1 Convergence of FE solver and slopes

We examined a superconvergence for finite element approximation using linear/quadratic/cubic basis functions and obtained the following results:

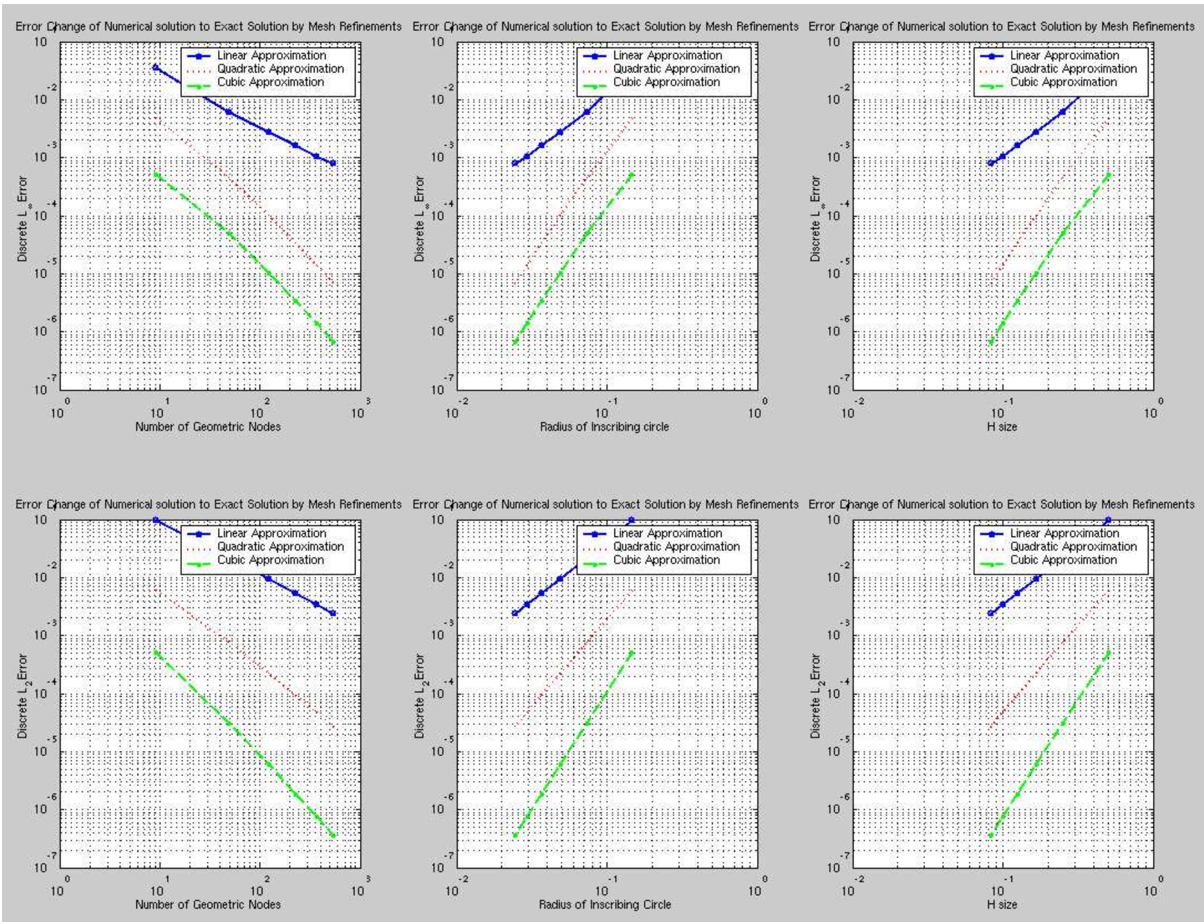

Figure 1: Convergence Test for the Method where N = 4,8,12,16,20, and 24 to the exact solution $u(x,y) = e^{-x^2-y^2}$

The slopes for convergence in terms of 2 kinds of norm are as follows:

Norm	Linear	Quadratic	Cubic
L^∞	1.7717	3.6939	3.9453
L^2	2.0243	3.0282	4.0368

The example solution was

$$u(x,y) = e^{-x^2-y^2}. \tag{1}$$

Conductivity Matrix A was defined as:

$$A(1,1) = \cos^2(\pi r^2) + \sin^2(\pi r^2) \tag{2}$$
$$A(1,2) = A(2,1) = \cos(\pi r^2) + \sin(\pi r^2) + 2 \tag{3}$$
$$A(2,2) = (\sin(\pi r^2) + 2)^2 \tag{4}$$

We subdivide the domain more finely to obtained small size of errors:

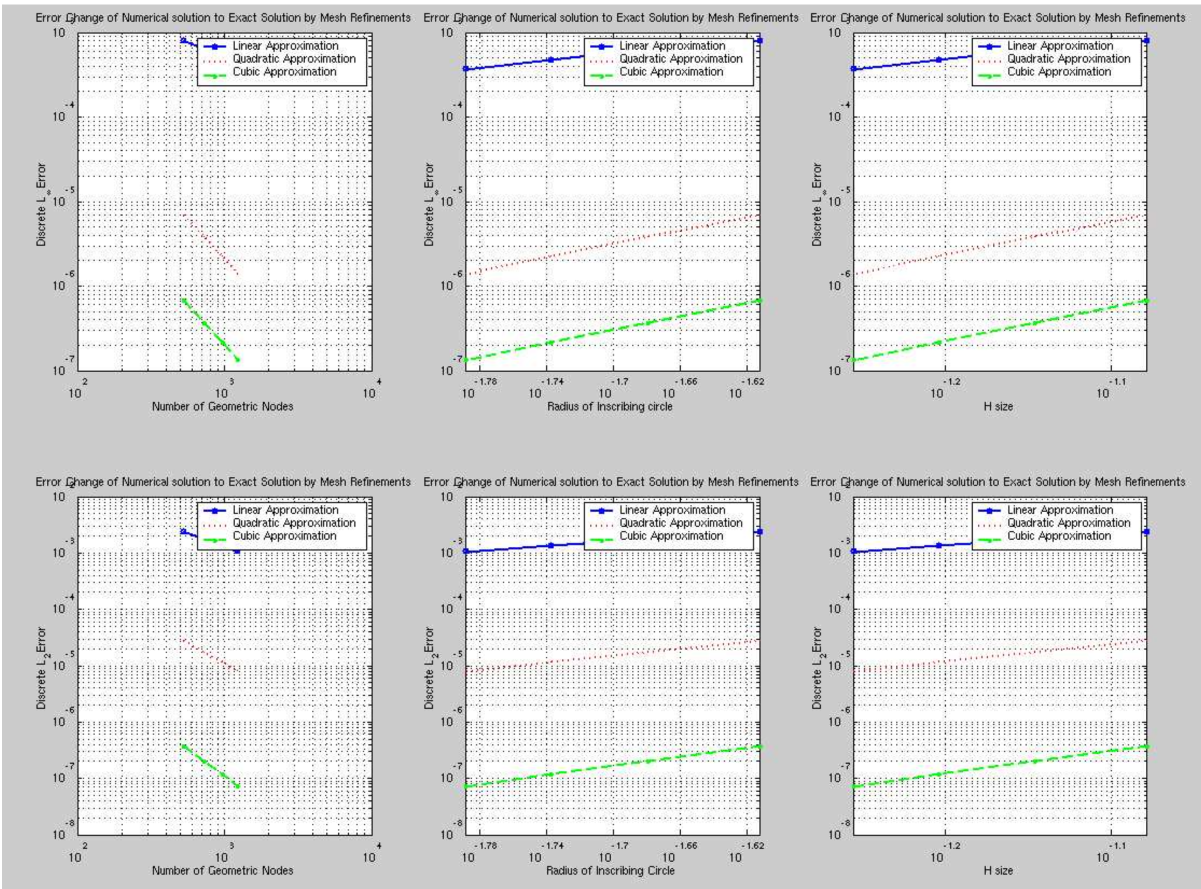

Figure 2: Continued Convergence Test N = 24,28,32,36, and 40 $u(x,y) = e^{-x^2-y^2}$

The following table shows slopes for convergence in terms of 2 kinds of norm:

Norm	Linear	Quadratic	Cubic
L^∞	1.8389	3.9372	3.9843
L^2	1.9992	3.0074	4.0099

This graph shows the result again which combines previous results.

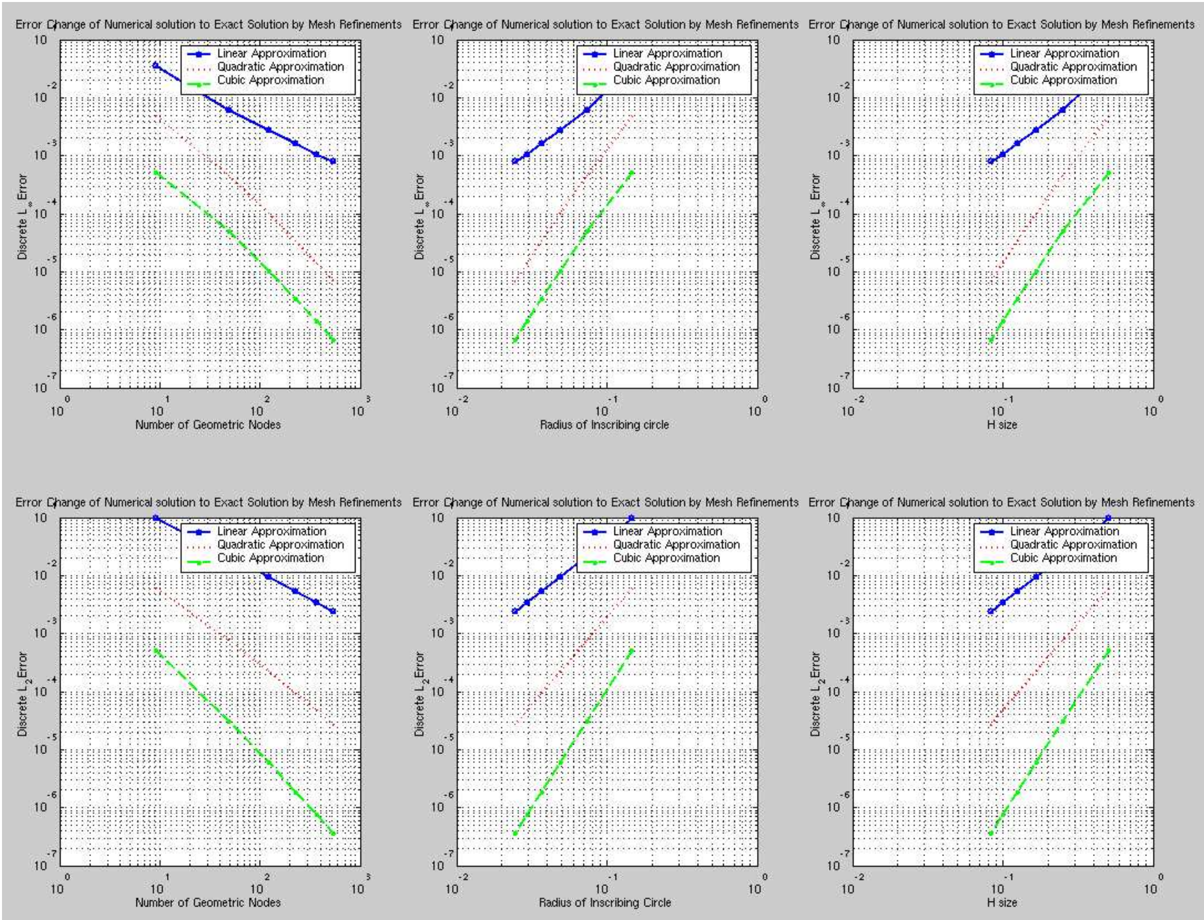

Figure 3: N = 4, 8, 12, 16, 20, 24,28,32 $u(x, y) = e^{-x^2-y^2}$

The following table shows slopes for convergence in terms of 2 kinds of norm:

Norm	Linear	Quadratic	Cubic
L^∞	1.4306	2.7583	4.0664
L^2	1.8903	3.0051	4.1248

All together : total 9 experiments.

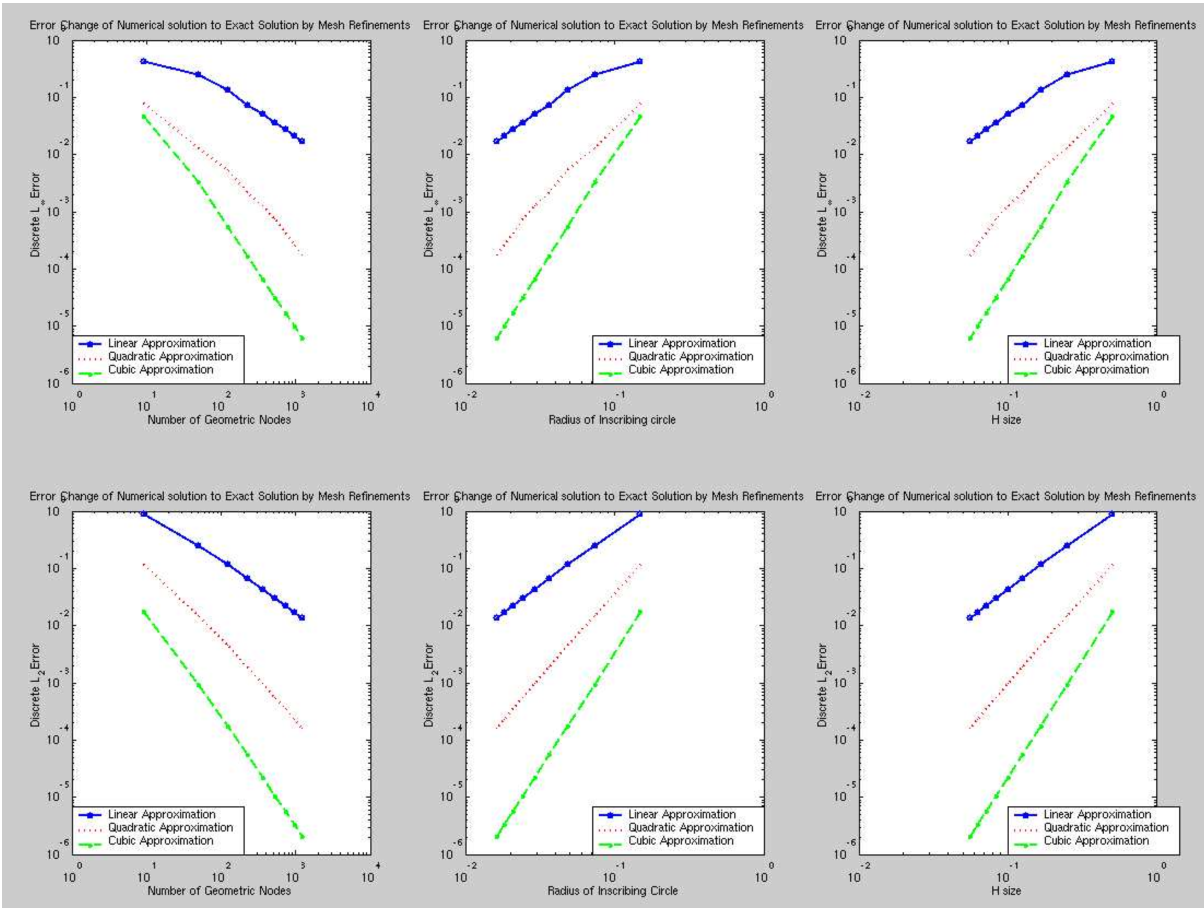

Figure 4: N = 4, 8, 12, 16, 20, 24,28,32, 36 $u(x, y) = e^{-x^2-y^2}$

The following table shows slopes for convergence in terms of 2 kinds of norm:

Norm	Linear	Quadratic	Cubic
L^∞	1.4616	2.7922	4.0640
L^2	1.8950	3.0055	4.1194