

For example: $a+b$

where, '+' operator is written before the operands 'a' & 'b'.

The prefix notation is also called polish notation.

③ Postfix notation:-

The expression where the operation written after the operands is called postfix notation

For example: $ab+$

where '+' operator is written after operands a & b .

The postfix notation is also called reverse polish notation.

Q Convert the following infix notation into postfix notation.

$$= A + [(B+C) + (D+E) * F / G]$$

$$= A + [\underline{BC+} + \underline{DE+} * \underline{FG/}]$$

$$= A + [BC+ + \underline{DE+FG/*}]$$

$$= A + [BC+ + DE+FG/* +]$$

$$= ABC+ + DE+FG/*++ //$$

Alternative way

$$\begin{aligned}
 &= A + [(B+C) + (D+E) * F/G] \\
 &= A + [P + Q * R] \quad P = BC+, \quad Q = DE+, \quad R = FG/ \\
 &= A + [P + S] \quad S = QR* \\
 &= A + T \quad T = PS+ \\
 &\cancel{=} \cancel{AT+} \\
 &= APS++ \\
 &= ABC + QR * ++ \\
 &= ABC + DE + FG/*++ //
 \end{aligned}$$

Operator precedence

$\$, ^ \Rightarrow$ Highest precedence

$/, *$ \Rightarrow medium precedence

$+, - \Rightarrow$ lowest precedence.

$$\begin{aligned}
 &= a * (b+c) - (d/e + f*g) \\
 &\Rightarrow a * \cancel{bc} - \cancel{de} + \cancel{fg}* \\
 &\equiv abc + * \\
 &= a * \cancel{bc} - \cancel{de} + * fg \\
 &= * a + bc - \cancel{de} + * fg \\
 &= - * a + bc \cancel{de} + * fg \\
 &= + - * a + bc / de * fg
 \end{aligned}$$

ii) $A + (B * C - (D / E \$ F) * G) * H$

$$\begin{aligned} &= A + (\underline{BC*} - (\underline{D/E\$F}) * G) * H \\ &= A + (\underline{BC*} - \underline{DEF\$} / G) * H \\ &= A + (\underline{BC*} - \underline{DEF\$} / \underline{G*}) * H \\ &= A + \underline{BC*DEF\$} / \underline{G*} - * H \\ &= A + \underline{BC*DEF\$} / \underline{G*} - H * \\ &= ABC*DEF\$ / G* - H * + \text{(post fix)} \end{aligned}$$

iii) $A + (B * C - (D / E \$ F) * G) * H$

$$\begin{aligned} &= A + (\underline{*BC} - (\underline{D/E\$F}) * G) * H \\ &= A + (\underline{*BC} - \underline{/D\$EF} * G) * H \\ &= A + (\underline{*BC} - \underline{* / D\$EF}) * H \\ &= A + - * BC * / D\$EF * H \\ &= A + * - * BC * / D\$EFH \\ &= + A * - * BC * / D\$EFGH \text{ (prefix)} \end{aligned}$$

Q Convert the following infix notation to postfix notation using stack.

$$\begin{aligned}&= (A+B) * ((C-D)+E)/F \\&= \underline{AB+} * (\underline{CD-} + E) / F \\&= \underline{AB+} * \underline{CD-E+} / F \\&= \underline{AB+} * \underline{CD-E+} / F \\&= AB+CD-E+* / F \\&= AB+CD-E+*F/\end{aligned}$$

Solution:-

Enclosing the given infix expression using parentheses we get as:

$$(A+B) * ((C-D)+E)/F)$$

Converting the given infix notation to postfix using stack step by step we get as follows:

step	scanned symbol	stack	postfix notation
1.	((
2.	(((
3.	A	((A	A
4.	+	((+A	A
5.	B	((+(B	AB
6.)	((+(B)	AB+
7.	*	((+(B)*	AB+
8.	(((+(B)*()	AB+
9.	(((+(B)*()	AB+
10.	C	((+(B)*C	AB+C
11.	-	((+(B)*C(-	AB+C
12.	D	((+(B)*C(-D	AB+C(D
13.)	((+(B)*C(-D)	AB+(D-
14.	+	((+(B)*C(-D)+	AB+CD-
15.	E	((+(B)*C(-D)+E	AB+CD-E
16.)	((+(B)*C(-D)+E)	AB+CD-E+
17.	/	((+(B)*C(-D)+E)/	AB+CD-E+*
18.	F	((+(B)*C(-D)+E)/F	AB+CD-E+*F
19.)	Empty	AB+CD-E+*F/

This is the required postfix notation of given expression.

$(A + (B * C - (D / E) * F) * G) * H)$

Step	Scanned Symbol	Stack	Post fix expression
1.	((
2.	A	(A
3.	+	(+	A
4.	((+(A
5.	B	(+(C	AB
6.	*	(+(C*	AB
7.	C	(+(C*	ABC
8.	-	(+(C(-	ABC*
9.	((+(C(-C	ABC*
10.	D	(+(C(-C	ABC*D
11.	/	(+(C(-C	ABC*D
12.	E	(+(C(-C	ABC*DE
13.)	(+(C(-	ABC*DE
14.	*	(+(C(-*	ABC*DE
15.	G	(+(C(-*	ABC*DE G
16.)	(+	ABC*DE G*-
17.	*	(+ *	ABC*DE G*-
18.	H	(+ *	ABC*DE G*-H
19.)	empty	ABC*DE G*-H*+

$$(A + (B * C - (D / E * F) * G) * H)$$



step	scanned symbol	stack	post fix expression
1.	((
2.	A	(A
3.	+	(+	A
4.	((+(A
5.	B	(+C	AB
6.	*	(+(*	AB
7.	C	(+(*	ABC
8.	-	(+(*-	ABC*
9.	((+(-C	ABC*
10.	D	(+(-C	ABC*D
11.	I	(+(-CI	ABC*D
12.	E	(+(-CI	ABC*DE
13.	\$	(+(-C\$	ABC*DEI
14.	F	(+(-C\$	ABC*DEIF
15.)	(+(-	ABC*DEIF\$
16.	*	(+(-*	ABC*DEIF\$
17.	G	(+(-*	ABC*DEIF\$G
18.)	(+	ABC*DEIF\$G*-
19.	*	(+*	ABC*DEIF\$G*-
20.	H	(+*	ABC*DEIF\$G*-H
21.)	empty	ABC*DEIF\$G*-H*+

Prefix

$$(A+B*C\$D)/(C(E+F-G)*H)\$IJ$$
Solution

Writing the given infix expression in reverse order
& enclosing by parenthesis we get

$$(J/I\$.H*(G-F+E))/(D\$C*B+A)$$

converting this infix notation into postfix notation
using stack step by step we get as;

1.	(C	A	B	*	D	+	H	-	E	*	G	+	F	*	I	.	J	.
2.	J																		
3.	I																		
4.	I																		
5.	\$																		
6.	(
7.	H																		
8.	*																		
9.	*																		
10.	(
11.)																		
12.	G																		
13.	-																		
14.	F																		
15.	+																		

14.	ϵ	$C/\$ C * (+ J I H G F - E$
15.)	$C / \$ C *$ $J I H G F - E +$
16.)	$C / \$$ $J I H G F - E + *$
17.	/	$C /$ $J I H G F - E + * \$ /$
18.	(C / C $J I H G F - E + * \$ /$
19.	D	C / C $J I H G F - E + * \$ / D$
20.	\$	$C / C \$$ $J I H G F - E + * \$ / D$
21.	C	$C / C \$$ $J I H G F - E + * \$ / DC$
22.	*	$C / C *$ $J I H G F - E + * \$ / DC \$$
23.	B	$C / C *$ $J I H G F - E + * \$ / DC \$ B$
24.	+	$C / C +$ $J I H G F - E + * \$ / DC \$ B *$
25.	A	$C / C +$ $J I H G F - E + * \$ / DC \$ B * A$
26.)	$($ $J I H G F - E + * \$ / DC \$ B * A +$
27.	(empty $J I H G F - E + * \$ / DC \$ B * A + /$

reversing the expression to get
prefix expression

$$= / + A * B \$ C D / \$ * + E - F G H I J$$

$$(A+B)* ((C-D)+E)/F$$

$$(F) (E+(D-C)) * (B+A))$$

1. $($ $)$
 2. F C F
 3. I C F
 4. $($ C/C F
 5. E C/C FE
 6. $+$ $C/C +$ FE
 7. $($ $C/C + C$ FE
 8. D $C/C + C$ FED
 9. $-$ $C/C + C -$ FED
 10. C $C/C + C -$ $FEDC$
 11. $)$ $C/C +$ $FEDC -$
 12. $)$ C/I $FEDC - +$
 13. $*$ $C*$ $FEDC - + /$
 14. C $C*C$ $FEDC - + /$
 15. B $C*C$ $FEDC - + / B$
 16. $+$ $C*C +$ $FEDC - + / B$
 17. A $C*C +$ $FEDC - + / BA$
 18. $)$ $C*$ $FEDC - + / BA +$
 19. $)$ empty $FEDC - + / BA + *$

reverse to get prefix.

$$= * + AB / + - CDEF$$

Evaluate the given postfix expression using stack.

③ $623+-3821+*2\$3+$

solution

Evaluating the given postfix expression using stack step by step we get as follows.

Step	Scanned symbol	Operation	Stack
1.	6	push onto stack	6
2.	2.	push onto stack	6 2
3.	3.	push onto stack	6 2 3
4.	+	pop two items from stack & calculate $2+3=5$ push result onto stack	6 5
5.	-	pop two items from stack & calculate $6-5=1$ push result onto stack	1
6.	3	push onto stack	1 3
7.	8	push onto stack	1 3 8
8.	2	push onto stack	1 3 8 2
9.	1	pop two items from stack & calculate $8/2=4$ push result onto stack	1 3 4
10.	+	pop two items from stack & calculate $3+4=7$ push result onto stack	1 7

11.	*	pop two items from stack & calculate $1 * 7 = 7$ push result into stack	7
12.	2	push two onto stack	7 2
13.	\$	pop two items from stack & calculate $7 \$ 2 = 49$ push result into stack	49
14.	3.	push onto stack	49 3
15.	+	pop two items from stack & calculate $99 + 3 = 52$ push result into stack	52

The result of given postfix result is 52.

(Q) Evaluate the given postfix expression using stack

$8\ 2\ 2\ +\ -\ 2\ 8\ *\ \bullet\ +\ 4\ 2\ \$\ 4\ /-$

Step	Scanned symbol	operation	stack
1.	8	push onto stack	8
2.	2	push onto stack	8 2
3.	2	push onto stack	8 2 2
4.	+	pop two items into stack & calculate $2+2=4$ push result onto stack	8 4
5.	-	pop two items into stack & calculate $8-4=4$ push result onto stack	4
6.	2	push onto stack	4 2
7.	8	push onto stack	4 2 8
8.	*	pop two items from stack & calculate $2*8=16$ push result onto stack	4 16
9.	+	pop two items from stack & calculate $4+16=20$ push result onto stack	20
10.	4	push onto stack	20 4
11	2	push onto stack	20 4 2
12.	\$	pop two items from stack & calculate $4\$2=16$ push result onto stack	20 16

13.	4	push onto stack	20	16	4
14.	1	pop two items from stack & calculate $18/4 = 4$ push result onto stack	20	4	
18.	-	pop two items from stack & calculate $20 - 4 = 16$ push result onto stack	16		

The result of postfix is 16.

$A \oplus B \rightarrow$ postfix

$B \oplus A \rightarrow$ prefix

Q) Convert the following postfix expression into equivalent infix expression.

$ABC - + DE - FG I - H + / *$

Converting given postfix operation from equivalent infix expression step by step using stack, we get as;

Step	Scanned symbol	Stack
1.	A	A
2.	B	A B
3.	C	A B C
4.	-	A B - C
5.	+	A + B - C
6.	D	A + B - C D
7.	E	A + B - C D E
8.	-	A + B - C D - E
9.	F	A + B - C D - E F
10.	G	A + B - C D - E F G
11.	-	A + B - C D - E F - G
12.	H	A + B - C D - E F - G H
13.	+	A + B - C D - E F - G + H
14.	/	A + B - C (D - E) / (F - G + H)
15.	*	(A + B - C) * [(D - E) / (F - G + H)]

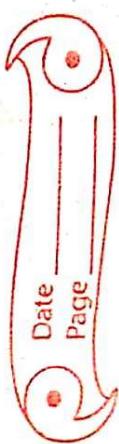
(Q) Convert the following prefix expression into equivalent infix expression.

$* + A - B C / - D E + - F G H$

Writing the given expression into reverse order;

$H G F - + E D - / C B - A + *$

Step	Scanned Symbol	Stack
1.	H	H
2.	G	H G
3.	F	H G - F
4.	-	H F - G
5.	+	F - G + H
6.	E	F - G + H E
7.	D	F - G + H E D
8.	-	F - G + H D - E
9.	/	(D - E) / (F - G + H)
10.	C	(D - E) / (F - G + H) C
11.	B	(D - E) / (F - G + H) C B
12.	-	(D - E) / (F - G + H) B - C
13.	A	(B - C) (D - E) / (F - G + H) B - C A
14.	+	(A + B - C) [(D - E) / (F - G + H)] C A + B - C



$$15 \quad * \quad | \quad ((A+B-C) * [(D-E) / (F-G+H)])$$

Q) Evaluate the prefix expression Using Stack.

- + * 142 / 18 42 * 723

Reversing the given expression.

$327 * 248 / / 24 ^ + -$

Step	Scanned symbol	Operation	Stack
1.	3	push;	3
2.	2	push;	2 3 2 7
3.	7	push;	3 2 7
4.	*	pop, $7 * 2 = 14$, push;	3 14
5.	2	push;	3 14 2
6.	4	push;	3 14 2 4
7.	8	push;	3 14 2 4 8
8.	/	pop, $8 / 4 = 2$, push;	3 14 2 2
9.	/	pop, $2 / 2 = 1$, push;	3 14 1
10.	2	push;	3 14 1 2
11.	4	push;	3 14 1 2 4
12.	\wedge	pop, $4 \wedge 2 = 16$, push;	3 14 1 16
13.	+	pop, $16 + 1 = 17$, push;	3 14 17
14.	-	pop, $17 - 14 = 3$, push;	3 3
15.	-	pop, $3 - 3 = 0$, push;	0