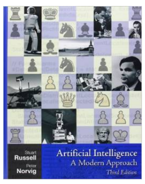
TDT4171 Artificial Intelligence Methods

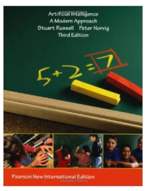
Exercise 2

Feb. 2018

- **Delivery deadline: Feb 28, 2018** by 23:59.
- Required reading for this assignment: Chapter 15, up to and including 15.3
- Deliver your solution on *It's Learning*.
- Please upload your report as a **PDF file**, and pack everything else (such as code) into an archive (zip, rar, gz, ...). Please **do not** put the pdf into the archive, but upload two separate files: pdf + archive. If you use Word, you can convert to pdf on the IDI terminal servers. If you want to use plain ASCII, then you should also convert that to PDF (e.g., by typing in Word and following the procedure just mentioned).
- Students can NOT work in groups. Each student can only submit solution individually.
- This homework counts up to 3% of the final grade.
- The homework is graded on a pass/fail basis. A pass grade will only be given when a decent attempt has been made to solve each question in the exercise.
- Cribbing from other students (*"koking"*) is not accepted, and if detected will lead to the assignment being failed.
- The textbook, *Artificial Intelligence: A Modern Approach, 3rd edition*, exists in two versions:



"Blue version"



"Green version"

References to chapters, figures and pages in the textbook are given as 1.2.3 / 2.3.4, for the blue and the green version respectively, whenever they differ.

Exercise

There is only one exercise in this assignment. You are going to implement the Forward-Backward algorithm for Hidden Markov (HMM) models, and try it out on the *"Umbrella-world"* described in Figure 15.2 (p. 569 / 579) in the book. Your code is a part of your delivery, so *please put some effort in making your code as readable as possible.* It is completely up to you which programming language you want to use to implement the Forward-Backward algorithm, but it might be a good tip to select a programming language, which offers access to powerful matrix operation (like Matlab for instance). Recall that all required calculations can be performed as matrix operations in a HMM.

An alternative for those who are not comfortable with Matlab is to use Python to solve the programming part of the assignment. The Numpy library provides a set of matrix algebra routines and types that are just as easy to use as those in Matlab. Using a matrix type, multiplications between matrix objects acts as ordinary matrix multiplication, and thus you don't need to reimplement matrix multiplication just to solve this assignment. See `http://docs.scipy.org/doc/numpy/reference/generated/numpy.matrix.html` for help on how to use the Numpy matrix type. Numpy: `http://www.numpy.org`

Java is also an option, although due to lack of operator overloading the syntax for matrix operations is less elegant than in Matlab or Python with Numpy. Among Java libraries that provide matrix operations we recommend Jama (`http://math.nist.gov/javanumerics/jama/`).

The Forward-Backward is described in (Figure 15.4, page 576 / 586) in the book, but the description is maybe a bit superficial. It might be beneficial to consider the lecture notes from Chapter 15, too (downloadable from It's Learning).

Make sure to read the whole exercise before you start coding.

Part A

Describe the *"Umbrella domain"* as an HMM:

- What is the set of unobserved variable(s) for a given time-slice t (denoted $\mathbf{X}_t$ in the book)?
- What is the set of observable variable(s) for a given time-slice t (denoted $\mathbf{E}_t$ in the book)?

- Present the *dynamic model* $\mathbf{P}(\mathbf{X}_t|\mathbf{X}_{t-1})$ and the *observation model* $\mathbf{P}(\mathbf{E}_t|\mathbf{X}_t)$ as matrices.
- Which assumptions are encoded in this model? (*Hint:* Read page 568 / 578). Are the assumptions reasonable for this particular domain?

Part B

Implement filtering using the FORWARD operation (Equation 15.5 and Equation 15.12). Note that this can be done with simple matrix operations in the HMM.

- Verify your implementation by calculating $\mathbf{P}(\mathbf{X}_2|\mathbf{e}_{1:2})$, where $\mathbf{e}_{1:2}$ is the evidence that the Umbrella was used both on day 1 and day 2. The desired result is (confer the slides from the lecture available on It's Learning) is that the probability of rain at day 2 (after the observations) is 0.883.
- Use your program to calculate the probability of rain at day 5 given the sequence of observations $\mathbf{e}_{1:5} = \{\texttt{Umbrella}_1 = \texttt{true}, \texttt{Umbrella}_2 = \texttt{true}, \texttt{Umbrella}_3 = \texttt{false}, \texttt{Umbrella}_4 = \texttt{true}, \texttt{Umbrella}_5 = \texttt{true}\}$. Document your answer by showing all *normalized* forward messages (in the book the un-normalized forward messages are denoted $\mathbf{f}_{1:k}$ for $k = 1, 2, \ldots, 5$).

Part C

Implement smoothing using the FORWARD-BACKWARD algorithm (Figure 15.4, page 576 / 586). Note that also this can be done with simple matrix operations.

- Verify your implementation by calculating $\mathbf{P}(\mathbf{X}_1|\mathbf{e}_{1:2})$ where $\mathbf{e}_{1:2}$ is the evidence that the umbrella was used the first two days (as in Part (b) of the assignment). The desired result is $\mathbf{P}(\mathbf{X}_1|\mathbf{e}_{1:2}) = \langle 0.883, 0.117 \rangle$.
- Use your FORWARD-BACKWARD algorithm to calculate the probability of rain at day 1 given the sequence of observations $\mathbf{e}_{1:5} = \{\texttt{Umbrella}_1 = \texttt{true}, \texttt{Umbrella}_2 = \texttt{true}, \texttt{Umbrella}_3 = \texttt{false}, \texttt{Umbrella}_4 = \texttt{true}, \texttt{Umbrella}_5 = \texttt{true}\}$, i.e., $\mathbf{P}(\mathbf{X}_1|\mathbf{e}_{1:5})$. Document your answer by showing all backward messages ($\mathbf{b}_{k+1:t}$ for $k = 1, 2, \ldots, 5$).

Part D

Make sure your code is readable (add comments where appropriate), and deliver it as part of your solution. Delivery procedure is described in the header of this assignment.