Vector

- A ~~phy~~ vector is a physical quantity which has (i) magnitude as well as specific direction
  (ii) obeys commutative law of addition.
  $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
  (iii) obeys law of parallalogram of addition of vector or triangle rule of vector addition.

TERMS (1) If any of the above condition is not satisfied then physical quantity ~~can~~ cannot be a vector.

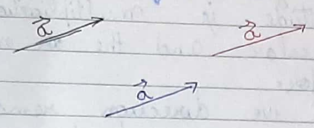
(2) If a physical quantity is a vector it has direction but converse may or may not be true i.e, a physical quantity has a direction it may or may not be vector
Eg: time, pressure, surface tension, current have direction but ~~they~~ are not vector because they don't obey llgm law of addition of vector.

Force vector is represented by $\vec{F}$ whose magnitude is donoted by $|\vec{F}|$ or F

$\vec{a}$ $|\vec{a}|$ or a

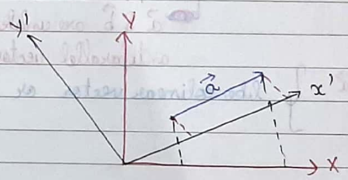
Geometrically vector is represented by directed line segment ending with a arrow. Arrow represent direction & length of line segment ~~represents~~ is proportional to its magnitude.

- Vector is independent of their spacial position. If vector is displaced parallel to itself it does ~~not~~ not change.



- If a vector is rotated through an angle other than $2\pi\ (360°)$, it changes.

- If the frame of refrence is translated or rotated, vector does not change though its component may change.



- Two vectors are said to be equal if their magnitude & direction are same and they represent same physical quantity.

→ Colinear vector - Two or more vector are said to be collinear if their support are either same or parallel.

→ S.

→ Scalar multiple to a vector:

- When a vector is multiplied with scalar a new vector physical quantity is obtained whose magnitude is multiplication of magnitude of scalar and the magnitude of original vector.
If scalar is +ve direction remains same & if scalar is -ve direction becomes opposite.

$\vec{b} = \lambda\vec{a}$

$|\vec{b}| = |\lambda\vec{a}|$

$|\vec{b}| = |\lambda|\,|\vec{a}|$

→ $\lambda > 0$
$|\vec{b}| = \lambda|\vec{a}|$
$\vec{a}$ & $\vec{b}$ are like colinear or || vector

→ $\lambda < 0$
$|\vec{b}| = -\lambda|\vec{a}|$
$\vec{a}$ & $\vec{b}$ are unlike coliner or antiparallel vector.

$\vec{a}$

$2\vec{a}$, $\frac{1}{2}\vec{a}$ } like colinear vector or parallel vector

anti-parallel or unlike colinear vector { $\frac{1}{2}\vec{a}$, $-\vec{a}$

$-\frac{1}{2}\vec{a}$, $-3\vec{a}$ } like colinear vector or || vector

Eg: $\vec{P} = m\vec{v}$ $\quad \vec{P} \parallel \vec{v}$

$\vec{F} = m\vec{a}$ $\quad \vec{F} \parallel \vec{a}$ $\quad \{ \because m > 0$

NOTE: If $\vec{a}$ & $\vec{b}$ are two colinear vector then $|\vec{b}| = \lambda|\vec{a}|$

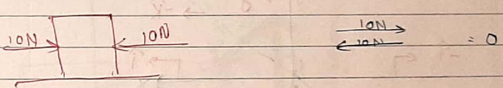
→ Negative of a vector:

- A vector whose magnitude is same as that of original vector but direction is exactly opposite of original vector.

$\vec{a}$, $-\vec{a}$ } -ve of each other

→ Null vector:

- A vector with 0 magnitude and any arbitrary direction. It's role is to justify vector addition



→ Unit vector:

- A unit vector which has unit magnitude & points in particular direction. It is represented as $\hat{a}$.

$|\hat{a}| = 1$

- Any vector $\vec{a}$ can be written as product of unit vector in that direction and magnitude of given vector.

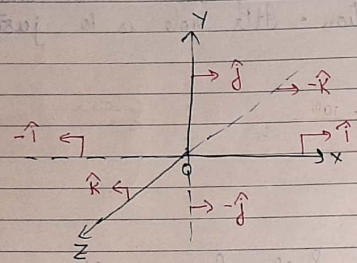
$\vec{a} = |\vec{a}|\hat{a}$ → direction
↓
magnitude

$$\boxed{\hat{a} = \frac{\vec{a}}{|\vec{a}|}}$$

$|\hat{a}| = \left|\frac{\vec{a}}{|\vec{a}|}\right|$

$= \frac{|\vec{a}|}{||\vec{a}||} = \frac{|\vec{a}|}{|\vec{a}|} = 1$

- A unit vector has no dimension and unit

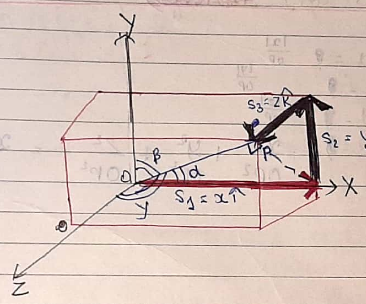


- Unit vector along positive +x, +y & +Z axis of a rectangular coordinate system (axis) is denoted by $\hat{i}, \hat{j}$ & $\hat{k}$ respectively. Similarly along -x, -y & -z is denoted by $-\hat{i}$, $-\hat{j}$ & $-\hat{k}$

$|\hat{i}| = |\hat{j}| = |\hat{k}| = |-\hat{i}| = |-\hat{j}| = |-\hat{k}| = 1$

→ Position vector:

- Position vector is a vector used to represent position of the particle. It is described by line segment joining the position of that particle with origin
- Position vector is fixed vector or localised vector
- For the rest of vector in graphical representation starting point is not fixed. It is arbitrarily chosen according to convenience. ie, free vector.



$\vec{OP}$ = position vector of point P (x, y, z)

$\vec{OP} = S_1 + S_2 + S_3$

$$\boxed{\vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}}$$

$|\vec{OP}| = \sqrt{x^2+y^2+z^2}$

$\hat{OP} = \dfrac{\vec{OP}}{|\vec{OP}|} = \dfrac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2+y^2+z^2}}$

~~Find: $l = \cos\alpha = ???$~~

~~$m = \cos\beta = ???$~~

~~$n = \cos\gamma = ???$~~

~~$l^2+m^2+n^2 =$~~

• If $\vec{OP}$ makes an angle α, β & γ with +ve x-axis, + y-axis & + z-axis respectively then $l = \cos\alpha$, $m = \cos\beta$ & $n = \cos\gamma$ is called directional cosine.

Q Find:

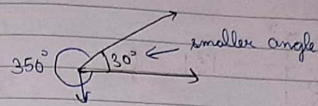
$l = \cos\alpha = \frac{|x|}{OP}$

$m = \cos\beta = \frac{|y|}{OP}$

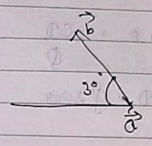
$n = \cos\gamma = \frac{|z|}{OP}$

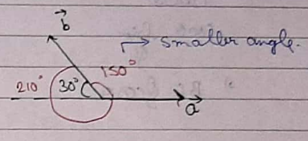
$l^2+m^2+n^2 = \frac{x^2}{OP^2} + \frac{y^2}{OP^2} + \frac{z^2}{OP^2} = \frac{x^2+y^2+z^2}{OP^2} = \frac{OP^2}{OP^2} = 1$

→ Angle between vector :



$\vec{a} \wedge \vec{b}$ = angle between $\vec{a}$ & $\vec{b}$
head
$= 30°$

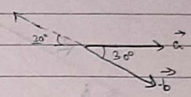


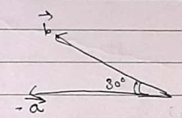


$\vec{a} \wedge \vec{b} = 150°$

$\vec{a} \wedge (-\vec{b}) = 30°$

$-\vec{a} \wedge \vec{b} = 30°$

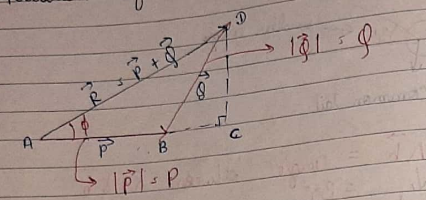




• Angle between two vectors means smaller of the two angle between the vectors when they are placed tail to tail by displacing either of the vectors parallel to itself.

$\boxed{0 \le \theta \le \pi}$

→ Addition of vectors :



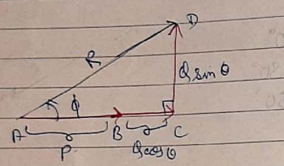
ΔBCD

$\cos\theta = \frac{BC}{Q}$

$\Rightarrow BC = Q\cos\theta$

$\sin\theta = \frac{CD}{Q}$

$\Rightarrow CD = Q\sin\theta$



$R = \sqrt{(P + Q\cos\theta)^2 + (Q\sin\theta)^2}$

$R = \sqrt{P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta}$

$= \sqrt{P^2 + Q^2(\cos^2\theta + \sin^2\theta) + 2PQ\cos\theta}$

$\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$

If resultant, makes an angle α with $\vec{P}$

~~tan~~ $\boxed{\tan\phi = \frac{Q\sin\theta}{P + Q\cos\theta}}$

Q Solve following special cases:
(i) $\theta = 0°$ (ii) $\theta = 90°$ (iii) $\theta = 180°$
(iv) $P = Q$

Ans (i) $\vec{R} = \sqrt{A^2 + B^2 + 2 \cdot A \cdot B \cdot \cos 0}$
$= \sqrt{A^2 + B^2 + 2AB} = (A+B)$

$\tan\phi = \frac{Q \times 0}{P + Q} = 0$

$\phi = 0°$

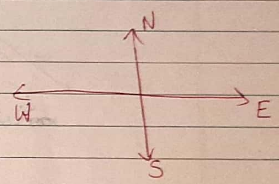
(ii) $\vec{R} = \sqrt{A^2 + B^2 + 2AB\cos 90°} = \sqrt{A^2 + B^2 + 0} = \sqrt{A^2 + B^2}$

(iii) $\vec{R} = \sqrt{A^2 + B^2 + 2AB\cos 180°} = \sqrt{A^2 + B^2 - 2AB} = (A - B)$

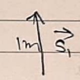
(iv) $\vec{R} = \cancel{\sqrt{P^2 + Q^2 + 2AB\cos\theta}} \cdot \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$= \sqrt{2P^2 + 2P^2\cos\theta} = \sqrt{2}P\sqrt{1 + \cos\theta}$

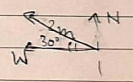
→ Successive displacement



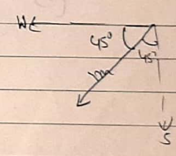
- 1m due north
  OR
  1m towards north

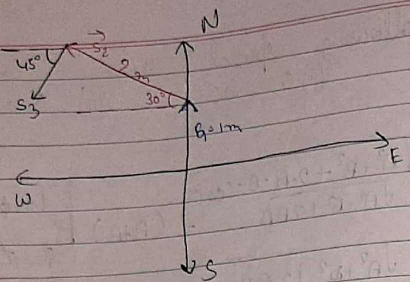


- 2m 30° North of west

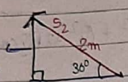


- 1m south west
  OR
  1m 45° south of west
  OR
  1m 45° ~~south~~ west of south

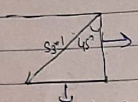




$\vec{S_1} = 1\hat{j}$



$\vec{S_2} = -\sqrt{3}\,\hat{i} + 1\hat{j}$



$\vec{S_3} = -\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$

$S_{net} = \vec{S_1} + \vec{S_2} + \vec{S_3}$

$= -\left(\sqrt{3} + \frac{1}{\sqrt{2}}\right)\hat{i} + \left(2 - \frac{1}{\sqrt{2}}\right)\hat{j}$

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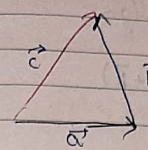
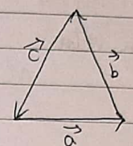


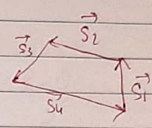
$\vec{C} = \vec{a} + \vec{b}$

#



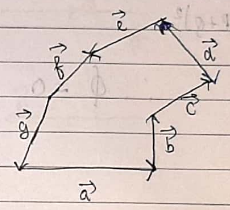
$\vec{a} + \vec{b} + \vec{c} = 0$

#

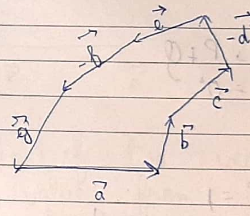


$= \vec{S_1} + \vec{S_2} + \vec{S_3} + \vec{S_4} = 0$

Q



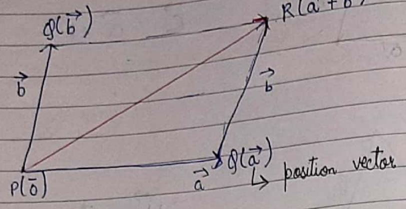
Ans



$\vec{a} + \vec{b} + \vec{c} - \vec{d} + \vec{e} - \vec{f} + \vec{g} = 0$

$\therefore \vec{a} + \vec{b} + \vec{c} + \vec{e} + \vec{g} = \vec{d} + \vec{f}$

Q $\vec{a}$ & $\vec{b}$ are two sides of a llgm



→ Special case:

① $\theta = 0°$

$\sin\theta = 0$; $\cos\theta = 1$.

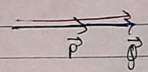
M I:

$R = \sqrt{P^2+Q^2+2PQ} = \sqrt{(P+Q)^2}$

$\tan\phi = \dfrac{Q\times 0}{P+Q} = 0$ $\therefore \phi = 0$

M II:

$\theta = 0°$



$R = P+Q$

$\phi = 0°$

② $\theta = 90°$

$\cos 90° = 0$; $\sin 90° = 1$

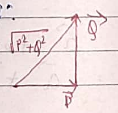
M I:

$R = \sqrt{R^2+P^2+2PQ\cos\theta} = \sqrt{P^2+Q^2}$

$\tan\phi = \dfrac{Q\times 1}{P+Q\times 0} = \dfrac{Q}{P}$

M II:

$|\vec{R}| = \sqrt{P^2+Q^2}$



$\tan\phi = \dfrac{Q}{P}$

③ $\theta = 180°$

$\cos\theta = \cos 180° = -1$. ; $\sin 180° = 0$.

M I:

$R = \sqrt{P^2+Q^2+2PQ(-1)} = \sqrt{(P-Q)^2} = |P-Q|$

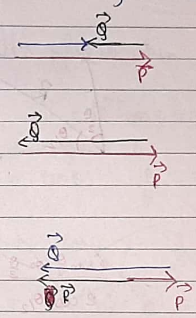
$\tan\phi = \dfrac{Q\times 0}{P+Q(-1)} = 0$

$= \begin{cases} P-Q & \text{if } P>Q \\ 0 & \text{if } P=Q \\ Q-P & \text{if } Q>P \end{cases}$

$\phi = 0$

M II:

$|\vec{R}| = P-Q$



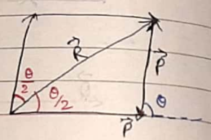
NOTE: Resultant of two vector $(\vec{R})$ can have any value from $|\vec{P}-\vec{Q}|$ to $P+Q$ depending on the angle between them and the magnitude of resultant decreases from 0 to 180°

Q. 10 m & 6 m displacement can be added to get
(i) 17 m
~~(ii)~~ 16 m
~~(iii)~~ 15.4 m
~~(iv)~~ 14 m
~~(v)~~ 10 m
~~(6)~~ 9 m
~~(7)~~ 7 m
~~(8)~~ 5 m
~~(9)~~ 4 m
(10) 3.49 m.

$|10-6| \le \vec{R} \le 10+6$
$4 \le \vec{R} \le 16.$

If two vectors having same magnitude are added then

$$\vec{R} = \sqrt{P^2+P^2+2P^2\cos\theta}$$
$$= \sqrt{2P^2+2P^2\cos\theta}$$
$$= P\sqrt{2(1+\cos\theta)}$$
$$= P\sqrt{2\times 2\cos^2\frac{\theta}{2}}$$



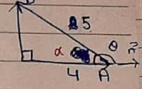
$$\boxed{\vec{R} = 2P\cos\frac{\theta}{2}}$$

$$\tan\phi = \frac{P\sin\theta}{P+P\cos\theta} = \frac{\sin\theta}{1+\cos\theta} = \frac{\cancel{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cancel{2}\cos^{\cancel{2}}\theta/2}$$
$$= \tan\frac{\theta}{2}$$

$$\boxed{\phi = \frac{\theta}{2}}$$

$$\boxed{\tan 35.5^\circ = \frac{5}{7}}$$

Q. Resultant of two vectors ~~$\vec{OA}$ & $\vec{OB}$~~ $\vec{A}$ & $\vec{B}$ is $\perp$ to vector A. Given: $|\vec{A}| = 4$, $|\vec{B}| = 5$. Find θ.



Ans. ~~$\tan\phi = \frac{|\vec{A}|\sin\theta}{|\vec{A}|+|\vec{B}|\cos\theta}$~~

~~$\Rightarrow \tan 90^\circ = \frac{4\sin\theta}{2(2+3\cos\theta)} = \frac{2\sin\theta}{2+3\cos\theta}$~~

$$\tan\phi = \frac{|\vec{B}|\sin\theta}{|\vec{A}|+|\vec{B}|\cos\theta}$$

$$\Rightarrow \tan 90^\circ = \frac{|\vec{B}|\sin\theta}{|\vec{A}|+|\vec{B}|\cos\theta}$$

not defined

It's possible when $D^r = 0$.

$$|\vec{A}| + |\vec{B}|\cos\theta = 0$$
$$\Rightarrow B\cos\theta = -A$$
$$\Rightarrow \cos\theta = \frac{-A}{B} = \frac{-3}{5}$$

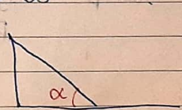
~~$\Rightarrow -\cos\theta = \frac{3}{5}$~~

~~$\Rightarrow \sin(90^\circ + \theta) = \frac{3}{5}$~~

~~$\therefore \theta = 143^\circ$~~

$$\alpha + \theta = 180^\circ$$
$$\alpha = 180^\circ - \theta$$
$$\cos\alpha = \cos(180^\circ - \theta)$$
$$= -\cos\theta$$
$$= -\left(-\frac{3}{5}\right) = \frac{3}{5}$$
$$\alpha = 53^\circ$$

$$\theta = 180^\circ - \alpha$$
$$= 180^\circ - 53^\circ = 127^\circ$$



1) If two vectors of equal magnitude at an angle such that resultant is having same magnitude, find the angle between two vectors

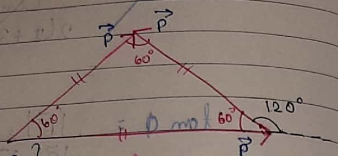
An $\vec{R} = 2P\cos\frac{\theta}{2}$

$\Rightarrow P = 2P\cos\frac{\theta}{2}$

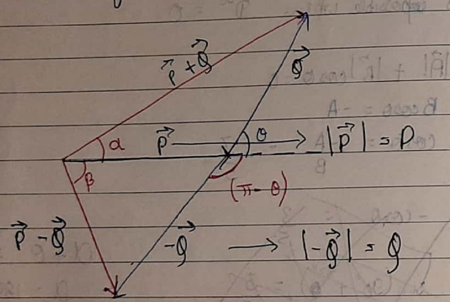
$\Rightarrow \frac{1}{2} = \cos\frac{\theta}{2}$

$\Rightarrow \frac{\theta}{2} = 60^\circ \quad \left\{ \begin{array}{l} 0 \le \theta \le 180^\circ \\ 0 \le \frac{\theta}{2} \le 90^\circ \end{array} \right\}$

$\therefore \theta = 120^\circ$



→ Subtraction of vector :

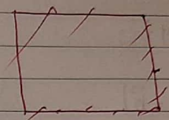


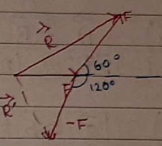
$|\vec{P}-\vec{Q}| = \sqrt{P^2 + Q^2 + 2PQ\cos(\pi-\theta)}$

$\Rightarrow |\vec{P}-\vec{Q}| = \sqrt{P^2 + Q^2 - 2PQ\cos\theta}$

$\tan\beta = \frac{Q\sin(\pi-\theta)}{P + Q\cos(\pi-\theta)} = \frac{Q\sin\theta}{P - Q\cos\theta}$

Q Two forces each of magnitude F are applied on a block at an angle of 60° with each other. If the direction of one of the force is reversed, keeping magnitude same, find the ratio of acceleration in two cases





$\vec{R} = 2F\cos\frac{\theta}{2} = 2F\cos\frac{60^\circ}{2}$

$= 2F\frac{\sqrt{3}}{2} = \sqrt{3}F$

$\cancel{\vec{R} = \sqrt{2F^2 - 2F^2\cos\theta}}$

$\vec{R'} = 2F\cos\frac{120^\circ}{2} = 2F\cos 60^\circ = F.$

$a = \frac{\sqrt{3}F}{m}$

$a' = \frac{F}{m}$

$\frac{a}{a'} = \frac{\sqrt{3}F/m}{F/m} = \frac{\sqrt{3}}{1}$

Q. If $\vec{A}$ & $\vec{B}$ are non-zero vector, find the angle between $\vec{A}$ & $\vec{B}$ such that,

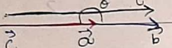
(i) $\vec{A} + \vec{B} = \vec{C}$
& $|\vec{A}| + |\vec{B}| = |\vec{C}|$

(ii) $\vec{A} + \vec{B} = \vec{C}$
& $|\vec{A}|^2 + |\vec{B}|^2 = |\vec{C}|^2$

(iii) $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

(iv) $\vec{a} + \vec{b} = \vec{a} - \vec{b}$

Ans (i) $\theta = 0°$



M2: $\vec{a} + \vec{b} = \vec{C}$

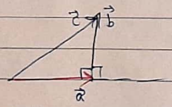
$\Rightarrow |\vec{a} + \vec{b}| = |\vec{C}| \Rightarrow |\vec{a} + \vec{b}|^2 = |\vec{C}|^2$

$\Rightarrow (\vec{a} + \vec{b})(\vec{a} + \vec{b}) = c^2 \Rightarrow a^2 + 2\vec{a}\cdot\vec{b} + b^2 = c^2$ $(a+b)^2$

$\Rightarrow \cancel{a^2} + 2\vec{a}\cdot\vec{b} + \cancel{b^2} = \cancel{a^2} + \cancel{b^2} + 2ab$

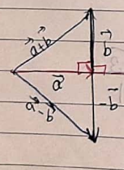
$\Rightarrow \cos\theta = 1 \quad \therefore \theta = 0°$

(ii) M1:



$\theta = 90°$

(iii) $\theta = \frac{\pi}{2}$



$|\vec{a} + \vec{b}| = |\vec{a} - b|$

$\Rightarrow \sqrt{a^2 + b^2 + 2ab\cos\theta} = \sqrt{a^2 + b^2 - 2ab\cos\theta}$

$\Rightarrow \frac{AB\cos\theta}{4} = 0$

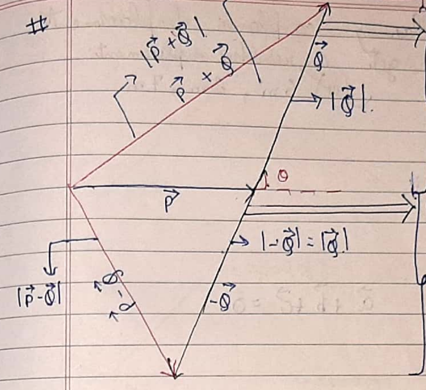
$\therefore \cos\theta = 0 \quad$ Thus, $\theta = 90°$

(iv) $\vec{a} + \vec{b} = \vec{a} - \vec{b}$

$2\vec{b} = 0 \Rightarrow \vec{b} = 0$

But $\vec{b}$ is non zero,
Thus it's not possible.

$$\boxed{||\vec{P}| - |\vec{Q}|| = |\vec{P} - \vec{Q}|}$$



In a Δ, sum of two sides is greater than 3rd side

$|\vec{P}| + |\vec{Q}| > |\vec{P} + \vec{Q}|$ — (i)

From (i) & (ii)

$|\vec{P}| + |\vec{Q}| \geq |\vec{P} + \vec{Q}|$

equality hold when $\theta = 0°$, i.e collinear

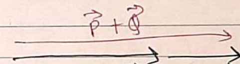
In a Δ difference of 2 sides less than 3rd side.

$||\vec{P}| - |\vec{Q}|| < |\vec{P} - \vec{Q}|$ — (iii)

From (iii) & (iv)

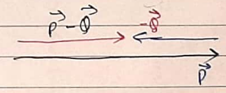
$||\vec{P}| - |\vec{Q}|| \leq |\vec{P} - \vec{Q}|$.

$\theta = 0°$



$|\vec{P}| + |\vec{Q}| = |\vec{P} + \vec{Q}|$ — (ii)

$\theta = \pi$



$||\vec{P}| - |\vec{Q}|| = |\vec{P} - \vec{Q}|$ — (iv)

Q Which of the following co-planer displacement can be added to get 0 resultant displacement.

(i) 12 m, 2 m, 8 m

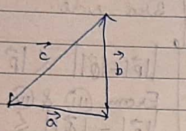
(ii) 20 m, 30 m, 5 m

✓(iii) 2 m, 1 m, 3 m

✓(iv) 6 m, 3 m, 7 m

Ans: Possibilities.

(i)



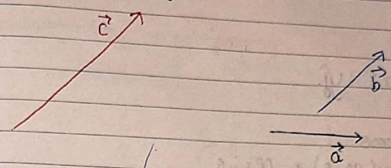
$\vec{a} + \vec{b} + \vec{c} = 0$

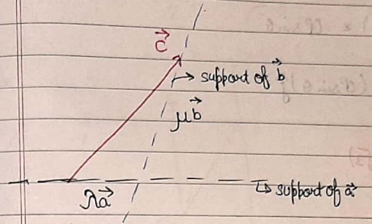
(ii)

→ Minimum no. of coplaner, unequal vector whose sum can be 0 is 3.

This is possible when sum of two smaller is either greater or equal to third one.

→ Resolution of vector :



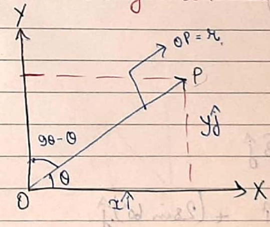


λ & μ are scalar (real no.)

$$\boxed{\vec{C} = \mu\vec{b} + \lambda\vec{a}}$$

↳ C is resolved in terms of $\vec{a}$ & $\vec{b}$

• Resolution ~~along~~ in two dimension :



M I : (r, θ)

MI:

P(x, y)

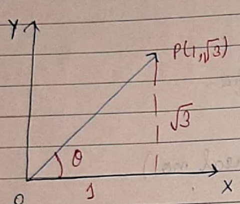
MII: $\vec{OP} = x\hat{i} + y\hat{j}$

MIV: $x = OP\cos\theta$

$y = OP\cos(90-\theta) = OP\sin\theta$

$\vec{OP} = (OP\cos\theta)\hat{i} + (OP\sin\theta)\hat{j}$

Q



Ans→1 MI:

$OP = \sqrt{x^2 + y^2} = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$

$\tan\theta = \sqrt{3}$ $\boxed{\theta = 60^\circ}$

$P(r, \theta) = P\left(2, \frac{\pi}{3}\right)$

MII: P(1,3)

MII: $\vec{OP} = 1\hat{i} + \sqrt{3}\hat{j}$

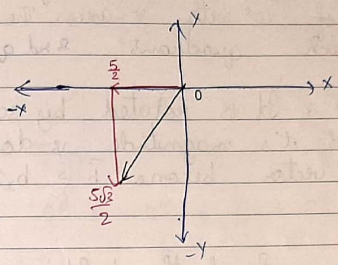
MII: $\vec{OP} = (2\cos 60^\circ)\hat{i} + (2\sin 60^\circ)\hat{j}$

Q Find net force in each of the following cases.

(i) ~~$5\hat{i}$ +~~ ~~$\vec{F} = -\frac{5}{2}\hat{i} + 5\sqrt{3}\hat{j}$~~

(i) $\vec{F} = -\frac{5}{2}\hat{i} - \frac{5}{\sqrt{3}}\hat{j}$

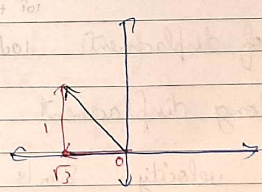
A



$|\vec{F}| = \sqrt{\frac{25}{4} + \frac{75}{4}} = \sqrt{\frac{100}{4}} = \sqrt{25} = 5$

$\tan\theta = \frac{5\sqrt{3}}{2} \times \frac{2}{5} \Rightarrow \theta = 60.$

(ii) $\vec{F} = -\sqrt{3}\hat{i} + \hat{j}$



$|\vec{F}| = \sqrt{1^2 + (\sqrt{3})^2} = 2$

~~tanθ~~ $\tan\theta = \frac{1}{\sqrt{3}}$ $\therefore \theta = 30^\circ$

Q1 A particle starts from origin & moves to 10m along east & then 20m, 60° north of east. Then get back to initial position. Find the displacement of position in 3rd part of motion.

Q2 $\vec{A_1}$ is in 2nd quadrant making an angle of 30° with y-axis. $\vec{A_2}$ is in 4th quadrant making an angle of 30° with x-axis. The resultant cannot be in which quadrant. 3rd quadrant

Q3 $\vec{A} = 5\hat{i} + 5\sqrt{3}\hat{j}$. It is rotated by 30° in clockwise sense & it's magnitude is doubled such that new vector become $\vec{b} = b_x\hat{i} + b_y\hat{j}$. Find b_x & b_y.

Q4 A man on the roof of the building of height 10 m walks 10 m due east & then 10 m due north & then fall vertically on the ground. Set a cartesian coordinate system taking origin on the roof east as +ve x-axis & north as +ve y axis.

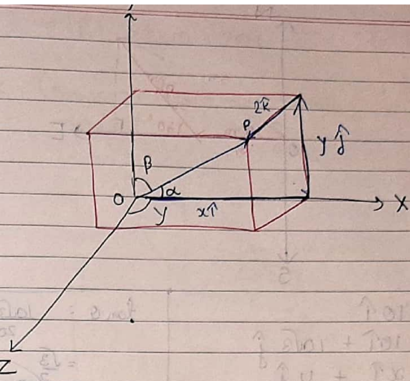
(i) Write the displacement vector for the man taking origin to be it's initial position. ~~10√3~~

$10\hat{i} + 10\hat{j} - 10\hat{k}$

(ii) Magnitude of displacement $10\sqrt{3}$

(iii) unit vector along displacement

(iv) if ~~man~~ bird has velocity 2 m/s along direction of displacement, find velocity vector of bird.



$\vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}$

$|\vec{OP}| = \sqrt{x^2+y^2+z^2}$

$\hat{OP} = \frac{\vec{OP}}{|\vec{OP}|}$

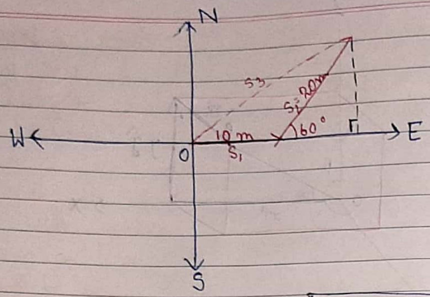
$x = OP\cos\alpha$, $y = OP\cos\beta$, $z = OP\cos\gamma$

NOTE: The resultant of three non-coplaner vector can never be 0.

or Minimum no. of non, co planer vector whose sum can be zero is 4.

Thursday : 8 - 12

Ans→1



$S_1 = 10\hat{i}$

$S_2 = 10\hat{i} + 10\sqrt{3}\,\hat{j}$

$S_3 = x\hat{i} + y\hat{j}$

Aq

$S_1 + S_2 + S_3 = 0$

$10\hat{i} + 10\hat{i} + 10\sqrt{3}\hat{j} + x\hat{i} + y\hat{j} = 0$

$\Rightarrow x\hat{i} + y\hat{j} = -20\hat{i} - 10\sqrt{3}\hat{j}$

$|S_3| = \sqrt{(20)^2 + (10\sqrt{3})^2} = \sqrt{700} = 10\sqrt{7}$ m

$$\boxed{\begin{aligned} \tan\theta &= \frac{10\sqrt{3}}{20} \\ &= \frac{\sqrt{3}}{2} \\ \theta &= \tan^{-1}\left(\frac{\sqrt{3}}{2}\right) \end{aligned}}$$

Ans→3 $\vec{A} = 5\hat{i} + 5\sqrt{3}\hat{j}$

Now,

$\cancel{5\hat{j} = A\sin\theta}$

$A\cos\theta = 5 \Rightarrow A = \frac{5}{\cos\theta}$ —— (i)

$A\sin\theta = 5\sqrt{3} \Rightarrow A = \frac{5\sqrt{3}}{\sin\theta}$ —— (ii)

For (i) = (ii),

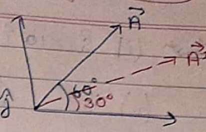
$\frac{5}{\cos\theta} = \frac{5\sqrt{3}}{\sin\theta}$

$\Rightarrow \tan\theta = \sqrt{3}$

$\therefore \theta = 60°$

$A = \frac{5}{\cos 60°} = 10$.

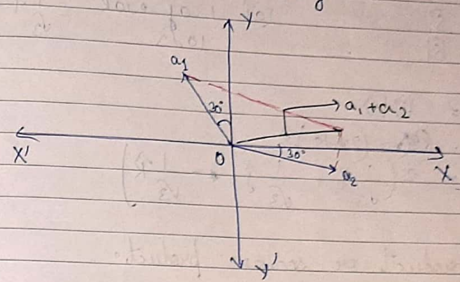
$|\vec{A'}| = 2|\vec{A}| = 2 \times 10 = 20$



$\vec{A'} = (20\cos 30°)\hat{i} + (20\sin 30°)\hat{j}$

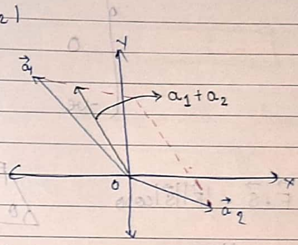
$= 10\sqrt{3}\,\hat{i} + 10\,\hat{j}$

Ans→2



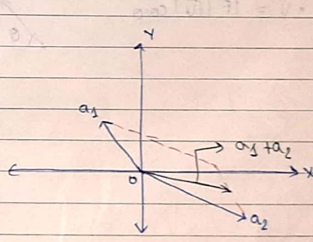
when,
$a_1 = a_2$

When,
$|a_1| > |a_2|$



(contribution of a_1 is more so $\vec{r}$ is ~~inclined~~ close to it.)

when,
$|a_1| < |a_2|$



Ans (i) $\vec{S} = 10\hat{i} + 10\hat{j} - 10\hat{k}$

(ii) $|\vec{S}| = \sqrt{(10)^2 + (10)^2 + (-10)^2} = 10\sqrt{3}$

(iii) $\hat{S} = \frac{\vec{S}}{|\vec{S}|} = \frac{10\hat{i} + 10\hat{j} - 10\hat{k}}{10\sqrt{3}} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$

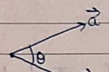
(iv) $\vec{v} = |\vec{v}|\hat{v}$

$= (2m/s)\hat{S}$

$= 2\left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}\right)$

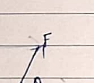
→ Dot product or scalar product.

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

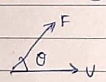


$$\begin{cases} +ve & 0° \le \theta < \frac{\pi}{2}, \cos\theta > 0 \\ 0 & \theta = \frac{\pi}{2}, \cos\theta = 0 \\ -ve & \frac{\pi}{2} < \theta \le \pi, \cos\theta < 0 \end{cases}$$

Work done $= \vec{F} \cdot \vec{S} = |\vec{F}||\vec{S}|\cos\theta$



Power $= \vec{F} \cdot \vec{v} = |\vec{F}||\vec{v}|\cos\theta$



Q Under the application of force of 10N in north direction a block displaces $\sqrt{2}$ m in north east direction. Find work done by force.

Ans $|\vec{F}| = 10$ N , $|\vec{S}| = \sqrt{2}$

$\theta = 45°$

$WD = FS\cos\theta = (10)(\sqrt{2})\cos\left(\frac{\pi}{4}\right) = 10\sqrt{2} \times \frac{1}{\sqrt{2}} = 10$ J

NOTE: If $\vec{a}$ & $\vec{b}$ are non zero vectors & $\vec{a} \cdot \vec{b} = 0$ then the angle between vector $\vec{a}$ & $\vec{b}$ is 90°.

$\left.\begin{aligned} \vec{a} \cdot \vec{b} &= ab\cos\theta \\ \vec{b} \cdot \vec{a} &= ba\cos\theta \end{aligned}\right\}$ $\boxed{\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}}$ (Dot product is commutative)

$\boxed{\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}}$ (dot product is distributive)

$\vec{a} \cdot \vec{b}\big|_{max} = ab\cos\theta\big|_{max} = ab$ when $\cos\theta = 1$, $\theta = 0°$ → like parallel vector.

$\vec{a} \cdot \vec{b}\big|_{min} = ab\cos\theta\big|_{min} = -ab$ when $\cos\theta = -1$, $\theta = 180°$ → unlike or antiparallel vector

$\vec{a} \cdot \vec{a} = |\vec{a}||\vec{a}|\cos 0° = |\vec{a}|^2 = a^2$

Similarly,

$\hat{i} \cdot \hat{i} = |\hat{i}| \cdot |\hat{i}| \cos 0 = 1$

$\hat{j} \cdot \hat{j} = |\hat{j}| \cdot |\hat{j}| \cos 0 = 1$

$\hat{k} \cdot \hat{k} = |\hat{k}| \cdot |\hat{k}| \cos 0 = 1$

$(\vec{a}+\vec{b})^2 = (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = \vec{a}\cdot\vec{a} + \vec{b}\cdot\vec{a} + \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{b}$

or $\cancel{(\vec{a}+\vec{b})^2}\ |a+b|^2$. $= a^2 + 2a\cdot b + b^2$

$= a^2 + b^2 + 2ab\cos\theta$

$|\vec{a}+\vec{b}| = \sqrt{a^2+b^2+2ab\cos\theta}$.

$\Rightarrow |\vec{a}+\vec{b}|_{max} \xrightarrow{\theta = 0^\circ} |\vec{a}| + |\vec{b}|$ i.e., like parallel vector

$(\vec{a}-\vec{b})^2 = (\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b}) = \vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{b}$

or $|a-b|^2$ $= a^2 + b^2 - 2ab\cos\theta$

$|\vec{a}-\vec{b}| = \sqrt{a^2+b^2-2ab\cos\theta}$

$|\vec{a}-\vec{b}|_{min} \xrightarrow{\theta = 0} \big||\vec{a}| - |\vec{b}|\big|$

$\boxed{|a+b+c|^2 = \sqrt{a^2+b^2+c^2+2\vec{a}\cdot\vec{b}+2\vec{b}\cdot\vec{c}+2\vec{c}\cdot\vec{a}}}$

$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

$\hat{i}\cdot\hat{j} = |\hat{i}|\cdot|\hat{j}|\cos 90^\circ = 0 = \hat{j}\cdot\hat{i}$

$\hat{j}\cdot\hat{k} = 0 = \hat{k}\cdot\hat{j}$

$\hat{k}\cdot\hat{i} = 0 = \hat{i}\cdot\hat{k}$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ $\quad |\vec{a}| = \sqrt{a_x^2+a_y^2+a_z^2}$

$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$ $\quad |\vec{b}| = \sqrt{b_x^2+b_y^2+b_z^2}$

$\vec{a}\cdot\vec{b} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k})\cdot(b_x\hat{i} + b_y\hat{j} + b_z\hat{k})$

$= a_x b_x(\hat{i}\cdot\hat{i})^{1} + (a_y b_x)(\hat{i}\cdot\hat{j})^{0} + (a_z b_x)(\hat{i}\cdot\hat{k})^{0} +$

$a_x b_y(\hat{i}\cdot\hat{j})^{0} + (a_y b_y)(\hat{j}\cdot\hat{j})^{1} + (a_z b_y)(\hat{k}\cdot\hat{j})^{0} +$

$a_x b_z(\hat{i}\cdot\hat{k})^{0} + (a_y b_z)(\hat{j}\cdot\hat{k})^{0} + a_z b_z(\hat{k}\cdot\hat{k})^{1}$

$\boxed{\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z}$

$\boxed{\cos\theta = \dfrac{a_x b_x + a_y b_y + a_z b_z}{(\sqrt{a_x^2+a_y^2+a_z^2})(\sqrt{b_x^2+b_y^2+b_z^2})}}$

Q Find angle between vector:

$\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$

$\vec{b} = (-\hat{i}) + \hat{j} + 2\hat{k}$.

A $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \dfrac{(1)(-1) + (-1)(1) + 3\times 2}{(\sqrt{1^2+(-1)^2+3^2})(\sqrt{(-1)^2+(1)^2+2^2})}$

$= \dfrac{-2+6}{(\sqrt{11})(\sqrt{6})} = \dfrac{4}{\sqrt{66}}$

$\theta = \cos^{-1} = \dfrac{4}{\sqrt{66}}$

Q Momentum vector of a particle in x-y plane is given by $\vec{P} = 2\sin(\pi t)\hat{i} + 2\cos(\pi t)\hat{j}$. Find angle between $\vec{P}$ & $\vec{F}$.

Ans $F = \dfrac{d\vec{P}}{dt} = 2\pi\cos\pi t - 2\pi\sin\pi t$

$= 2\pi(\cos\pi t - \sin\pi t)$

$\vec{F} = 2\pi\cos\pi t\,\hat{i} - 2\pi\sin\pi t\,\hat{j}$

$\cos\theta = \dfrac{4\pi\cos\pi t\sin\pi t - 4\pi\cos\pi t\sin\pi t}{|\vec{F}||\vec{P}|}$

$= 0$

$\theta = 90^\circ$

Q A particle is moving with velocity $u = 3\hat{i} + 4\hat{j}$. After time t, final velocity become, $3\hat{i} + (4-t)\hat{j}$. Find time t when velocity of particle is $\perp$ to initial velocity.

An $\cos 90° = \dfrac{(3\times3)+[4\times(4-t)]}{|\vec{u}|\cdot|\vec{v}|}$

$\Rightarrow 0 \times [|\vec{u}|\cdot|\vec{v}|] = 9 + 16 - 4t$

$\Rightarrow 4t = 25$

$\therefore t = 6.25s$

Q $\vec{A} = 3\hat{i} + 4\hat{j}$ is perpendicular to, → broken into components so we do not need $\cos\theta$ directly multiply.

(a) $2\hat{i} + 3\hat{j}$ — $a\cdot b = 3\times2 + 4\times2 = 6+12 \neq 0$

(b) $4\hat{i} - 3\hat{j} + 8\hat{k}$ — $a\cdot b = 3\times4 + (4\times-3) + 0\times8 = 0$

(c) $4\hat{i} - 3\hat{j}$ — $a\cdot b = 4\times3 + 4\times(-3) = 0.$

(d) $8\hat{k}$ — $a\cdot b = 3\times0 + 4\times0 + 0\times8 = 0$

Q If $|\vec{A}| = 5$ unit & $|\vec{B}| = 2$ unit. Angle between these two vector is $45°$. Find:

(i) $(\vec{a}+\vec{b})(\vec{a}-\vec{b})$

(ii) $|a+b|^2$ (iii) $|a+b|^2 + |a-b|^2$

An $\cos 45° = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{A}||\vec{B}|}$ $\Rightarrow \dfrac{1}{\sqrt{2}} = \dfrac{\vec{a}\cdot\vec{b}}{10}$

$\therefore \vec{a}\cdot\vec{b} = 5\sqrt{2}$

(ii) $(\vec{a}+\vec{b}) = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2\cdot\vec{A}\cdot\vec{B}} = \sqrt{25+4+5\sqrt{2}}$

$\therefore (\vec{a}+\vec{b})^2 = |29 + 10\sqrt{2}|$

$|\vec{a}-\vec{b}|^2 = |29 - 10\sqrt{2}|$

~~(i) $(\vec{a}+\vec{b})(\vec{a}-\vec{b}) = (29)^2 - (5\sqrt{2})^2 = 841 - 50 = 791$~~

~~(ii) $|a+b|^2 + |a-b|^2 =$~~

(i) $(\vec{a}+\vec{b})(\vec{a}-\vec{b}) = a^2 - b^2 = 5^2 - 2^2 = 21$

(iii) $|a+b|^2 + |a-b|^2 = 2\times29 = 58.$

(ii) $(\vec{a}+\vec{b})^2 = 29 + 10\sqrt{2}$

Q $\vec{a}\cdot\vec{b} = \vec{c}\cdot\vec{b}$ $\vec{a} \neq \vec{c}$ & a, b, c → non-zero vector

Ans $\vec{a}\cdot\vec{b} = \vec{c}\cdot\vec{b}$ $\Rightarrow \vec{a}\cdot\vec{b} - \vec{c}\cdot\vec{b} = 0.$

$(\vec{a}-\vec{c})\cdot\vec{b} = 0$

$\vec{a}-\vec{c} \perp \vec{b}$

→ Application of dot product:

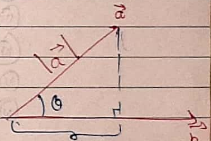
projection of $\vec{a}$ on $\vec{b}$ (परछाई) $= |\vec{a}|\cos\theta$

$= \cancel{|\vec{a}||\vec{b}|\cos\theta}$

$= |\vec{a}|\dfrac{|\vec{b}|}{|\vec{b}|}\cos\theta$

$= \vec{a}\cdot\hat{b}$

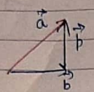
$= \vec{a}\cdot\dfrac{\vec{b}}{|\vec{b}|} = (\vec{a}\cdot\vec{b})\cdot\dfrac{1}{|\vec{b}|}$ ↳ scalar



component of $\vec{a}$ on $\vec{b}$ $= (|\vec{a}|\cos\theta)\hat{b} = (\vec{a}\cdot\hat{b})\hat{b}$

$= (\vec{a}\cdot\vec{b})\dfrac{\vec{b}}{|\vec{b}|^2}$

component of $\vec{a}$ perpendicular to $\vec{b} = \vec{p}$



$$= \vec{a} - (\vec{a}\cdot\vec{b})\frac{\vec{b}}{|\vec{b}|^2}$$

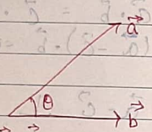
projection of $\vec{b}$ on $\vec{a}$ $= |\vec{b}|\cos\theta = (\hat{a}\cdot\vec{b})$

$$= (\vec{a}\cdot\vec{b})\frac{1}{|\vec{a}|}$$

Component of $\vec{b}$ on $\vec{a}$ $= |\vec{b}|\cos\theta\,\hat{a}$

$$= (\hat{a}\cdot\vec{b})\hat{a} = (\vec{a}\cdot\vec{b})\frac{\vec{a}}{|\vec{a}|^2}$$

→ <u>Cross product or vector product:</u>



$$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$$
$$= |\vec{a}\times\vec{b}|\,\hat{n}$$

$\hat{n}$ is a unit vector in direction of $\vec{a}\times\vec{b}$.

$\hat{n}$ is
(i) perpendicular ($\perp^r$) to $\vec{a}$
(ii) $\perp^r$ to $\vec{b}$
(iii) $\perp^r$ to plane containing $\vec{a}$ & $\vec{b}$
(iv) $\perp^r$ to $\lambda\vec{a}$
(v) $\perp^r$ to $\mu\vec{b}$
(vi) $\perp^r$ to $\lambda\vec{a} + \mu\vec{b}$

→ Place thumb of right hand in direction of 1st vector & fingers in direction of 2nd vector. then palm gives the direction of 1st × 2nd (cross product).

$\vec{a}$	$\vec{b}$	$\hat{n} \parallel \vec{a}\times\vec{b}$	$\vec{b}\times\vec{a}$
N	E	$\otimes$	$\odot$
$\otimes$	E	S	N
N-E	$\odot$	S-E	NW
30° north of west	15° west of south	$\odot$	$\otimes$
N	30° south of east	$\otimes$	$\odot$

$\boxed{\vec{a}\times\vec{b} = -(\vec{b}\times\vec{a})}$ (cross product is not commutative)

unit vector $\perp^r$ to plane containing $\vec{a}$ & $\vec{b}$ & in in the direction of $\vec{a}\times\vec{b}$ $= \boxed{\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}}$

unit vector $\perp^r$ to plane containing $\vec{a}$ & $\vec{b}$ $= \boxed{\pm\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}}$

$|\vec{a}\times\vec{b}| = |\vec{b}\times\vec{a}| = ab\sin\theta$

$\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b} + \vec{a}\times\vec{c}$

$(\vec{b}+\vec{c})\times\vec{a} = \vec{b}\times\vec{a} + \vec{c}\times\vec{a}$

Vector product is distributive when the order of vector is strictly maintained.

Special case: (a & b are non-zero)

(1) $\theta = 0^\circ$ or 180° $\sin\theta = 0$

$\boxed{\vec{a}\times\vec{b} = 0} \longrightarrow \boxed{\vec{b} = \lambda\vec{a}}$

$|\vec{a}\times\vec{b}| = 0 \rightarrow$ minimum

(2) $\vec{a}\times\vec{a} = |\vec{a}||\vec{a}|\sin 0^\circ\,\hat{n} = \vec{0}$

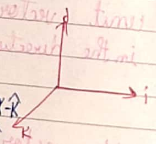
$\hat{i}\times\hat{i} = 0$
$\hat{j}\times\hat{j} = 0$
$\hat{k}\times\hat{k} = 0$

(3) $\theta = 90^\circ$ $\sin 90^\circ = 1$

$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin 90^\circ\,\hat{n} = |\vec{a}||\vec{b}|\,\hat{n}$

$\boxed{|\vec{a}\times\vec{b}|_{max} = ab}$

$\hat{i}\times\hat{j} = |\hat{i}||\hat{j}|\sin 90^\circ\,\hat{k} = \hat{k}$ $\hat{j}\times\hat{i} = -\hat{k}$
$\hat{j}\times\hat{k} = |\hat{j}||\hat{k}|\sin 90^\circ\,\hat{i} = \hat{i}$ $\hat{k}\times\hat{j} = -\hat{i}$
$\hat{k}\times\hat{i} = \hat{j}$ $\hat{i}\times\hat{k} = -\hat{j}$



#
$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$
$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$

$(\vec{a}\times\vec{b}) = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k})\times(b_x\hat{i} + b_y\hat{j} + b_z\hat{k})$

$= a_xb_x\cancel{(\hat{i}\times\hat{i})}^{0} + a_yb_x\cancel{(\hat{j}\times\hat{i})}^{-\hat{k}}$
$+ a_xb_y\cancel{(\hat{i}\times\hat{j})}^{\hat{k}} + a_yb_y\cancel{(\hat{j}\times\hat{j})}^{0} + a_zb_x\cancel{(\hat{k}\times\hat{i})}^{\hat{j}}$
$+ a_xb_z\cancel{(\hat{i}\times\hat{k})}^{-\hat{j}} + a_yb_z\cancel{(\hat{j}\times\hat{k})}^{\hat{i}} + a_zb_y\cancel{(\hat{k}\times\hat{j})}^{-\hat{i}} + a_zb_z\cancel{(\hat{k}\times\hat{k})}^{0}$

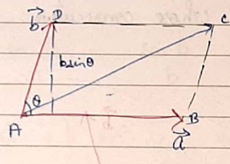
$$\boxed{\vec{a}\times\vec{b} = (a_yb_z - a_zb_y)\hat{i} - (a_xb_z - a_zb_x)\hat{j} + (a_xb_y - a_yb_x)\hat{k}}$$

$$= \hat{i}\begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \hat{j}\begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \hat{k}\begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}$$

$$\boxed{\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}}$$

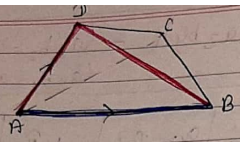
→ Geometrical application of cross product:

$\vec{a}\times\vec{b}$ represent area of ||gm whose two consecu[tive] sides are $\vec{a}$ & $\vec{b}$.



ar of ||gm ABCD = $a \times b\sin\theta = ab\sin\theta = |\vec{a}\times\vec{b}|$

ar of $\Delta ABC = \frac{1}{2}ab\sin\theta = \frac{1}{2}|\vec{a}\times\vec{b}|$

Q 

$\vec{AD} + \vec{DB} = \vec{AB}$

$\vec{DB} = \vec{AB} - \vec{AD}$

Ans ar. of quad. ABCD = ar(ΔABC) + ar(ΔACD)

$= \left| \frac{1}{2}(\vec{AB} \times \vec{AC}) + \frac{1}{2}(\vec{AC} \times \vec{AD}) \right|$

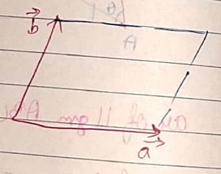
$= \left| \frac{1}{2}(\vec{AB} \times \vec{AC}) - \frac{1}{2}(\vec{AD} \times \vec{AC}) \right|$

$= \frac{1}{2} \left| (\vec{AB} - \vec{AB}) \times \vec{AC} \right|$

$= \frac{1}{2} |\vec{DB} \times \vec{AC}| = \frac{1}{2} |d_1 \times d_2|$

Q Find area of ||gm whose consecutive sides are
$\vec{a} = \hat{i} - 2\hat{j}$
$\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$

A ar(||gm) = $\vec{a} \times \vec{b}$



$= \begin{vmatrix} i & j & k \\ 1 & -2 & 0 \\ 2 & -1 & 2 \end{vmatrix}$

$= \hat{i}(-4-1) - \hat{j}(2-0) + \hat{k}(-1+4)$

$= -4\hat{i} - 2\hat{j} + 3\hat{k}$

Magnitude = $|\vec{a} \times \vec{b}| = |-4\hat{i} - 2\hat{j} + 3\hat{k}| = \sqrt{(-4)^2 + (-2)^2 + 3^2}$

$= \sqrt{29}$ units sq.

Unit vector $\perp^r$ to plane containing a & b

$= \pm \hat{n} = \pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \pm \frac{(-4\hat{i} - 2\hat{j} + 3\hat{k})}{\sqrt{29}}$

Q $\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$

A $\perp^r$ to $\vec{a}$

$\perp^r$ to $\vec{b}$

Q If $\vec{a}, \vec{b}, \vec{c}$ are mutually $\perp$

$(\vec{a} \times \vec{b}) \times \vec{c} = 0$

$\theta = 0°$

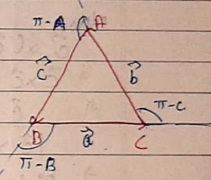
A $\vec{a} \times \vec{b}$ is $\perp$ to plane containing $\vec{a}$ & $\vec{b}$

ie, parallel to $\vec{c}$

$\boxed{(\hat{i} \times \hat{j}) \times \hat{k} = 0}$

Q Prove sine law:

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



Ans $\vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow \vec{a} + \vec{b} = -\vec{c}$

cross with $\vec{a}$ $\times \vec{b}$ $\times \vec{c}$

$\mapsto a \times (\vec{a} + \vec{b}) = \vec{a} \times (-\vec{c})$

$\Rightarrow \vec{a} \times \vec{a} + \vec{a} \times \vec{b} = \vec{c} \times a$

$\Rightarrow ab \sin C = ac \sin B$

$\Rightarrow \frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{\sin B}{b} = \frac{\sin C}{c}$

$b \times (\vec{a} + \vec{b}) = \vec{a} \times (\vec{c})$ $c \times (\vec{a}$

$\Rightarrow \vec{b} \times \vec{a} + 0 = \vec{a} \times \vec{c}$

Q. Prove cosine law & projection law.

Q. Prove that angle in semicircle is 90°.

Ans 1

$\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{a} \times (\vec{a} + \vec{b} + \vec{c}) = 0$

$\Rightarrow \cancel{\vec{a} \times \vec{a}}^{\,0} + \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = 0$

$\Rightarrow \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = 0$

$\Rightarrow \vec{a} \times \vec{b} = -\vec{a} \times \vec{c}$

$\Rightarrow |\vec{a} \times \vec{b}| = |-\vec{a} \times \vec{c}|$

$\Rightarrow ab \sin(\pi - C) = ac \sin(\pi - B)$

$\Rightarrow ab \sin C = ac \sin B$

$\Rightarrow \frac{\sin B}{B} = \frac{\sin C}{C}.$

Now,

$\vec{c} \times (\vec{c} + \vec{b} + \vec{a}) = 0$

$\Rightarrow \vec{c} \times \vec{c} + \vec{c} \times \vec{b} + \vec{c} \times \vec{a} = 0$

$\Rightarrow \vec{c} \times \vec{b} = -\vec{c} \times \vec{a}$

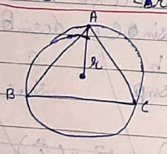
$\Rightarrow |\vec{c} \times \vec{b}| = |-\vec{c} \times \vec{a}|$

$\Rightarrow bc \sin(\pi - A) = ac \sin(\pi - B)$

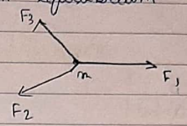
$\Rightarrow bc \sin A = ac \sin B$

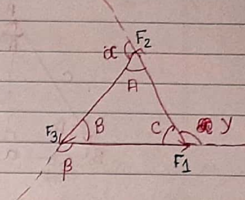
$\Rightarrow \frac{\sin A}{a} = \frac{\sin B}{b}$

Thus, $\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{2r}}$



A body is in equilibrium





$\frac{F_1}{\sin A} = \frac{F_2}{\sin B} = \frac{F_3}{\sin C}$

$\downarrow$

$\sin A = \sin(\pi - \alpha) = \sin \alpha$

$\boxed{\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma}}$ $\Rightarrow$ Lami's theorem.

Ans 2

$\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{b} + \vec{c} = -a$

$\Rightarrow |\vec{b} + \vec{c}| = |-\vec{a}|$

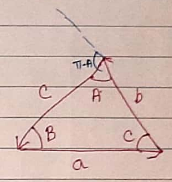
$\Rightarrow |\vec{b} + \vec{c}|^2 = |\vec{a}|^2$

$\Rightarrow b^2 + c^2 + 2\vec{b}\cdot\vec{c} = a^2$

$\Rightarrow b^2 + c^2 + 2bc\cos(\pi - a) = a^2$

$\Rightarrow b^2 + c^2 - 2bc\cos a = a^2$

$\Rightarrow b^2 + c^2 - a = 2bc \cos A$

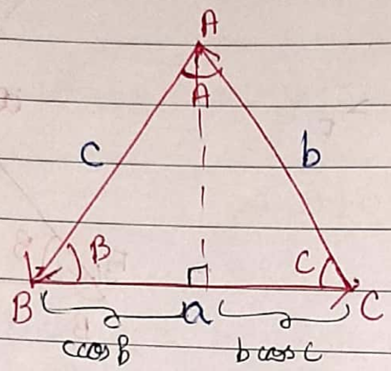


$\therefore \boxed{\cos A = \frac{b^2 + c^2 - a^2}{2bc}}$

$\boxed{\cos B = \frac{a^2 + c^2 - b^2}{2ac}}$

$\boxed{\cos C = \frac{a^2 + b^2 - c^2}{2ab}}$

Projection rule:



$a = c\cos B + b\cos C$

$b = a\cos C + c\cos A$

$c = b\cos A + a\cos B$