26 Discrete Time Markov Chain

$\{X_n : n = 0, 1, 2, 3\}$ is a stochastic process that takes on a finite # of states

- If $X_n = i$, then the process is in state i at time n
- when process is in state i, there is a fixed prob. of transitioning from state i to state j - P_{ij}

$$P(X_{n+1} = j \mid X_n = i, X_{n-1} = i_{n-1}, X_{n-2} = i_{n-2}, \dots, X_0 = i_0)$$

Markov Property

$$= P(X_{n+1} = j \mid X_n = i) = P_{ij}$$

The probability of going from one state to another can be represented by a state transition diagram and matrix.

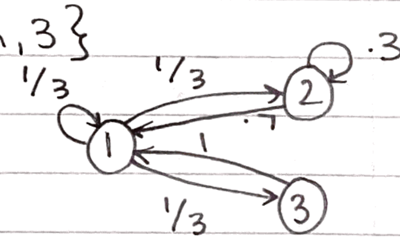
- rows represent the current state
- columns represent the future state

EX: 8.2 state variable $X_n \in \{1, 2, 3\}$

$X_n = 1$ $X_{n+1} = 1, 2, 3$ $1/3$

$X_n = 2$ $X_n = 1$ $.7$ $X_{n+1} = 2$ $.3$

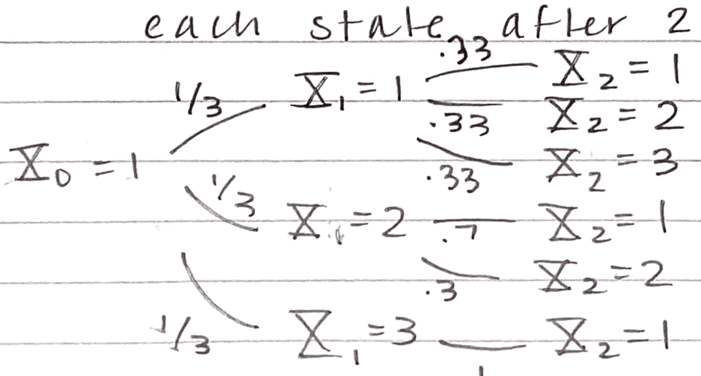
$X_n = 3$: $X_{n+1} = 1$ 1



matrix:

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} .33 & .33 & .33 \\ .7 & .3 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

Suppose $X_0 = 1$ - calculate the prob of being in each state after 2 steps



$P(X_2 = 1) = .6777$

$P(X_2 = 2) = .2111$

$P(X_2 = 3) = .1115$

$\vec{\pi}_2 \equiv$ the probability vector for each state at step 2

$\vec{\pi}_2 = [.6777, .2111, .1115]$

$X_0 = 1$

$\vec{\pi}_0 = [1\ 0\ 0]$

$$\vec{\pi}_1 = [1\ 0\ 0] \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ .7 & 0.3 & 0 \\ 1 & 0 & 0 \end{bmatrix} = [1/3\ 1/3\ 1/3] = \vec{\pi}_1$$

then do again for state 2

SEE R   matrix Power

There is some limiting probability...

- Steady State Probability

$\lim_{n\to\infty} \vec{\pi}_0 P^n = \vec{\pi}_*$ - steady state probability

See R LSN 28