Optimal Control using Physics Informed Neural Networks (PINNs)

Sambhu Karumanchi and Shubham Kedia

Proposal Description

Optimal control theory aims to determine the control signal that will cause a dynamical system to obey its physical constraints and at the same time minimize (or maximize) some performance metric. It has applications in diverse areas like trajectory optimization and tracking, inventory optimization, control of biomedical systems, *etc.* However, in most of the optimal control problems (OCPs) it is difficult to find the analytical solution and it needs to be approximated by numerical methods. Closed-loop solutions of OCPs can be computed by applying the Bellman Theory of Optimality (BPO) [1]. BPO provide the necessary and sufficient conditions,which result in the Hamilton–Jacobi–Bellman (HJB) partial differential equation [1]. The HJB PDE needs to be solved numerically in many practical applications. The HJB equation doesn't scale well with higher dimensional state spaces. Recently, Bellman Neural Networks [2] provides a promising alternative to solve optimal control problems with closed-form solutions and to reduce the computational burden of solving the HJB equation.

BeNNs are trained to learn the solutions of OCPs via the application of the BPO. The authors [2] implemented the BeNNs framework using vanilla PINNs [3] and PINNs combined with Theory of Functional Connections (TFC) [4]. The BeNNs are trained to solve two classes of OCPs with integral quadratic cost: finite horizon problems and infinite horizon problems. The trained network or closed form solution allows deployment and computation of optimal trajectories in real-time.

In this project, we propose to analyze Bellman Neural Networks(BeNNs) and the feasibility of using PINNs-based methods to learn closed-loop optimal control solutions. The optimal control problem we will be targeting is trajectory optimization for linear and non-linear systems. We also propose to compare the PINNs method with analytical techniques for constrained numerical optimization such as shooting method and direct transcription. The application of BeNNs to higher dimensional state spaces has not been studied in the literature so far. We propose to evaluate the scalability of BeNNs and numerical optimization techniques for higher dimensions. Specifically, we want to accomplish the following during the course of the project:

- Evaluate BeNNs framework using vanilla PINNs on low dimension linear and non-linear systems with known closed form solution
- Compare the performance with constrained numerical optimization techniques such as shooting method and direct transcription
- Evaluate BeNNs framework on higher dimension dynamical systems for trajectory optimization.

References

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