Results of 1 Dimensional Spectral Polynomial Element Solver

Seung-Keol Choe

Sep 21th, 2003

Abstract

This document is for specifying the result of spectral method in 1 dimensional case. For given 3 questions, we present 2 solutions for question 1, and 2. And show test result to setup best order and element that shows the feasibility of high order methods. This is just for showing the result currently working on. Sections have convergence graphs and their explanations.

In section 1, 2, we present the result of convergence in both h refinement and p refinement with the following steady-state Poisson differential equation:

$$\frac{d^2}{dx^2}u(x) = \sin(\pi x),$$

for all x in $[0, 1]$.

1 Result of H-Convergence of 1-D Spectral Method

This test is to validate the relation between size of element and the accuracy of approximation. We apply equidistance element and investigate the movement of error scale. As shown in Figure 1, the smaller are the elements, the more exact is the solution. Moreover by testing with different order of basis, we also could see the fact that the higher are orders, the faster do they converge.

Figure 2 is showing the convergence of error bound when the order of basis goes close to 10. With the higher order than it, the error shows to value small enough of order -15.

Table 1: Specification of Figure1 and their errors

Polynomial order	Color	Err(h=1/32)	Err(h=1/512)
1	Blue	$1.2179e-004$	$1.9073e-006$
4	Red	$1.6362e-011$	$2.4647e-014$
7	Magenta	$7.3275e-015$	$3.6859e-014$

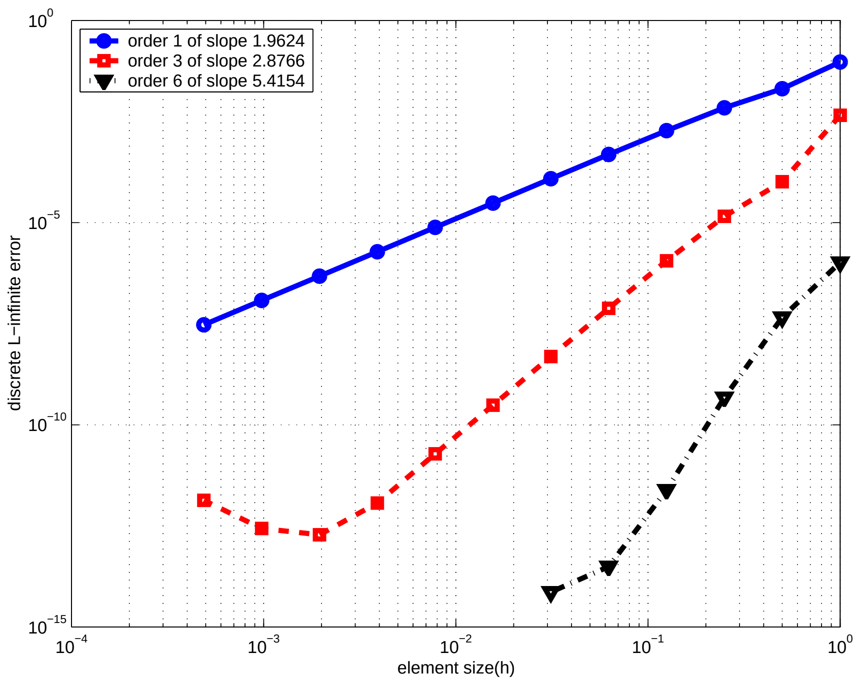

Figure 1: Graph showing change of errors by the size of elements

2 Result of P-Convergence of 1-D Spectral Method

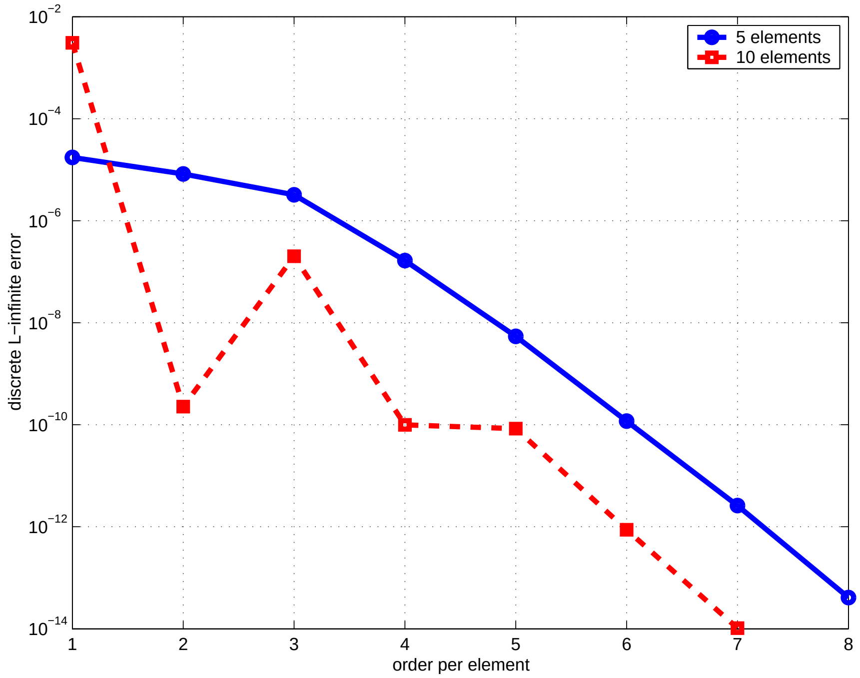

Figure 2: Graph showing change of errors by the increase of order Dirichlet-Neumann

Table 2: Specification of Figure2 and its error

Element Size	Err
0.2	$6.6613e - 016$
0.1	4.9960e-016

Table 3: Specification of Figure3 and its error

Element Size	Err
0.2	$7.0777e - 016$
0.1	$1.1796e - 016$

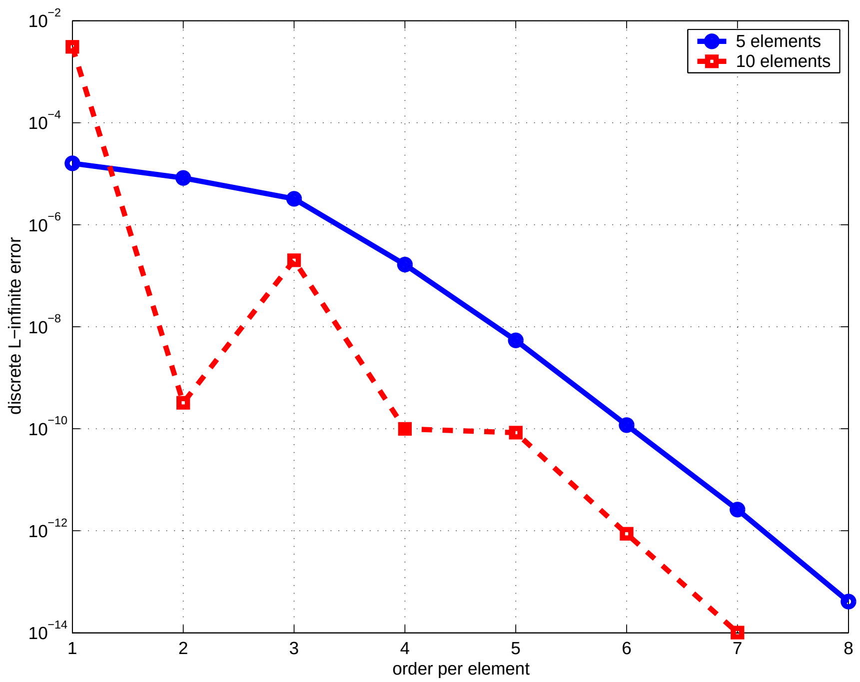

Figure 3: Graph showing change of errors by the increase of order Dirichlet-Dirichlet

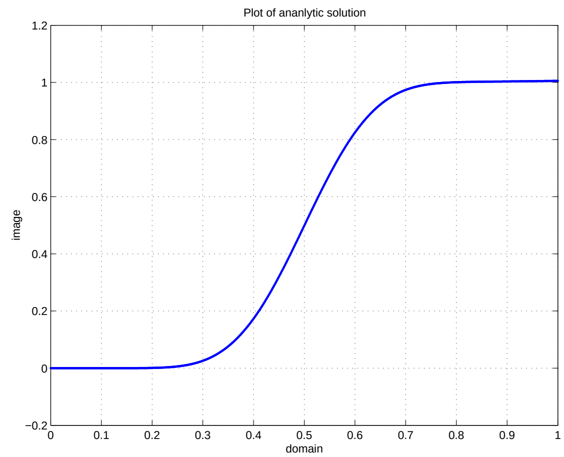

Figure 4: Solution polynomial of order 21

3 Approximation of High order Polynomial solving 1-D Poisson Equation

In this section we construct a polynomial P_n of order n defined on $[0, 1]$, which satisfies the following.

$$P_n(0) = 0, \qquad P_n(1) = 1$$
$$\frac{d^k}{dx^k}P_n(0) = 0, \quad \tfrac{d^k}{dx^k}P_n(1) = 0$$

for all $k = 1, \cdots, n-2$.
Then for each n, we obtain a polynomial P_n by solving a system of linear equations having unique solution which determines the set of coefficients of P_n. We will apply the spectral polynomial solver to approximate the second derivative Q_{n-2} of P_n.

Figure 4, 5, and 6 are showing some samples of solution of order(n) 21, 13, and 13. respectively.

Problem 3.1 *Consider the following differential equation for $u(x)$ such that*

$$\frac{d^2}{dx^2}u(x) = Q_{n-2},$$

for all x in $[0, 1]$. Then the problem is to find approximation $p(x)$ of $u(x)$ using spectral polynomial method.

3.1 Existence of Approximate Solution

The Figure 7 and 8 are showing the result of spectral polynomial method approximating the solution of Problem 3.1. The maximum error value in Table 4 is showing the approximation is within numerically exact solution tolerance.

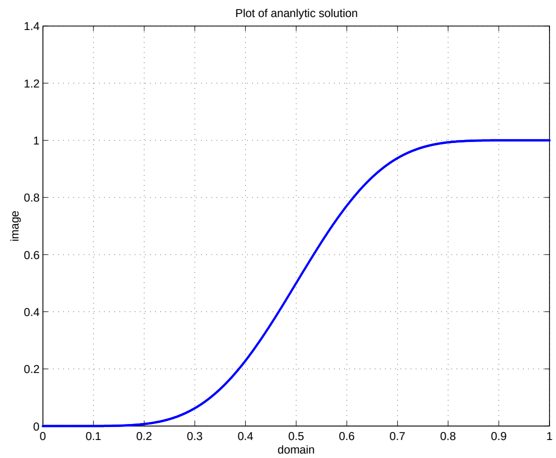

Figure 5: Solution of order 13

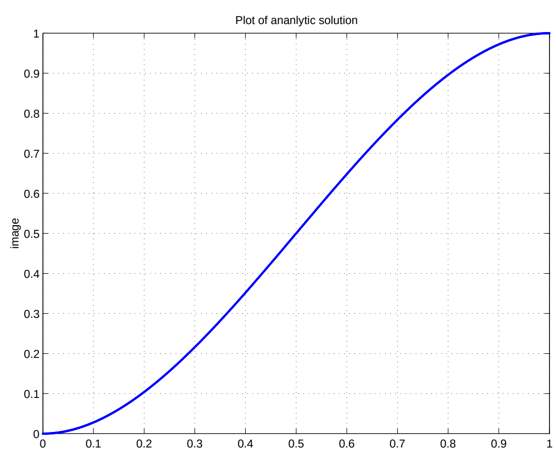

Figure 6: Solution of Of order 3

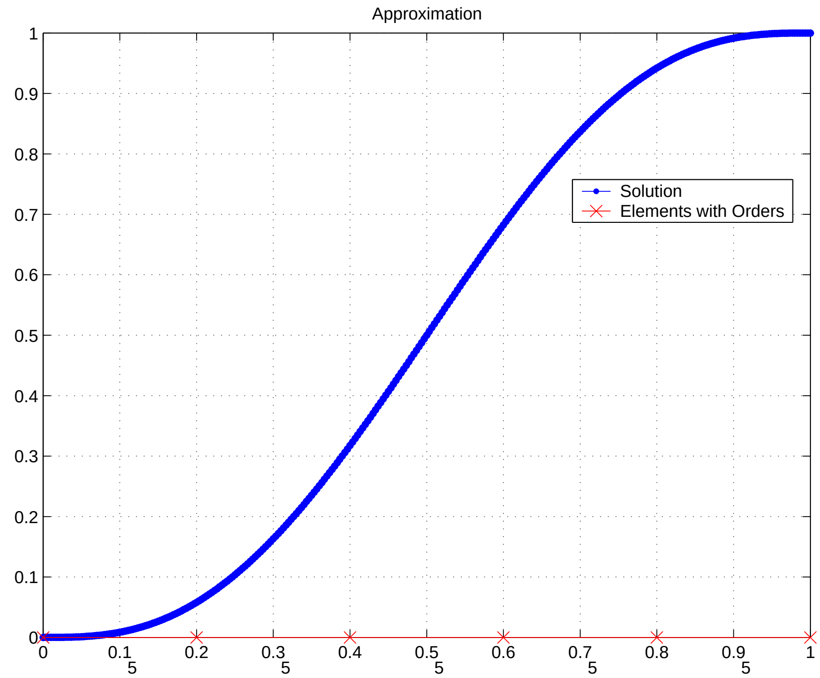

Figure 7: Graph showing spectral approximation satisfying Problem 3.1

Table 4: Specification of Figure 7 and its error

Element Size	Num. of Element	Orders	Err
0.2	5	$5, 5, 5, 5, 5$	$5.7732e-015$

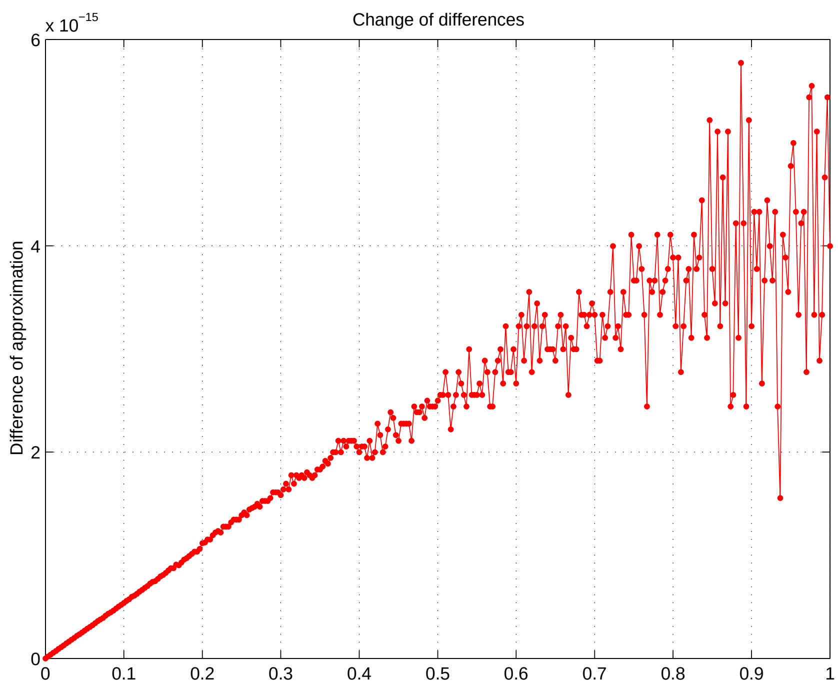

Figure 8: Graph showing the error of approximation in Figure 7

3.2 Convergence of Solution in Equidistance and Uniformly Ordered Elements

In this section we show the convergence of solutions obtained by handling orders of basis on each element. We fix the element to be same size(length) and divide the domain $[0, 1]$ by 5 elements.

Figure 9 is the result of convergence to approximating to a solution of order 5 in Problem 3.1. It shows monotonic decreasing with same similar slope until the order reaches from 1 to 4, and the slope get stiff between order 4 and 5.

Figure 10 is that of solution of order 7. In this case, the error stops to decrease after the order is larger than 7. This part should be considered carefully and need to be made sure the applicable range of the numerical method.

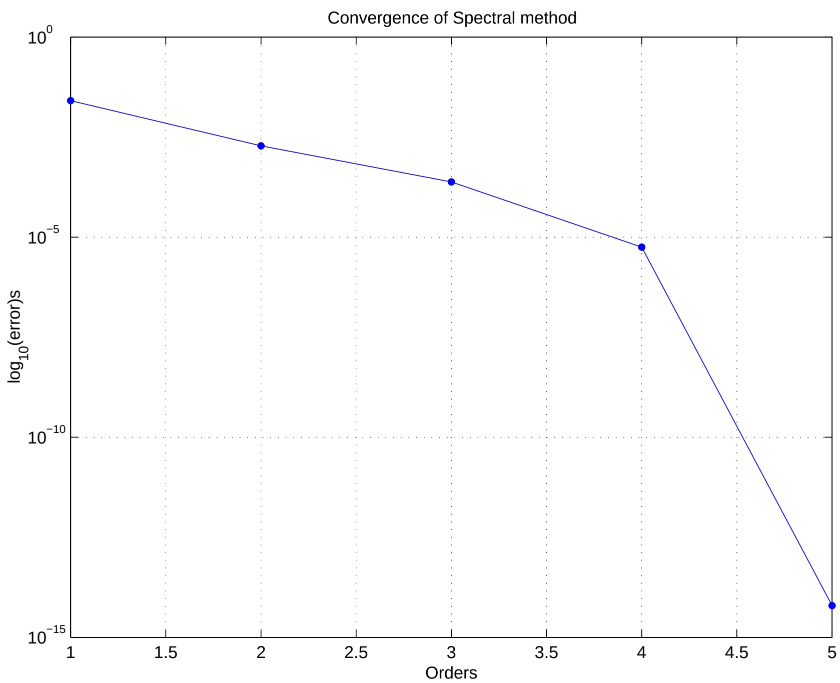

Figure 9: Graph showing convergence of order 5 problem

3.3 Test of Convergence of Solution in Variable Ordered Elements

According to the idea that the solution need to be carefully approximated specially in the center of the curve, we assign different orders by the position of elements. I tested 2 cases. The one is to variate 3 elements in center among 5 elements. The other is to variate 1 element in the exact center of all elements.

Figure 11 is the first case with 3 different orders at the 2 ends of elements. Since it is based on the solution of order 5, the orders in 3 center elements moves from 1 to 5.

Figure 12 is the same as Figure 11 except that it is based on solution of order 7 problem and the orders at the center elements varies from 1 to 7.

Figure 13 and Figure 14 are the same as 11 and 12 except that these have the element that varies only a center element. The 2 different control of orders on each element doesn't give out much different error movement. This means choosing wise orders in each element can save time of computing since the lower is the order, the faster does the system solve.

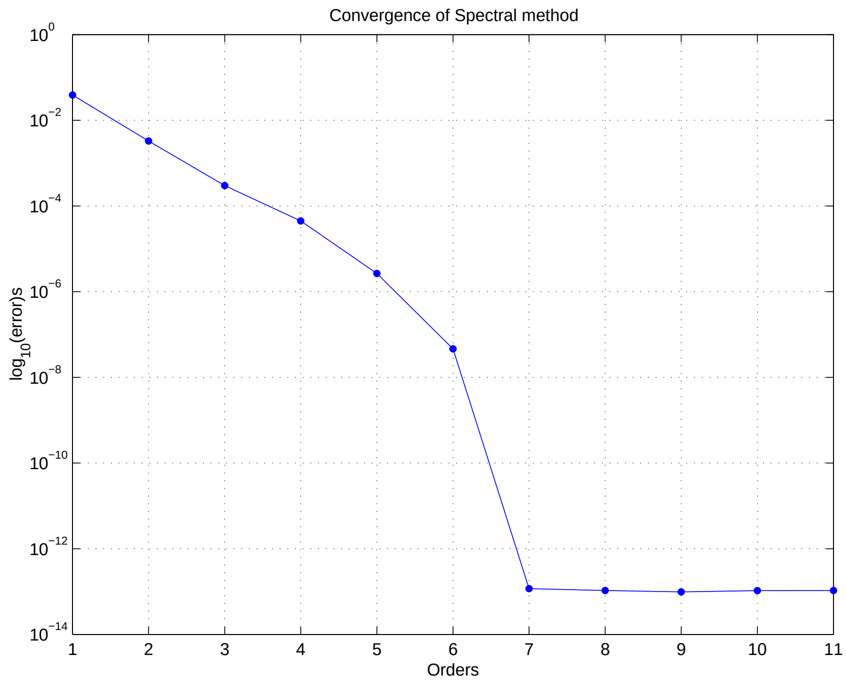

Figure 10: Graph showing convergence of order 7 problem

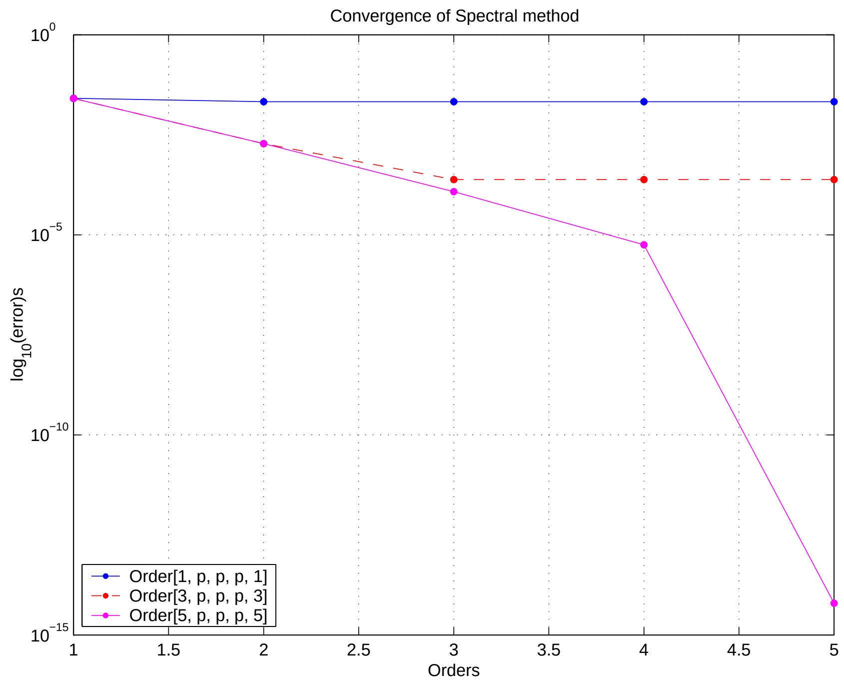

Figure 11: Graph showing convergence of order 5 problem

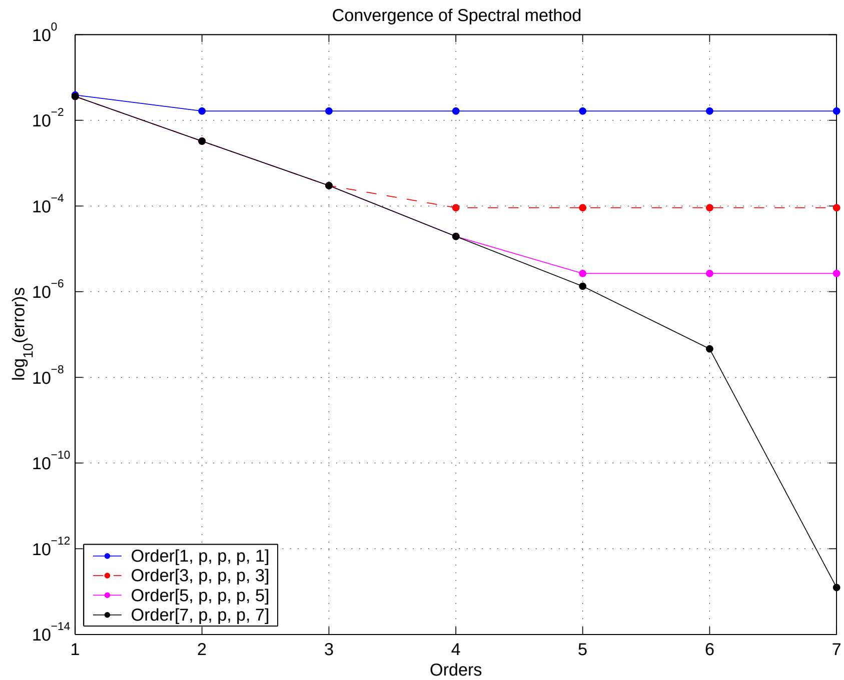

Figure 12: Graph showing convergence of order 7 problem

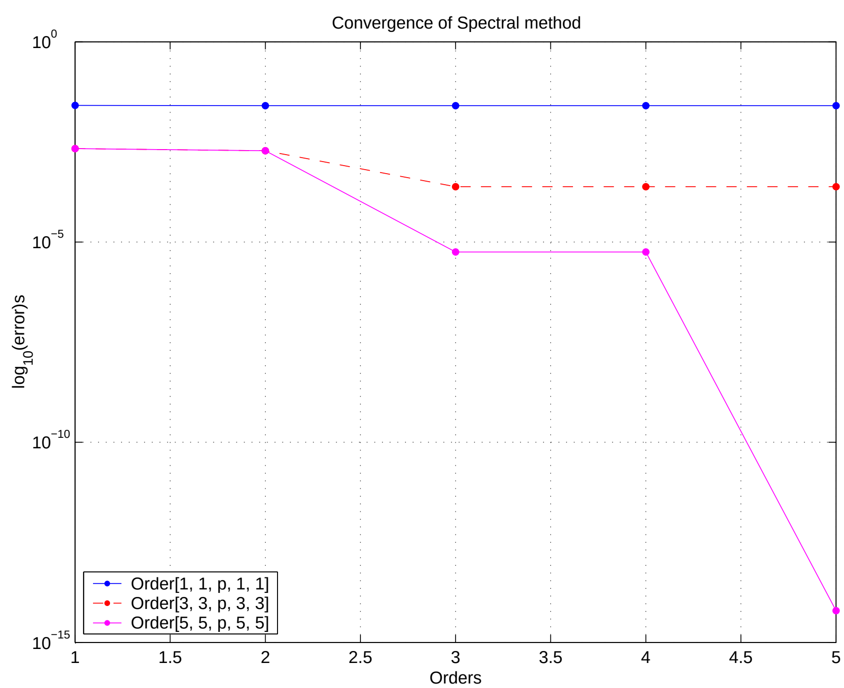

Figure 13: Graph showing convergence of order 5 problem

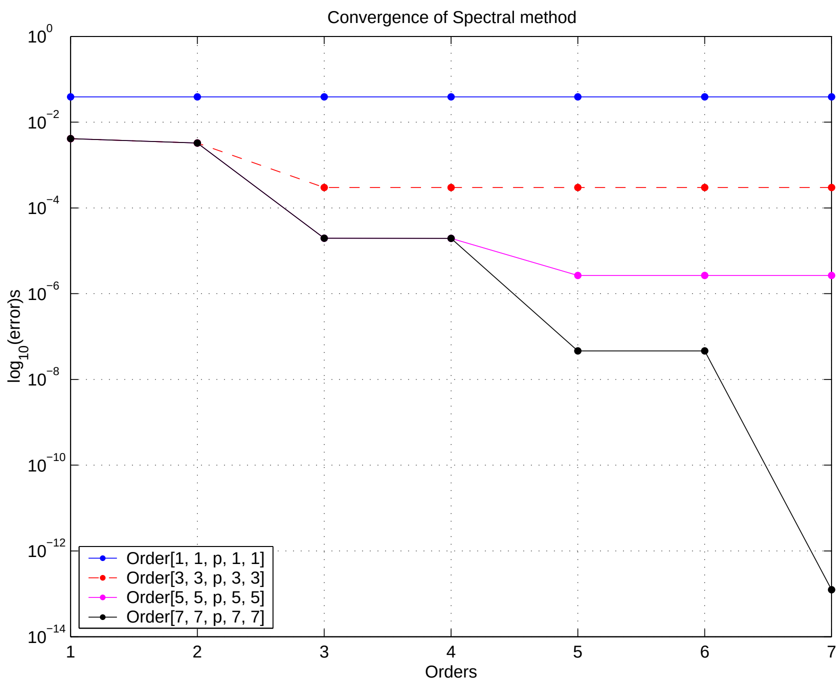

Figure 14: Graph showing convergence of order 7 problem