28 Continuous Time Markov chains

State Trans. Matrix $\begin{bmatrix} 0 & 1/3 & 2/3 \\ 1/2 & 0 & 1/2 \\ 3/4 & 1/4 & 0 \end{bmatrix}$

Find long term avg. time in each state, find S.S. probs.

$\pi^* = \begin{bmatrix} .3962264 \\ .2264151 \\ .3773585 \end{bmatrix}$ ≡ is S1: 39.6% S2: 22.6% S3: 37.7%

sec R

Amt of time in each state is not discrete... but a r.v. dist. exponentially

For state 1 $T_1 \equiv$ the time spent in State 1 $T_1 \sim \exp(1/1)\ \lambda_1 = 1$

$E(T_1) = 1$

State 2 $T_2 \equiv$ " " State 2 $T_2 \sim \exp(1/2)\ \lambda_2 = \frac{1}{2}$

$E(T_2) = 2$

State 3 $T_3 \equiv$ " " 3 $T_3 \sim \exp(1/3)\ \lambda_3 = \frac{1}{3}$

$E(T_3) = 3$

In continuous it is prob · rate = $p \cdot \lambda$

rate diagram and rate matrix

$P_i \equiv$ the proportion of time in state $i = P(X_t = i)$

$P_1'(t) = -1/3\ P_1 - 2/3\ P_1 + 1/4\ P_3 + 1/4\ P_2 = -1\ P_1 + 1/4\ P_2 + 1/4\ P_3$

$P_2'(t) = 1/3\ P_1 - 1/2\ P_2 + 1/12\ P_3$

$P_3'(t) = 2/3\ P_1 + 1/4\ P_2 - 1/3\ P_3$

$$\begin{bmatrix} P_1' \\ P_2' \\ P_3' \end{bmatrix} = \begin{bmatrix} -1 & 1/4 & 1/4 \\ 1/3 & -1/2 & 1/12 \\ 2/3 & 1/4 & -1/3 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

equilibrium all P_i' are 0 so looking for nullspace

$\sum_i P_i = 1$

$$\left[\begin{array}{ccc|c} -1 & 1/4 & 1/4 & 0 \\ 1/3 & -1/2 & 1/12 & 0 \\ 2/3 & 1/4 & -1/3 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right] \rightarrow PI^T PI \vec{x} = PI^T \vec{b}$$

PI $\quad$ $\vec{b}$

solve(t(PI) %*% PI, PI^T %*% b)

answer

$$\begin{bmatrix} .2 \\ .2285714 \\ .5714286 \end{bmatrix}$$

state 1
state 2
state 3