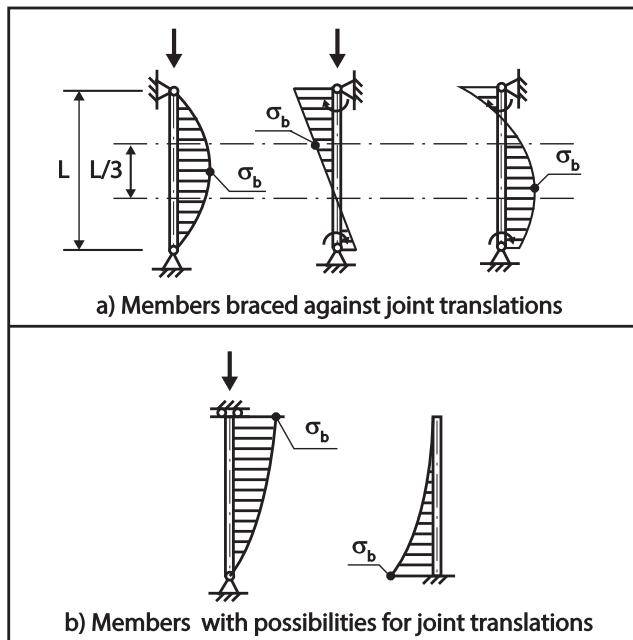


Figure 5 Effective bending stress for beam-columns

The allowable utilisation factor, η , is defined in DNVGL-RU-SHIP Pt.3 Ch.8 Sec.1 [3.4].

When the buckling analyses of a beam-column is carried out by use of the effective bending stress, σ_b , it is in addition necessary to check the beam with respect to yield at the position of maximum bending stress.

5.8 Overall buckling of built-up members

A built-up member assumed to be composed of two or more sections (chords) and intermittent transverse connecting elements (bracings), see Figure 6. It is assumed that all connections are welded.

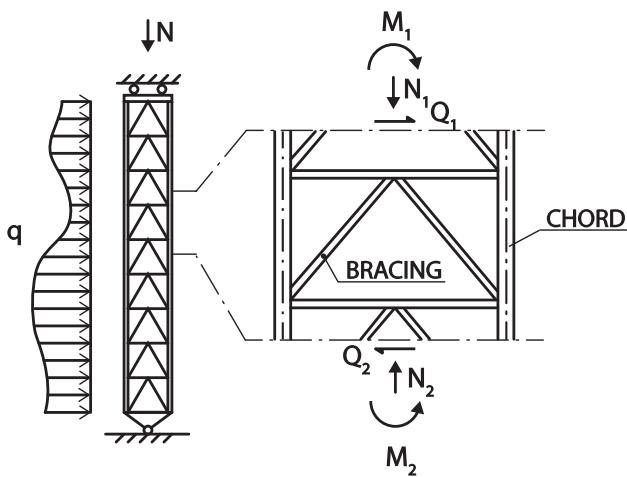


Figure 6 Built-up compression member

Overall buckling of a built-up member corresponds to column buckling of a homogeneous member as given in [5.5]. The column buckling must take into account the shear stiffness factor into the elastic buckling stress as following:

$$\sigma_{Em} = \frac{\pi^2 E}{(1 + \omega)\lambda_k^2} \quad = \text{elastic buckling stress, in N/mm}^2, \text{ for the built up member, where}$$

$$\omega = 2\pi^2 \left(1 + \nu\right) \frac{I}{\ell_m^2 A_Q} \quad = \text{shear stiffness modification factor of the built-up member}$$

where:

- A = net cross sectional area, in cm^2 , for the built-up member, see [Table 12](#)
- A_Q = net effective shear area, in cm^2 , for the built-up member, see [Table 11](#) and [Table 12](#)
- I = net moment of inertia, in cm^2 , of the built-up member, see [Table 12](#)
- ℓ_m = effective length of the built-up member, in m.

In addition to the overall buckling control, it is required to check the buckling capacity of each single element in the built-up member.

If the characteristic buckling stress of a single chord element is less than the yield stress, the overall buckling analysis of the built-up member is to be based on a reduced yield stress equal to the characteristic buckling stress of the chord element.

Bracing members in the built-up member, see [Figure 6](#), shall be designed for an overall shear force, Q_d , in kN, given by:

$$Q_d = Q + Q_o$$

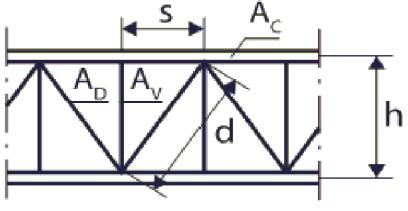
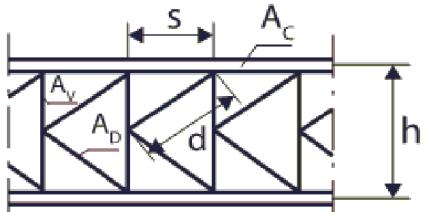
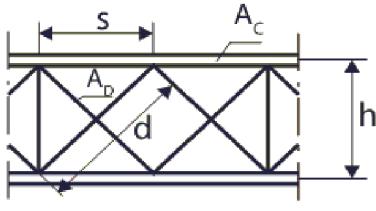
where Q is the overall shear force, in kN, in the built-up member due to design loading and Q_o is the local shear force, in kN, in each leg defined by:

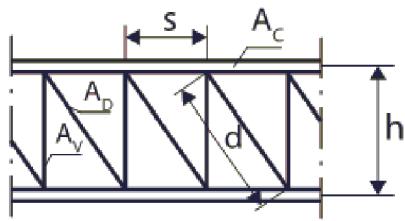
$$Q_0 = \pi \frac{P}{P_E - P} \frac{\sqrt{f_{end}} M_{\max}}{\ell_{pill}} \cos \frac{\sqrt{f_{end}} \pi}{\ell_{pill}} x$$

where:

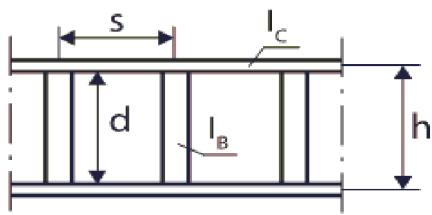
- P = average axial force in each leg, in kN
- P_E = elastic buckling force for the beam column, in kN
- M_{\max} = maximum 1st order bending moment, in kN, i.e. due to:
 - lateral load
 - eccentric axial load
 - initial deformation, out of straightness
- x = distance from zero bending moment, in m.

Table 11 Equivalent shear area of plane built-up members.

Build-up member	Equivalent net shear area, in cm ²
	$A_Q = \frac{(1 + \nu)sh^2}{\frac{d^3}{2A_D} + \frac{s^3}{3A_C}}$
	$A_Q = \frac{(1 + \nu)sh^2}{\frac{d^3}{A_D} + \frac{h^3}{8A_V} - \frac{s^3}{NA_C} \left(\frac{N^3}{3} - \sum_{i=1}^N i^2 \right)}$ <p>N = No of active bays</p>
	$A_Q = \frac{(1 + \nu)sh^2}{\frac{d^3}{A_D} + \frac{s^3}{12A_C}}$



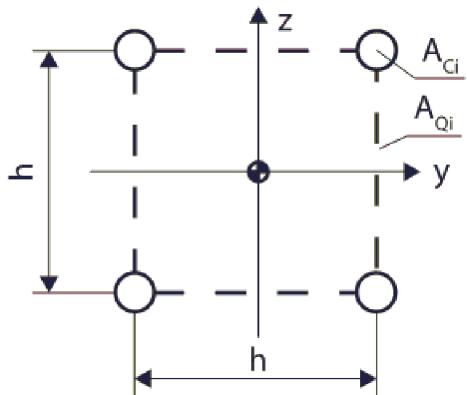
$$A_Q = \frac{(1 + \nu)sh^2}{\frac{d^3}{2A_D} + \frac{h^3}{2A_V} + \frac{s^3}{12A_C}}$$



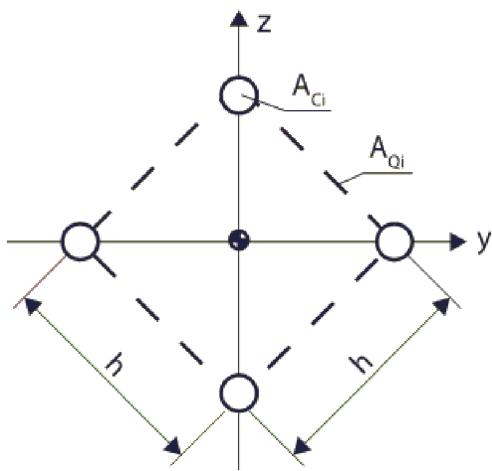
$$A_Q = \frac{48(1 + \nu)I_C}{s^2 \left(1 + 2 \frac{d}{s} \frac{I_C}{I_B} \right)}$$

Table 12 Cross section properties of three-dimensional built-up members.

Build-up member type	Equivalent net shear area, in cm ²
	$A = 3A_{Cl}$ $A_{Qy} = A_{Qz} = \frac{3}{2} \cdot A_{Qi}$ $I_y = I_z = \frac{3}{2} \cdot A_{Cl} \cdot h^2$ $I_T = \frac{1}{4} \cdot A_{Qi} \cdot h^2$



$$\begin{aligned}A &= 4A_{ci} \\A_{qi} &= A_{qz} = 2 \cdot A_{qi} \\I_y &= I_z = A_{ci} \cdot h^2 \\I_T &= A_{qi} \cdot h^2\end{aligned}$$



$$\begin{aligned}A &= 4A_{ci} \\A_{qy} &= A_{qz} = 2 \cdot A_{qi} \\I_y &= I_z = A_{ci} \cdot h^2 \\I_T &= A_{qi} \cdot h^2\end{aligned}$$

6 Spherical Shells

6.1 Introduction

This chapter handled buckling of un-stiffened spherical shells and dished end closures.

The following symbols are used without a specific definition in the text where they appear:

- N = axial load, in kN
- P = lateral pressure, in kN/m^2
- r = middle radius of the shell, in m
- t = net thickness of shell, in mm.

6.2 Stresses

Spherical shells are usually designed to resist lateral pressure. For a complete spherical shell subjected to uniform lateral pressure the state of stress is defined by the principal membrane stresses, σ_1 and σ_2 , in N/mm², defined by:

$$\sigma_1 = \sigma_2 = \frac{Pr}{2t}$$

For the spherical shell segment shown in [Figure 7](#), the meridional membrane stress, in N/mm², is given by:

$$\sigma_\Phi = \frac{Pr \sin(\varphi + \alpha) \sin(\varphi - \alpha)}{(sin\varphi)^2} + \frac{N}{2\pi rt(\sin\varphi)^2}$$

The circumferential membrane stress, in N/mm², is given by:

$$\sigma_\theta = \frac{Pr}{t} - \sigma_\varphi$$

If the axial force, N , in kN, is due to end pressure alone, the stresses, in N/mm², are given by:

$$\sigma_\varphi = \sigma_\theta = \frac{Pr}{2t}$$

These equations are only valid if the edges are reinforced. If the axial force is due to end pressure only, the net cross-sectional area, in cm², of the reinforcement as:

$$A = \frac{5\sin 2\alpha}{(1-\nu)} rt$$

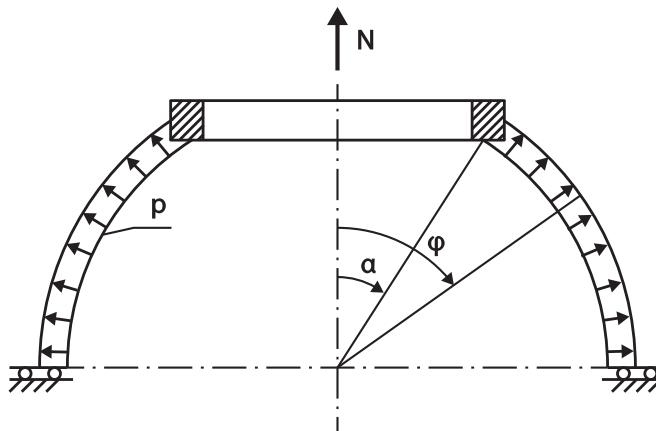


Figure 7 Spherical shell segment

6.3 Shell buckling, general

Buckling of an unstiffened spherical shell occurs when the largest compressive principal membrane stress, σ_1 , reaches a critical value, σ_{cr} . The critical stress, in N/mm², may be taken as:

$$\sigma_{cr} = \frac{R_e H}{\sqrt{1 - \psi + \psi^2 + \lambda^4}}$$

where:

$$\psi = \frac{\sigma_1}{\sigma_2} \text{ stress ratio } (-1 \leq \psi \leq 1)$$

$$\lambda = \sqrt{\frac{R_{eH}}{\sigma_E}} \text{ reduced slenderness}$$

σ_1 = largest compressive principal membrane stress, in N/mm²

σ_2 = principal membrane stress normal to σ_1 (compressive or tensile), in N/mm².

The allowable buckling utilisation factor for spherical shell is defined by:

$$\eta = \frac{\sigma_1}{\sigma_{cr}}$$

The allowable buckling utilisation factor, η , is given in [DNVGL-RU-SHIP Pt.3 Ch.8 Sec.1 \[3.4\]](#).

The elastic buckling stress σ_E may be taken as:

$$\sigma_E = 0.606\rho E \frac{t}{r} 10^{-3}$$

In lieu of more detailed information ρ may be taken as:

$$\rho = \frac{0.5}{\sqrt{1 + \frac{10}{(3 - 2\psi)} \frac{r}{t}}}$$

For a complete sphere subjected to uniform external pressure the stress ratio is $\psi = 1$, and the expressions given above for σ_{cr} and ρ may be taken as:

$$\sigma_{cr} = \frac{R_{eH}}{\sqrt{1 + \lambda^4}}$$

$$\rho = \frac{0.5}{\sqrt{1 + 10 \frac{r}{t}}}$$

6.4 Buckling of dished ends convex to pressure

Hemispherical ends are to be designed as a complete sphere under uniform pressure.

Torispherical ends are to be designed as a complete sphere with radius equal to the crown radius. The thickness should not be less than 1.2 times the thickness required for a structure of the same shape subjected to maximum internal pressure.

Ellipsoidal ends are to be designed as a complete sphere with radius equal to r^2/H , where H is the short axis and r is the long axis of the ellipsoid. The thickness should not be less than 1.2 times the thickness required for a structure of the same shape subjected to maximum internal pressure.

SECTION 4 SEMI-ANALYTICAL BUCKLING MODELS (PULS)

1 General

PULS is a computerized buckling code recognized by DNV GL and IACS for strength assessment of stiffened thin plate elements as used in ship and offshore structures.

The code is based on a direct semi-analytical approach using a recognized non-linear plate theory which is capable of predicting both the elastic buckling limit (eigenvalues) and the post-buckling behaviour up to the ultimate strength limit.

Several types of elements are available and they are all implemented into an easy and intuitive stand-alone software (Visual Basic Advanced Viewer (AV) interface). The two basic plate elements (U3, S3) are also available in Excel format and as automatic code checks in DNV GL's NAUTICUS Hull/GeniE programs.

2 Objective

The code is developed and tailor-made for the purpose of doing fast buckling and ultimate strength assessments of local stiffened panels with an accuracy close to what is available using more advanced non-linear FE tools. The elements available covers the same structural layout as more standard closed formed formulas (CFM, Sec.3), i.e. regular stiffening arrangements etc., but have some extra features. Special elements coping with non-regular stiffening and corrugations are also included in the library.

By applying directly a non-linear plate theory additional information are available which will give the user some insight and understanding of the failure mechanisms, i.e. the weakest failure mode, stress redistribution patterns etc. using available 3D graphics of buckling deflections etc.

The approach has the same theoretical basis as used non-linear FE programs, which means that the results have focus on concepts such as load bearing capacities, elastic buckling (eigenvalues), plate deformations at collapse etc. For the purpose of code checking the results are transformed to concepts such as utilization factors and buckling modes which designers and approval engineers are familiar with from rules.

3 Theory fundamentals

3.1 General

The buckling models can be classified as semi-analytical in the sense that they are based on a recognized non-linear plate theory in combination with a Rayleigh-Ritz discretization's of the deflections. The latter imply global trigonometric functions over the plate and stiffener surfaces and by using sufficiently many terms in the series, relatively complex deflection shapes such as inclined shear buckling patterns etc. can be described. For problems which can be analysed with using few or only a single degree of freedom, the buckling and post-buckling behaviour resemble classical analytical solutions found in the literature and standard textbooks such as found in /1/ and /2/.

The set of non-linear equilibrium equations are derived based on energy methods and they are solved using an incremental numerical perturbation scheme with arc length control. The equivalent von Mises yield stress in hot spot locations is controlled against the material yield condition for each load step along the equilibrium path and used as indicators for when the ultimate strength limit is reached.

The fundamental non-linear plate theory including initial curvature was first published by Marguerre /3/. More details on the fundamental stability theory and incremental perturbation approach used for solving the non-linear equilibrium equation can be found in /4/.

The different buckling models are documented in series of publications /5-16/.

3.2 Basic theory

The theory is given detailed treatment elsewhere, so only some of the basics are given here. The fundamental is the non-linear plate theory due to Marguerre formulating first the well known kinematic and compatibility equations for plates with geometrical imperfections i.e. the membrane strain-displacement relations are

$$\begin{aligned}\varepsilon_{11} &= u_{1,1} + \frac{1}{2}w_{,1}^2 + w_{,1}w_{0,1} \\ \varepsilon_{22} &= u_{2,2} + \frac{1}{2}w_{,2}^2 + w_{,2}w_{0,2} \\ \varepsilon_{12} &= \frac{1}{2}(u_{1,2} + u_{2,1}) + \frac{1}{2}(w_{,1}w_{,2} + w_{,1}w_{0,2} + w_{,2}w_{0,1})\end{aligned}\quad (1)$$

The corresponding compatibility equation reads

$$(\nabla^4 F = E[w_{,12}^2 - w_{,11}w_{,22} + 2w_{0,12}w_{,12} - w_{0,11}w_{,22} - w_{0,22}w_{,11}]) \quad (2)$$

The $\varepsilon_{\alpha\beta}$ are the membrane strain tensor, $u_{\alpha,\beta}$ the in-plane displacement gradients and $w_{,\alpha}$, $w_{,\alpha\beta}$, $w_{0,\alpha}$, $w_{0,\alpha\beta}$ the additional and initial out-of plane plate deflection gradients respectively ($\alpha, \beta = 1, 2$). F is the Airy's stress function.

Energy methods

The equilibrium equations are derived using energy principles, i.e. a virtual work or potential energy approach whichever is most convenient. The most general is a virtual work approach written on total form as

$$\int_V \sigma_{\alpha\beta} \delta \varepsilon_{\alpha\beta} = \delta E \quad (3)$$

The left side is the internal virtual work over the volume V and the right side is the external virtual work (δE) done by the external forces. The subscript summation convention is used here for convenience where the Greek letters runs over 1, 2.

For derivation of equilibrium equations on incremental form (1th order perturbation rate) the corresponding virtual work equation is derived as

$$\int_V \dot{\sigma}_{\alpha\beta} \delta \dot{\varepsilon}_{\alpha\beta} + \int \sigma_{\alpha\beta} \delta \dot{\varepsilon}_{\alpha\beta} = \dot{\delta E} \quad (4)$$

Here a dot over the symbol symbolize incremental properties (1th order rates in a perturbation approach)

Discretizations of buckling deflections

In the present Rayleigh-Ritz semi-analytical approach the lateral buckling deflections w due to applied loads and initial stress free geometrical imperfections w_0 are discretized by double Fourier series ($i, j = 1, 2, \dots, N$).

$$\begin{aligned}w &= q_i f_i(x_1, x_2) = \sum_m \sum_n A_{mn} \sin\left(\frac{m\pi}{a}x_1\right) \sin\left(\frac{n\pi}{b}x_2\right) \\ w_0 &= q_{i0} f_i(x_1, x_2) = \sum_m \sum_n A_{mn}^0 \sin\left(\frac{m\pi}{a}x_1\right) \sin\left(\frac{n\pi}{b}x_2\right)\end{aligned}\quad (5)$$

For convenience of notation the set of unknown Fourier parameters A_{mn} (A_{mn}^0) are given whenever practical the single subscript notation q_i (q_{i0}) where the Latin subscript letters i, j runs from 1 to N where N is the total number of degrees of freedom. This means that the rates of the deflection coefficient parameters A_{mn} (or

q_i) with respect to the arc length variable η are the unknown parameters. The rates are given the symbol \dot{A}_{mn} (or \dot{q}_i) etc., i.e. a dot above the symbol indicates an incremental property.

Numerical approach solving equilibrium equations

The first order perturbation expansion provides a linearized set of N equations in $N+1$ unknowns (N deflection parameters + one load parameter),

$$K_{ij}\dot{q}_j + G_{i\Lambda}\dot{\Lambda} = 0 \quad i, j = 1, 2, \dots, N \quad (6a)$$

The last equation is based on the definition of the arc length parameter η and gives a single quadratic equation in the unknown rate coefficients (e.g. 6b). Summarized the final set of first order (linear) algebraic equations are compactly written as

$$K_{ij}\dot{q}_j + G_{i\Lambda}\dot{\Lambda} = 0 \quad i, j = 1, 2, \dots, N \quad (6a)$$

$$\dot{q}_i\dot{q}_i + \dot{\Lambda}^2 = 1 \quad (6b)$$

Here K_{ij} , $G_{i\Lambda}$ are the state dependent geometrical stiffness matrix and incremental load vector, respectively.

The \dot{q}_i 's are the unknown deflection rate parameters and $\dot{\Lambda}$ is the load rate respectively. The solution for deflections and load parameter in state $s+1$ can be written as a Taylor series expanded around state s , i.e. mathematically written as

$$\begin{aligned} q_{1,s+1} &= A_{11,s+1} = A_{11,s} + \dot{A}_{11,s}\Delta\eta + \dots \\ q_{2,s+1} &= A_{12,s+1} = A_{12,s} + \dot{A}_{12,s}\Delta\eta + \dots \\ &\dots \\ \Lambda_{s+1} &= \Lambda_s + \dot{\Lambda}_s\Delta\eta + \dots \end{aligned} \quad (7)$$

The Latin letter s as subscript on the variables indicates a general state s , index $s+1$ the next neighbouring state $s+1$ and $\Delta\eta$ is the incremental prescribed arc length parameter between the two states.

The approach implies that the non-linear equilibrium path is traced solving the equilibrium equations in an incremental scheme. The parameter rates (\dot{A}_{ij} , $\dot{\Lambda}$) in state s are directly used for finding deflections and the loads for the next state $s+1$.

Moreover, only the first order rates in the perturbation expansion are used since a very small value of the incremental perturbation parameter $\Delta\eta$ is assumed. It is generally found that $\Delta\eta \approx 0.01$ gives sufficiently accurate solutions for practical applications implying in the range of 50 to 100 steps along up to the ultimate capacity. The incremental approach is illustrated in [Figure 1](#).

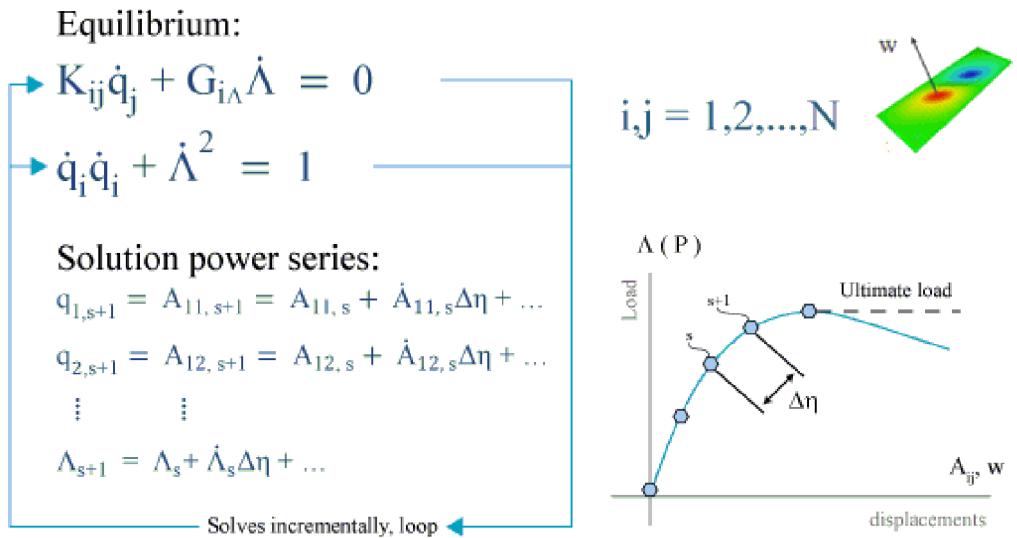


Figure 1 Incremental solution algorithm stepping along a non-linear equilibrium path; ultimate load

By checking the redistributed stress pattern against the von Mises yield criterion in critical locations inside the panel and along plate edges the localized critical stresses is under control at each load step. When the "first major onset" of material yield is reached the incremental loading is stopped and the corresponding total load is defined as the ultimate capacity.

Using the present approach the ultimate capacity is expected to be on the conservative side, i.e. some more capacity may be expected for stocky sections in particular, but at the expense of some plastic redistribution of stresses and possibly some minor permanent sets.

3.3 Load history

The approach can handle any general load history path defined in load space (or displacement space) as illustrated in Figure 2.

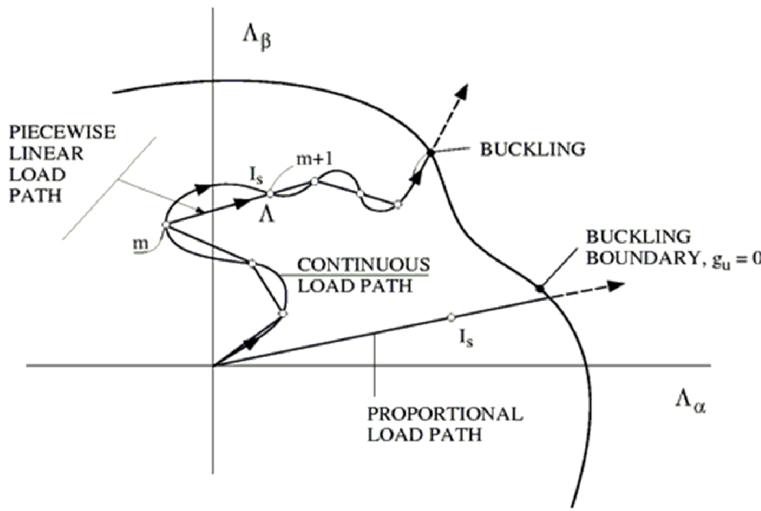


Figure 2 Load history: Schematically illustration of load paths in load space crossing buckling (ultimate) strength boundary

The most general case is a continuous turning and twisting path in load space which can be approximated as a multi-linear path using sufficiently small steps by using a single load factor ($LF \equiv \Lambda$) with value ranging from 0 to unity (1) for each "linear load step" in load space, i.e.

$$\begin{aligned}\sigma_1 &= \sigma_{1,m} + \Lambda(\sigma_{1,m+1} - \sigma_{1,m}) \\ \sigma_2 &= \sigma_{2,m} + \Lambda(\sigma_{2,m+1} - \sigma_{2,m}) \\ &\dots \\ \sigma_K &= \sigma_{K,m} + \Lambda(\sigma_{K,m+1} - \sigma_{K,m})\end{aligned}\tag{8}$$

The subscript m indicates the load reference value for point m in load space for load s_i etc. The general load history above is not very practical as detailed information on real load-histories is not available or not known for a given case. Thus no more discussion of this general case is given here.

A more practical case is to assume a single step proportional loading history, i.e. all in-plane loads are scaled in the same proportion from zero load until buckling or collapse is identified. Mathematically written all external loads $\sigma_1, \sigma_2, \dots, \sigma_K$ are scaled in the same proportion in relation to a load reference level (0 as subscript), i.e.

$$\begin{aligned}\sigma_1 &= \Lambda\sigma_{10} \\ \sigma_2 &= \Lambda\sigma_{20} \\ &\dots \\ \sigma_K &= \Lambda\sigma_{K0}\end{aligned}\tag{8}$$

For a basic case of stiffened panel/girder structure the component σ_1 is typically the axial load, σ_2 the transverse load acting perpendicular to σ_1 and σ_3 is in-plane shear stress.

The proportional load concept compare to a convenient utilization factor measure as used in buckling codes, rules etc., see [3.4]. The case of a single fixed pre-load is discussed below ($p=p_{fixed}$; ex. ship bottom panel subjected to lateral sea pressure and hull girder longitudinal stresses).

3.4 Margins to collapse - utilization factor

For proportional loadings the non-linear solution algorithm identifies the value of load parameter Λ to be Λ_u at the ultimate capacity limit. The corresponding ultimate strength values of the external loads are accordingly

$$\begin{aligned}\sigma_{1u} &= \Lambda_u \sigma_{10} \\ \sigma_{2u} &= \Lambda_u \sigma_{20} \\ &\vdots \\ &\vdots \\ \sigma_{Ku} &= \Lambda_u \sigma_{K0}\end{aligned}\tag{7}$$

The ultimate capacity factor Λ_u is directly a measure of the safety margin against collapse. The inverse of this factor is the actual utilization factor commonly used in ship classification rules, i.e.

$$\eta_{actual} = \frac{1}{\Lambda_u} = \frac{\sqrt{\sigma_{10}^2 + \sigma_{20}^2 + \dots + \sigma_{K0}^2}}{\sqrt{\sigma_{1U}^2 + \sigma_{2U}^2 + \dots + \sigma_{KU}^2}}\tag{8}$$

The concept of utilization factor is illustrated schematically in load space ($\sigma_1, \sigma_2, \sigma_3$) in Figure 3. The design loads (reference load) corresponds a load factor of $\Lambda = 1$.

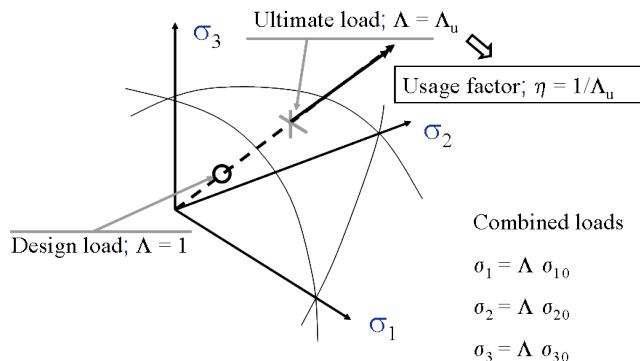


Figure 3 Ultimate Capacity failure surface in load space. Proportional load history - definition of utilization factor η_{actual}

The same concept can be used for the case of having one “fixed” (pre-) load component while the rest of the load parameters are scaled to collapse. An example of such a case is a bottom panel in a ship hull for which the acting sea pressure is constant while the longitudinal hull girder stress, being the integrated effect along the ship length, varies more and thus the natural load to be scaled to collapse. The loading history is illustrated schematically in two steps, see Figure 4.

- step 1) Lateral pressure p increased to p_{fixed} while in-plane stress σ_1 kept at zero

- step 2) $p = p_{fixed}$ kept as fixed value, while σ_1 is increased to collapse through ref stress σ_{10}

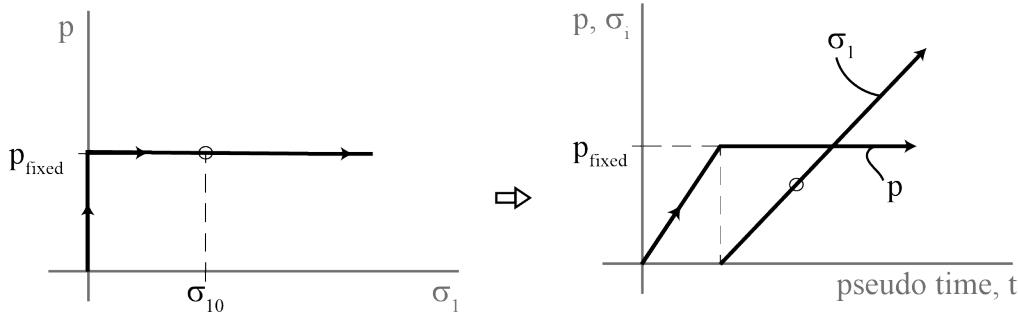


Figure 4 Schematically illustration of load history; one fixed load (pre-load) and one load scaled to collapse

This load history can be compactly written as

$$\begin{aligned}
 \sigma_{1u} &= \Lambda_u \sigma_{10} \\
 \sigma_{2u} &= \Lambda_u \sigma_{20} \\
 &\vdots \\
 &\vdots \\
 \sigma_{Ku} &= \Lambda_u \sigma_{K0} \\
 P &= P_{fixed}
 \end{aligned} \tag{7}$$

For such a case it is convenient to define two utilization factors, i.e. one measuring the margin to buckling (ultimate) collapse limit, i.e.

$$\eta_{actual}^{buckling} = \frac{1}{\Lambda_u} = \frac{\sqrt{\sigma_{10}^2 + \sigma_{20}^2 + \dots + \sigma_{K0}^2}}{\sqrt{\sigma_{1U}^2 + \sigma_{2U}^2 + \dots + \sigma_{KU}^2}}$$

and the other measuring the margin to the maximum (yielding) pressure capacity

$$\eta_{actual}^{pressure} = \frac{p_{fixed}}{p_{capacity}}$$

It is illustrative to show these two utilization factors in the load space together with buckling/collapse and other design (yielding) limit surfaces, see [Figure 5](#).

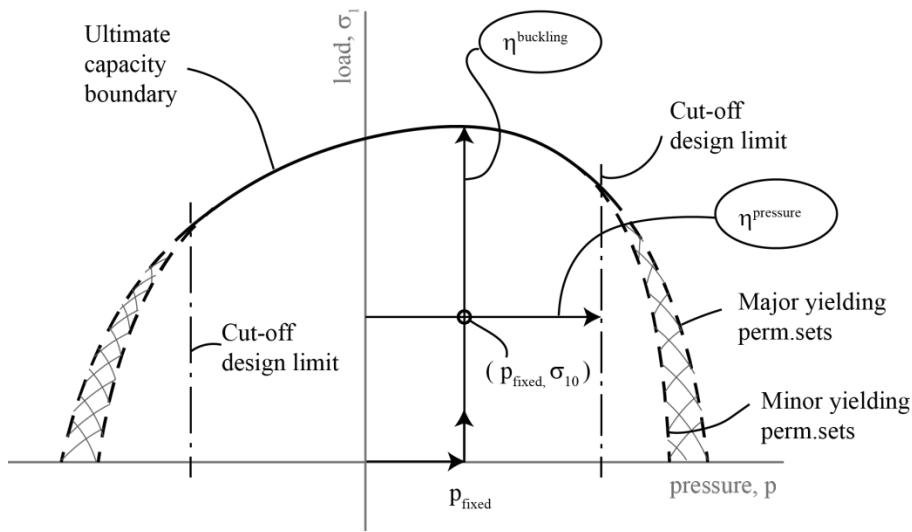


Figure 5 Definition of utilization factors for case (e.g. stiffened panel in ship hull) with one pre-load and one variable load)

For panels subjected mainly to in-plane compressive loads a clear ultimate value (peak) is identified in load space and it follows that the buckling utilization factor $\eta^{buckling}$ has unique definition. This ultimate strength limit then includes the possible knock down effect of lateral pressure on the in-plane load bearing capacity. For stiffened panels, as used in ship structures, this knock down effect is moderate, i.e. in the range of 5 to 10% for pressures up to say 0.2 - 0.4 MPa is typical.

For panels subjected to predominantly lateral pressure the characteristic behaviour is that no distinct ultimate capacity is identified i.e. the pressure can seemingly be increased beyond any reasonable limit. This is mainly due to the capability of plates to develop significant non-linear membrane stretching and develop large strains. The final failure mode will be controlled by localized large plastic straining in plates and stiffeners and involve fracture/material cracking due to tension stresses rather than buckling. The lateral pressure levels at which such failure limits are reached are far beyond normal design limits and thus not of practical value.

Thus simple "design" cut-offs are introduced for ensuring robust and solid stiffeners and plating. These cut-offs are typical limiting the stresses in the stiffeners at support (bending and shear) to be less than the yield stress. The purpose is to limit the possibility of permanent sets and damages.

They are similar in form as rule minimum requirements to stiffener section modulus Z. The rule criteria will overrule the present PULS cut-off criteria if otherwise not agreed with Society.

It is also worth noting that stiffened panels in ships are hanging on girder/bulkhead structures and the lateral pressure is transferred into these elements. Thus, the focus will normally be to ensure sufficient shear area in the girder webs to avoid local high stresses and possibly local plastic buckling damages here. Such local web girder checks are not part of the buckling codes or the present PULS models.

4 Elements and their validity

4.1 General

The current element library count four (4) different structural elements

- 1) U3; unstiffened rectangular plate
- 2) S3; uni-axially or orthogonally stiffened rectangular plate
- 3) T1; non-regular stiffened plate

- 4) K3; symmetric trapezoidal open corrugation.

A brief description of each of these elements is given in the following with some recommendations for practical use with limitations.

Details on validity limits, outside which the semi-analytical buckling models are not valid, are given in the User's manual /17/. These validity limits are similar to the rules slenderness requirements in DNVGL-RU-SHIP Pt.3 Ch.8 Sec.2 but generally more relaxed as they represent the theoretical limits of the semi-analytical models and not practical design limits. Thus rules max slenderness requirements overrules the PULS limits.

Applied stresses in buckling assessment with PULS are to be based on plate theory, not beam theory. Hence, the reference stresses σ_x and σ_y obtained by FE analysis according to DNVGL-Pt.3-Ch.8 Sec.4 [4.2] are to be used, i.e. correction as given in Sec.3 [2.3.6] shall not be done. In case of grillage analysis where the stresses are obtained based on beam theory, correction of stresses according to Sec.3 [2.2.7] shall be done.

4.2 Unstiffened regular plate; U3

The unstiffened plate model (U3) is single-bayed and applicable for plates where the edges are known to have rigid support in the lateral direction, Figure 6.

The element assesses elastic buckling (eigenvalues) as well as non-linear post-critical strength and the ultimate capacity.

Rotational boundary support can be varied from simply supported to fully clamped using rotational springs.

Two in-plane boundary restraints are available

- a) Integrated plating. The edges forced to be on a straight line (method A)
- b) Girder panel. The unloaded plate edges are free to pull in (method B).

The in-plane loads can be uniform or linearly varying along the edges while they are assumed to be constant in the direction they act, i.e. the in-plane loads go directly through the plate. Any combination bi-axial stresses in compression or tension combined with in-plane shear stresses can be used.

Lateral pressure effects can be simulated but for a single-bay panel this is not a very practical case. In ship hull structures the panels are integrated across many bays and the symmetric deflection pattern imposed by the lateral pressure is not considered in single-bayed models. This limits the practical usefulness of this feature of combined in-plane loads and lateral pressure. It may be interesting for special cases.

The ultimate capacity prediction is consistent with normal production tolerances used by yards and builders.

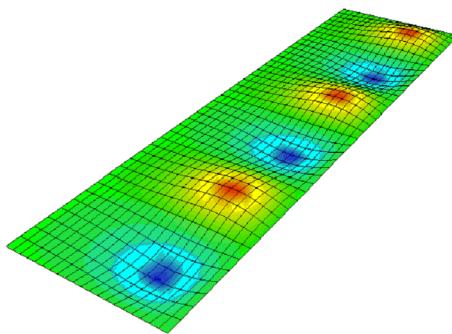


Figure 6 PULS unstiffened plate model (U3) – buckling mode for linearly varying loads in x_1 direction.

4.3 Uni-axially or orthogonally stiffened regular plate; S3

The regularly stiffened plate model (S3) is multi-bayed and assume the outer plate edges to have rigid support in the lateral direction as typical for panels framed inside a rigid girder/floor in hull structures, [Figure 7](#). The stiffening system may be uni-directional or orthogonal, with the continuous plating and primary stiffeners as the main load bearing elements. Secondary stiffeners running perpendicular to the primary direction act as buckling stiffeners.

The element assesses elastic buckling (eigenvalues) as well as non-linear post-critical strength and the ultimate capacity.

Two in-plane boundary restraints are available

- a) Integrated plating. The edges forced to be on a straight line (method A)
- b) Girder panel. The unloaded plate edges are free to pull in (method B).

The in-plane loads is uniform in the primary stiffener direction while can vary or linearly in the orthogonal direction. The in-plane shear loads are constant across the panel. Any combination bi-axial stresses in compression or tension combined with in-plane shear stresses can be used.

Uniform lateral pressure can be simulated and is assumed constant across several bays in all bay directions. The lateral pressure is carried by the primary stiffeners. The lateral pressure amplifies the local plate deflections between primary stiffeners and it is modelled as an extra imperfection on top production model imperfection.

A code constraint is introduced preventing overall buckling (out-of the plate plane) of the primary stiffeners. This is implemented as a cut-off at the global elastic buckling load (GEB eigenvalue) and should ensure robust stiffeners supporting the plating.

The failure of the panel will generally be visualized as a hybrid mix of buckling modes in plating and stiffeners, one more dominating than others depending on geometrical layout and loading type. For this element the failure modes is split in four (4) categories consistent with prescriptive rules notation

1) Plate buckling between stiffeners (plate buckling)	x %
2) Lateral buckling of primary stiffener (global/overall buckling)	y %
3) Sideways buckling of stiffener top (torsional buckling)	z %
4) Local buckling of stiffener web (web buckling)	xy%
SUM	100%

Using this element for buckling control implies an integrated check of all failure modes and load shedding between plating and primary stiffeners is coped with. There is thus no need to check plate buckling (U3) in addition. This is a principal difference to the CFM approach.

Note that load shedding to neighbouring girders/floors outside the actual S3 model (running in x_2 -direction) is a separate issue as discussed in [App.A \[1.4\]](#).

The ultimate capacity prediction is consistent with normal production tolerances used by yards and builders.

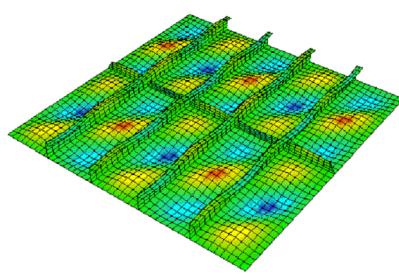


Figure 7 PULS regularly stiffened plate model (S3) - buckling mode for in-plane shear stress.

4.4 Non-regular stiffened plate; T1

The non-regularly stiffened plate model (T1) is single-bayed and assume the outer plate edges to have rigid support in the lateral direction, [Figure 8](#). The stiffening system may be arbitrary oriented and stiffeners with different proportions running in different directions can be modelled. The stiffeners are modelled as secondary buckling stiffener and have to be proportioned such that local buckling of their web or flange is not possible. This is ensured by setting reasonable slenderness requirements.

The element assesses elastic buckling (eigenvalues) and plasticity correction is included for buckling strength assessment in the moderate to low slenderness range. The model apply a linearized version of the present plate buckling theory meaning the loads beyond the elastic buckling level (eigenvalue) is not possible.

Rotational boundary support can be varied from simply supported to fully clamped using rotational springs. A stiffener has by default no rotational stiffness but rotational springs can be modelled along the plate/stiffener junction line.

The in-plane loads can vary linearly in both directions while the in-plane shear loads are constant across the panel. Any combination bi-axial stresses in compression or tension combined with in-plane shear stresses can be used.

The buckling capacity prediction is consistent with normal production tolerances used by yards and builders.

Relevant areas for application can be web plating on girders/brackets etc. in fore or aft ship where the layout of plating and stiffeners is non-regular.

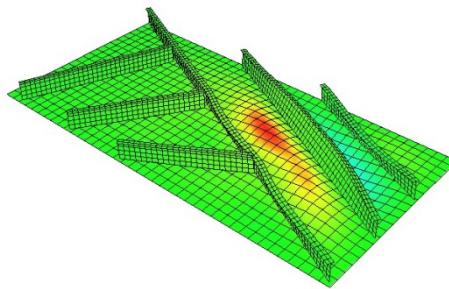


Figure 8 PULS non-regular geometry model (T1)

4.5 Open corrugated trapezoidal panel; K3

The corrugated plate model (K3) is single-bayed and assume the outer plate edges to have rigid support in the lateral direction. The corrugation is open and has a regular symmetric trapezoidal shape as typical for bulkheads in ship hulls, [Figure 9](#).

The element assesses elastic buckling (eigenvalues) as well as non-linear post-critical strength and the ultimate capacity. Buckling failure modes cover local flange, local web buckling and overall (global) panel buckling.

Rotational boundary support across the corrugation height (hw) can be varied from simply supported to fully clamped using rotational springs.

The in-plane stress is uniform in the axial direction and the in-plane shear loads are constant across the panel. Any combination of axial and in-plane shear stresses can be modelled. Stresses perpendicular to the corrugation are in practise very low and not a parameter for buckling assessment.

Uniform lateral pressure can be simulated and can be varied linearly across the length of the corrugation. The lateral pressure is carried by the corrugation in bending and out-of plane shear.

The load input can be either taken from a linear FE analyses of the corrugation bulkhead (e.g. FE cargo hold model) or directly as edge loads and lateral pressure (simplified).

The ultimate capacity prediction is consistent with normal production tolerances used by yards and builders.

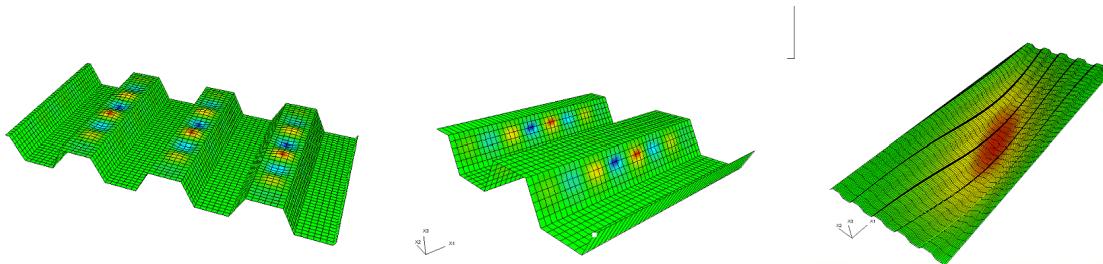


Figure 9 PULS open trapezoidal corrugation element (K3); left; local flange buckling, middle; local shear web buckling, right; overall panel buckling

4.6 FRP Composite plate buckling element – C1

An unstiffened FRP composite plate buckling model (C1) is available for plates where the edges have rigid support in the lateral direction.

The plate can be built up of many equal FRP layers, each layer can have different angle orientation of orthotropic material properties E_1 , E_2 , ν_{12} and G_{12} . Thus any anisotropic plate can be assessed wrt elastic buckling limit as well as first material failure.

The element assesses elastic buckling (eigenvalues) in combination with material failure according to Tsai – Wu criteria.

Rotational boundary support can be varied from simply supported to fully clamped using rotational springs.

The in-plane loads can be uniform or linearly varying along the edges while they are assumed to be constant in the direction they act, i.e. the in-plane loads goes directly through the plate. Any combination bi-axial stresses in compression or tension combined with in-plane shear stresses can be used.

A semi-analytical buckling model for steel sandwich plates (SPS) can be found in [0154](#).

5 Acceptance criteria

The acceptance levels for ships follows the rules as given in [DNVGL-RU-SHIP Pt.3 Ch.8 Sec.1 Table 1](#)

For general buckling assessments of other structures, the acceptance criteria needs to reflect the redundancy and consequence of failure and has to be based on a case by case evaluation by Society.

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SECTION 5 ULTIMATE HULL GIRDER CAPACITY

1 General

The present chapter gives a general description of the ultimate hull girder capacity models (HGUS) as part of the ULS criteria given in [DNVGL-RU-SHIP Pt.3 Ch.5 Sec.2](#). This criterion is supplementary to hull girder yielding strength criteria [DNVGL-RU-SHIP Pt.3 Ch.5 Sec.1](#) and prescriptive longitudinal buckling requirements in [DNVGL-RU-SHIP Pt.3 Ch.8 Sec.3](#).

HGUS control of a ship represents an overall safety limit state that has the purpose of ensuring sufficient margins against total ship collapse thus avoiding loss of cargo, personnel and ship.

2 Objective

The HGUS girder criteria are to ensure sufficient operational safety margins against overall hull girder collapse under extreme wave load conditions.

For an intact ship hull the vessel shall stay afloat after the extreme wave incident. Buckling, yielding and development of permanent sets/buckles locally in the hull are accepted as long as the hull girder does not collapse or break in two with possible plate cracking, compartment flooding and capsizing as consequence.

For ships suffering collision or grounding damages similar capacity models for hull girder collapse can be used. However such analysis is not mandatory.

3 Scope

The hull sections are to be checked for an extreme sagging and hogging condition separately, [Figure 1](#).

Guidance note:

The hull girder strength criteria in this note include only vertical bending response. Horizontal bending and hull twisting of hull sections typical for ships in inclined waves are not included. Similarly, capacity models for large hull asymmetrical collision damages for which the hull sections will twist, warp and bend about an inclined axis even for head waves, is not part of present description.

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

The hull girder criteria shall be applied to all relevant hull sections along the ship length, also out-side 0.4 L for ship designs in which buckling and hull girder failure is considered to be a possible failure mode.

Guidance note:

Hull girder sections subjected to pure or dominating vertical moments will typically be in the mid-ship area, while sections towards the ship ends will have a combination of bending moment and shear forces.

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

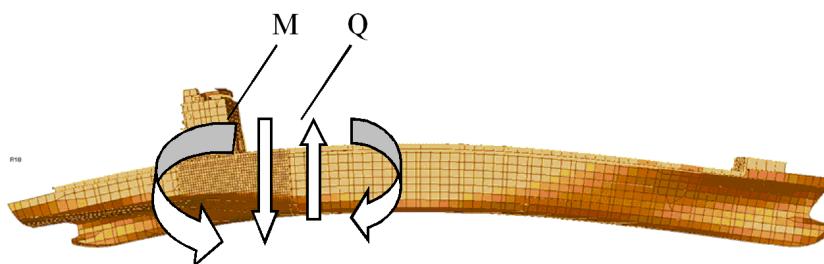


Figure 1 Example; Ship hull girder in a hogging state, M, Q global hull sectional loads

4 Methods

Ultimate hull girder capacity involves assessing the overall strength by summing up load-bearing capacities of individual elements such as plates, stiffeners, girder s in the cross-section.

Several rule models can be used for assessing the ultimate moment, M_u :

- 1) Multi-step model (Smith method - iterations).
- 2) Single-step model (reduced effective area of compressed members).
- 3) Non-linear FE analyses .

The multi-step model is described in [6.2] and involves integrating load-shortening curves of all elements in the hull section for finding the ultimate moment capacity M_u . Iterations are necessary in order solve the equilibrium condition at each load step of having no axial force through the section and thus adjusting the neutral axis accordingly.

The single-step model is described in [6.3] and involves assessing the effective load bearing area of each element a priori. This gives an effective hull section with adjusted neutral axis. The ultimate moment capacity M_u is found by summing the moment contribution from all elements.

Non-linear FE analyses are accepted following the guidelines in Sec.6. However, approval using such a direct approach is based on case by case evaluation by the Society and comes at extra costs (approval fee).

5 Design safety formats

The design format is given as two uncoupled criteria for moment and shear forces respectively

$$\begin{aligned} M_s + \gamma_w^M M_w &< \frac{M_u}{\gamma_m^M} \\ Q_s + \gamma_w^Q Q_w &< \frac{Q_u}{\gamma_m^Q} \end{aligned}$$

The load and safety parameters are given in the rules DNVGL-RU-SHIP Pt.3 Ch.5 Sec.4. The ultimate capacity models for M_u (and Q_u) are described in [6]

Special considerations are needed for ship types for which double bottom stresses can be an issue with respect to hull girder capacity in hogging.

If non-linear FE models are used the safety factors will specially considered on case by case basis by Society.

6 Simple models

6.1 General

The Society allows basically two types of models for assessing the ultimate capacity; the multi-step and single-step respectively.

6.2 Multi-step method

Multi-step model

The ultimate capacity of the total hull girder section is based on integrating load-shortening curves of the individual plate and stiffeners for both compressive and tension elements. The necessary input for this approach is the full load-shortening characteristics of all load-carrying members valid in the elastic (pre-collapse) as well as in the plastic (post-collapse) range.

Applying an iterative approach incrementing the hull girder curvature in small steps (with equilibrium correction if necessary) the total $M - \kappa$ relationship is traced. The maximum load identified as the peak moment along this curved, called M_U , is then the maximum moment the actual hull girder section can carry. The general way to find the M_U value will be to solve the non-linear physical problem (equilibrium equations) by stepping along the $M - \kappa$ curve using an incremental-iterative numerical approach. This means that the ultimate capacity can be found by summing up the incremental moments along the curve until the peak value is reached, i.e. from knowing the moment in state i the next state $i+1$ can be found:

$$M_U^{i+1} = M_U^i + \Delta M^i$$

Here the ΔM^i is an incremental moment corresponding to an incremental curvature $\Delta \kappa_i$ and N is the number of steps used in order to reach the peak value M_U , beyond which the incremental moments become negative (post-collapse region).

The incremental moment, ΔM^i , is related to the incremental curvature, $\Delta \kappa_i$, through the tangent stiffness relation

$$\Delta M^i = (EI)_{red-i} \Delta \kappa_i$$

Here $(EI)_{red-i}$ represent the incremental bending stiffness of the hull girder. The $(EI)_{red-i}$ stiffness is state (load) dependent and will be gradually lower along the $M-\kappa$ curve and zero at the global hull collapse level (M_U). The $(EI)_{red-i}$ parameter shall include all important effects such as:

- 1) geometrical and material non-linear effects
- 2) buckling, post-buckling and yielding of individual hull section members
- 3) geometrical imperfections/tolerances - size and shape; trigger of critical modes
- 4) interaction between buckling modes
- 5) bi-axial compression/tension and/or shear stresses acting simultaneously with the longitudinal stresses
- 6) double bottom bending effects (hogging)
- 7) shift in neutral axis due to buckling/collapse and consequent load shedding between elements in the cross-section
- 8) boundary conditions and interactions/restraints between elements
- 9) global shear loads (vertical bending)
- 10) lateral pressure effects
- 11) local patch loads (crane loads, equipments etc.)
- 12) for damaged hull cases special consideration are to be given to flooding effects, non-symmetric deformations, warping, horizontal bending, residual stresses from the collision/ grounding.

One version of the multi-step method is the Smith method which is based on integrating simplified semi-empirical load-shortening ($P - \varepsilon$, load-strain) curves across the hull section to give the total moment $M - \kappa$ relation. The maximum value M_U along the $M - \kappa$ curve is found by incrementing the curvature κ of the hull section in steps and then calculated the corresponding increase in moment. When the moment starts to drop the maximum moment M_U is identified.

The Smith method is based on load-shortening $P - \varepsilon$ curves for the compressed and collapsing elements and how the listed effects i) - xii) above are embedded into these relations.

Details for this method are shown in [Appendix E Hull girder ultimate bending capacity by multi-step method](#).

6.3 Single-step

The ultimate capacity of the total hull girder section is based on assessing the ultimate capacity of the individual plate and stiffeners buckling/tension capacity (PULS or CFM) and then summarized/integrate up to the total capacity. The individual capacity plates and stiffeners are each given an effective area and the total hull girder capacity is in effect the moment load bearing capacity of the effective hull girder section (i.e. simple beam theory).

The basis for the single step method is to summarize the moments carried by each individual element across the hull section at the point of hull girder collapse, i.e.

$$M_U = \int_{\text{hull-section}} \sigma z \, dA = \sum_{i=1}^K P_i z_i = \sum (EA)_{\text{eff}-i} \varepsilon_i z_i$$

where:

- P_i = axial load in element no. i at hull girder collapse ($P_i = (EA)_{\text{eff}-i} \varepsilon_i^{\text{g-collapse}}$)
- z_i = distance from hull-section neutral axis to centre of area of element no. i at hull girder collapse. The neutral axis position is to be shifted due to local buckling and collapse of individual elements in the hull-section.
- $(EA)_{\text{eff}-i}$ = axial stiffness of element no. i accounting for buckling of plating and stiffeners (pre-collapse stiffness)
- K = total number of assumed elements in hull section (typical stiffened panels, girders etc.)
- ε_i = axial strain of centre of area of element no. i at hull girder collapse ($\varepsilon_i = \varepsilon_i^{\text{g-collapse}}$); the collapse strain for each element follows the displacement hypothesis assumed for the hull section
- σ = axial stress in hull-section
- z = vertical co-ordinate in hull-section measured from neutral axis

It is generally accepted for intact vessels that the hull sections rotate under the assumption of Navier's hypothesis, i.e. plane sections remain plane and normal to "instantaneous" neutral axis, i.e.

$$\varepsilon_i = z_i \kappa, \kappa = \theta L_S$$

where:

- ε_i = axial strain of centre of area of element no. i (relative end-shortening)
- κ = curvature of the hull section between two transverse frames (across hull section length L)
- L_S = length of considered hull section
- θ = relative rotation angle of hull section end planes (across hull section length L)

This gives the following formula for the Ultimate moment capacity

$$M_U = (EI)_{\text{eff}} \kappa_U$$

where

$$(EI)_{\text{eff}} = \sum_{i=1}^K (EA)_{\text{eff}-i} (z_i)^2$$

Effective bending stiffness of the hull section accounting for reduced axial stiffness $(EA)_{\text{eff}-i}$ of individual elements due to local buckling and collapse of stiffeners, plates etc.

$(EA)_{eff-i}$

Effective axial stiffness of individual elements/stiffened panels accounting for local buckling of plates and stiffeners and interactions between them. Effects from geometrical imperfections and out-of flatness to be included

$$\kappa_U = \min\left(\frac{\varepsilon_U^{deck}}{z_{deck}}; \frac{\varepsilon_U^{bottom}}{z_{bottom}}\right)$$

Hull curvature at global collapse

$$\varepsilon_U^{deck}$$

Average axial strain in deck at global collapse. $\varepsilon_U^{deck} = \varepsilon_F^{deck} = \sigma_F/E$ is accepted

$$\varepsilon_U^{bottom}$$

Average axial strain in bottom at global collapse. $\varepsilon_U^{bottom} = \varepsilon_F^{bottom} = \sigma_F/E$ is accepted

$$\varepsilon_F^{deck}$$

Weighted yield strain of deck elements if material class differences (uni-axial linear material law: $\varepsilon_F = \sigma_F/E$)

$$\varepsilon_F^{bottom}$$

Weighted yield strain of the bottom elements if material class differences (uni-axial linear material law $\varepsilon_F = \sigma_F/E$) (corrections to be considered if inner bottom has lower yield stress than bottom)

This gives

$$M_U = \min(W_{eff}^{deck} \sigma_{eff}^{deck}; W_{eff}^{bottom} \sigma_F^{bottom})$$

with the following definitions

$$W_{eff}^{deck} = \frac{l_{eff}}{z_{deck}}$$

Effective section modulus of the hull section in the deck

$$W_{eff}^{bottom} = \frac{l_{eff}}{z_{bottom}}$$

Effective section modulus of the hull section in the bottom

7 Direct approach; non-linear FE

A direct method using “state of the art” tools such as non-linear FE codes will represent the most comprehensive approach for assessing the overall hull girder capacity of the vessel.

Depending on the case to be assessed the FE model can include the full ship or cut-out sections within which the hull girder is most likely to collapse.

A large FE ship hull model, extending many frame spacings or even several cargo holds in the longitudinal direction, has the benefit of covering complicated effects which are generally missed when applying short prismatic sections of the hull girder. This can typically be bi-axial stress effects in bottom plating, in-plane shear stresses, double hull bending effects, stress variations around hatches/opening in decks, local loadings and elimination of boundary effects.

The FE mesh density is to be fine enough to capture all relevant types of local buckling deformations and localized plastic collapse behaviour in plating, stiffeners and primary support members.

The following requirements apply when using 4 node shell element (thin-shell element is sufficient):

- 1) Min. 5 elements across the plating between stiffeners/girders
- 2) Min 3 elements across stiffener web height
- 3) One element across stiffener flange is acceptable
- 4) Longitudinal girders: min. 5 elements between local secondary stiffeners
- 5) Element aspect ratio 2 or less in critical areas susceptible to buckling
- 6) For transverse girders a coarser meshing is acceptable. The girder modelling should represent a realistic stiffness and restraint for the longitudinal stiffeners, ship hull plating, tank top plating etc.
- 7) Man holes and large cut-outs in girder web frames and stringers shall be modelled
- 8) Secondary stiffener on web frames prone to buckling shall be modelled
- 9) Plated and shell elements shall be used in all structural elements and areas susceptible to buckling and localized collapse
- 10) Stiffeners can be modelled as beam-elements in areas not critical from a local buckling and collapse point of view.

When using non-linear FE analyses the accept criteria and partial safety factors in strength format need special consideration. The Society will accept non-linear FE methods based on a case by case evaluation and an independent assessment is to be carried out.

SECTION 6 STRUCTURAL CAPACITY USING NON-LINEAR FE

1 General

In this present guideline the focus is on buckling and collapse analyses applying the quasi-static approach, i.e. structural problems in which the load varies slowly with time (normal wave encounters etc.) such that vibration and short duration dynamic effects can be neglected.

For design assessments the Society will only accept well established and recognized non-linear FE programs. Additional information is available in related guideline for application of non-linear FE for offshore structures, see /1.

2 Independent assessment by the Society

If the designer chose to apply non-linear FE or equivalent methods for documenting new innovative design solutions not covered by rules, or intend to use more advanced methods than standard rule requirement for design of specific details, an independent non-linear FE simulation will be required.

3 Objectives

The user needs to analyse the actual case under different (initial) conditions in order to identify the sensitivity to important parameters and variables. Such trial and error testing is needed for structures which show particular unstable response, e.g. displaying numerical convergence problems due to violent mode snapping, localized material yielding, progressive collapse behaviour.

4 Scope

The present chapter gives guidelines on how to model and analyse structural problems for plated and framed beam-column structures. Focus is on buckling and collapse failure modes covering material and geometrical non-linear effects, geometrical out-of flatness and boundary conditions.

5 Non-linear FE methods

5.1 General

Non-linear FE is capable of considering the following non-linear basic effects

- a) Non-linear geometrical behaviour
- b) Non-linear and inelastic material behaviour
- c) Contact problems
- d) Fracture.

In ultimate strength assessments of ship structures the a) and b) effects are always to be included while c) and d) are normally not relevant unless for special considerations.

For ultimate capacity models the following items are of special importance:

- 1) Geometrical out-of flatness of plate, stiffeners, shells, beams (geometrical imperfections)
- 2) Load shedding between structural elements; plates, stiffeners, girders etc.
- 3) Load histories for simultaneous acting loads: ex. bi-axial compression/tension, shear and lateral pressure
- 4) Element mesh
- 5) Extent of FE model
- 6) Boundary conditions

5.2 Non-linear geometrical behaviour

This effect results from large deflections and changes of the geometrical shape due to the acting external load. The effect leads to a non-linear relation between load and deflection when the deflections become large enough compared to the structural dimensions.

For compressive loads acting on a plate, the effect is pronounced already for deflections of the order half the plate thickness and it is typically visualized as local buckles in the plating. For very unstable shell buckling problems the non-linear effect is significant already for deflections being a fraction of the shell thickness. In framed structures such as jack-up platforms etc., large sideways sway deflections will be governed by non-linear effects in general ($P-\Delta$ effect amplifying linear response).

Non-linear geometrical effects are most pronounced for slender structures, i.e. large deflections will be present before material non-linearity starts.

Non-linear plate theories based on the thin-walled concepts of von Karman and Marguerre are sufficiently accurate for assessing elastic buckling, ultimate strength and initial post-collapse of steel, aluminium or FRP/GRP composite structures. For other structures such as based on the sandwich concept etc., more accurate plate theories are needed which includes shear deformations through the thickness etc. Alternatively solid elements can be used but that is normally beyond normal design analyses practise.

Recognized FE codes have plate shell elements that account for first order shear deformations through the thickness as part of the standard library (Mindlin or Reissner plate theory or equivalent).

5.3 Non-linear material behaviour

Non-linear material behaviour is characterized by:

- Stress - strain curve
- Yield criteria for metallic materials (von Mises)
- Failure criteria for composites (Tsai-Wu)
- A hardening model for metallic material (isotropic or kinematic)
- A flow rule for metallic materials (defines constitutive material coefficients for plastic flow).

1) Stress – strain curve

The real stress strain curve can be approximated as a multi-linear curve. For steel material with a marked yield plateau a bi-linear approximation is also acceptable with a low plastic tangent module ($E_t = 1000$ MPa) or ideally a perfect elastic-plastic curve ($E_t = 0$)

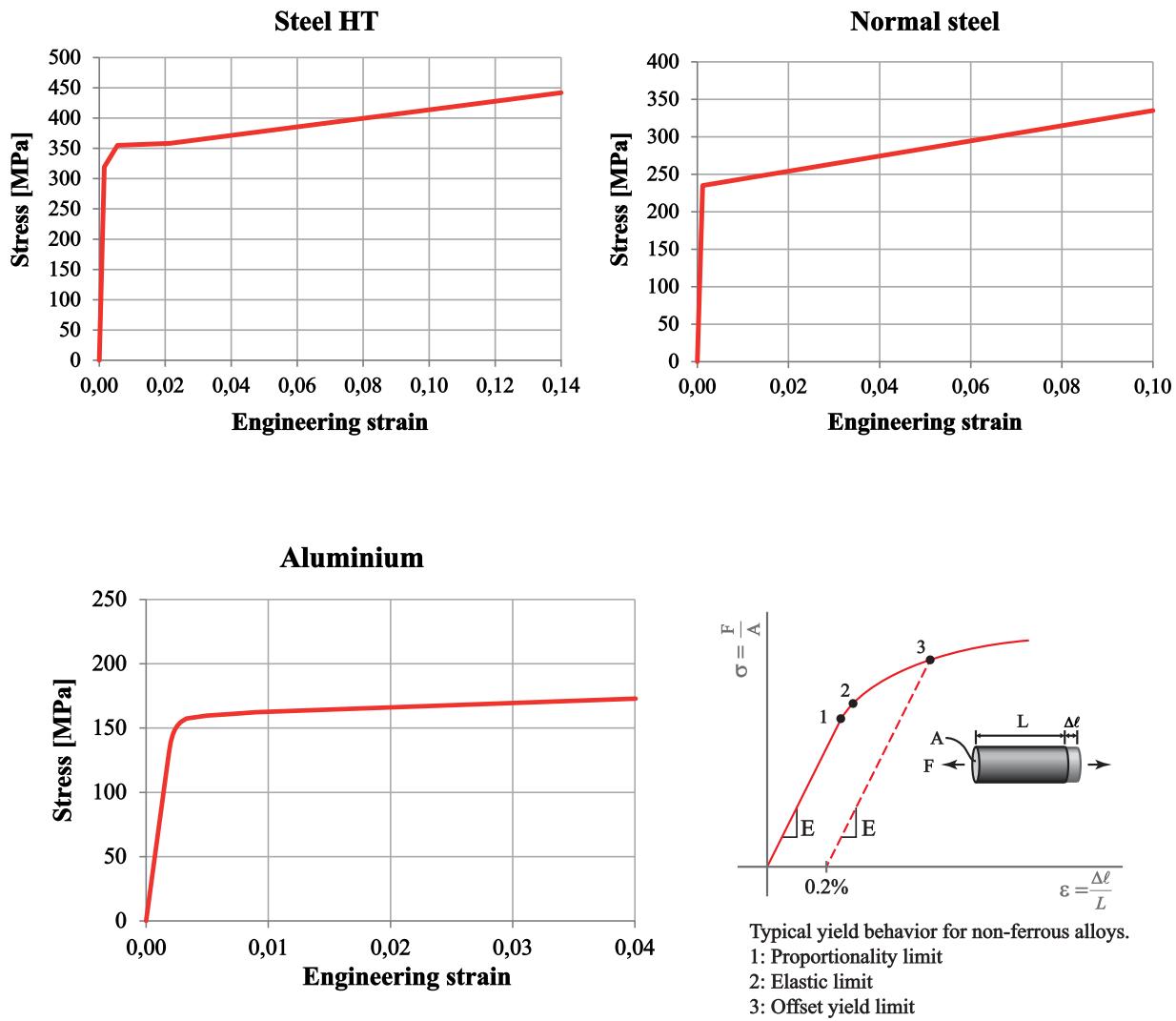


Figure 1 Illustration of different material curves (engineering stress-strain)

For aluminium the stress-strain curves are typically more gradual with a continuously changing slope, i.e. the material characteristics is not well defined, see Figure 1. The elastic limit (2) is defined as the limit beyond which permanent (plastic) straining will occur. The point at which the curve starts to flatten/levels off (3) major plastic flow starts can be defined as 0.2% plastic strain if detailed information is not available.

FE programs are using different material definitions and the material input must correspond with this definition. For instance, in many FE programs the material input must be in true stress-strain. For buckling analyses the strains are small and the distinction between engineering strain and true strain are negligible.

For ultimate strength characteristics the lower bound stress-strain curve should be used. The yield and material fracture stress are to be lower bound with typical 95% probability of exceedance (rule values if not more accurate data is available).

2) Yield criteria for metallic materials (von Mises)

The von Mises yield criterion is generally accepted for all metallic materials and is the standard “default” model available in FE programs. For new materials verification with the von Mises criterion are to be documented unless a more specific material model has to be implemented. The input to the model is a single input material characteristic, i.e. the uni-axial tensile yield stress.

3) Failure criteria for composites (Tsai-Wu)

For composites materials the Tsai-Wu criterion is generally accepted and available in most FE programs. The criterion needs a set of material strength parameters (F_{ij}) that are experimentally determined. Hashin fracture models are recognized alternatives.

4) A hardening model for metallic material (isotropic or kinematic)

The hardening model describes how the yield condition changes when the stresses reaches and moves outside the yield condition. The two most used models are isotropic and kinematic hardening models and both are available in most FE programs.

The isotropic model implies that the shape of the yield stress is the same but it expands equally in all directions. The kinematic model implies that the yield surface is fixed but moves in load space. The latter includes the Bauchinger effect, i.e. after first yielding the tension and compression yield stress will deviate.

For buckling and ultimate strength assessments both models are equally acceptable.

5) A flow rule for metallic materials (defines constitutive material coefficients for plastic flow)

The flow rule relates the incremental stress to the incremental plastic strains defining a set of constitutive coefficients for plastic loading. The J2 flow theory is default in most FE programs and are generally acceptable in combination with metallic materials, von Mises yield criterion and isotropic or kinematic hardening.

5.4 Geometrical out-of flatness

Real structures do always have deviations from the perfect form due to welding and production etc. These imperfection patterns will be rather random and exact information is never available. In practise ship yards and class societies have quality standards defining simple and local maximum tolerance limits to be measured over defined gauge lengths.

Moreover, the ultimate capacity is known to be imperfection sensitive to different degrees depending on the structural configuration, load type, slenderness, redundancy level etc. The issue is of particular concern for non-redundant structures in which the possibility of load shedding is limited or not possible. For larger and redundant plate structures the issue is of less importance, i.e. the focus is rather on the FE model to be able to trigger the most critical (preferred modes) such that progressive collapse and control of the numerical solution are achieved.

Non-linear analyses need to consider the “imperfect” geometry in some way and since real life imperfections patterns (shapes and sizes) are not available simplified regular model imperfections will be the natural approach. There is no exact and unique way to construct model imperfections covering all cases, however, the natural first step will be to use the minimum eigenmode shape for the “perfect” structure, or a combination of eigenmodes if a cluster of eigenvalues around the lowest are found.

Alternative approaches may be used as found relevant, e.g. for rather slender structures a pure non-linear geometrical analyses on the “perfect” geometry may be able to identify “higher” and critical modes not otherwise identified (assume linear elastic material). The computed deformed geometry can then be used as “stress free” imperfection in a fully non-linear analysis with both material and geometrical non-linearities included.

The regular model imperfection amplitude to be used in the FE model should be balanced in relation to how this fictive model shape compares to the typical production shapes. Regular model shapes will be harmonic over a large area while real life deviations are much more localized and random in shape (e.g. “hungry horse” shape for plate panels etc.). Thus the former may be taken less than the maximum tolerance specified in the survey Standards. It is generally required that a sensitivity study is carried out in order to document a safe lower bound strength limit.

For cases in which some information is available on the imperfection pattern, or for residual strength assessments based on "measured damages", it may be relevant to directly model these deviations from perfect form in a reasonable realistic manner.

However, in any case it is crucial to study and vary parameters such that the main features and degree of instability and imperfection sensitivity are identified. The solutions and strength values found should be verified against other relevant analyses models, text book theories and other published work in order to document the validity of the results.

In [App.B](#) and [App.C](#) respectively two examples are given for illustration; the first being an extremely unstable and imperfection sensitive shell buckling problem and latter being a more moderately sensitive stiffened plate buckling case.

5.5 Load shedding between elements

Analysing structures having an instability problem involves considerations of all relevant buckling modes and their interactions. Mode interactions may be detrimental to the ultimate load bearing capacity and will typically be pronounced in structures where two or more buckling modes will be triggered at or close to the same external load level.

Examples are stiffened plates or built up columns in which buckling typical is possible into a local and overall mode. The triggering of the local mode, like plate buckling between stiffeners, involves loss of stiffness and rigidity for the stiffener, which then triggers an overall mode which next amplifies the local mode etc.

The same type of mode interaction can relevant in built-up columns (jack up legs) where chord buckling represents the local mode and global leg mode the overall mode. In normal jack up leg design the local mode is stronger than the overall mode.

Mode interactions are also linked to load shedding between elements, e.g. in plated structures local plate buckling implies that the stresses sheds to the plate edges/stiffeners/girders (effective width) and the extra compressive loads then to be carried by stiffeners/edges amplifies the plate deflections further etc.

In order to cope with these effects in non-linear FE models it is important to have sufficiently fine mesh so that the buckling modes, especially the local modes, are developing realistically and material yielding are coped with in narrow and "hard corner" zones. Some guidance on element meshing is given in [\[5.9\]](#).

5.6 Load history

Load histories for simultaneous acting loads: For structural problems in which more than one load acts simultaneously a load history is needed. Often the exact relation between the combined loads does not exist and for such cases it is convenient to assume proportional loading, i.e. all loads are scaled in same proportion. For a one step case the loads are scaled as:

$$\left[\begin{array}{l} P_1 = \Lambda P_{1, ref} \\ P_2 = \Lambda P_{2, ref} \\ \vdots \\ \vdots \\ P_i = \Lambda P_{i, ref} \\ \vdots \\ \vdots \\ P_N = \Lambda P_{N, ref} \end{array} \right] \quad i = 1, 2, \dots, N: \text{N-Number of independent loads } P_i$$

In this way N independent loads are reduced to one, i.e. the load factor Λ , which is to be multiplied with the reference loads $P_{i,\text{ref}}$ to find each any load at any state. For displacement control, the reference loads must be replaced by reference displacements.

For general description of load histories non-linear FE codes provides full flexibilities by pre-defining all load components as a function of a single and always continuously increasing pseudo time variable t . This can be needed in the case of e.g. prestressed structures, i.e. a set of load parameters are first increased to a fixed level and then kept constant while the other load components are then increased up to collapse (see also Sec.4 [3.3]). This is mathematically expressed as:

$$\begin{aligned} P_1 &= P_1(t) \\ P_2 &= P_2(t) \\ \vdots & \\ P_i &= P_i(t) \\ \vdots & \\ P_N &= P_N(t) \end{aligned}$$

In this way any load history can be described in load-space as illustrated schematically in Figure 2.

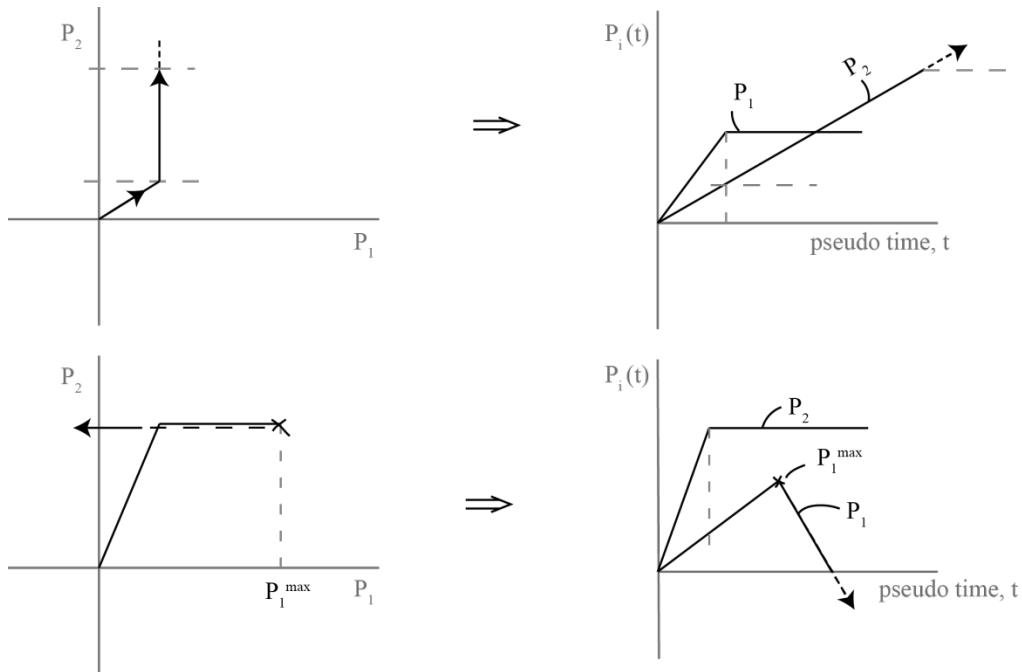


Figure 2 Schematic illustration of load histories;

$P_i = P_i(t)$ (t -pseudo time) and in load space (P_1, P_2, \dots, P_N).

5.7 Boundary conditions

Attention on boundary conditions is important in order to allow all relevant failure modes to develop at the same time as unrealistic edge effects are to be eliminated.

For non-linear analyses focus is both on rotational out of-plane rotations as well as in-plane translational displacements. The former has effects of the linearized eigenvalues while the latter have influence on the post-buckling and non-linear moderate to large deflection effects.

5.8 Extent of model

The extent of the model should be large enough to allow the relevant failure modes to develop without interference from boundary effects. As an example of this, a stiffened plate field should be modelled with two whole frame spacings plus one half frame spacing on either side in the longitudinal direction and a minimum of 6 stiffener spacings in the transverse direction. In this way, the relevant failure modes including stiffener buckling and local and global plate buckling can be captured by the model.

5.9 Element and mesh

The element mesh should be sufficiently detailed to capture all relevant failure modes. Some relevant guiding is;

- Depending on the element type, 3 to 6 elements are needed across a typical half wave in order to have a correct description of local stiffness and buckling strength.
- Element aspect ratio should be ideally as close to square as possible. Aspect ratio beyond three (3) should be avoided.
- Gradual meshing from fine to courses meshed areas are to be used avoiding non-physical and abrupt stiffness changes.

5.10 Residual stresses

Welding stresses are not needed in the FE shells/plate models for which the ultimate strength analyses are main interest. For very localized behaviour in which the behaviour of on the welds are the main focus they may be included using preferably solid elements.

Structures suffering large plastic straining in local areas will also have residual stresses when unloaded (to zero load). This may be topic to be considered if reloading or cyclic response is of relevance.

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APPENDIX A BUCKLING OF PLATES AND STIFFENERS

1 Stiffened flat plated Structures

1.1 General

Being the main load bearing element in ship hulls, buckling characteristics of stiffened flat panels are given some extra focus and descriptions herein. The topics discussed covers ultimate strength, elastic buckling and post-buckling, load-shedding/stress redistributions, imperfection sensitivity, load-shortening behaviour and boundary supports effects.

1.2 Ultimate strength – plastic buckling

The ultimate capacity is defined as the maximum load the stiffened panel can carry without collapsing. Compression beyond the ultimate capacity limit is not accepted as plastic buckles and permanent sets will be likely (see also Sec.2 [2.1])

Guidance note:

For most plated structures subjected to predominantly in-plane compressive and shear loads, some marginal and localized material yielding (plate surface yielding due to second order stresses) will always be initiated before the ultimate load limit is reached. However, such marginal material yielding will not lead to major permanent sets/buckling damages and is in this respect harmless.

Figure 1 and Figure 2 illustrate typical permanent sets and buckling damages for stiffened plates compressed beyond the ultimate strength limit. Figure 1 is a case where the torsional stiffener failure (SI) is the critical mode and compression beyond this level triggers a tripping failure with localized sideways buckling damages of the stiffener. Figure 1 illustrates a case were the plate (PI) is the weakest element and localized plastic plate buckling takes in case of "overloading".

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

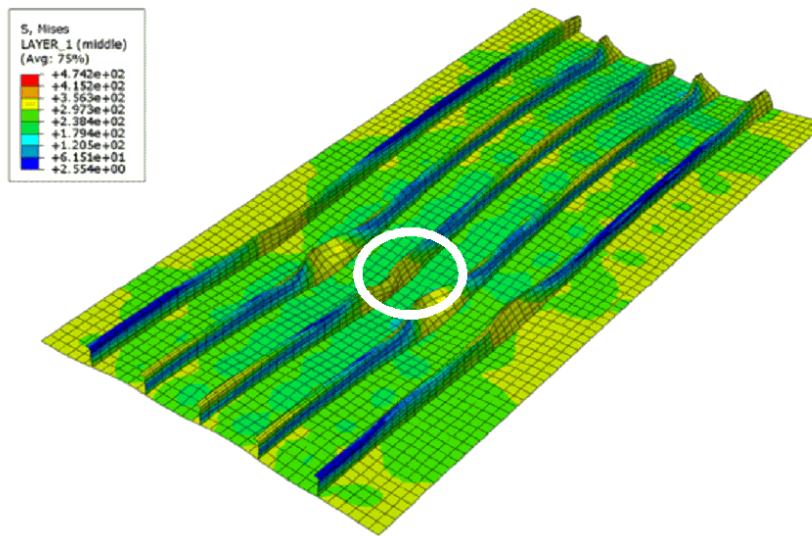


Figure 1 Localized buckling pattern in the plastic (post-collapse) range, e.g. typical stiffener induced tripping failure.

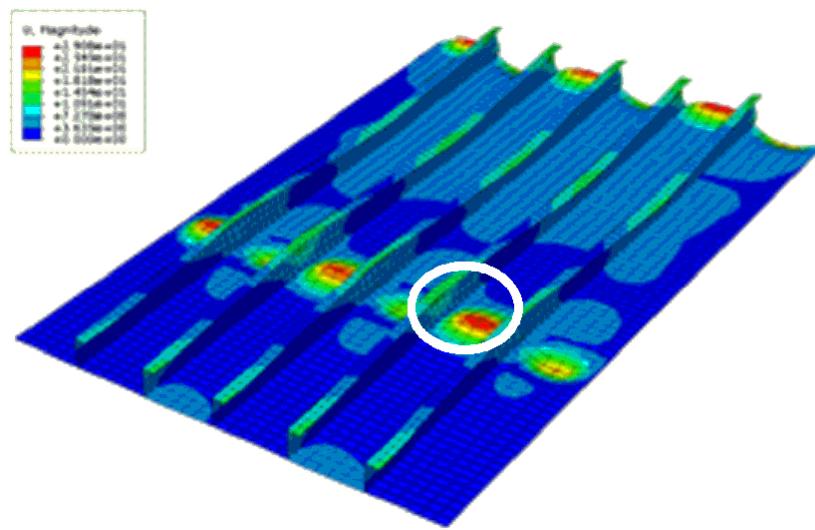


Figure 2 Localized buckling pattern in the plastic (post-collapse) range, e.g. typical plate induced failure.

1.3 Elastic buckling and postbuckling – second order stresses

Elastic buckling of plates is not critical since higher loads than the eigenvalue can be carried due to the positive postbuckling strength being provided from support along the edges.

Accordingly, elastic plate buckling is not classified as a failure mode. The deflections are recoverable and do not lead to damages or permanent sets.

Guidance note:

For stiffened plated areas such as decks, bottom areas etc with relatively uniform stress levels, the elastic buckling modes are normally harmonic in shape and span over a relative large area, i.e. typically several plate and stiffener bays, see [Figure 3](#).

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

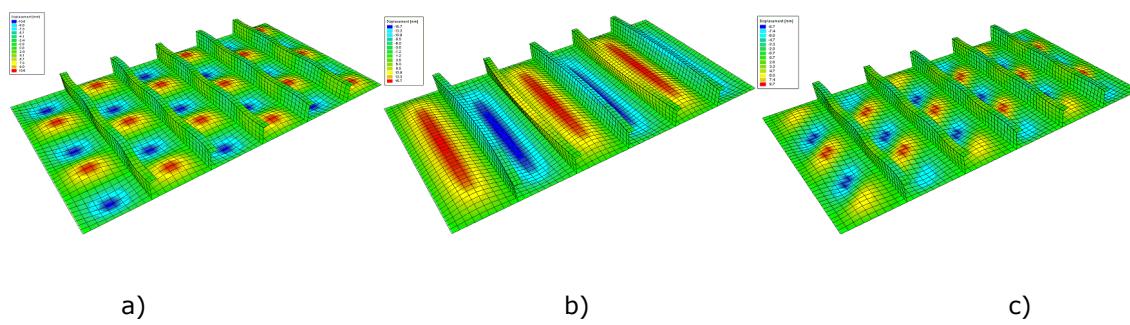


Figure 3 Harmonic elastic buckling patterns for a) axial compression, b) transverse compression, c) shear loading

Guidance note:

A characteristic feature of elastic buckling is that a "second order stress field" is induced in the structure (self-equilibrium set) adding on top of the direct applied stresses. This is visualised in [Figure 4](#) below where the von Mises membrane stresses vary in periodic pattern following the harmonic buckling mode shape ([Figure 3](#)).

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

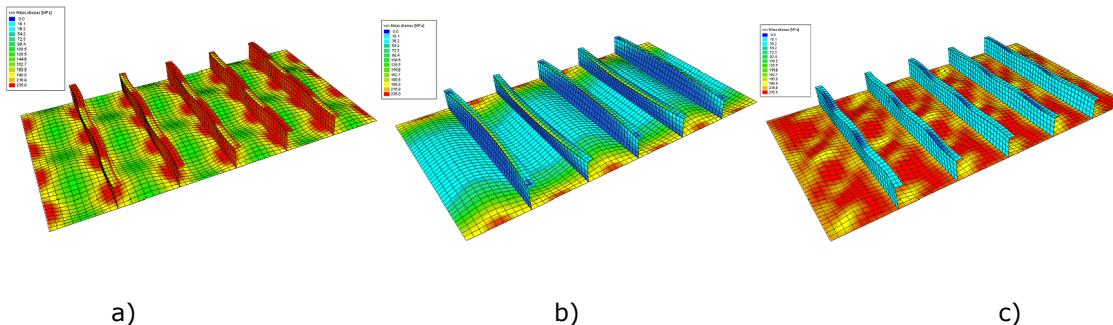


Figure 4 Illustration of von Mises membrane stresses that vary due to harmonic and regular buckling shapes; valid up to the ultimate capacity limit.

Guidance note:

For compressed stiffened plates the rate of grow of second order stresses depends on mainly two parameters; the slenderness (buckling) and the degree of geometrical imperfections in the plating/stiffeners respectively.

The second order stresses can be high especially for slender structures. The highest "second order" membrane stresses accumulate along supported plate edges and where stiffeners/girders are attached (hard corner, red zones in figures). When the sum direct + second order stresses are close to the yield stress (first onset of the material yield limit), it is considered that the ultimate capacity is imminent and that panel plastic collapse will be triggered subsequently.

The non-linear response of compressed stiffened plates is conveniently presented as load-deflection curves and the format of load-shortening (stress-strain) is particularly useful as the tangent to the curves represent directly the instantaneous membrane stiffness. The membrane stiffness is of particular interest as it dictates the degree of load-shedding (stress re-distributions) from the actual panel to surrounding structures.

The linear membrane stiffness is Young's modulus E (Hooke's law) and any deviation from this upper stiffness value will be due to the combined effect of elastic buckling and geometrical imperfections. The characteristic behaviour for imperfect plates will be that the membrane stiffness drop below the Young's modulus already at zero load (initial stiffness). This drop depends on the imperfection size and shape and it is thus most pronounced for thin to moderately thin plates. The initial stiffness is the same for compressed as for stretched plating as the load-shortening curve is continuous also at zero load.

By compressing the plating further the drop in membrane stiffness will be rather gradual but with a marked change when the eigenvalue is passed. Passing this limit means that the plates enter the elastic post-buckling region, and for near initially perfect plates the characteristics behaviour will be that the post-buckling stiffness is constant. The stiffness drop is significant, i.e. 50% or more is typical for pure plating.

Load-shortening curves for some typical plate configurations and load-directions are given in [\[1.4\]](#).

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

1.4 Load-shortening stress-strain curves

It is illustrative to show different aspect of non-linear behaviour of plated structures by plotting the load-shortening relation, i.e. the in-plane load carried as a function of the corresponding shortening of the panel (stress-strain). They are here conveniently compared with the linear elastic material curve (Young Modulus) for illustrating membrane stiffness differences before the ultimate capacity is reached. These stiffness deviations imply load shedding characteristics in the elastic range (see also [Figure 12](#)).

Load-shortening curves are also the main input to hull girder capacity models of multi-step (Smith) type. For such applications the full curve is used, i.e. pre and beyond the ultimate load into the plastic post-collapse region is needed (HGUS, Sec.5).

For the curves shown here the geometrical imperfections have been modelled as harmonizing with the critical eigenmode shapes. This is a conservative assumption and it will normally predict a safe lower bound strength. In real welded structures, i.e. multi-bayed panels etc., other more random and complex imperfection pattern of "hungry horse" shape, in combination with harmonic buckling mode shapes, will be typical.

Figure 5 and **Figure 6** show such curves for two different slenderness categories for the case of axial compression of unstiffened plates. **Figure 5** is for $t = 10$ mm plating with varying geometrical imperfection amplitude (x -factor; w_{max}) in the range 3.8 to 45.9 mm, the latter being an extreme (damage) case. **Figure 6** is for $t = 20$ mm plating with varying geometrical imperfection amplitude (w_{max}) in the range 1.9 to 22.9 mm.

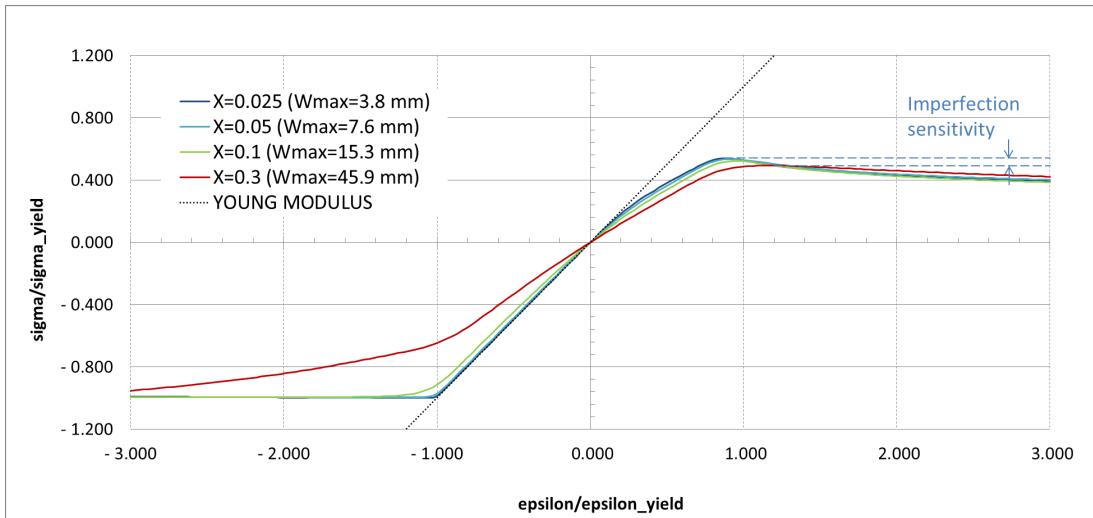


Figure 5 Load-shortening curves for plates with different slenderness (ABAQUS). Axial compression of plate 4000 mm x 1000 mm x 10 mm; $\text{sig}_y = 315 \text{ MPa}; \lambda = 1.94$ ($t = 10 \text{ mm}$).

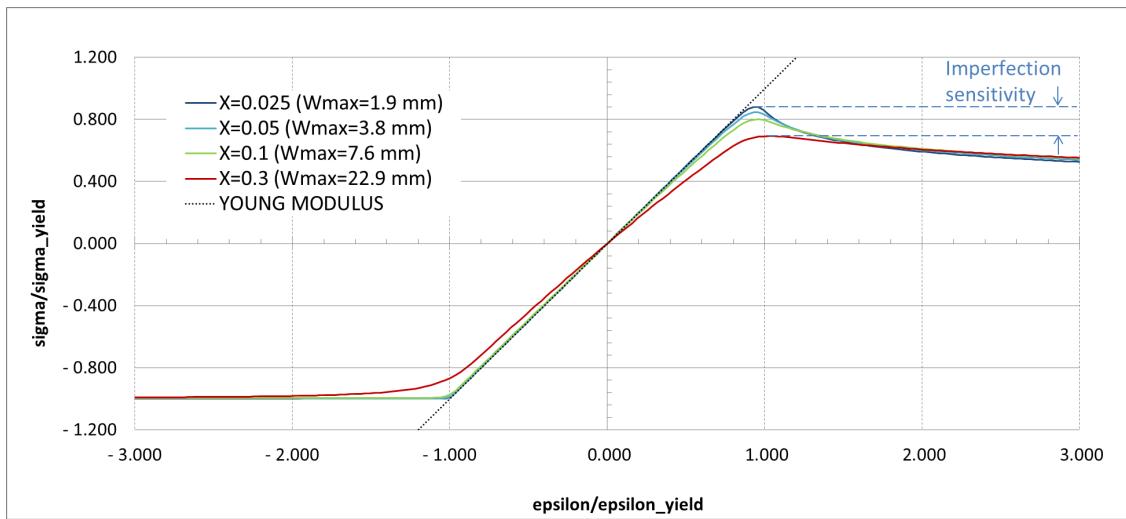


Figure 6 Load-shortening curves for plates with different slenderness (ABAQUS). Axial compression of plate 4000 mm x 1000 mm x 20 mm; $\sigma_y = 315$ MPa; $\lambda = 1.02$ ($t = 20$ mm).

The curves illustrate that for thin plates ($t = 10$ mm) the tangent stiffness drops off the linear Young's modulus stiffness (E) much before the ultimate capacity is reached. This indicates that load-shedding will take place to supporting edges/stiffeners/girders. Moreover the ultimate capacity itself is seen to be rather insensitive to the size of the imperfection amplitude in the range of normal tolerances, ca. 4 to 15 mm.

The curves for the thicker plates ($t = 20$ mm) shows no drop in the linear Young's modulus stiffness (E) up to the ultimate capacity limit. This indicates that load-shedding will not take place unless the plate is compressed beyond the peak load and into the plastic range which is not an option for properly dimensioned ships following the rules. Moreover it is noted that the ultimate capacity itself is seen to be moderate sensitive to the size of the imperfection amplitude, ca. 2 to 8 mm.

Figure 7 and Figure 8 give similar curves for transverse compression of the same plates ($t = 10$ mm; $t = 20$ mm).

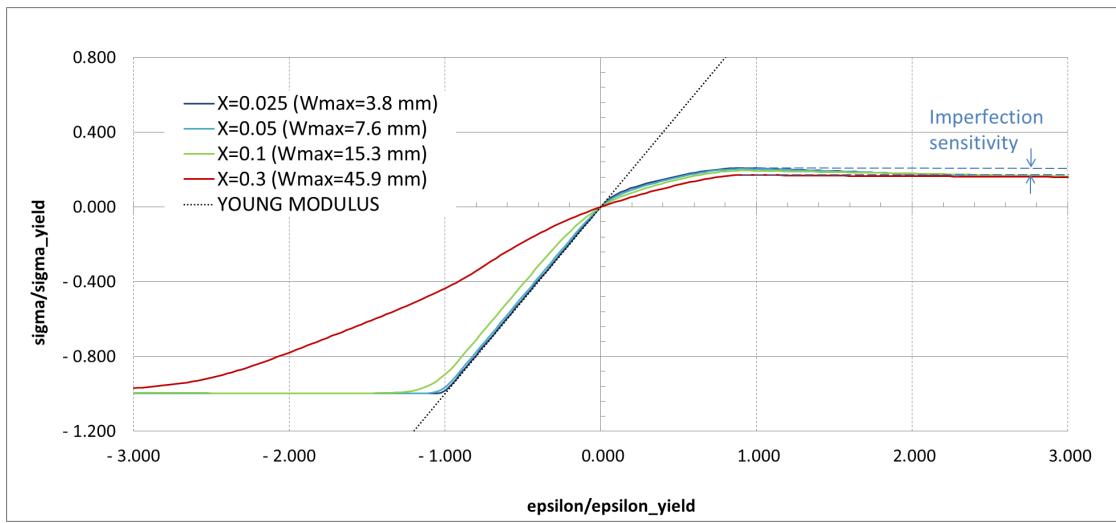


Figure 7 Load-shortening curves for plates with different slenderness (ABAQUS). Transverse compression of plate 4000 mm x 1000 mm x 10 mm; $\sigma_y = 315$ MPa; $\lambda = 3.86$ ($t = 10$ mm).

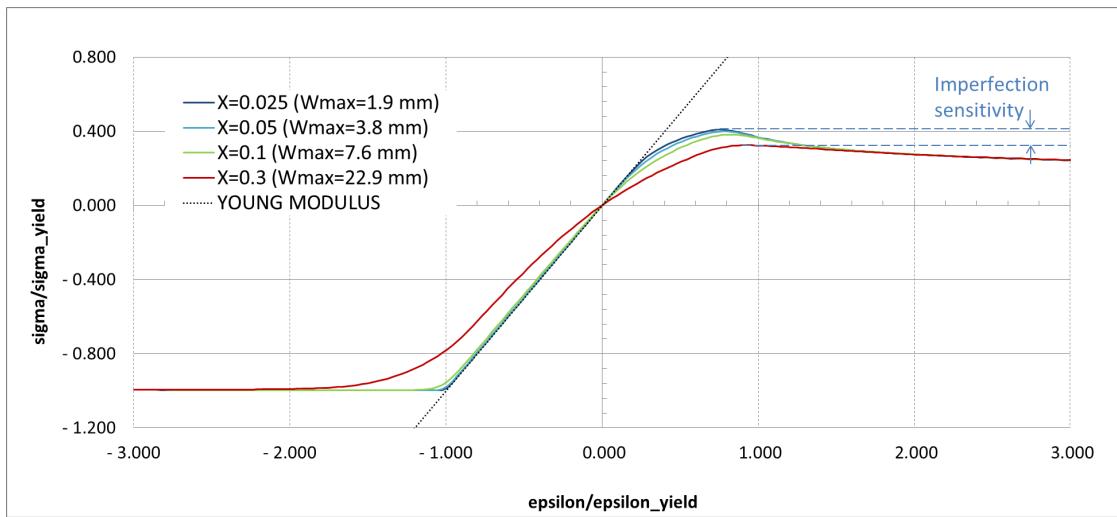


Figure 8 Load-shortening curves for plates with different slenderness (ABAQUS). Transverse compression of plate 4000 mm x 1000 mm x 20 mm; $\sigma_y = 315$ MPa; $\lambda = 1.93$ ($t = 20$ mm).

The transverse compression curves clearly illustrates that both thin plates (10 mm) and thicker plates (20 mm) drops off the linear Young's modulus stiffness (E) well before the ultimate capacity is reached. This indicates that load-shedding will be a dominating feature for such structures. An example can be double bottom structures and high transverse stresses from cargo/sea pressure/waves. Moreover the ultimate capacity itself is seen to be rather insensitive to the size of the imperfection amplitude particularly so for thin plates.

Guidance note:

Depending on the slenderness the plate buckling characteristics deviate in the pre-and post-buckling region while in the post-collapse region the curves follow basically the same trend.

It is characteristic that the ultimate capacity is reached for a shortening strain in the range of 0.9 to 1.0 of the yield strain, i.e. closer to 1.0 for most of the axial compression cases and closer to 0.9 for transverse compression cases.

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

Two examples of load-shortening curves for axial compression/tension of stiffened panels are included illustrating the main characteristics of the two most typical localized collapse modes;

- 1) Stiffener induced failure/tripping mode, see [Figure 1](#) and [Figure 9](#).
- 2) Plate induced failure, see [Figure 2](#) and [Figure 10](#).

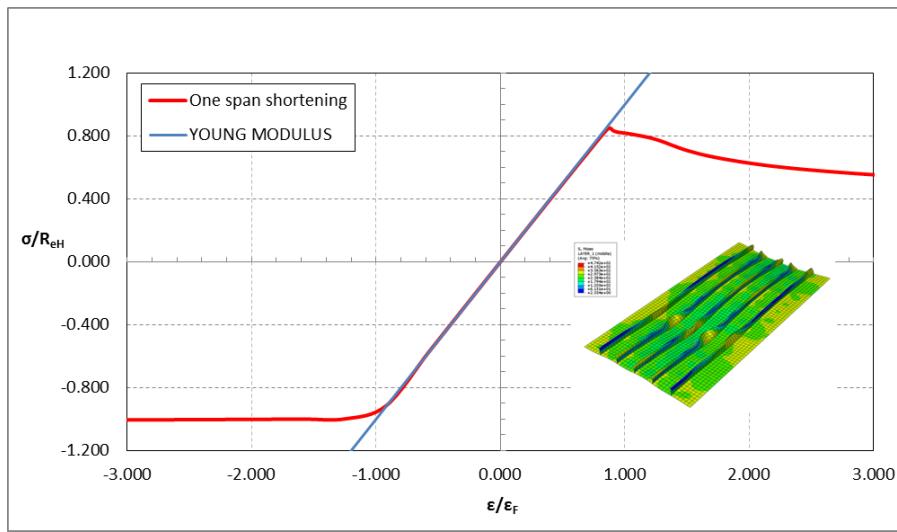


Figure 9 Load-shortening curves for representative ship hull stiffened panel (Angel/Bulb profile).
Axial load; collapsing in a stiffener tripping mode see [Figure 1 \(ABAQUS\)](#).

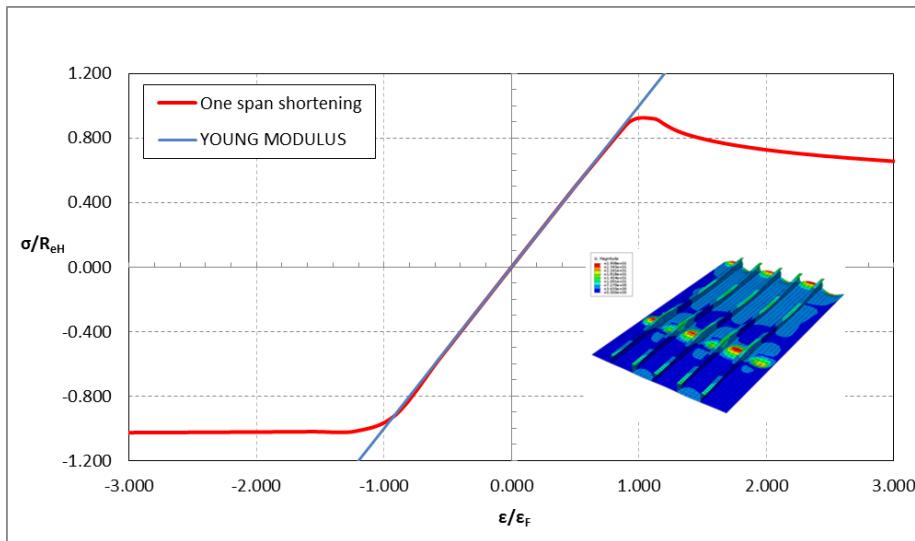


Figure 10 Load-shortening curves for representative ship hull stiffened panel (T profile).
Axial load; collapsing in a plate induced local mode see [Figure 2 \(ABAQUS\)](#).

From these load-shortening curves it is observed that the stiffness follows very closely the Young's modulus (E) up to the peak load. However, note that since the nominal stress (stiffness) here also includes the stiffener cross-sectional area, the load shedding between plating and stiffeners is not so easily observed as for the unstiffened plate curves in [Figure 5](#) and [Figure 6](#).

It is also shown that the characteristics of the post-collapse behaviour deviate somewhat for the two failure modes but the trend of the rate of change of the negative slope into the large axial shortening range is rather similar.

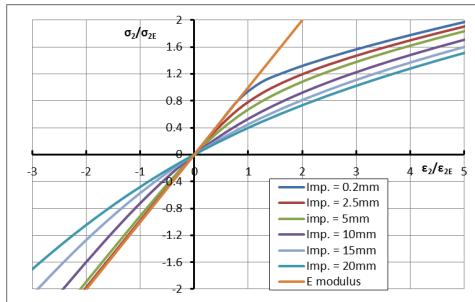
1.5 Elastic buckling and post-buckling, reduced membrane stiffness – general

By only focusing on the elastic region of the load-shortening curves, the stiffness characteristics as function of the buckling strength (eigenvalue) and geometrical imperfections are more easily observed. Note that the assumption in these discussions is that the shape of the imperfections is harmonized with the lowest eigenmode.

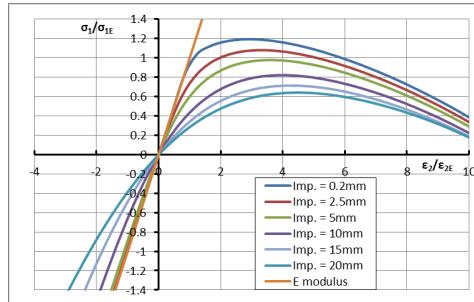
Figure 11 below shows typical results for transversely compressed plating. It is observed that the “perfect” case (0.2 mm imperfection) has a clear bi-linear characteristic; i.e. a linear pre-buckling curve (E) with an abrupt change of tangent stiffness beyond the eigenvalue with a reduced linear stiffness post-buckling curve (E^*).

By adding imperfection amplitudes into the model the drop in tangent stiffness become evident also from the very onset of load application, i.e. for zero load the tangent (initial) stiffness is clearly influenced by the size of imperfections. From the curves, see **Figure 11 a**, it can be observed that the initial stiffness converges against the post-buckling stiffness for large imperfection amplitudes (damages), and the reduced tangent stiffness is also evident for tension loading.

a)



b)



c)

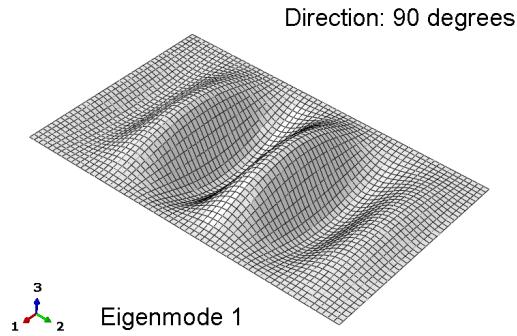


Figure 11 Load-shortening curves illustrating stiffness changes due to varying size of imperfection amplitude in lowest Eigenmode shape ($a=4000$ mm, $b=6000$ mm, $t=10$ mm, $E=206\,000$ MPa, $\nu = 0.3$; Loading: In-plane shortening control in ε_2 prescribed while $\varepsilon_1=0$).

As shown a main characteristic of the post-buckling response of "perfect" plates is that it follows a linear curve from the eigenvalue and well into the compressed region. It follows that the non-linear behaviour can be linearized in an incremental sense and the load-shortening can simply be written in incremental macro material form as

$$\Delta\sigma = E^* \Delta\varepsilon$$

The E^* is the reduced tangent stiffness of the elastically buckled plate to be compared with the linear Hooke's law value E . For integrated plates, as in ship structures following axial strain (ε) compatibility, the stiffness difference is a measure of load to be shed to neighbouring linearly behaving elements (stiffeners, girders) (see Figure 12).

These characteristics of elastic plate buckling behaviour and imperfection dependence can be utilized in modern rule procedures that apply linear Cargo hold/global FE models and Ultimate Limit State (ULS) principles for buckling control ([1.6]).

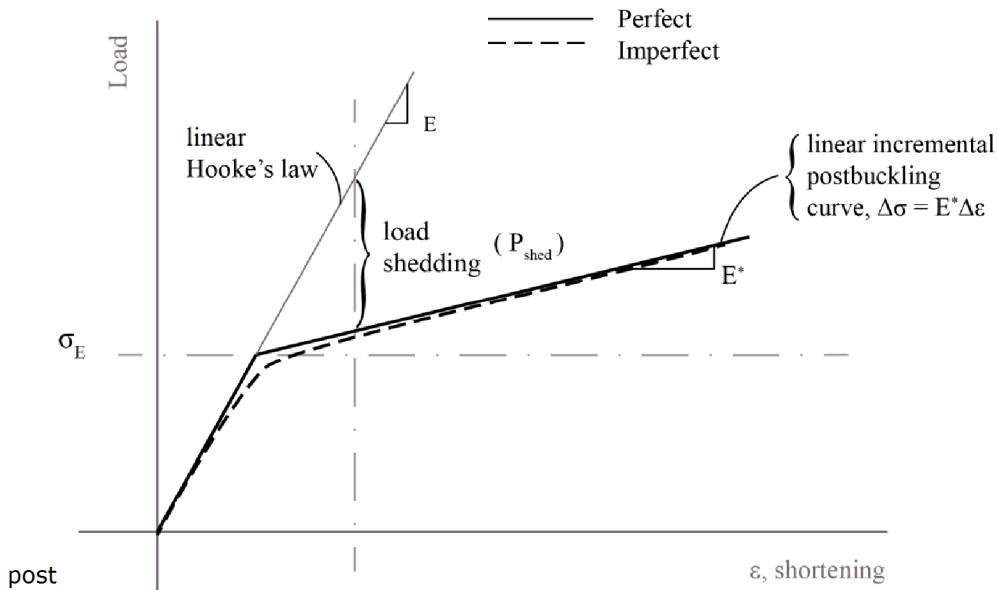


Figure 12 Drop in effective E module (E^*) beyond the eigenvalue; Load shedding to plate edges/neighbouring elements

For a plate with cross-sectional area A_c the load to be redistributed to stiffeners/girders/floors (assuming these to behave linearly) are accordingly

$$P_{shed} = (E - E^*)A_c(\varepsilon - \varepsilon_E)$$

where

- P_{shed} = Load to be redistributed to stiffeners or girders along edges
- A_c = Plate cross sectional area (plate width * plate thickness)
- ε = actual average engineering strain of plate (shortening)

If $\varepsilon < \varepsilon_E$ it is assumed that no load shedding takes place for "perfect plate"

$$\begin{aligned}\varepsilon_E &= \text{strain at elastic buckling } \varepsilon_E = \sigma_E / E \\ \sigma_E &= \text{elastic buckling stress (eigenvalue)}\end{aligned}$$

For pure axial compression

$$E^* = 0.5E$$

$$\sigma_E = \frac{E\pi^2}{3(1-\nu^2)} \left(\frac{t}{b}\right)^2$$

For pure transverse compression

$$E^* = C_{22} - \frac{(C_{12})^2}{C_{11}}$$

$$\sigma_E = \frac{E\pi^2}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2 \left(1 + \left(\frac{b}{a}\right)^2\right)^2$$

1.6 Load-shedding – linear anisotropic FE modelling – imperfect plates

For plates subjected to combined in-plane loads (bi-axial, shear), as typical for ship hull plating, the load-shortening relations can be written in a very compact macro material form linking the loads to the corresponding edge straining as follows (matrix notation)

$$\begin{bmatrix} \Delta\sigma_1 \\ \Delta\sigma_2 \\ \Delta\sigma_3 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{33} \end{bmatrix} \begin{bmatrix} \Delta\varepsilon_1 \\ \Delta\varepsilon_2 \\ \Delta\varepsilon_3 \end{bmatrix} \quad \text{and inverted} \quad \begin{bmatrix} \Delta\varepsilon_1 \\ \Delta\varepsilon_2 \\ \Delta\varepsilon_3 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{33} \end{bmatrix} \begin{bmatrix} \Delta\sigma_1 \\ \Delta\sigma_2 \\ \Delta\sigma_3 \end{bmatrix}$$

The incremental material form is needed since the C_{ij} coefficients are load dependent (state dependent).

Here a delta symbol (Δ) before the stress (σ) and strain (ε) notations imply incremental properties.

Then, as illustrated in [Figure 11](#), i.e. the tangent stiffness at zero load (the initial stiffness) is observed to be dependent on the imperfection size. It can also be seen that the tangent is rather unchanged for loads up to around the eigenvalue. These “linear” observations suggest that the drop in membrane stiffness can be modelled by manipulating the material coefficient in a linear behaving model, e.g. FE Cargo Hold models.

From a non-linear plate buckling model the following simple expressions for the initial tangent stiffness coefficients are derived

$$\begin{aligned}C_{11} &= \frac{E}{(1-\nu^2)} \left[1 - 6 \frac{(k_1 + \nu k_2)^2}{A} \right] \\ C_{12} &= \frac{Ev}{(1-\nu^2)} \left[1 - \frac{6(k_1 + \nu k_2)(k_2 + \nu k_1)}{\nu A} \right] (= C_{21}) \\ C_{22} &= \frac{E}{(1-\nu^2)} \left[1 - 6 \frac{(k_2 + \nu k_1)^2}{A} \right]\end{aligned}$$

$$C_{33} = \frac{E}{2(1+\nu)}$$

Where the involved parameter are defined as

$$A = \frac{2(k_1 + k_2)^2}{q_{10}^2} + 3(3 - \nu^2)(k_1^2 + k_2^2) + 12\nu k_1 k_2 \quad (\text{imperfeccr plates})$$

$$A = 3(3 - \nu^2)(k_1^2 + k_2^2) + 12\nu k_1 k_2 \quad (\text{perfect plates})$$

$$k_1 = \left(\frac{t}{a}\right)^2; k_2 = \left(\frac{t}{b}\right)^2, q_{10} = \frac{\delta}{t}$$

$$q_{10} = x \left(\frac{b}{t}\right)^2 \frac{R_{eH}}{E} \quad (\text{imperfection model; Faulkner})$$

x = imperfection factor, see Guidance Note in [1.7].

For modelling in standard FE programs the material anisotropic/orthotropic coefficients ($E_1, E_2, \nu_{12}, G_{12}$) are different than in the notation (C_{ij}) used describing buckling models. However, there is a simple relation between them as given below.

In FE tools the orthotropic material model is given as:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_1} & \frac{-\nu_{12}}{E_1} & 0 \\ \frac{-\nu_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{bmatrix}$$

The relation between the $E_1, E_2, \nu_{12}, G_{12}$ and the C_{ij} coefficients above is:

$$E_1 = \frac{c_{11}c_{22} - (c_{12})^2}{c_{22}}$$

$$E_2 = \frac{c_{11}c_{22} - (c_{12})^2}{c_{11}}$$

$$\nu_{12} = \frac{c_{12}}{c_{22}}; \nu_{21} = \frac{c_{12}}{c_{11}} \left(= \frac{E_2}{E_1} \nu_{12} \right)$$

$$G_{12} = C_{33}$$

In practise for rectangular plates with aspect ratio say > 2 , the E_2 and ν_{12} may be significantly lower than the corresponding linear values while the E_1 is almost unchanged ($\approx E$). For plates with aspect ratio < 2 the E_1 will also be reduced accordingly and in the limit of a square plate the stiffness reductions are equal in both directions. The shear stiffness G_{12} (C_{33}) is reasonably approximated to be the full linear value.

1.7 Geometrical imperfection effects - knock down

The ultimate capacity of compressed stiffened plates will be reduced due to the presence of geometrical imperfections from production and welding. This effect is often called imperfection sensitivity or knock-down effect if the actual level production imperfections are included.

Imperfections here refer to of out-of-flatness/straightness of plate and stiffeners, load eccentricities, misalignments and welding residual stresses.

The degree of imperfection sensitivity will be highest for moderate slender plates ($0.6 < \lambda < 1.4$) and less for slender plates. For stocky plates, the capacity will be governed by material yielding and the non-linear effect due to the imperfection is small.

For plates the imperfection sensitivity is moderate in general. Scatter in ultimate capacity for normal plates can be expected to be in the range of 5 to 15% considering typical normal production levels as used in yards for ship hulls. [Figure 5](#) and [Figure 6](#) include some data which support these levels.

Guidance note:

Geometrical imperfections in welded steel plates have a rather random characteristic across multiple bays, i.e. they will vary both in size and shape in the different plating directions (x, y). From a buckling strength point of view the imperfections coinciding in shape with natural/preferred modes (typical eigenmodes) are most critical. Thus imperfections deviating from this idealized shape (e.g. hungry horse etc.) will not be very detrimental and rather add on the upper side of the strength.

When analysing the ultimate capacity of flat plated structures (e.g. in non-linear FE etc.) a model imperfection approach is used rather than the real life imperfections since the latter are not available nor practical to use in computer codes. The model imperfection amplitude is normally taken less than maximum local tolerance limits given in e.g. fabrication standards and IACS Rec.47.

The ultimate strength predictions from CFM (Sec.3) and PULS (Sec.4) are consistent with the normal tolerance limits as given in IACS and DNV GL fabrication standards/Instruction to Surveyors.

An imperfection model for plates frequently referred to in connection with buckling and ultimate strength is due to Faulkner /1/. This model is based on some full scale measurements on ship plating and simply relates the maximum imperfection amplitude (w_{max}) to the plate slenderness in the following format

$$\frac{w_{max}}{t} = x \left(\frac{b}{t} \right)^2 \frac{R_e H}{E} ; \text{Faulkner imperfection model}$$

Where a typical range for x -factor is given as

$x = 0.025$ slight imperfections

$x = 0.05$ representative imperfections

$x = 0.1$ average imperfections

$x = 0.3$ severe imperfections

The measured plate data following Faulkner's original paper is plotted in a diagram Figure 13 below.

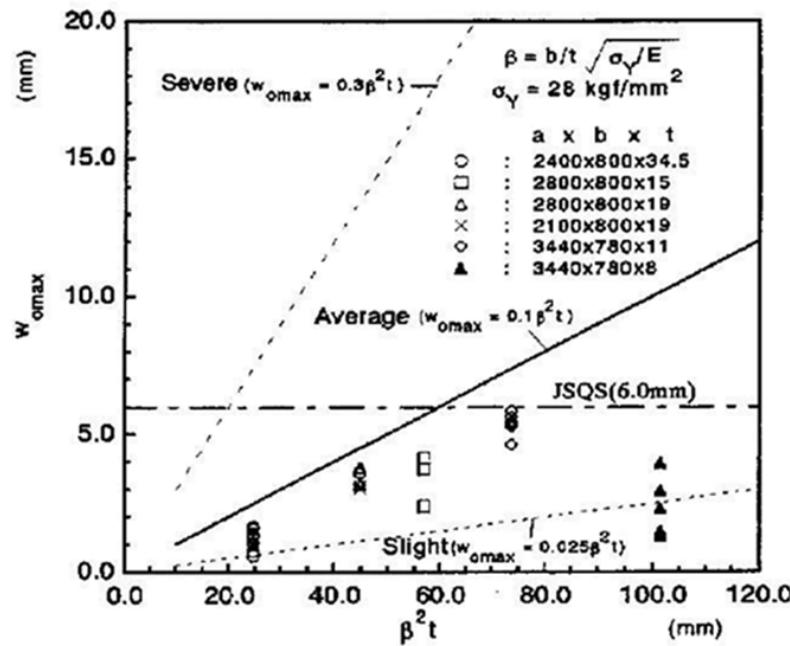


Figure 13 Measured full scale maximum plate imperfections (w_{max} ; Faulkner imperfection model)

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

1.8 Boundary support

Boundary support of plates will directly influence on the buckling, post-buckling and ultimate strength characteristics. With reference to buckling features it is convenient to split the boundary conditions in two categories, rotational edge and in-plane support.

Rotational edge support (1th order)

- 1) Rotational edge supports spans typically from free to rotate (simply supported) to a fully rotationally fixation (clamped support).

The degree of rotational restraint is a “first order effect” and influences significantly on the elastic buckling stress (eigenvalue) and thereby also the ultimate capacity. Typically a clamped plate compressed perpendicular to the long edges will have an elastic buckling stress of the order of four (4) times the simply supported plate.

An example is the longitudinal buckling strength of transversely framed ships for which the rotational stiffness of the web plating in the floors will effectively improve the buckling strength of the bottom/tank top plating as compared to assume no restraint (simply supported).

- 2) In-plane membrane support (2nd order)

The in-plane support is a “second order effect” which influence on the post-buckling behaviour and not the elastic buckling load. The impact on the ultimate strength is thus small in general and negligible for normal ship hull scantlings.

In the rules two types of in-plane boundary supports are defined

- Method A: All plate edges are forced to remain straight but are free to move in the plate plane. This corresponds to boundary conditions as typical for integrated (multi-bayed) plates in ship hulls like decks, bottom plating, bulkheads etc.
- Method B: Plate edges are free to pull in-plane, i.e. no restraints from surrounding structures. This corresponds to boundary conditions typical for floors in double bottoms, girder structures with free in-plane but laterally supported edges.

Guidance note:

Method B is the option in the rules for checking buckling strength of typical girders, stringers, floor etc for which the plating (e.g. girder web plating) has no real in-plane restraint from surrounding structures while supported rigidly out-of the plate plane along edges. The stress level/gradients in the web plating will depend on the actual external loading, being uniform loads (from sea pressure) or more localized loads (as from container cell guides). The plate buckling models need to consider the degree of stress localizations being a combination of shear and bi-axial stresses across rather local areas.

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

1.9 Design principles - summary

The overall buckling design principles of stiffened panels in the present rules are summarized as:

- 1) Elastic buckling of plating between stiffeners and girders is accepted as long as the stress redistributions (load shedding) to neighbouring structures such as stiffeners and girders are ensured.

Guidance note:

For stiffeners an effective approach is generally used for this purpose.

For transverse girders a more general approach is given in [1.6].

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

- 2) Elastic overall buckling of stiffeners are not accepted, i.e. the stiffeners should be sufficiently strong out-of the plate so as to provide support to plating.
- 3) The Ultimate capacity is not be exceed by a safety margin as defined in the rules

Guidance note:

This prevents local plastic collapse and permanent sets and buckle damages.

---e-n-d---o-f---g-u-i-d-a-n-c-e---n-o-t-e---

The present analyses shows that it is crucial that the unstable postbuckling behaviour is identified and that geometrical imperfections are included in the FE analyses.

2 General ULS design check formats

2.1 Implicit limit state formulations – general

A limit state function $G(x_1, x_2, \dots)$, in its most general form, is a non-linear function in the design parameters $x_1, x_2, x_3 \dots$ etc.

In the ULS terminology the function describe a buckling failure mode and have the following properties

$G(x_1, x_2, x_3, \dots) > 0$; OK safe region

$G(x_1, x_2, x_3, \dots) = 0$; OK on the limit of safe/unsafe region

$G(x_1, x_2, x_3, \dots) < 0$; NOT OK unsafe region

where the parameter vector $(x_1, x_2, x_3 \dots)$ typically has components such as

(x_1, x_2, \dots) load/stress components, s_1, s_2, \dots etc

(y_1, y_2, \dots) geometrical parameters; plate thickness etc

(z_1, z_2, \dots) material parameters; yield stress etc

$(\gamma_1, \gamma_2, \dots)$ partial safety coefficients, load factors, material factors etc.

A limit state function $G(x_1, x_2, \dots)$ involves normally several load/stress component. For integrated plates the loadings are typical three simultaneously acting in-plane stresses; bi-axial stresses in perpendicular directions with simultaneously acting in-plane shear and lateral pressure.

The G function is generally written in terms of loads $(\sigma_1, \sigma_2, \sigma_3, p)$ and design parameters (y_1, y_2, \dots) (plate thickness etc.) as:

$$G(\sigma_1, \sigma_2, \sigma_3, p, y_1, y_2, \dots)$$

The solution to:

$$G(\sigma_1, \sigma_2, \sigma_3, p, y_1, y_2, \dots) = 0$$

will be a surface in three dimensional load space $(\sigma_1, \sigma_2, \sigma_3)$ as illustrated in Figure 14.

Assuming a proportional load history in the in-plane loads, i.e. introducing a load proportional factor Λ (scaling factor) defined as:

$$\sigma_1 = \Lambda \sigma_{10}$$

$$p = p_0 = \text{fixed},$$

$$\sigma_2 = \Lambda \sigma_{20}$$

$$(\sigma_{11}, \sigma_{20}, \sigma_{30}) = \text{reference fixed in-plane design stresses}$$

$$\sigma_3 = \Lambda \sigma_{30}$$

The limit state function $G(x_1, x_2, x_3, \dots) > 0$ can then be written as a non-linear function in the single load parameter Λ , i.e. $G(\Lambda, y_1, y_2, y_3, \dots) = 0$. This equation solved explicitly in Λ gives the value for the load factor at the ultimate capacity, i.e.

$$\Lambda = \Lambda_u$$

And the ultimate capacity of the in plane stresses are

$$\sigma_{1u} = \Lambda_u \sigma_{10}$$

$$\sigma_{2u} = \Lambda_u \sigma_{20}$$

$$\sigma_{3u} = \Lambda_u \sigma_{30}$$

It follows from the definition of the utilization factor

$$\eta_{actual} = \frac{\text{stress}}{\text{capacity}}$$

that

$$\eta = \frac{\text{square load norm} - \text{load effect}}{\text{square capacity norm} - \text{capacity}} = \frac{\sqrt{\sigma_{10}^2 + \sigma_{20}^2 + \sigma_{30}^2}}{\sqrt{\sigma_{1u}^2 + \sigma_{2u}^2 + \sigma_{3u}^2}} = \frac{1}{\Lambda_u}$$

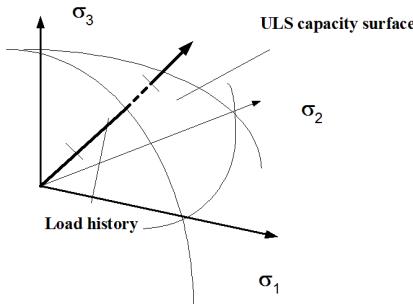


Figure 14 Schematic illustration of limit failure surface $G = 0$ and utilization factors for proportional load history

The solution of $G(\Lambda, y_1, y_2, y_3) = 0$ with respect to load parameter Λ can be facilitated by simple load incrementing checking left side against right side, or applying other more general approaches based on Newton-Raphson iterations or equivalent.

2.2 Explicit limit state formulations

Limit state functions describing buckling of elements can under certain conditions be written in a simplified linear form with respect to the load (S) and resistance parameters (R), i.e. it takes the form

$$G(y_1, y_2, y_3, \dots) = R(y_1, y_2, y_3, \dots) - S(y_1, y_2, y_3, \dots)$$

where

$R(y_1, y_2, y_3, \dots)$ is the resistance (ultimate capacity)

$S(y_1, y_2, y_3, \dots)$ is the load effect (external load/stress)

From $G(y_1, y_2, y_3, \dots) > 0$ it follows that the limit state criterion simplifies to:

$$S(y_1, y_2, y_3, \dots) \leq R(y_1, y_2, y_3, \dots)$$

This linear format gives directly the utilization factor as used in rule terminology, i.e. inverse of safety factor.

3 References

- 1) Faulkner, D. (1975). A review of Effective Plating to be Used in the Analysis of Stiffened Plating in Bending and Compression. *Journal of Ship Research*, 19(1), 1-17.

APPENDIX B NON-LINEAR FE – TORISPERICAL TANK

Buckling of a toro-spherical tank shell subjected to external pressure constitute a special and demanding case in that spherical shells is known to behave very unstable and thus shows a high degree of imperfection sensitivity.

The dimension of the shell is shown in [Figure 1](#) where the shell thickness is 8 mm with the largest radius of 4328 mm. The yield stress of the tank is taken as 480 MPa.

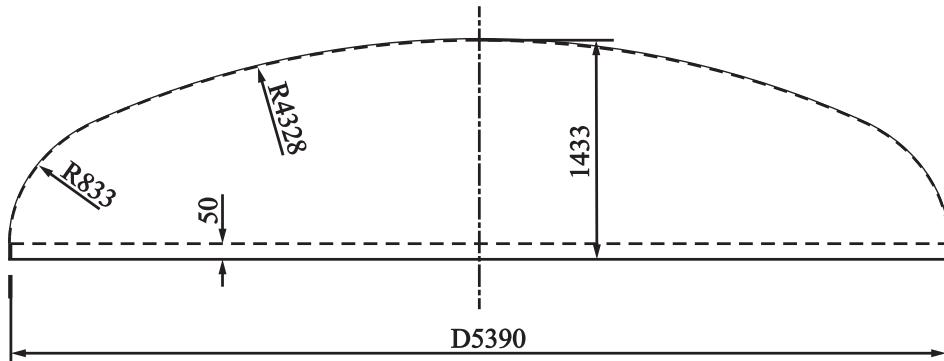


Figure 1 Dimension of the shell

The tank has been modelled with a very fine mesh so that any local and short waved buckling pattern is described with sufficient accuracy. The FE model is shown in [Figure 2](#).

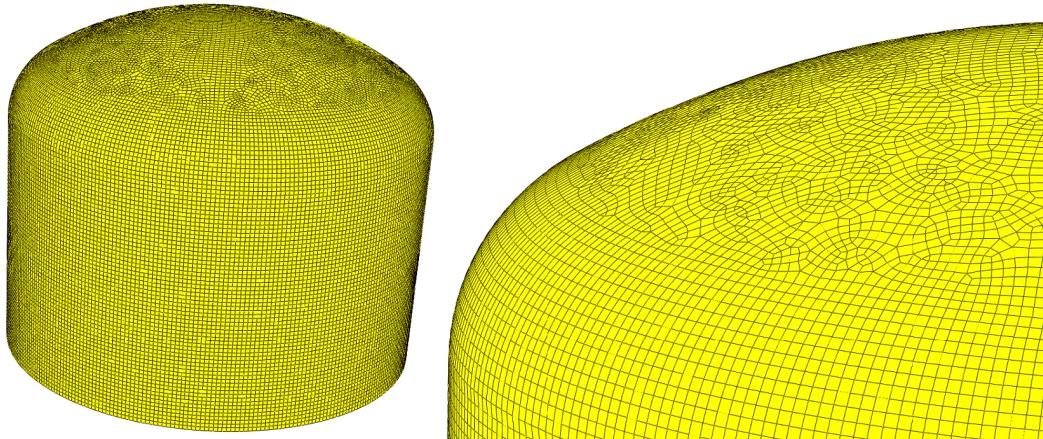


Figure 2 FE Model of toro-spherical shell – Very fine mesh close up

For having a reference strength estimate, a linear elastic eigenvalue calculation has been carried out. The eigenvalue found from ABAQUS was $P_E = 0.84$ MPa (8.4 bar).

As a verification of this value the text book solution from /2,3/ is checked in terms of the principal stress

$$\sigma_{CL} = \frac{E}{\sqrt{3(1-\nu^2)}} \frac{t}{r} = 0.606 E \frac{t}{r}$$

For a complete a sphere the membrane stress and surface pressure is related as

$$P = 2\sigma \frac{t}{r}$$

giving

$$P_{CL} = \frac{2E}{\sqrt{3(1-\nu^2)}} \left(\frac{t}{r}\right)^2 = 0.87$$

MPa (8.7 bar)

The principal membrane stress in the shell spherical part is for buckling pressure

$$\sigma_{CL} = \frac{1}{2} P_{CL} \frac{r}{t} = 235$$

MPa (<480 yield stress)

This confirms that the actual tank geometry very closely behaves and buckles as an comparative complete sphere with dominantly elastic response.

Buckling of spherical shells are known to behave very unstable and the eigenvalue is far from a realistic design capacity. In [Sec.8 \[3.3\]](#) a knock down value due to unstable postbuckling and presence of geometrical imperfections is predicted as

$$\rho = \frac{0.5}{\sqrt{1 + \frac{r}{t} \frac{1}{100}}} = 0.2$$

i.e meaning that design buckling capacity is $0.87 \cdot 0.2 = 0.17$ MPa (excluding safety factors)

Non-linear FE calculations were carried out systematically by varying the imperfection amplitude. It was clearly observed that the imperfection sensitivity was extreme and that buckling capacity was rapidly falling from the "perfect" shell of 0.84 MPa to 0.24 for imperfection amplitudes less than the shell thickness. The results are shown in [Figure 1](#) were the capacity computed using the formula in [Sec.8 \[3.3\]](#) is included. The latter and simple assessment is slightly conservative compared to fully non-linear finite element analysis.

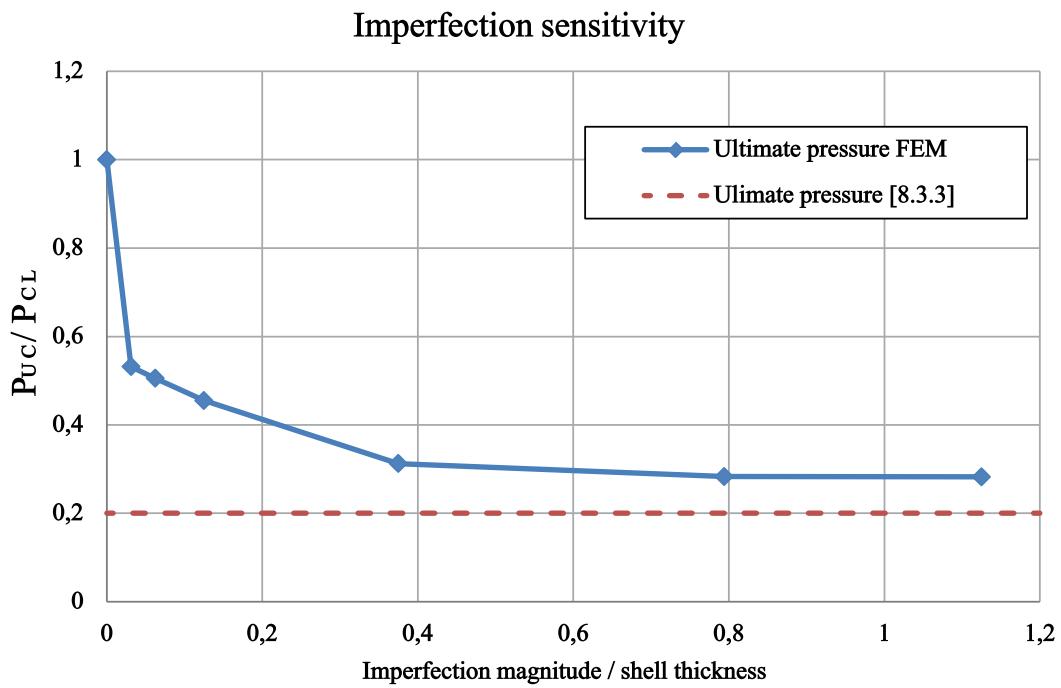


Figure 3 Sensitivity study for imperfection with the shape of the first eigenmode where the capacity is normalised wrt the classical eigenvalue P_{CL}

Load-displacement curves for some different imperfection amplitudes is shown in [Figure 4 a\), b\) c\) d\)](#)

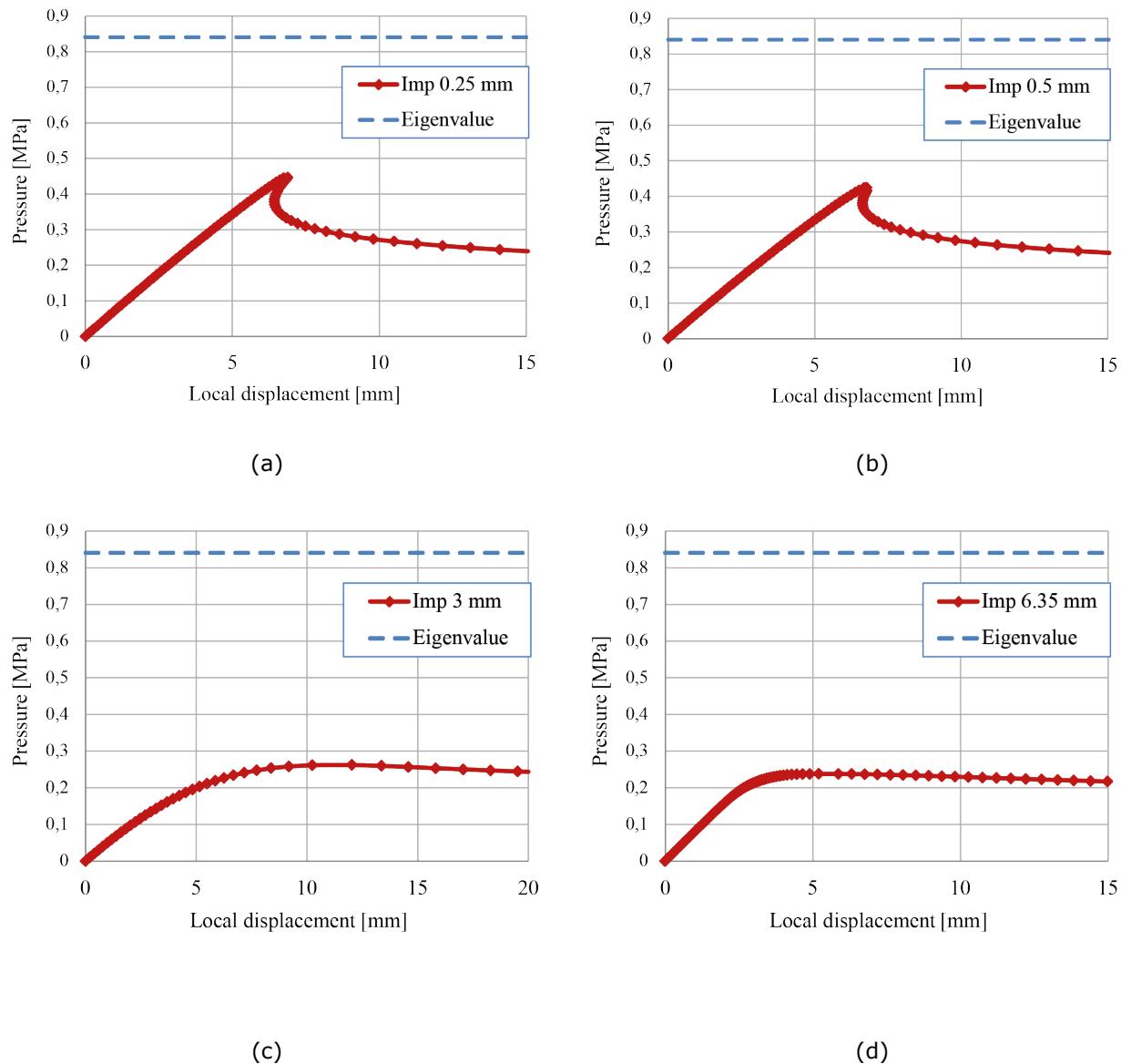


Figure 4 Response curves for different imperfection magnitudes

The non-linear analyses were carried out using model imperfections in the shape of the minimum eigenmode. It was evident that the non-linear collapse was unstable and that mode snapping occurred around the ultimate pressure. The shape of the imperfection and the collapse mode are shown in [Figure 5](#).

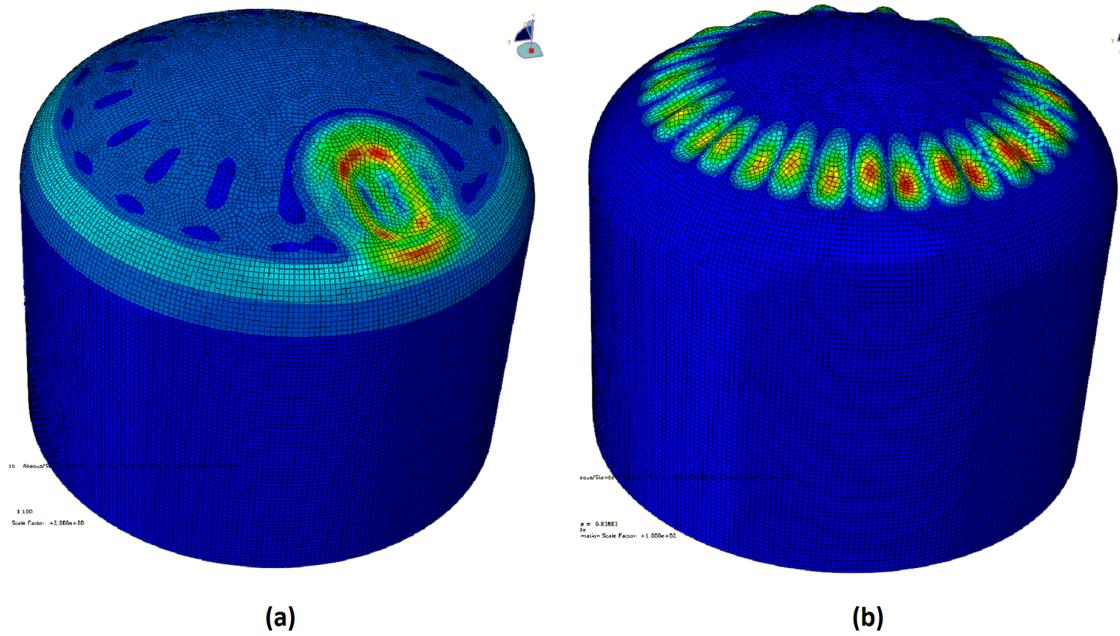


Figure 5 (a) Post-collapse mode and (b) eigenmode used as imperfection

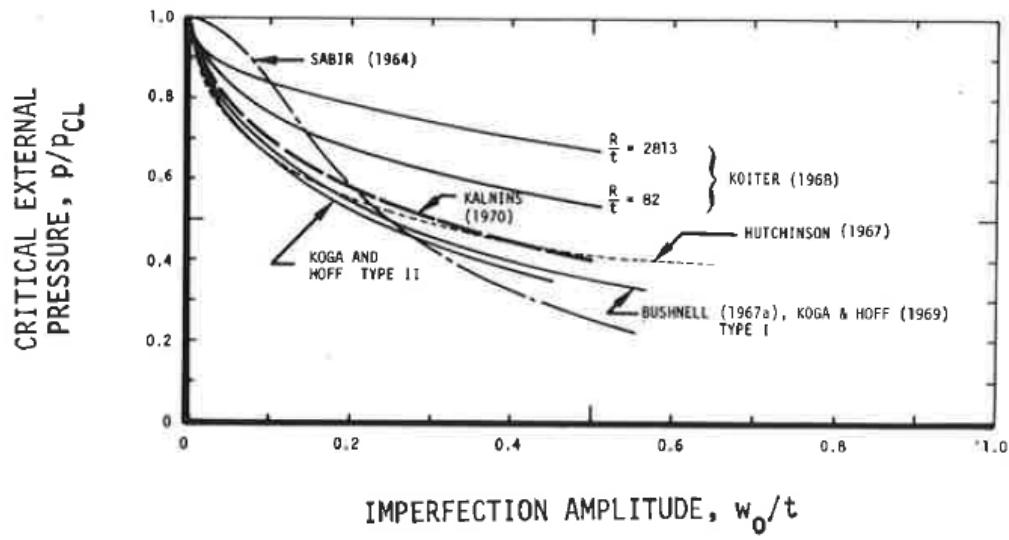


Figure 6 Analytical predictions for sensitivity of complete spherical

Discussions

The present toro-spherical LNG tank enclosure shows extreme unstable buckling response which is according to known and documented behaviour in the literature /1-3/ and [Figure 6](#). The knock-down factor, defined as

the buckling capacity of the “imperfect” shell in relation to the perfect shell, is found to be (P_U/P_{CL}) 0.24/0.84 = 0.28, i.e. the capacity of the real shell is the range of 70% lower than the perfect shell. The response is dominantly elastic up to the ultimate pressure and only marginal surface yield towards the toro-spherical part is identified.

The present analyses shows that it is crucial that the unstable postbuckling behaviour is identified and that geometrical imperfections are included in the FE analyses.

1 References

- 1) Bushnell, D. (1985). *Computerized Buckling Analysis of Shells* (Mechanics of Elastic Stability) 9, Nijhoff.
- 2) Flügge, W. (1973). *Stresses in Shells*. Springer Verlag, 2. ed.
- 3) Brush, D.O. & Almroth, B.O. (1975). *Buckling of Bars, Plates and Shells*, Student edition. McGrawHill.

APPENDIX C NON-LINEAR FE – GIRDER WITH CUT-OUTS

Non-linear finite element analysis has been used to investigate the effect of local buckling stiffeners at the free edges of cut-outs in a girder web subjected to a pure axial load. The girder is shown in [Figure 1](#).

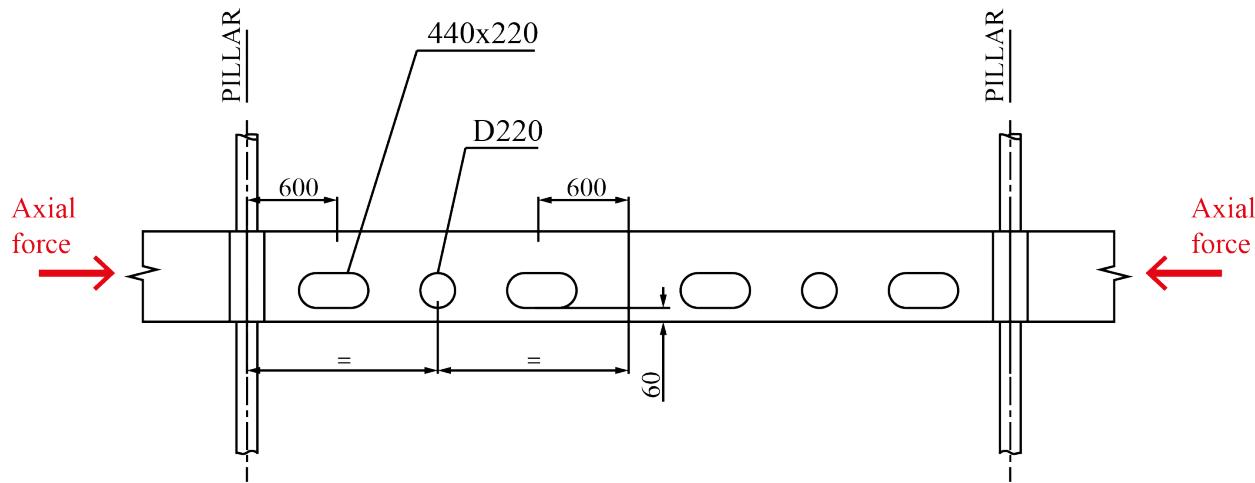


Figure 1 Dimensions of the girder

The geometry and boundary conditions are shown for the unstiffened and stiffened web plate in [Figure 2](#) and [Figure 3](#) respectively. The initial imperfection is taken equal to the first eigenmode as shown in [Figure 4](#).

A similar imperfection mode is used for the web with buckling stiffener as shown in [Figure 5](#).

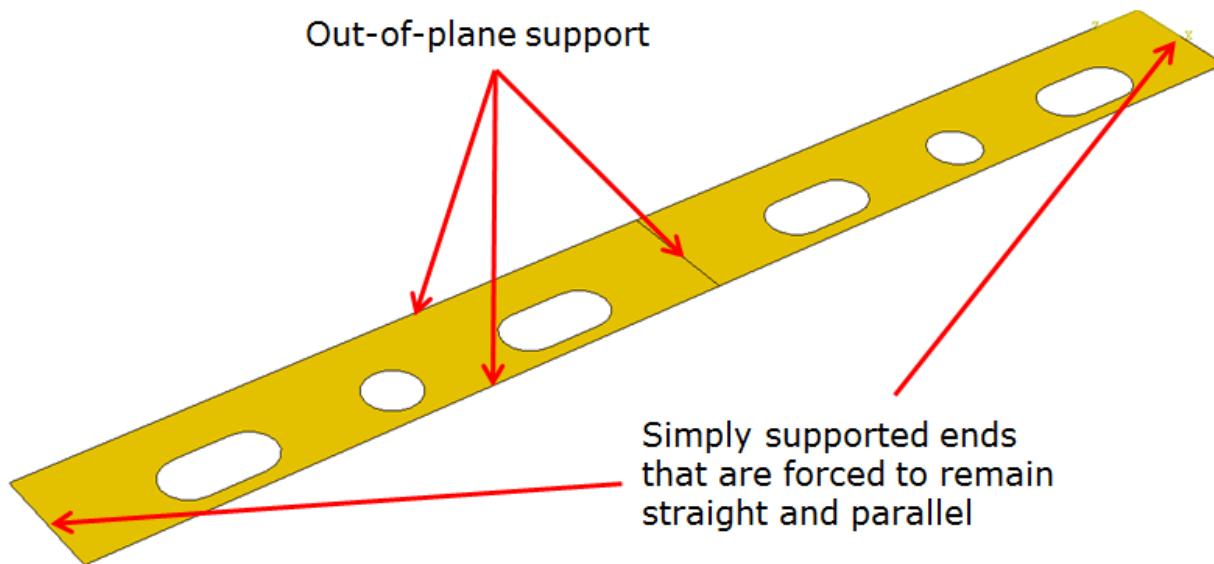


Figure 2 Geometry and boundary conditions of the unstiffened web plate

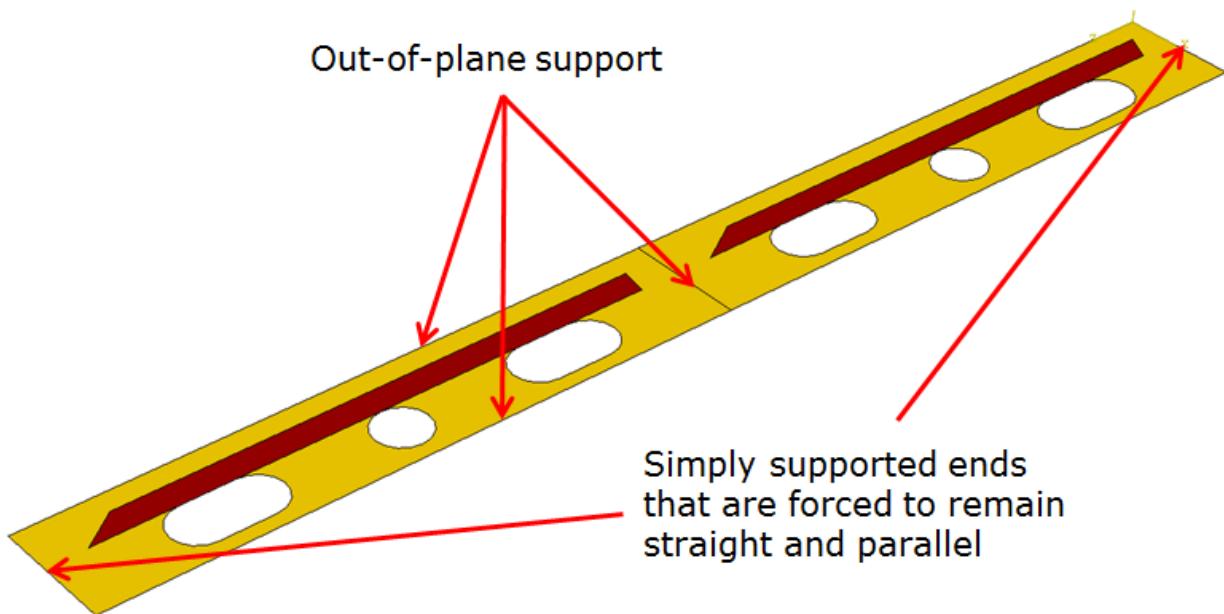


Figure 3 Geometry and boundary conditions of the web plate with local buckling stiffeners

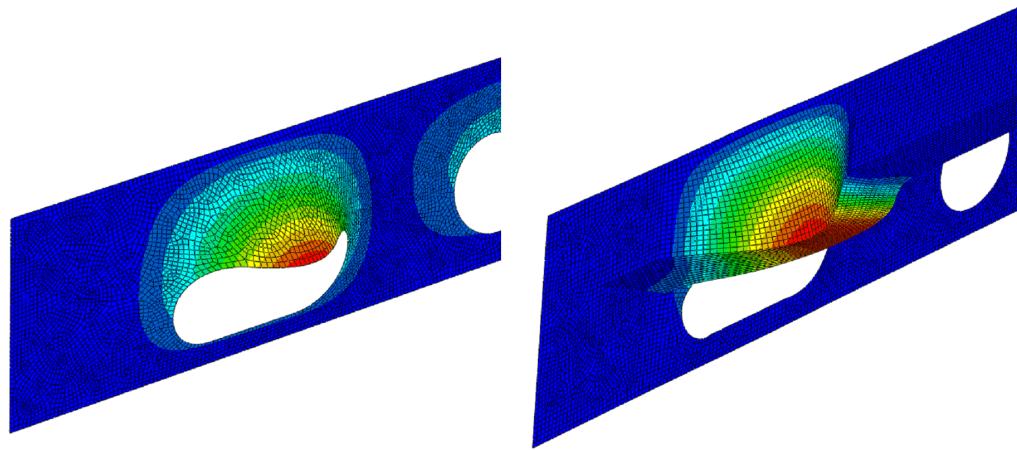


Figure 4 Imperfection and eigenmode for the web plate without buckling stiffeners and with buckling stiffeners.

For the plate without buckling stiffener, response curves obtained by non-linear FE are shown in [Figure 5](#)/[Figure 6](#). The ultimate capacity is 140 MPa which is the maximum point of the response curve and the corresponding capacity mode is shown in [Figure 7](#). In this case, the ultimate capacity is larger than the linear elastic buckling load (eigenvalue) at 100 MPa. This means that the plate has some reserve strength beyond elastic buckling.

In [Figure 5](#)/[Figure 6](#), unloading curves are included from three different levels. It can be seen that the permanent deformation is zero when unloading from elastic buckling in this case. After unloading from the ultimate capacity, the plate will have some, but very small (2 mm), permanent deformations (purple curve). An unloading curve from post-collapse is also included in the figures (green curve). In this case the permanent deformation is more significant (14 mm) and the shape is shown in [Figure 8](#).

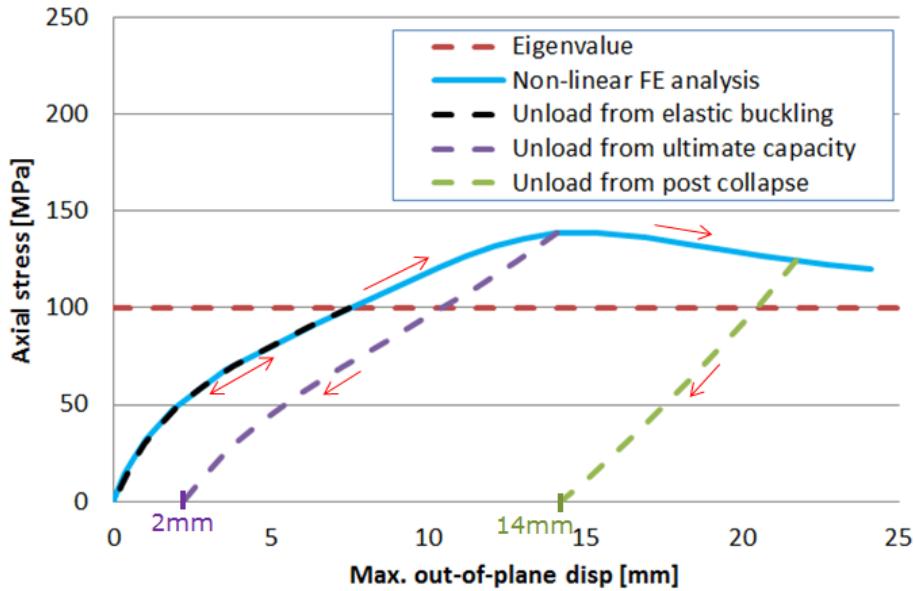


Figure 5 Plate response for the web plate with cut-outs; loading and unloading

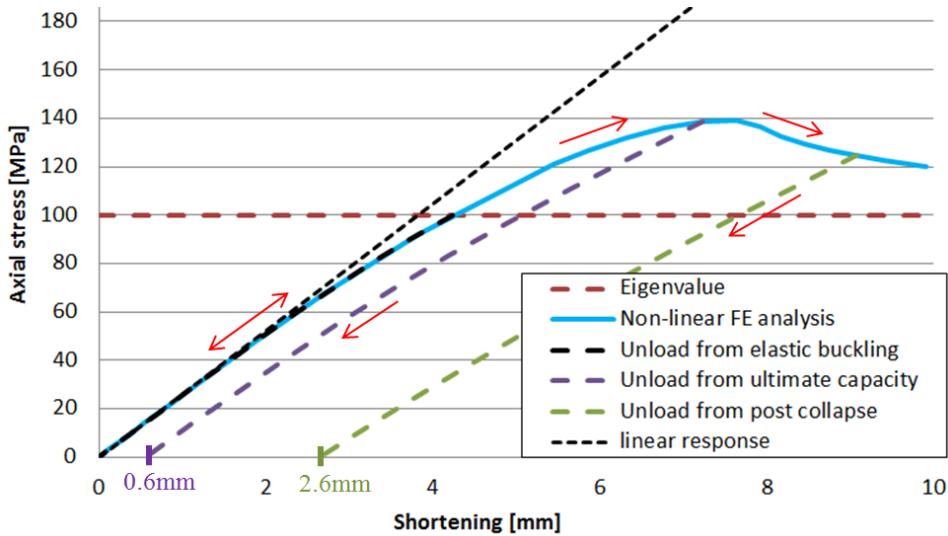


Figure 6 Plate response for the web plate with cut-outs; loading and unloading

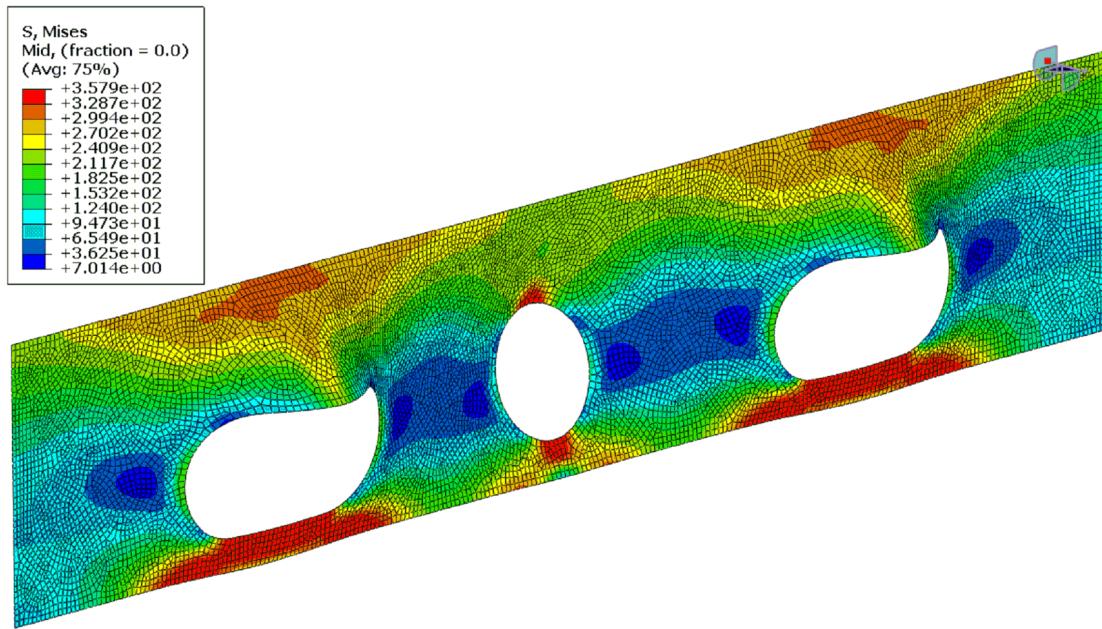


Figure 7 von Mises membrane stresses at ultimate load for the web plate with cut-outs

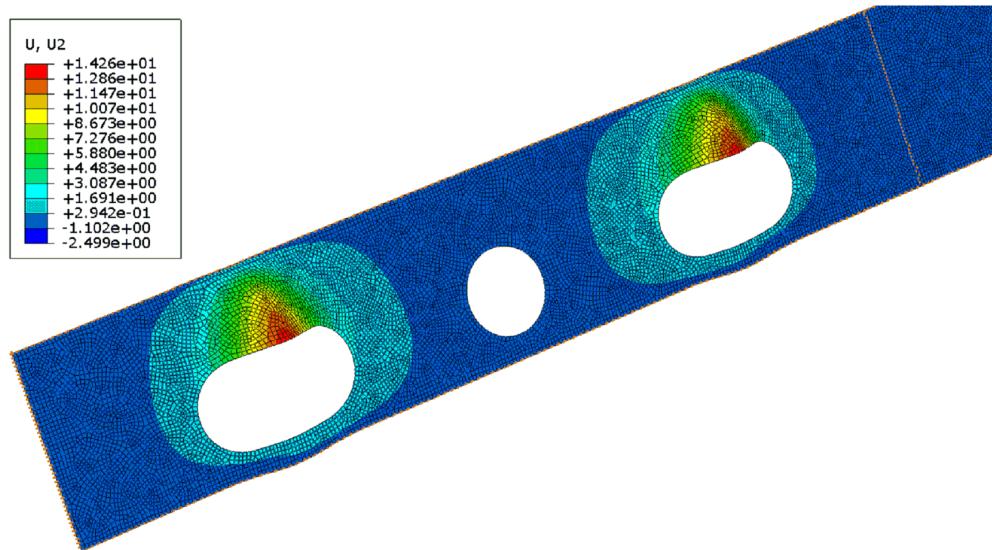


Figure 8 Permanent deformations (max. 14 mm) when unloading from post collapse

For the plate with buckling stiffeners, the response curve obtained by non-linear finite element is shown in [Figure 9](#). The ultimate capacity is 180 MPa which is 30% larger than for the plate without buckling stiffeners. In addition, elastic buckling does not occur before the ultimate strength is reached. The von Mises membrane stresses at ultimate load is shown in [Figure 10](#).

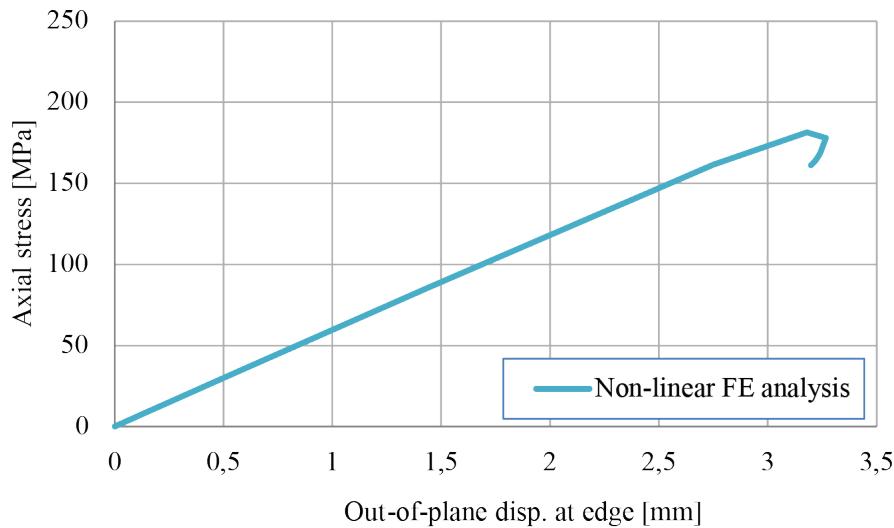


Figure 9 Plate response for the web plate with buckling stiffeners

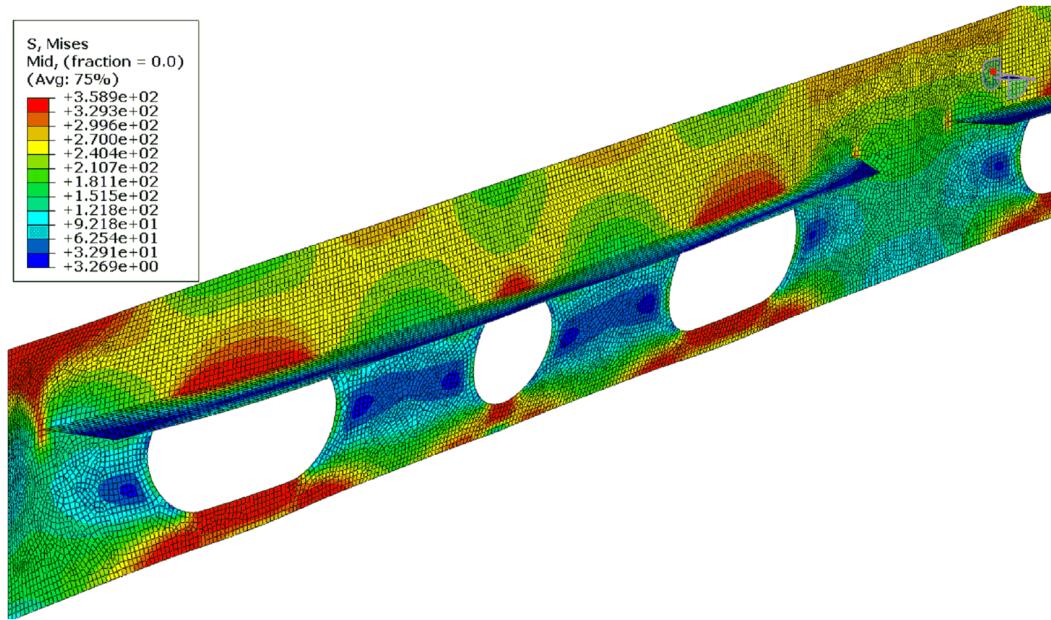


Figure 10 von Mises membrane stresses at ultimate load for the web plate with cut-outs and buckling stiffener

Imperfection Sensitivity study

A sensitivity study with respect to the imperfection magnitude has also been performed and the results are presented for both a plate with and without local edge stiffening in [Figure 11](#). It can be seen that the effect of the imperfection magnitude is small for both cases.

For the plate without edge stiffener, this is because the slenderness is rather large and for such cases the imperfection sensitivity is usually small. For the plate with edge stiffener, the imperfection sensitivity is small since the imperfection with maximum amplitude at the edge is not coincident with the critical mode.

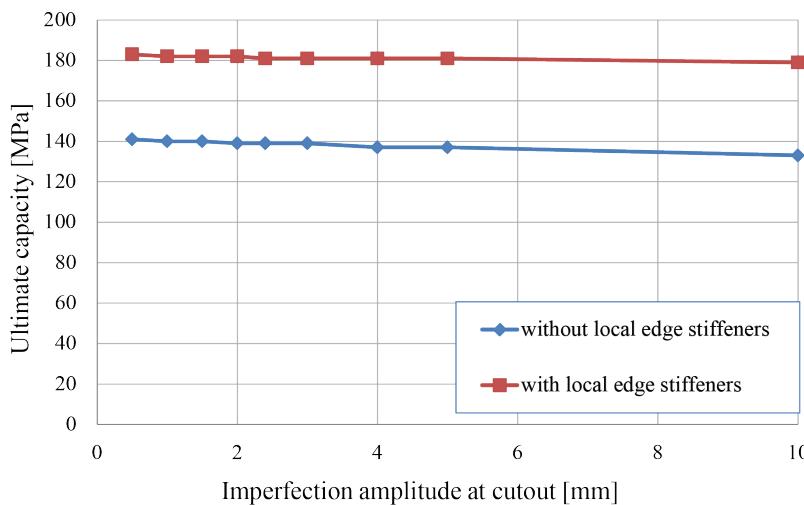


Figure 11 Longitudinal membrane stresses for the web plate with cut-outs and buckling stiffener

Comparisons with CFM calculations

The computed FE-results by Abaqus are compared with the CFM for a simply supported plate ([Sec.3](#)). In the CFM, the ultimate capacity is computed for the plates below and above the cut-out as shown in [Figure 12](#). The nominal stress for the capacity $S_{cx,tot}$ of the total girder web with height h_{tot} is found by adding the contribution from Panel 1 and 2 (weighted area approximation):

$$S_{cx,tot} = \frac{h_1 S_{cx,1} + h_2 S_{cx,2}}{h_{tot}}$$

where $S_{cx,1}$ and $S_{cx,2}$ are the capacity for Panel 1 and 2, respectively, and h_1 and h_2 are the corresponding height of the panels.

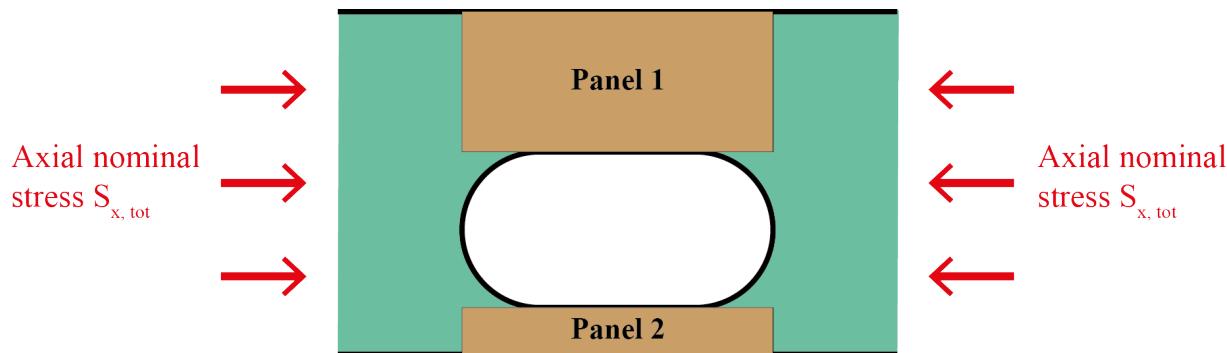


Figure 12 Local panels that are used in buckling check with the CFM

For the plate web without local buckling stiffeners the formulas for plates with free edge is used as illustrated in [Figure 13](#). For the plate web with buckling stiffeners, the CFM formulas for a simply supported plate is used for panel 1.

Case	Stress ratio ψ	Aspect ratio α	Buckling factor K	Reduction factor C
3	O N Ψ A I	1	$K_x = \frac{4(0.425 + 1/\alpha^2)}{3\psi + 1}$	$C_x = 1 \text{ for } \lambda \leq 0.7$
	1 - A Ψ A O	1	$K_x = 4(0.425 + 1/\alpha^2)(1 + \psi) - 5\psi(1 - 3.42\psi)$	
4		1 - A Ψ A 1	$K_x = \left(0.425 + \frac{1}{\alpha^2}\right) \frac{3-\psi}{2}$	$C_x = \frac{1}{\lambda^2 + 0.51} \text{ for } \lambda > 0.7$

Figure 13 Formulas from Sec.3 for capacity of plates with a free edge.

The capacities computed by the formulas in [Sec.3](#) are given in [Table 1](#). The FE-results are 48% larger than the CFM calculations for the plate without buckling stiffeners, and 6% smaller for plate with buckling stiffeners.

Table 1 Capacity computed by the formulas in Sec.3

	$S_{cx,1} / h_1$	$S_{cx,2} / h_2$	$S_{cx,tot}$
Plate without buckling stiffeners	119 MPa / 200 mm	355 MPa / 60 mm	94 MPa
Plate with buckling stiffeners	355 MPa / 200 mm	355 MPa / 60 mm	192 MPa

Discussion

The present detailed analyses show that the capacity of a typical ship girder with cut-out depends strongly on whether stiffener edge reinforcements are attached or not. The results with free unstiffened edges shows that the elastic buckling is triggered around 100 MPa nominal stress and for loads beyond this level and up the ultimate capacity the axial stiffness has dropped ca 40% (60% left of net web area stiffness). The nominal ultimate capacity is found for this example to be around 130 MPa. If the girder is compressed beyond this level (overload) significant permanent sets (damages) is identified and the free edge has suffered a severe localized plastic collapse ([Figure 8](#)).

By adding edge stiffener at cut-out upper edge the nominal ultimate capacity is increased to around 180 MPa and failure mode is moved to be away from the cut-out cross-section. The elastic buckling stress higher than the ultimate capacity (217 MPa).

The nominal load bearing capacity of 190 MPa in the way of the cut-out cross-section corresponds very closely to assuming the net web girder area to carry the yield stress ([Table 1](#)).

The imperfection sensitivity is also documented to be minimal for the present structure, i.e. this is in contrast to the spherical shell case in [App.B](#).

The comparison between CFM and non-lin FE are reasonable.

APPENDIX D REFERENCE STRESSES FOR DIRECT STRENGTH ANALYSIS

Symbols

For symbols not defined in this section, refer to [DNVGL-RU-SHIP Pt.3 Ch.1 Sec.4](#).

A_i	Area, in mm^2 , of the i -th plate element of the buckling panel
n	Number of plate elements in the buckling panel
σ_{xi}	Actual stress, in N/mm^2 , at the centroid of the i -th plate element in x direction, applied along the shorter edge of the buckling panel
σ_{yi}	Actual stress, in N/mm^2 , at the centroid of the i -th plate element in y direction, applied along the longer edge of the buckling panel
Ψ	Edge stress ratio $= \frac{\sigma_2}{\sigma_1}$
σ_1	Maximum stress, in N/mm^2
σ_2	Minimum stress, in N/mm^2
τ_i	Actual membrane shear stress, in N/mm^2 , at the centroid of the i -th plate element of the buckling panel.

1 Stress based method

1.1 Introduction

1.1.1

This section provides a method to determine stress distribution along edges of the considered buckling panel by 2nd order polynomial curve, by linear distribution using least square method and by weighted average approach. This method is called stress based method. The reference stress is the stress components at centre of plate element transferred into the local system of the considered buckling panel.

1.1.2 Definition

A regular panel is a plate panel of rectangular shape. An irregular panel is plate panel which is not regular, see [DNVGL-RU-SHIP Pt.3 Ch.8 Sec.4 \[2.3\]](#).

1.2 Stress application

1.2.1 Regular panel

The reference stresses are to be taken as defined in [\[2.1\]](#) for a regular panel when the following conditions are satisfied:

- At least, one plate element centre is located in each third part of the long edge a of a regular panel, and
- This element centre is located at a distance in the panel local x direction not less than $a/4$ to at least one of the element centres in the adjacent third part of the panel.

Otherwise, the reference stresses are to be taken as defined in [\[2.2\]](#) for an irregular panel.

1.2.2 Irregular panel and curved panel

The reference stresses of an irregular panel or of a curved panel are to be taken as defined in [\[2.2\]](#).

2 Reference stresses

2.1 Regular panel

2.1.1 Longitudinal stress for plate buckling assessment

The longitudinal stress σ_x applied on the shorter edge of the buckling panel for plate buckling assessment according to Sec.3 [2] is to be calculated from distribution of $\sigma_x(x)$ assumed as 2nd order polynomial curve as:

$$\sigma_x(x) = C \cdot x^2 + D \cdot x + E$$

The best fitting curve $\sigma_x(x)$ is to be obtained by minimising the square error Π considering the area of each element as a weighting factor.

$$\Pi = \sum_{i=1}^n A_i [\sigma_{ix} - (Cx_i^2 + Dx_i + E)]^2$$

The unknown coefficients C , D and E must yield zero first derivatives, $\partial\Pi$ with respect to C , D and E respectively.

$$\begin{cases} \frac{\partial\Pi}{\partial C} = 2 \sum_{i=1}^n A_i x_i^2 [\sigma_{ix} - (Cx_i^2 + Dx_i + E)] = 0 \\ \frac{\partial\Pi}{\partial D} = 2 \sum_{i=1}^n A_i x_i [\sigma_{ix} - (Cx_i^2 + Dx_i + E)] = 0 \\ \frac{\partial\Pi}{\partial E} = 2 \sum_{i=1}^n A_i [\sigma_{ix} - (Cx_i^2 + Dx_i + E)] = 0 \end{cases}$$

The unknown coefficients C , D and E can be obtained by solving the 3 above equations.

$$\sigma_{x1} = \frac{1}{b} \int_0^b \sigma_x(x) dx = \frac{b^2}{3} C + \frac{b}{2} D + E$$

$$\sigma_{x2} = \frac{1}{b} \int_{a-b}^a \sigma_x(x) dx = \left(a^2 - ab + \frac{b^2}{3} \right) C + \left(a - \frac{b}{2} \right) D + E$$

If $-D/2C < b/2$ or $-D/2C > a-b/2$, σ_{x3} is to be ignored. Otherwise, σ_{x3} is taken as:

$$\sigma_{x3} = \frac{1}{b} \int_{x_{\min}}^{x_{\max}} \sigma_x(x) dx = \frac{b^2}{12} C - \frac{D^2}{4C} + E$$

where:

$$x_{\min} = -\frac{b}{2} - \frac{D}{2C}$$

$$x_{\max} = \frac{b}{2} - \frac{D}{2C}$$

The longitudinal stress and the edge stress ratio are to be taken as:

$$\sigma_x = \max(\sigma_{x1}; \sigma_{x2}; \sigma_{x3})$$

$$\Psi_x = 1$$

2.1.2 Longitudinal stress for stiffener buckling assessment

For stiffener buckling assessment according to Sec.3 [2] and stiffened panel buckling assessment with PULS (S3) according to Sec.4, $\sigma_x(x)$ applied on the shorter edge of the attached plate is to be taken as:

$$\sigma_x = \frac{\sum_1^n A_i \sigma_{xi}}{\sum_1^n A_i}$$

The edge stress ratio Ψ_x for the stress σ_x is equal to 1.0.

2.1.3 Transverse stress for plate buckling assessment

The transverse stress σ_y applied along the longer edges of the buckling panel is to be calculated by extrapolation of the transverse stresses of all elements up to the shorter edges of the considered buckling panel.

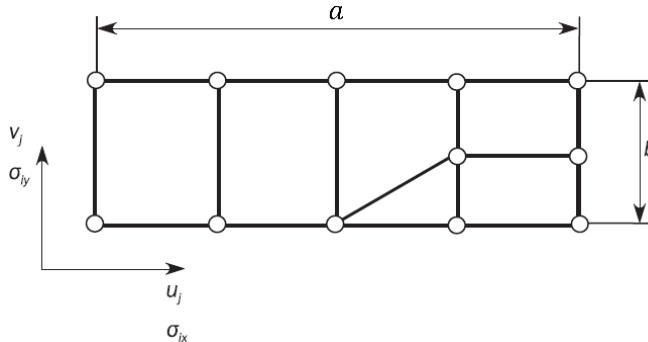


Figure 1 Buckling panel

The distribution of $\sigma_y(x)$ is assumed as straight line. Therefore:

$$\sigma_y(x) = A + Bx$$

The best fitting curve $\sigma_y(x)$ is to be obtained by the least square method minimising the square error Π considering area of each element as a weighting factor.

$$\Pi = \sum_{i=1}^n A_i [\sigma_{iy} - (A + Bx_i)]^2$$

The unknown coefficients C and D must yield zero first partial derivatives, $\partial\Pi$ with respect to C and D , respectively.

$$\begin{cases} \frac{\partial \Pi}{\partial A} = 2 \sum_{i=1}^n A_i [\sigma_{iy} - (A + Bx_i)] = 0 \\ \frac{\partial \Pi}{\partial B} = 2 \sum_{i=1}^n A_i x_i [\sigma_{iy} - (A + Bx_i)] = 0 \end{cases}$$

The unknown coefficients A and B are obtained by solving the 2 above equations and are given as follow:

$$\begin{cases} A = \frac{\left(\sum_{i=1}^n A_i \sigma_{iy} \right) \left(\sum_{i=1}^n A_i x_i^2 \right) - \left(\sum_{i=1}^n A_i x_i \right) \left(\sum_{i=1}^n A_i x_i \sigma_{iy} \right)}{\left(\sum_{i=1}^n A_i \right) \left(\sum_{i=1}^n A_i x_i^2 \right) - \left(\sum_{i=1}^n A_i x_i \right)^2} \\ B = \frac{\left(\sum_{i=1}^n A_i \right) \left(\sum_{i=1}^n A_i x_i \sigma_{iy} \right) - \left(\sum_{i=1}^n A_i x_i \right) \left(\sum_{i=1}^n A_i \sigma_{iy} \right)}{\left(\sum_{i=1}^n A_i \right) \left(\sum_{i=1}^n A_i x_i^2 \right) - \left(\sum_{i=1}^n A_i x_i \right)^2} \end{cases}$$

$$\sigma_y = \max(A, A + Ba)$$

$$\Psi_y = \frac{\min(A, A + Ba)}{\max(A, A + Ba)} \quad \text{for } \sigma_y \geq 0$$

$$\Psi_y = 1 \quad \text{for } \sigma_y < 0$$

2.1.4 Transverse stress for stiffener buckling assessment

For stiffener buckling assessment according to Sec.3 [2] and stiffened panel buckling assessment with PULS (S3) according to Sec.4, σ_y applied on the longer edge of the attached plate shall be taken as:

$$\sigma_y = \frac{\sum_{i=1}^n A_i \sigma_{yi}}{\sum_{i=1}^n A_i}$$

The edge stress ratio Ψ_y for the stress σ_y shall be taken equal to 1.0.

2.1.5 Shear stress

The shear stress τ shall be calculated using a weighted average approach, and shall be taken as:

$$\tau = \frac{\sum_{i=1}^n A_i \tau_i}{\sum_{i=1}^n A_i}$$

2.2 Irregular panel and curved panel

2.2.1 Reference stresses

The longitudinal, transverse and shear stresses are to be calculated using a weighted average approach. They shall be taken as:

$$\sigma_x = \frac{\sum_1^n A_i \sigma_{xi}}{\sum_1^n A_i}$$

$$\sigma_y = \frac{\sum_1^n A_i \sigma_{yi}}{\sum_1^n A_i}$$

$$\tau = \frac{\sum_1^n A_i \tau_i}{\sum_1^n A_i}$$

The edge stress ratios shall be taken as:

$$\Psi_x = 1$$

$$\Psi_y = 1$$

APPENDIX E HULL GIRDER ULTIMATE BENDING CAPACITY BY MULTI-STEP METHOD

Symbols

I_{y-n50} :	Net moment of inertia, in m^4 , of the hull transverse section around its horizontal neutral axis
Z_{B-n50}, Z_{D-n50} :	Section moduli, in m^3 , at bottom and deck, respectively
R_{eH_S} :	Minimum specified yield stress, in N/mm^2 , of the material of the considered stiffener
R_{eH_P} :	Minimum specified yield stress, in N/mm^2 , of the material of the considered plate
A_{s-n50} :	Net sectional area, in cm^2 , of stiffener, without attached plating
A_{p-n50} :	Net sectional area, in cm^2 , of attached plating

1 General Assumptions

1.1

The method for calculating the ultimate hull girder capacity is to identify the critical failure modes of all main longitudinal structural elements.

1.2

Structures compressed beyond their buckling limit have reduced load carrying capacity. All relevant failure modes for individual structural elements, such as plate buckling, torsional stiffener buckling, stiffener web buckling, lateral or global stiffener buckling and their interactions, are to be considered in order to identify the weakest inter-frame failure mode.

1.3

The members contributing to the hull girder longitudinal strength are to be considered using the net offered scantlings based on gross offered thickness reduced by 50% of corrosion addition.

2 Incremental-iterative method

2.1 Assumptions

In applying the incremental-iterative method, the following assumptions shall generally be made:

- 1) The ultimate strength is calculated at hull transverse sections between two adjacent transverse webs.
- 2) The hull girder transverse section remains plane during each curvature increment.
- 3) The hull material has an elasto-plastic behaviour.
- 4) The hull girder transverse section is divided into a set of elements, see [2.2.2], which are considered to act independently.

According to the iterative procedure, the bending moment M_i acting on the transverse section at each curvature value χ_i is obtained by summing the contribution given by the stress σ acting on each element.

The stress σ corresponding to the element strain, ε shall be obtained for each curvature increment from the non-linear load-end shortening curves $\sigma - \varepsilon$ of the element.

These curves shall be calculated, for the failure mechanisms of the element, from the formulae specified in [2.3]. The stress σ is selected as the lowest among the values obtained from each of the considered load-end shortening curves $\sigma - \varepsilon$.

The procedure shall be repeated until the value of the imposed curvature reaches the value, χ_F in m^{-1} , in hogging and sagging condition, obtained from the following formula:

$$\chi_F = \pm 0.003 \frac{M_y}{EI_{y-n50}}$$

where:

M_Y = lesser of the values M_{Y1} and M_{Y2} , in kNm

$M_{Y1} = 10^3 R_{eH} Z_{B-n50}$

$M_{Y2} = 10^3 R_{eH} Z_{D-n50}$.

If the value χ_F is not sufficient to evaluate the peaks of the curve $M - \chi$, the procedure shall be repeated until the value of the imposed curvature permits the calculation of the maximum bending moments of the curve.

2.2 Procedure

2.2.1 General

The curve $M - \chi$ shall be obtained by means of an incremental-iterative approach, summarised in the flow chart in Figure 1.

In this procedure, the ultimate hull girder bending moment capacity, M_U is defined as the peak value of the curve with vertical bending moment M versus the curvature χ of the ship cross section as shown in Figure 1. The curve is to be obtained through an incremental-iterative approach.

Each step of the incremental procedure is represented by the calculation of the bending moment M_i which acts on the hull transverse section as the effect of an imposed curvature χ_i .

For each step, the value χ_i shall be obtained by summing an increment of curvature, $\Delta\chi$ to the value relevant to the previous step χ_{i-1} . This increment of curvature corresponds to an increment of the rotation angle of the hull girder transverse section around its horizontal neutral axis.

This rotation increment induces axial strains ε in each hull structural element, whose value depends on the position of the element. In hogging condition, the structural elements above the neutral axis are lengthened, while the elements below the neutral axis are shortened, and vice-versa in sagging condition.

The stress σ induced in each structural element by the strain ε is to be obtained from the load-end shortening curve $\sigma - \varepsilon$ of the element, which takes into account the behaviour of the element in the non-linear elasto-plastic domain.

The distribution of the stresses induced in all the elements composing the hull transverse section determines, for each step, a variation of the neutral axis position due to the nonlinear $\sigma - \varepsilon$, relationship. The new position of the neutral axis relevant to the step considered shall be obtained by means of an iterative process, imposing the equilibrium among the stresses acting in all the hull elements on the transverse section.

Once the position of the neutral axis is known and the relevant element stress distribution in the section is obtained, the bending moment of the section M_i around the new position of the neutral axis, which corresponds to the curvature χ_i imposed in the step considered, shall be obtained by summing the contribution given by each element stress.

The main steps of the incremental-iterative approach described above are summarised as follows (see also Figure 1):

- Step 1: Divide the transverse section of hull into stiffened plate elements.
- Step 2: Define stress-strain relationships for all elements as shown in [Table 1](#).
- Step 3: Initialise curvature χ_1 and neutral axis for the first incremental step with the value of incremental curvature (i.e. curvature that induces a stress equal to 1% of yield strength in strength deck) as:

$$\chi_1 = \Delta\chi = 0.01 \frac{R_{eH}}{E} \frac{1}{z_D - z_n}$$

where:

z_D = Z coordinate, in m, of strength deck at side

z_n = Z coordinate, in m, of horizontal neutral axis of the hull transverse section with respect to the reference coordinate system defined [DNVGL-RU-SHIP Pt.3 Ch.1 Sec. 4 \[3.6\]](#).

- Step 4: Calculate for each element the corresponding strain, $\varepsilon_i = \chi(z_i - z_n)$ and the corresponding stress σ_i
- Step 5: Determine the neutral axis z_{NA_cur} at each incremental step by establishing force equilibrium over the whole transverse section as:

$$\sum A_{i-n50} \sigma_i = \sum A_{j-n50} \sigma_j$$

(i-th element is under compression, j-th element under tension).

- Step 6: Calculate the corresponding moment by summing the contributions of all elements as:

$$M_U = \sum \sigma_{Ui} A_{i-n50} |(z_i - z_{NA_cur})|$$

- Step 7: Compare the moment in the current incremental step with the moment in the previous incremental step. If the slope in $M - \chi$ relationship is less than a negative fixed value, terminate the process and define the peak value M_U . Otherwise, increase the curvature by the amount of $\Delta\chi$ and go to step 4.

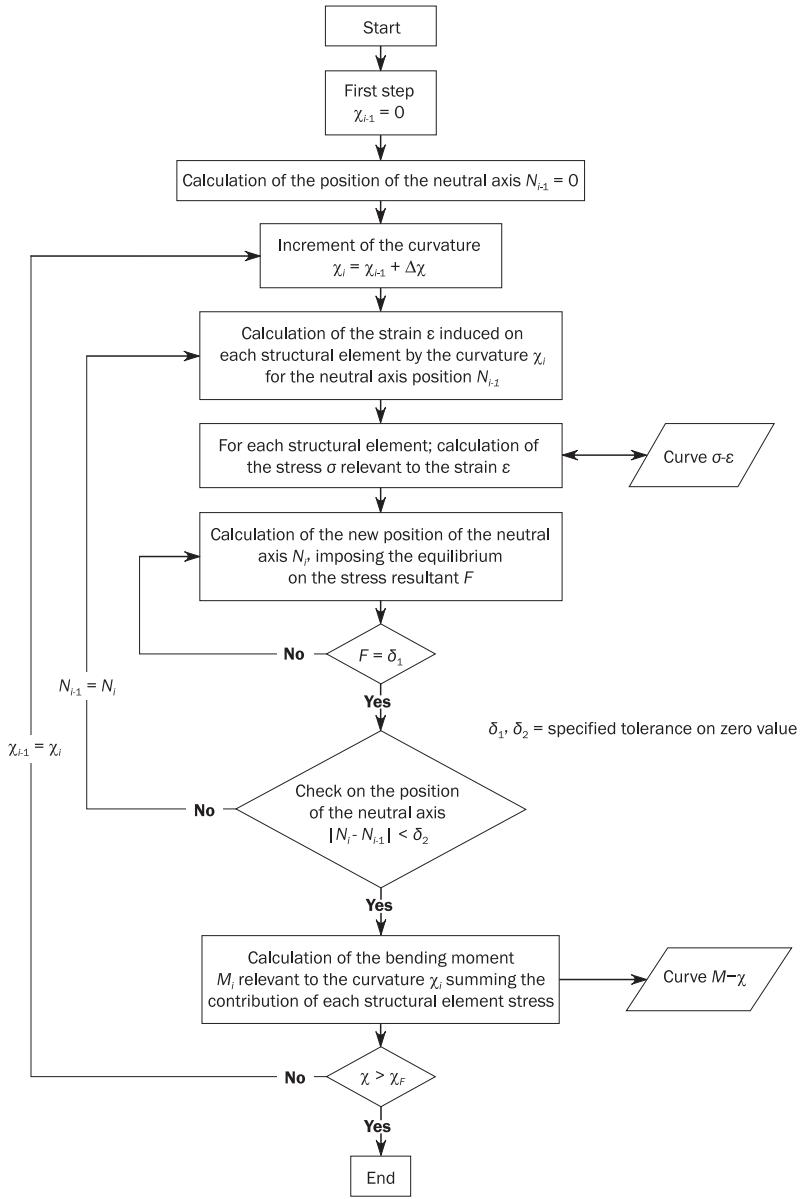


Figure 1 Flow chart of the procedure for the evaluation of the curve $M - \chi$

2.2.2 Modelling of the hull girder cross section

Hull girder transverse sections are to be considered as being constituted by the members contributing to the hull girder ultimate strength.

Sniped stiffeners are also to be modelled, taking account that they do not contribute to the hull girder strength.

The structural members are categorised into a stiffener element, a stiffened plate element or a hard corner element.

The plate panel including web plate of girder or side stringer is idealised into a stiffened plate element, an attached plate of a stiffener element or a hard corner element.

The plate panel is categorised into the following two kinds:

- 1) longitudinally stiffened panel of which the longer side is in ship's longitudinal direction, and
- 2) transversely stiffened panel of which the longer side is in the perpendicular direction to ship's longitudinal direction.

a) Hard corner element:

Hard corner elements are sturdier elements composing the hull girder transverse section, which collapse mainly according to an elasto-plastic mode of failure (material yielding); they are generally constituted by two plates not lying in the same plane.

The extent of a hard corner element from the point of intersection of the plates is taken equal to $20 t_{n50}$ on a transversely stiffened panel and to $0.5 s$ on a longitudinally stiffened panel, see [Figure 2](#).

where:

t_{n50} = net thickness of the plate, in mm.

s = spacing of the adjacent longitudinal stiffener, in m.

Bilge, sheer strake-deck stringer elements, girder-deck connections and face plate-web connections on large girders are typical hard corners.

b) Stiffener element:

The stiffener constitutes a stiffener element together with the attached plate.

The attached plate width is in principle:

- equal to the mean spacing of the stiffener when the panels on both sides of the stiffener are longitudinally stiffened, or
- equal to the width of the longitudinally stiffened panel when the panel on one side of the stiffener is longitudinally stiffened and the other panel is of the transversely stiffened, see [Figure 2](#).

c) Stiffened plate element:

The plate between stiffener elements, between a stiffener element and a hard corner element or between hard corner elements shall be treated as a stiffened plate element, see [Figure 2](#).

The typical examples of modelling of hull girder section are illustrated in [Figure 3](#) and [Figure 4](#).

Notwithstanding the foregoing principle, these figures are to be applied to the modelling in the vicinity of upper deck, sheer strake and hatch coaming.

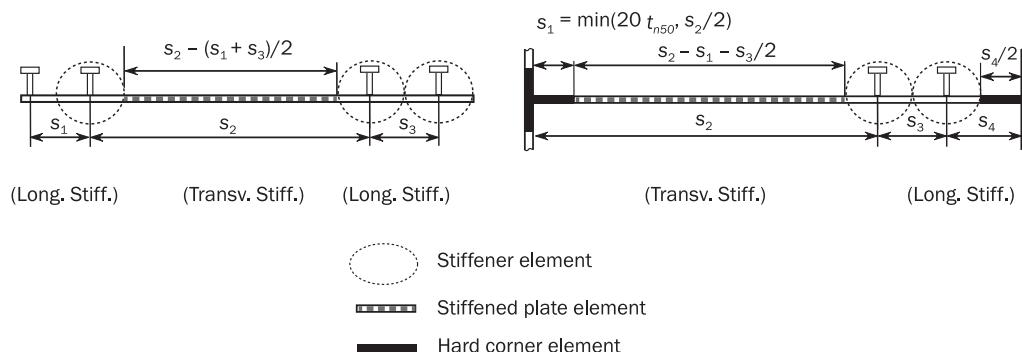


Figure 2 Extension of the breadth of the attached plating and hard corner element

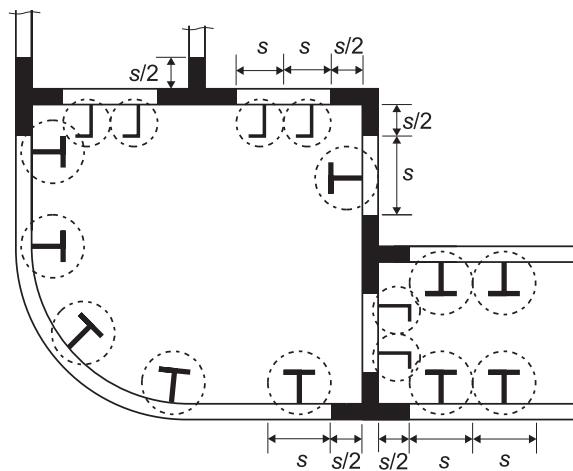
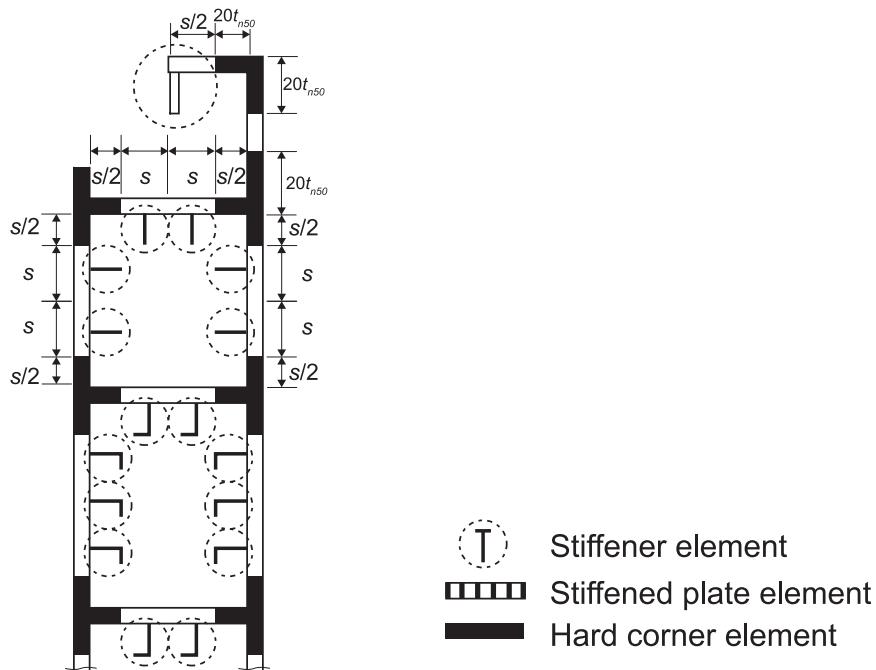


Figure 3 Examples of the configuration of stiffened plate elements, stiffener elements and hard corner elements on a hull section for a container ship

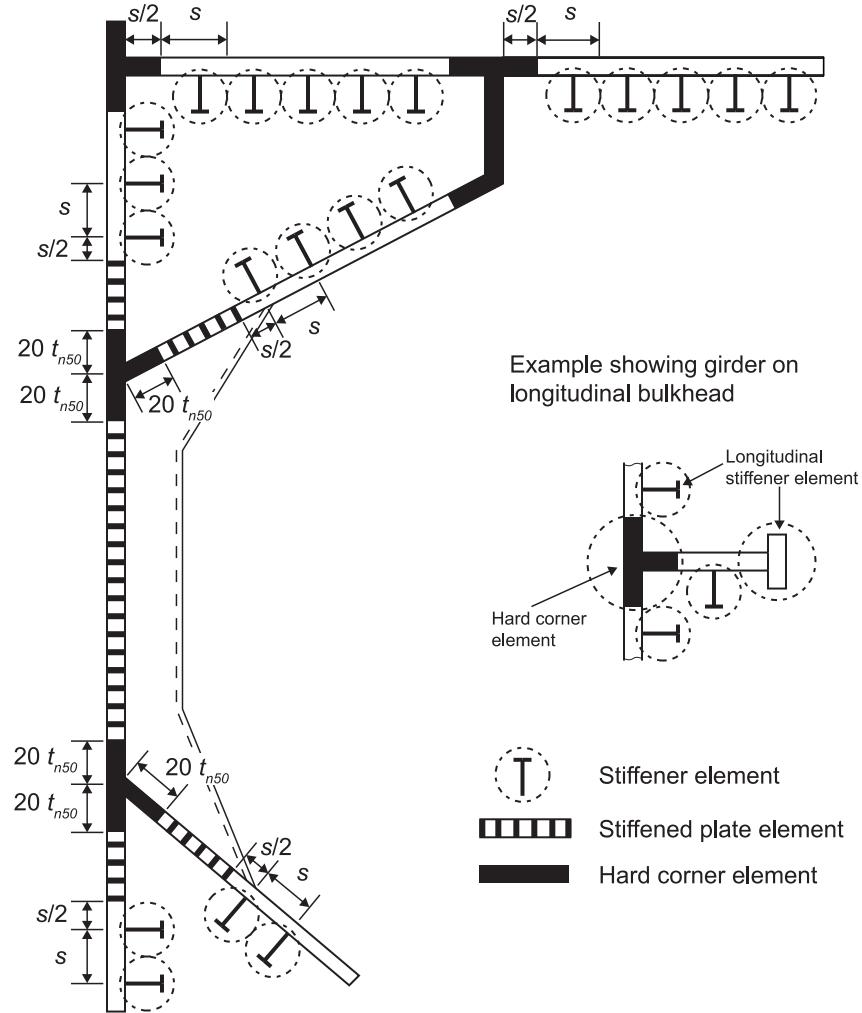


Figure 4 Examples of the configuration of stiffened plate elements, stiffener elements and hard corner elements on a hull section for a gas carrier

- In case of the knuckle point as shown in Figure 5, the plating area adjacent to knuckles in the plating with an angle greater than 30 degrees is defined as a hard corner. The extent of one side of the corner is taken equal to $20t_{n50}$ on transversely framed panels and to $0.5s$ on longitudinally framed panels from the knuckle point.
- Where the plate members are stiffened by non-continuous longitudinal stiffeners, the non- continuous stiffeners are as dividing a plate into various elementary plate panels.
- Where the opening is provided in the stiffened plate element, the openings shall be considered in accordance with the requirements of the Society.
- Where attached plating is made of steels having different thicknesses and/or yield stresses, an average thickness and/or average yield stress obtained from the following formula shall be used for the calculation.

$$t_{n50} = \frac{t_1 - n50 + t_2 - n50}{s} \quad R_{eH_P} = \frac{R_{eH_P1} t_1 - n50 s_1 + R_{eH_P2} t_2 - n50 s_2}{t_{n50} s}$$

where R_{eH_P1} , R_{eH_P2} , t_{1-n50} , t_{2-n50} , s_1 , s_2 and s are shown in [Figure 6](#).

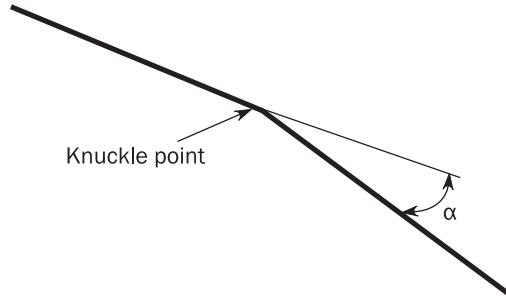


Figure 5 Plating with knuckle point

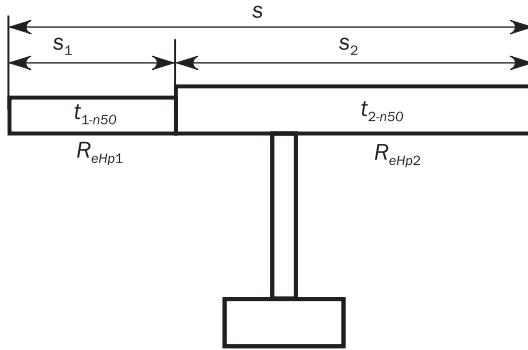


Figure 6 Element with different thickness and yield strength

2.3 Load-end shortening curves

2.3.1 Stiffened plate element and stiffener element

Stiffened plate element and stiffener element composing the hull girder transverse sections may collapse following one of the modes of failure specified in [Table 1](#).

- Where the plate members are stiffened by non-continuous longitudinal stiffeners, the stress of the element shall be obtained in accordance with [\[2.3.2\]](#) to [\[2.3.7\]](#), taking into account the non-continuous longitudinal stiffener.

In calculating the total forces for checking the hull girder ultimate strength, the area of non-continuous longitudinal stiffener is to be assumed as zero.

- Where the opening is provided in the stiffened plate element, the considered area of the stiffened plate element shall be obtained by deducting the opening area from the plating in calculating the total forces for checking the hull girder ultimate strength.
- For stiffened plate element, the effective width of plate for the load shortening portion of the stress-strain curve shall be taken as full plate width, i.e. to the intersection of other plate or longitudinal stiffener – neither from the end of the hard corner element nor from the attached plating of stiffener element, if any.

In calculating the total forces for checking the hull girder ultimate strength, the area of the stiffened plate element shall be taken between the hard corner element and the stiffener element or between the hard corner elements, as applicable.

Table 1 : Modes of failure of stiffened plate element and stiffener element

Element	Mode of failure	Load-end shortening curve defined in
Lengthened stiffened plate element or stiffener element	Elasto-plastic collapse	[2.3.2]
Shortened stiffener element	Beam column buckling Torsional buckling Web local buckling of flanged profiles Web local buckling of flat bars	[2.3.3], [2.3.4], [2.3.5], [2.3.6]
Shortened stiffened plate element	Plate buckling	[2.3.7]

2.3.2 Elasto-plastic collapse of structural elements (Hard corner element)

The equation describing the load-end shortening curve $\sigma - \varepsilon$ for the elasto-plastic collapse of structural elements composing the hull girder transverse section shall be obtained from the following formula.

$$\sigma = \Phi R_{eHA}$$

where:

R_{eHA} = equivalent minimum yield stress, in N/mm², of the considered element, obtained by the following formula:

$$R_{eHA} = \frac{R_{eH_P} A_p - n50 + R_{eH_S} A_s - n50}{A_p - n50 + A_s - n50}$$

Φ = edge function equal to:

$$\Phi = -1 \text{ for } \varepsilon < -1$$

$$\Phi = \varepsilon \text{ for } -1 \leq \varepsilon \leq 1$$

$$\Phi = 1 \text{ for } \varepsilon > 1$$

ε = relative strain, equal to:

$$\varepsilon = \frac{\varepsilon_E}{\varepsilon_Y}$$

ε_E = element strain

ε_Y = strain at yield stress in the element, equal to:

$$\varepsilon_y = \frac{R_{eHA}}{E}$$

2.3.3 Beam column buckling

The positive strain portion of the average stress - average strain curve $\sigma_{CR1} - \varepsilon$ based on beam column buckling of plate-stiffener combinations is described according to the following:

$$\sigma_{CR1} = \Phi \sigma_{C1} \frac{A_s - n50 + A_p E - n50}{A_s - n50 + A_p - n50}$$

where:

Φ = edge function, as defined in [2.3.2]

σ_{C1} = critical stress, in N/mm², equal to:

$$\sigma_{C1} = \frac{\sigma_{E1}}{\epsilon} \quad \text{for} \quad \sigma_{E1} \leq \frac{R_{eHB}}{2} \epsilon$$

$$R_{C1} = R_{eHB} \left(1 - \frac{R_{eHB}}{4\sigma_{E1}} \epsilon \right) \quad \text{for} \quad \sigma_{E1} > \frac{R_{eHB}}{2} \epsilon$$

R_{eHB} = equivalent minimum yield stress, in N/mm², of the considered element, obtained by the following formula:

$$R_{eHB} = \frac{R_{eH_P} A_{pEI} - n50 \ell_{pE} + R_{eH_S} A_s - n50 \ell_{sE}}{A_{pEI} - n50 \ell_{pE} + A_s - n50 \ell_{sE}}$$

A_{EI} = effective net area, in cm², equal to:

$$A_{pEI-n50} = 10 b_{E1} t_{n50}$$

ℓ_{pE} = distance, in mm, measured from the neutral axis of the stiffener with attached plate of width b_{E1} to the bottom of the attached plate.

ℓ_{sE} = distance, in mm, measured from the neutral axis of the stiffener with attached plate of width b_{E1} to the top of the stiffener.

ϵ = relative strain, as defined in [2.3.2]

σ_{E1} = euler column buckling stress, in N/mm², equal to:

$$\sigma_{E1} = \pi^2 E \frac{I_{E-n50}}{A_{E-n50} \ell^2} 10^{-4}$$

I_{E-n50} = net moment of inertia of stiffeners, in cm⁴, with attached plate of width b_{E1}

A_{E-n50} = net area, in cm², of stiffeners with attached plating of width b_{E1}

b_{E1} = effective width corrected for relative strain, in m, of the attached plating, equal to:

$$b_{E1} = \frac{s}{\beta_E} \quad \text{for} \quad \beta_E > 1.0$$

$$b_{E1} = s \quad \text{for} \quad \beta_E \leq 1.0$$

$$\beta_E = 10^3 \frac{s}{t_{n50}} \sqrt{\frac{R_{eH_P}}{E} \epsilon}$$

A_{pE} = net area, in cm², of attached plating of width b_E equal to:

$$A_{pE-n50} = 10 b_E t_{n50}$$

b_E = effective width, in m, of the attached plating, equal to:

$$b_E = \left(\frac{2.25}{\beta_E} - \frac{1.25}{\beta_E^2} \right) s \quad \text{for} \quad \beta_E > 1.25$$

$$b_E = s \quad \text{for} \quad \beta_E \leq 1.25$$

2.3.4 Torsional buckling

The load-end shortening curve $\sigma_{CR-2} - \epsilon$ for the flexural-torsional buckling of stiffeners composing the hull girder transverse section shall be obtained according to the following formula:

$$\sigma_{CR2} = \Phi \frac{A_s - n50 \sigma_{C2} + A_p - n50 \sigma_{CP}}{A_s - n50 + A_p - n50}$$

where:

Φ = edge function, as defined in [2.3.2]

σ_{C2} = critical stress, in N/mm², equal to:

$$\sigma_{C2} = \frac{\sigma_{E2}}{\varepsilon} \quad \text{for} \quad \sigma_{E2} \leq \frac{R_{eH_S}}{2} \varepsilon$$

$$\sigma_{C2} = R_{eH_S} \left(1 - \frac{R_{eH_S}}{4\sigma_{E2}} \varepsilon \right) \quad \text{for} \quad \sigma_{E2} > \frac{R_{eH_S}}{2} \varepsilon$$

ε = edge function, as defined in [2.3.2]

σ_{E2} = euler column buckling stress, in N/mm², taken as σ_{ET} defined in Sec. 3 [2.3.4]

σ_{CP} = buckling stress of the attached plating, in N/mm², equal to:

$$\sigma_{CP} = \left(\frac{2.25}{\beta_E} - \frac{1.25}{\beta_E^2} \right) R_{eH_P} \quad \text{for} \quad \beta_E > 1.25$$

$$\sigma_{CP} = R_{eH_P} \quad \text{for} \quad \beta_E \leq 1.25$$

β_E = coefficient, as defined in [2.3.3].

2.3.5 Web local buckling of stiffeners made of flanged profiles

The load-end shortening curve $\sigma_{CR3} - \varepsilon$ for the web local buckling of flanged stiffeners composing the hull girder transverse section is to be obtained from the following formula:

$$\sigma_{CR3} = \Phi \frac{10^3 b_E t_{n50} R_{eH_P} + (h_{we} t_w - n50 + b_f t_f - n50) R_{eH_S}}{10^3 s t_{n50} + h_{we} t_w - n50 + b_f t_f - n50}$$

where:

Φ = edge function, as defined in [2.3.2]

b_E = effective width, in m, of the attached plating, as defined in [2.3.3]

h_{we} = effective height, in mm, of the web, equal to:

$$h_{we} = \left(\frac{2.25}{\beta_w} - \frac{1.25}{\beta_w^2} \right) h_w \quad \text{for} \quad \beta_w \geq 1.25$$

$$h_{we} = h_w \quad \text{for} \quad \beta_w < 1.25$$

$$\beta_w = \frac{h_w}{t_w - n50} \sqrt{\frac{R_{eH_S}}{E}} \varepsilon$$

ε = relative strain, as defined in [2.3.2].

2.3.6 Web local buckling of stiffeners made of flat bars

The load-end shortening curve $\sigma_{CR4} - \varepsilon$ for the web local buckling of flat bar stiffeners composing the hull girder transverse section shall be obtained from the following formula:

$$\sigma_{CR4} = \Phi \frac{A_p - n50 \sigma_{CP} + A_s - n50 \sigma_{C4}}{A_p - n50 + A_s - n50}$$

where:

Φ = edge function, as defined in [2.3.2]

σ_{CP} = buckling stress of the attached plating, in N/mm², as defined in [2.3.4]

σ_{C4} = critical stress, in N/mm², equal to:

$$\sigma_{C4} = \frac{\sigma_{E4}}{\varepsilon} \quad \text{for} \quad \sigma_{E4} \leq \frac{R_{eH_S}}{2} \varepsilon$$

$$\sigma_{C4} = R_{eH_S} \left(1 - \frac{R_{eH_S}}{4\sigma_{E4}} \varepsilon \right) \quad \text{for} \quad \sigma_{E4} > \frac{R_{eH_S}}{2} \varepsilon$$

σ_{E4} = local Euler buckling stress, in N/mm², equal to:

$$\sigma_{E4} = 160000 \left(\frac{t_w - n50}{h_w} \right)^2$$

ε = relative strain, as defined in [2.3.2].

2.3.7 Plate buckling

The load-end shortening curve $\sigma_{CR5} - \varepsilon$ for the buckling of transversely stiffened panels composing the hull girder transverse section shall be obtained from the following formula:

$$\sigma_{CR5} = \min \left\{ R_{eH_P} \Phi; R_{eH_P} \Phi \left[\frac{s}{\ell} \left(\frac{2.25}{\beta_E} - \frac{1.25}{\beta_E^2} \right) + 0.1 \left(1 - \frac{s}{\ell} \right) \left(1 + \frac{1}{\beta_E^2} \right)^2 \right] \right\}$$

where:

Φ = edge function, as defined in [2.3.2]

β_E = coefficient as defined in [2.3.3]

s = plate breadth, in m, taken as the spacing between the stiffeners

ℓ = longer side of the plate, in m.

CHANGES – HISTORIC

October 2015 edition

Amendments April 2016

- Sec.3 Closed form method (CFM) - buckling capacity
 - [2.2.7]: Update on stress adjustment in case of plate buckling check based on beam element analysis (improvement/error)
- Sec.4 Semi-analytical buckling models (PULS)
 - [4.1]: Application of stress obtained by beam element analysis is described.
- App.D Reference stresses for direct strength analysis
 - This is a new appendix describing the method for FE buckling assessment based on CSR Pt.1 Ch.8 App.1

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