The general Poisson equation with conductivity is defined as follows:

$$-\nabla \cdot \nu \nabla u = f \tag{1}$$

on $\Omega \in$, where ν is a smooth function defined on Ω.

Between cartesian coordinate and polar coordinate, we have the following relation in differential operator.

$$\begin{aligned}
& \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial x}\rangle \cdot \langle \nu\frac{\partial}{\partial x}, \nu\frac{\partial}{\partial y}\rangle \qquad (2) \\
= \;& \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial x}\rangle \cdot \langle \nu\cos\theta\frac{\partial}{\partial r} + \nu(\frac{\sin\theta}{r})\frac{\partial}{\partial\theta}, \nu\sin\theta\frac{\partial}{\partial r} + \nu\frac{\cos\theta}{r}\frac{\partial}{\partial\theta}\rangle \qquad (3) \\
= \;& \cos\theta\frac{\partial}{\partial r}[\nu\cos\theta\frac{\partial}{\partial r} + \nu(-\frac{\sin\theta}{r})\frac{\partial}{\partial\theta}] - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}[\nu\cos\frac{\partial}{\partial r} + \nu(-\frac{\sin\theta}{r})\frac{\partial}{\partial\theta}] \qquad (4) \\
+ \;& \sin\theta\frac{\partial}{\partial r}[\nu\sin\theta\frac{\partial}{\partial r} + \nu(\frac{\cos\theta}{r})\frac{\partial}{\partial\theta}] + \frac{\cos\theta}{r}\frac{\partial}{\partial\theta}[\nu\sin\frac{\partial}{\partial r} + \nu(\frac{\cos\theta}{r})\frac{\partial}{\partial\theta}] \qquad (5) \\
= \;& \cos\theta[\frac{\partial}{\partial r}\nu\cos\theta\frac{\partial}{\partial r} + \nu\cos\theta\frac{\partial^2}{\partial r^2} + \frac{\partial}{\partial r}\nu(-\frac{\sin\theta}{r})\frac{\partial}{\partial\theta} + \nu(\frac{\sin\theta}{r^2})\frac{\partial}{\partial\theta}\nu(-\frac{\sin\theta}{r})\frac{\partial^2}{\partial r\partial\theta}] \qquad (6) \\
- \;& \frac{\sin\theta}{r}[\frac{\partial}{\partial\theta}\nu\cos\theta\frac{\partial}{\partial r} - \nu\sin\theta\frac{\partial}{\partial r} + \nu\cos\theta\frac{\partial^2}{\partial r\partial\theta} + \frac{\partial}{\partial\theta}\nu(-\frac{\sin\theta}{r})\frac{\partial}{\partial\theta} + \nu(-\frac{\cos\theta}{r})\frac{\partial}{\partial\theta} + \nu(-\frac{\sin\theta}{r})\frac{\partial^2}{\partial\theta^2}] \qquad (7) \\
+ \;& \sin\theta[\frac{\partial}{\partial r}\nu\sin\theta\frac{\partial}{\partial r} + \nu\sin\theta\frac{\partial^2}{\partial r^2} + \frac{\partial}{\partial r}\nu(\frac{\cos\theta}{r})\frac{\partial}{\partial\theta} + \nu(-\frac{\cos\theta}{r^2})\frac{\partial}{\partial\theta}\nu(\frac{\cos\theta}{r})\frac{\partial^2}{\partial r\partial\theta}] \qquad (8) \\
+ \;& \frac{\cos\theta}{r}[\frac{\partial}{\partial\theta}\nu\sin\theta\frac{\partial}{\partial r} + \nu\cos\theta\frac{\partial}{\partial r} + \nu\sin\theta\frac{\partial^2}{\partial r\partial\theta} + \frac{\partial}{\partial\theta}\nu(\frac{\cos\theta}{r})\frac{\partial}{\partial\theta} + \nu(-\frac{\sin\theta}{r})\frac{\partial}{\partial\theta} + \nu(\frac{\cos\theta}{r})\frac{\partial^2}{\partial\theta^2}] \qquad (9) \\
= \;& \frac{\partial}{\partial r}\nu\cos^2\theta\frac{\partial}{\partial r} + \nu\cos^2\theta\frac{\partial^2}{\partial r^2} + \frac{\partial}{\partial r}\nu(-\frac{\sin\theta\cos\theta}{r})\frac{\partial}{\partial\theta} + \nu(\frac{\sin\theta\cos\theta}{r^2})\frac{\partial}{\partial\theta}\nu(-\frac{\sin\theta\cos\theta}{r})\frac{\partial^2}{\partial r\partial\theta} \qquad (10) \\
+ \;& \frac{\partial}{\partial\theta}\nu(-\frac{\sin\theta\cos\theta}{r})\frac{\partial}{\partial r} + \nu\frac{\sin^2\theta}{r}\frac{\partial}{\partial r} + \nu(-\frac{\sin\theta\cos\theta}{r})\frac{\partial^2}{\partial r\partial\theta} \qquad (11) \\
+ \;& \frac{\partial}{\partial\theta}\nu(\frac{\sin^2\theta}{r^2})\frac{\partial}{\partial\theta} + \nu(\frac{\sin\theta\cos\theta}{r^2})\frac{\partial}{\partial\theta} + \nu(\frac{\sin^2\theta}{r^2})\frac{\partial^2}{\partial\theta^2} \qquad (12) \\
+ \;& \frac{\partial}{\partial r}\nu\sin^2\theta\frac{\partial}{\partial r} + \nu\sin^2\theta\frac{\partial^2}{\partial r^2} + \frac{\partial}{\partial r}\nu(\frac{\sin\theta\cos\theta}{r})\frac{\partial}{\partial\theta} + \nu(-\frac{\sin\theta\cos\theta}{r^2})\frac{\partial}{\partial\theta} + \nu(\frac{\sin\theta\cos\theta}{r})\frac{\partial^2}{\partial r\partial\theta} \qquad (13) \\
+ \;& \frac{\partial}{\partial\theta}\nu(\frac{\sin\theta\cos\theta}{r})\frac{\partial}{\partial r} + \nu\frac{\cos^2\theta}{r}\frac{\partial}{\partial r} + \nu\frac{\sin\theta\cos\theta}{r}\frac{\partial^2}{\partial r\partial\theta} \qquad (14) \\
+ \;& \frac{\partial}{\partial\theta}\nu(\frac{\cos^2\theta}{r^2})\frac{\partial}{\partial\theta} + \nu(-\frac{\sin\theta\cos\theta}{r^2})\frac{\partial}{\partial\theta} + \nu(\frac{\cos^2\theta}{r^2})\frac{\partial^2}{\partial\theta^2} \qquad (15) \\
= \;& \frac{\partial}{\partial r}\nu\cos^2\theta\frac{\partial}{\partial r} + \nu\cos^2\theta\frac{\partial^2}{\partial r^2} + \nu\frac{\sin^2\theta}{r}\frac{\partial}{\partial r} + \frac{\partial}{\partial\theta}\nu(\frac{\sin^2\theta}{r^2})\frac{\partial}{\partial\theta} + \nu(\frac{\sin^2\theta}{r^2})\frac{\partial^2}{\partial\theta^2} \qquad (16) \\
+ \;& \frac{\partial}{\partial r}\nu\sin^2\theta\frac{\partial}{\partial r} + \nu\sin^2\theta\frac{\partial^2}{\partial r^2} + \nu\frac{\cos^2\theta}{r}\frac{\partial}{\partial r} + \frac{\partial}{\partial\theta}\nu(\frac{\cos^2\theta}{r^2})\frac{\partial}{\partial\theta} + \nu(\frac{\cos^2\theta}{r^2})\frac{\partial^2}{\partial\theta^2} \qquad (17) \\
= \;& \frac{\partial}{\partial r}\nu\frac{\partial}{\partial r} + \nu\frac{\partial^2}{\partial r^2} + \nu\frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial}{\partial\theta}\nu\frac{1}{r^2}\frac{\partial}{\partial\theta} + \nu\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} \qquad (18) \\
= \;& \frac{\partial}{\partial r}(\nu\frac{\partial}{\partial r}) + \nu\frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial}{\partial\theta}(\nu\frac{1}{r^2}\frac{\partial}{\partial\theta}) \qquad (19) \\
= \;& \frac{\partial}{\partial r}(\nu\frac{\partial}{\partial r}) + \frac{1}{r}(\nu\frac{\partial}{\partial r}) + \frac{1}{r^2}\frac{\partial}{\partial\theta}(\nu\frac{\partial}{\partial\theta}) \qquad (20) \\
& \qquad (21)
\end{aligned}$$

Thus the equation becomes

$$-\frac{\partial}{\partial r}(\nu\frac{\partial}{\partial r}u) - \frac{1}{r}(\nu\frac{\partial}{\partial r}u) - \frac{1}{r^2}\frac{\partial}{\partial\theta}(\nu\frac{\partial}{\partial\theta}u) = f \tag{22}$$