The bilinear map B:

```
In[1]:= B[{v1_, w1_, x1_, y1_, z1_}, {v2_, w2_, x2_, y2_, z2_}] :=
    {(v1 + w1 + x1 + y1 + z1) (v2 + w2 + x2 + y2 + z2) - y1 z2 - z1 y2,
     (v1 + w1 + x1 + y1) v2 + v1 (v2 + w2 + x2 + y2), y1 z2 + z1 y2, v1 z2 + z1 v2, 0};
```

The 62 vectors:

```
In[2]:= v[1] = alpha^(-2) {1, 0, 0, 0, 0};
v[2] = alpha^(-6) {2 + (3446) / (Sqrt[R]), 2 + (3170) / (Sqrt[R]), 0, 0, 0};
v[3] = alpha^(-6) {4, 4, 0, 0, 0};
v[4] =
   alpha^(-21) {2016 + (3 380 832) / (Sqrt[R]), 2208 + (3 671 904) / (Sqrt[R]), 0, 0, 0};
v[5] = alpha^(-21) {4032, 4416, 0, 0, 0};
v[6] =
   alpha^(-21) {2016 + (3 367 584) / (Sqrt[R]), 2208 + (3 693 984) / (Sqrt[R]), 0, 0, 0};
v[7] = alpha^(-21) {(5 628 705 696) / R + (3 367 584) / (Sqrt[R]),
     (6 174 294 432) / R + (3 693 984) / (Sqrt[R]), 0, 0, 0};
v[8] = alpha^(-14) {(1 969 470 208) / (9 R) + (3 291 861 361 024) / (9 R^(3 / 2)),
     (2 179 611 976) / (9 R) + (3 643 134 552 760) / (9 R^(3 / 2)), 0, 0, 0};
v[9] = alpha^(-10) {12 + (20 676) / (Sqrt[R]), 14 + (22 466) / (Sqrt[R]), 0, 0, 0};
v[10] = alpha^(-3) {(143 768 593) / (108 R) + (85 015) / (108 Sqrt[R]),
     (105 898 423) / (72 R) + (66 145) / (72 Sqrt[R]), 0, 0, 0};
v[11] = alpha^(-3) {-(1625) / (108) + (2 887 105) / (108 Sqrt[R]),
     (1715) / (72) - (2 737 003) / (72 Sqrt[R]), 0, 0, 0};
v[12] = alpha^(-3) {(108 745) / (108) - (65 Sqrt[R]) / (108),
     (28 441) / (24) - (17 Sqrt[R]) / (24), 0, 0, 0};
v[13] = alpha^(-3) {1 / 2 + (1447) / (2 Sqrt[R]), 1 / 2 + (1999) / (2 Sqrt[R]), 0, 0, 0};
v[14] = alpha^(-14) {(201 818 664) / R + (120 744) / (Sqrt[R]),
     (267 240 144) / R + (159 888) / (Sqrt[R]), 0, 0, 0};
v[15] = alpha^(-14) {72 + (120 744) / (Sqrt[R]), 96 + (159 888) / (Sqrt[R]), 0, 0, 0};
v[16] = alpha^(-14) {144, 192, 0, 0, 0};
v[17] = alpha^(-7)
    {(31) / (18) + (116 617) / (18 Sqrt[R]), (281) / (54) + (208 991) / (54 Sqrt[R]), 0, 0, 0};
v[18] = alpha^(-7) {(108 745) / (18) - (65 Sqrt[R]) / (18),
     (219 163) / (27) - (131 Sqrt[R]) / (27), 0, 0, 0};
v[19] = {(8 142 156 817) / (23 328 R) + (3 615 127) / (23 328 Sqrt[R]),
    (2 689 931 479) / (11 664 R) + (4 131 265) / (11 664 Sqrt[R]), 0, 0, 0};
v[20] = {-(2 104 505) / (23 328) + (3 526 059 745) / (23 328 Sqrt[R]),
    (2 807 237) / (23 328) - (4 680 672 973) / (23 328 Sqrt[R]), 0, 0, 0};
v[21] = {(11 814 523 825) / (23 328) - (7 068 425 Sqrt[R]) / (23 328),
    (1 999 380 955) / (2916) - (1 196 195 Sqrt[R]) / (2916), 0, 0, 0};
v[22] = {(7345) / (432) - (12 119 705) / (432 Sqrt[R]),
    -(6197) / (288) + (10 499 773) / (288 Sqrt[R]), 0, 0, 0};
v[23] = {(2 443 777) / (8 R) + (1447) / (8 Sqrt[R]),
    (3 242 521) / (8 R) + (1999) / (8 Sqrt[R]), 0, 0, 0};
v[24] = {(32 805 161) / (108 R) + (54 881 040 239) / (108 R^(3 / 2)),
    (29 641 217) / (72 R) + (49 498 725 911) / (72 R^(3 / 2)), 0, 0, 0};
v[25] = {(1 237 818 669 070 513) / (1458 R^2) + (740 509 100 311) / (1458 R^(3 / 2)),
    (838 121 265 457 801) / (729 R^2) + (501 401 051 935) / (729 R^(3 / 2)), 0, 0, 0};
v[26] = alpha^(-15) {(71 754 261 114 955 960) / (81 R^2) +
      (119 933 657 317 951 854 472) / (81 R^(5 / 2)), (291 727 305 240 092 752) / (243 R^2) +
```

```
    (487 607 592 266 798 252 080) / (243 R^(5/2)), 0, 0, 0};
v[27] = alpha^(-8) {(140 725 263 328 052 732 114 393) / (1458 R^3) +
     (84 193 524 123 331 244 303) / (1458 R^(5/2)), (286 283 955 528 410 226 595 427) /
      (2187 R^3) + (171 278 806 193 160 289 301) / (2187 R^(5/2)), 0, 0, 0};
v[28] = alpha^(-4) {(10 512 907 255 001 141) / (1944 R^2) +
     (6 289 683 368 723) / (1944 R^(3/2)),
    (10 709 134 162 880 129) / (1458 R^2) + (6 407 069 939 495) / (1458 R^(3/2)), 0, 0, 0};
v[29] = {(794 956 577) / (2592 R) + (464 135) / (2592 Sqrt[R]),
   (524 137 043) / (1296 R) + (325 541) / (1296 Sqrt[R]), 0, 0, 0};
v[30] = alpha^(-15) {(17 072 730 067) / (54 R) + (28 535 661 518 197) / (54 R^(3/2)),
    (3 873 220 609) / (9 R) + (6 473 856 199 063) / (9 R^(3/2)), 0, 0, 0};
v[31] = alpha^(-8) {(803 579 524 335 268 753) / (23 328 R^2) +
     (480 751 987 830 295) / (23 328 R^(3/2)), (547 329 217 239 002 015) / (11 664 R^2) +
     (327 434 508 313 145) / (11 664 R^(3/2)), 0, 0, 0};
v[32] = alpha^(-8) {(293 738 136 445) / (23 328 R) + (470 526 609 087 163) / (23 328 R^(3/2)),
    (197 305 889 945) / (11 664 R) + (325 111 894 094 399) / (11 664 R^(3/2)), 0, 0, 0};
v[33] = alpha^(-8) {(8 133 848 923 273) / (23 328 R) - (4 522 191 905) / (23 328 Sqrt[R]),
    -(7 653 084 333 757) / (11 664 R) + (4 813 128 245) / (11 664 Sqrt[R]), 0, 0, 0};
v[34] = alpha^(-2) {(34 964 309) / (54 R) + (21 179) / (54 Sqrt[R]),
    (29 330 411) / (27 R) + (17 753) / (27 Sqrt[R]), 0, 0, 0};
v[35] = alpha^(-17) {(2 039 517 464) / (3 R) + (3 408 952 424 936) / (3 R^(3/2)),
    (10 267 634 576) / (9 R) + (17 161 853 188 592) / (9 R^(3/2)), 0, 0, 0};
v[36] = alpha^(-13) {38 + (63 818) / (Sqrt[R]), 64 + (106 960) / (Sqrt[R]), 0, 0, 0};
v[37] = alpha^(-6) {(892 769 641) / (216 R) + (538 495) / (216 Sqrt[R]),
    (62 416 754) / (9 R) + (37 790) / (9 Sqrt[R]), 0, 0, 0};
v[38] = alpha^(-6) {-(1355) / (216) + (3 337 411) / (216 Sqrt[R]),
    5/2 + (8339) / (2 Sqrt[R]), 0, 0, 0};
v[39] = alpha^(-4) {1, 2, 0, 0, 0};
v[40] = alpha^(-4) {(1895) / (216) - (2 436 799) / (216 Sqrt[R]),
    0, 0, -(1625) / (108) + (2 887 105) / (108 Sqrt[R]), 0};
v[41] = alpha^(-4) {(605 232 455) / (216 R) + (368 465) / (216 Sqrt[R]),
    0, 0, (143 768 593) / (108 R) + (85 015) / (108 Sqrt[R]), 0};
v[42] = alpha^(-11) {26 + (43 142) / (Sqrt[R]), 0, 0, 12 + (20 676) / (Sqrt[R]), 0};
v[43] = alpha^(-15) {(4 149 082 184) / (9 R) + (6 934 995 913 784) / (9 R^(3/2)),
    0, 0, (1 969 470 208) / (9 R) + (3 291 861 361 024) / (9 R^(3/2)), 0};
v[44] = {(23 696 513) / (54 R) + (14 327) / (54 Sqrt[R]), 0, 0,
   (1 877 966) / (9 R) + (1142) / (9 Sqrt[R]), 0};
v[45] = {1/8 + (2551) / (8 Sqrt[R]), 0, 0, 1/8 + (343) / (8 Sqrt[R]), 0};
v[46] =
  {(18 403) / (54) - (11 Sqrt[R]) / (54), 0, 0, (11 711) / (72) - (7 Sqrt[R]) / (72), 0};
v[47] = {(73) / (216) - (8081) / (216 Sqrt[R]), 0, 0,
   -(7) / (36) + (20 783) / (36 Sqrt[R]), 0};
v[48] = alpha^(-7) {8, 0, 0, 4, 0};
v[49] = alpha^(-7) {4 + (6616) / (Sqrt[R]), 0, 0, 2 + (3446) / (Sqrt[R]), 0};
v[50] = alpha^(-3) {1, 0, 0, 1, 0};
v[51] = alpha^(-5) {(1895) / (216) - (2 436 799) / (216 Sqrt[R]), 0, -(1625) / (108) +
     (2 887 105) / (108 Sqrt[R]), (1895) / (216) - (2 436 799) / (216 Sqrt[R]), 0};
v[52] = alpha^(-5) {(605 232 455) / (216 R) + (368 465) / (216 Sqrt[R]),
    0, (143 768 593) / (108 R) + (85 015) / (108 Sqrt[R]),
    (605 232 455) / (216 R) + (368 465) / (216 Sqrt[R]), 0};
v[53] = alpha^(-12) {26 + (43 142) / (Sqrt[R]), 0,
```

```
    12 + (20676) / (Sqrt[R]), 26 + (43142) / (Sqrt[R]), 0};
v[54] = alpha^(-16) {(4149082184) / (9 R) + (6934995913784) / (9 R^(3/2)),
    0, (1969470208) / (9 R) + (3291861361024) / (9 R^(3/2)),
    (4149082184) / (9 R) + (6934995913784) / (9 R^(3/2)), 0};
v[55] = alpha^(-1) {(23696513) / (54 R) + (14327) / (54 Sqrt[R]), 0, (1877966) / (9 R) +
     (1142) / (9 Sqrt[R]), (23696513) / (54 R) + (14327) / (54 Sqrt[R]), 0};
v[56] = alpha^(-1) {1/8 + (2551) / (8 Sqrt[R]), 0, 1/8 + (343) / (8 Sqrt[R]),
    1/8 + (2551) / (8 Sqrt[R]), 0};
v[57] = alpha^(-1) {(18403) / (54) - (11 Sqrt[R]) / (54), 0,
    (11711) / (72) - (7 Sqrt[R]) / (72), (18403) / (54) - (11 Sqrt[R]) / (54), 0};
v[58] = alpha^(-1) {(73) / (216) - (8081) / (216 Sqrt[R]), 0,
    -(7) / (36) + (20783) / (36 Sqrt[R]), (73) / (216) - (8081) / (216 Sqrt[R]), 0};
v[59] = alpha^(-8) {8, 0, 4, 8, 0};
v[60] = alpha^(-8)
   {4 + (6616) / (Sqrt[R]), 0, 2 + (3446) / (Sqrt[R]), 4 + (6616) / (Sqrt[R]), 0};
v[61] = alpha^(-4) {1, 0, 1, 1, 0};
v[62] = alpha^(-1) {0, 0, 0, 0, 1};
```

Generating all vectors for which we need to check that they lie in the convex hull:

```
In[64]:= ToCheck = Union[Flatten[Simplify[Table[B[v[i], v[j]], {i, 1, 62}, {j, 1, 62}]], 1]];
```

```
In[65]:= alpha = (13384 + 8 Sqrt[R])^(1/22); R = 2793745;
```

The set of vectors we need to check can be divided into four subsets each of which lies in a two-dimensional subspace of R^5:

Only first two coordinates nonzero:

```
In[66]:= ToCheck12 = Select[ToCheck, #[[3]] + #[[4]] + #[[5]] == 0 &];
```

Only first and fourth coordinate nonzero:

```
In[67]:= ToCheck14 = Select[ToCheck, #[[2]] + #[[3]] + #[[5]] == 0 &];
```

Only first, third and fourth coordinate nonzero, first and fourth equal:

```
In[68]:= ToCheck134A = Select[ToCheck, (#[[3]] > 0) && (#[[1]] == #[[4]]) &];
```

Only first, third and fourth coordinate nonzero, third and fourth equal:

```
In[69]:= ToCheck134B = Select[ToCheck, (#[[3]] > 0) && (#[[3]] == #[[4]]) &];
```

The union of these does indeed cover everything:

```
In[70]:= Complement[ToCheck, Union[ToCheck12, ToCheck14, ToCheck134A, ToCheck134B]]

Out[70]= {}
```

For the first of these sets, vectors v_1 to v_39 are sufficient:

```
In[71]:= v12 = Table[v[i], {i, 1, 39}];
```

In two dimensions, the verification for each vector amounts to testing whether it lies below a piecewise linear function, which is not difficult: the vectors v_1 to v_39 are sorted by their first coordinates. (Although it is not strictly necessary for the computational check, we note that the second coordinate is sorted in the opposite order. This is because, by construction, the vectors constitute a minimal set of vectors for the convex space; a vector that is out-of-order on the second coordinate could be generated by the others and would thus violate minimality). For each vector w, determine i such that its first coordinate lies between those of v_i and v_{i+1}. Then check whether w lies below the line segment determined by v_i and v_{i+1}.

In[72]:=
```
Check12[w_] :=
  Module[{i = 1}, If[(w[[1]] > v12[[1, 1]]) || (w[[1]] < v12[[-1, 1]]), Return[False]];
   If[w[[1]] == v12[[-1, 1]], Return[w[[2]] ≤ v12[[-1, 2]]]];
   While[v12[[i + 1, 1]] >= w[[1]], i++];
   (w[[1]] - v12[[i, 1]]) v12[[i + 1, 2]] / (v12[[i + 1, 1]] - v12[[i, 1]]) +
     (v12[[i + 1, 1]] - w[[1]]) v12[[i, 2]] / (v12[[i + 1, 1]] - v12[[i, 1]]) - w[[2]] ≥ 0];
```

The verification for the second set is similar, using vectors v_1 and v_40 to v_50:

In[73]:=
```
v14 = Join[{v[1]}, Table[v[i], {i, 40, 50}]];
```

In[74]:=
```
Check14[w_] :=
  Module[{i = 1}, If[(w[[1]] > v14[[1, 1]]) || (w[[1]] < v14[[-1, 1]]), Return[False]];
   If[w[[1]] == v14[[-1, 1]], Return[w[[4]] ≤ v14[[-1, 4]]]];
   While[v14[[i + 1, 1]] >= w[[1]], i++];
   (w[[1]] - v14[[i, 1]]) v14[[i + 1, 4]] / (v14[[i + 1, 1]] - v14[[i, 1]]) +
     (v14[[i + 1, 1]] - w[[1]]) v14[[i, 4]] / (v14[[i + 1, 1]] - v14[[i, 1]]) - w[[4]] ≥ 0];
```

The same principle also applies to the third set, using vectors v_50 to v_61:

In[75]:=
```
v134 = Table[v[i], {i, 50, 61}];
```

In[76]:=
```
Check134A[w_] := Module[{i = 1},
   If[(w[[1]] > v134[[1, 1]]) || (w[[1]] < v134[[-1, 1]]), Return[False]];
   If[w[[1]] == v134[[-1, 1]], Return[w[[3]] ≤ v134[[-1, 3]]]];
   While[v134[[i + 1, 1]] >= w[[1]], i++];
   (w[[1]] - v134[[i, 1]]) v134[[i + 1, 3]] / (v134[[i + 1, 1]] - v134[[i, 1]]) + (v134[[
         i + 1, 1]] - w[[1]]) v134[[i, 3]] / (v134[[i + 1, 1]] - v134[[i, 1]]) - w[[3]] ≥ 0];
```

The final set is particularly easy: one only needs vectors v_1 and v_61, so a single inequality suffices.

In[77]:=
```
Check134B[w_] := (w[[1]] - w[[4]] ≤ v[1][[1]] * (1 - w[[4]] / v[61][[4]]));
```

We apply the verification function to the first set. For some instances, numerical evaluation is not conclusive for the inequalities. Those are then dealt with by algebraic manipulations.

In[78]:=
```
Map[Check12, ToCheck12];
```

GreaterEqual: Internal precision limit $MaxExtraPrecision = 50.` reached while evaluating

$$\frac{5+\frac{8339}{\sqrt{2793745}}}{2\left(13384+8\sqrt{2793745}\right)^{3/11}} - \frac{2\left(-\frac{1355-\frac{3337411}{\sqrt{2793745}}}{216\,(13384+8\,\text{Power}[\ll 2\gg])^{3/11}} - \frac{-\frac{1355}{216}+\frac{3337411}{216\sqrt{2793745}}}{(13384+8\,\text{Power}[\ll 2\gg])^{3/11}}\right)}{\left(13384+8\sqrt{2793745}\right)^{2/11}\left(-\frac{-\frac{1355}{216}+\frac{3337411}{216}\text{Power}[\ll 2\gg]}{(13384+\text{Times}[\ll 2\gg])^{3/11}}+\frac{1}{(13384+8\ll 1\gg)^{2/11}}\right)} - \frac{\left(\frac{5}{2}+\frac{8339}{2\sqrt{2793745}}\right)\left(\frac{1355-\frac{3337411}{\sqrt{2793745}}}{216\,(13384+8\,\text{Power}[\ll 2\gg])^{3/11}}+\frac{1}{\left(13384+8\sqrt{2793745}\right)^{2/11}}\right)}{\left(13384+8\sqrt{2793745}\right)^{3/11}\left(-\frac{-\frac{1355}{216}+\frac{3337411}{216}\text{Power}[\ll 2\gg]}{(13384+\text{Times}[\ll 2\gg])^{3/11}}+\frac{1}{(13384+8\,\text{Power}[\ll 2\gg])^{2/11}}\right)}.$$

GreaterEqual: Internal precision limit $MaxExtraPrecision = 50.` reached while evaluating

$$\frac{6485042+3998\sqrt{2793745}}{44699920} - \frac{\left(\frac{3242521}{22349960}+\frac{1999}{8\sqrt{2793745}}\right)\left(\frac{32805161}{301724460}+\frac{54881040239}{301724460\sqrt{2793745}}-\left(\frac{1}{4}+\frac{1447}{4}\text{Power}[\ll 2\gg]\right)^2\right)}{-\frac{371657}{603448920}+\frac{613257073}{603448920\sqrt{2793745}}} - \frac{\left(\frac{29641217}{201149640}+\frac{49498725911}{201149640\sqrt{2793745}}\right)\left(-\frac{2443777}{22349960}-\frac{1447}{8\sqrt{2793745}}+\left(\frac{1}{4}+\frac{1447}{4\sqrt{2793745}}\right)^2\right)}{-\frac{371657}{603448920}+\frac{613257073}{603448920\sqrt{2793745}}}.$$

GreaterEqual: Internal precision limit $MaxExtraPrecision = 50.` reached while evaluating

$$\frac{1+\frac{1999}{\sqrt{2793745}}}{2\left(13384+8\sqrt{2793745}\right)^{3/22}} - \frac{\left(\frac{1}{2}+\frac{1999}{2\sqrt{2793745}}\right)\left(\frac{\frac{201818664}{2793745}+\frac{120744}{\sqrt{2793745}}}{(13384+8\,\text{Power}[\ll 2\gg])^{7/11}}-\frac{1+\frac{1447}{\sqrt{2793745}}}{2\,(13384+8\ll 1\gg)^{3/22}}\right)}{\left(13384+8\sqrt{2793745}\right)^{3/22}\left(\frac{\frac{201818664}{2793745}+120744\,\text{Power}[\ll 2\gg]}{(13384+\text{Times}[\ll 2\gg])^{7/11}}-\frac{\frac{1}{2}+\frac{1447}{2}\text{Power}[\ll 2\gg]}{(13384+\ll 1\gg)^{3/22}}\right)} - \frac{\left(\frac{267240144}{2793745}+\frac{159888}{\sqrt{2793745}}\right)\left(-\frac{\frac{1}{2}+\frac{1447}{2\sqrt{2793745}}}{(13384+8\,\text{Power}[\ll 2\gg])^{3/22}}+\frac{1+\frac{1447}{\sqrt{2793745}}}{2\,(13384+8\,\text{Power}[\ll 2\gg])^{3/22}}\right)}{\left(13384+8\sqrt{2793745}\right)^{7/11}\left(\frac{\frac{201818664}{2793745}+120744\,\text{Power}[\ll 2\gg]}{(13384+\text{Times}[\ll 2\gg])^{7/11}}-\frac{\frac{1}{2}+\frac{1447}{2}\text{Power}[\ll 2\gg]}{(13384+\text{Times}[\ll 2\gg])^{3/22}}\right)}.$$

General: Further output of GreaterEqual::meprec will be suppressed during this calculation.

In[79]:=
```
% // FullSimplify // Union
```

Out[79]=
```
{True}
```

All vectors in the first set pass the test. Now we repeat with the other three sets, all with the same (positive) outcome.

In[80]:= `Map[Check14, ToCheck14];`

GreaterEqual: Internal precision limit $MaxExtraPrecision = 50.` reached while evaluating

$$\frac{4032}{13384+8\sqrt{2793745}}-\frac{\left(\frac{11711}{72}-\frac{7\sqrt{2793745}}{72}\right)\left(\frac{73}{216}-\frac{8081}{216\sqrt{2793745}}-\frac{8448}{13384+8\,\text{Power}[\ll 2\gg]}\right)}{-\frac{8171}{24}-\frac{8081}{216\sqrt{2793745}}+\frac{11\sqrt{2793745}}{54}}-\frac{\left(-\frac{7}{36}+\frac{20783}{36\sqrt{2793745}}\right)\left(-\frac{18403}{54}+\frac{11\sqrt{2793745}}{54}+\frac{8448}{13384+8\,\text{Power}[\ll 2\gg]}\right)}{-\frac{8171}{24}-\frac{8081}{216\sqrt{2793745}}+\frac{11\sqrt{2793745}}{54}}.$$

GreaterEqual: Internal precision limit $MaxExtraPrecision = 50.` reached while evaluating

$$-\frac{65\left(25-\frac{44417}{\sqrt{2793745}}\right)}{108\left(13384+8\sqrt{2793745}\right)^{2/11}}-\frac{\left(-\frac{1625}{108}+\frac{577421\sqrt{\frac{5}{558749}}}{108}\right)\left(\frac{\frac{121046491}{120689784}+\frac{73693\sqrt{\ll 1\gg}}{216}}{(13384+8\ll 1\gg)^{2/11}}-\frac{1895-\frac{2436799}{\sqrt{2\ll 5\gg 5}}}{216(\ll 1\gg)^{2/11}}\right)}{\left(13384+8\sqrt{2793745}\right)^{2/11}\left(\frac{\frac{121046491}{120689784}+\frac{73693}{216}\text{Power}[\ll 2\gg]}{(13384+\text{Times}[\ll 2\gg])^{2/11}}-\frac{\frac{1895}{216}-\frac{\ll 7\gg}{\ll 3\gg}\ll 1\gg}{(\ll 1\gg)^{2/11}}\right)}-\frac{\left(\frac{143768593}{301724460}+\frac{17003\sqrt{\frac{5}{558749}}}{108}\right)\left(\frac{1895-\frac{2436799}{\sqrt{2793745}}}{216\,(13384+8\,\text{Power}[\ll 2\gg])^{2/11}}-\frac{\frac{1895}{216}-\frac{2436799}{216\sqrt{2793745}}}{(13384+8\,\text{Power}[\ll 2\gg])^{2/11}}\right)}{\left(13384+8\sqrt{2793745}\right)^{2/11}\left(\frac{\frac{121046491}{120689784}+\frac{73693}{216}\text{Power}[\ll 2\gg]}{(13384+\text{Times}[\ll 2\gg])^{2/11}}-\frac{\frac{1895}{216}-\frac{2436799}{216}\text{Power}[\ll 2\gg]}{(13384+\text{Times}[\ll 2\gg])^{2/11}}\right)}.$$

GreaterEqual: Internal precision limit $MaxExtraPrecision = 50.` reached while evaluating

$$\frac{2016\left(1+\frac{1677}{\sqrt{2793745}}\right)}{13384+8\sqrt{2793745}}-\frac{4\left(-\frac{73}{216}+\frac{8081}{216\sqrt{2793745}}+\frac{4224+\frac{7052736}{\sqrt{2793745}}}{13384+8\,\text{Power}[\ll 2\gg]}\right)}{\left(13384+8\sqrt{2793745}\right)^{7/22}\left(-\frac{73}{216}+\frac{8081}{216\sqrt{2793745}}+\frac{8}{(13384+\text{Times}[\ll 2\gg])^{7/22}}\right)}-\frac{\left(-\frac{7}{36}+\frac{20783}{36\sqrt{2793745}}\right)\left(-\frac{4224+\frac{7052736}{\sqrt{2793745}}}{13384+8\,\text{Power}[\ll 2\gg]}+\frac{8}{(13384+8\,\text{Power}[\ll 2\gg])^{7/22}}\right)}{-\frac{73}{216}+\frac{8081}{216\sqrt{2793745}}+\frac{8}{(13384+\text{Times}[\ll 2\gg])^{7/22}}}.$$

General: Further output of GreaterEqual::meprec will be suppressed during this calculation.

In[81]:= `% // FullSimplify // Union`

Out[81]= {True}

In[82]:= `Map[Check134A, ToCheck134A];`

... GreaterEqual: Internal precision limit \$MaxExtraPrecision = 50.` reached while evaluating

$$-\frac{65\left(25-\frac{44417}{\sqrt{2793745}}\right)}{108\left(13384+8\sqrt{2793745}\right)^{5/22}}-\frac{\left(-\frac{1625}{108}+\frac{577421\sqrt{\frac{5}{558749}}}{108}\right)\left(\frac{\frac{121046491}{120689784}+\frac{73693\sqrt{\ll 1\gg}}{216}}{(13384+8\ll 1\gg)^{5/22}}-\frac{1895-\frac{2436799}{\sqrt{2\ll 5\gg 5}}}{216(\ll 1\gg)^{5/22}}\right)}{\left(13384+8\sqrt{2793745}\right)^{5/22}\left(\frac{\frac{121046491}{120689784}+\frac{73693}{216}\text{Power}[\ll 2\gg]}{(13384+\text{Times}[\ll 2\gg])^{5/22}}-\frac{\frac{1895}{216}-\frac{\ll 7\gg}{\ll 3\gg}\ll 1\gg}{(\ll 1\gg)^{5/22}}\right)}-\frac{\left(\frac{143768593}{301724460}+\frac{17003\sqrt{\frac{5}{558749}}}{108}\right)\left(\frac{1895-\frac{2436799}{\sqrt{2793745}}}{216\,(13384+8\,\text{Power}[\ll 2\gg])^{5/22}}-\frac{\frac{1895}{216}-\frac{2436799}{216\sqrt{2793745}}}{(13384+8\,\text{Power}[\ll 2\gg])^{5/22}}\right)}{\left(13384+8\sqrt{2793745}\right)^{5/22}\left(\frac{\frac{121046491}{120689784}+\frac{73693}{216}\text{Power}[\ll 2\gg]}{(13384+\text{Times}[\ll 2\gg])^{5/22}}-\frac{\frac{1895}{216}-\frac{2436799}{216}\text{Power}[\ll 2\gg]}{(13384+\text{Times}[\ll 2\gg])^{5/22}}\right)}.$$

... GreaterEqual: Internal precision limit \$MaxExtraPrecision = 50.` reached while evaluating

$$\frac{7\left(-1+\frac{2969}{\sqrt{2793745}}\right)}{36\left(13384+8\sqrt{2793745}\right)^{1/22}}-\frac{\left(-\frac{7}{36}+\frac{20783}{36\sqrt{2793745}}\right)\left(\frac{8}{(13384+8\,\text{Power}[\ll 2\gg])^{4/11}}-\frac{73-\frac{8081}{\sqrt{2793745}}}{216\,(13384+8\,\text{Power}[\ll 2\gg])^{1/22}}\right)}{\left(13384+8\sqrt{2793745}\right)^{1/22}\left(\frac{8}{(13384+\text{Times}[\ll 2\gg])^{4/11}}-\frac{\frac{73}{216}-\frac{8081}{216}\text{Power}[\ll 2\gg]}{(13384+\text{Times}[\ll 2\gg])^{1/22}}\right)}-\frac{4\left(\frac{73-\frac{8081}{\sqrt{2793745}}}{216\,(13384+8\,\text{Power}[\ll 2\gg])^{1/22}}-\frac{\frac{73}{216}-\frac{8081}{216\sqrt{2793745}}}{(13384+8\,\text{Power}[\ll 2\gg])^{1/22}}\right)}{\left(13384+8\sqrt{2793745}\right)^{4/11}\left(\frac{8}{(13384+\text{Times}[\ll 2\gg])^{4/11}}-\frac{\frac{73}{216}-\frac{8081}{216}\text{Power}[\ll 2\gg]}{(13384+\text{Times}[\ll 2\gg])^{1/22}}\right)}.$$

... GreaterEqual: Internal precision limit \$MaxExtraPrecision = 50.` reached while evaluating

$$\frac{1+\frac{343}{\sqrt{2793745}}}{8\left(13384+8\sqrt{2793745}\right)^{1/22}}-\frac{\left(\frac{11711}{72}-\frac{7\sqrt{2793745}}{72}\right)\left(-\frac{\frac{1}{8}+\frac{2551}{8\sqrt{2793745}}}{(13384+8\,\text{Power}[\ll 2\gg])^{1/22}}+\frac{1+\frac{2551}{\sqrt{2793745}}}{8\,(13384+8\ll 1\gg)^{1/22}}\right)}{\left(13384+8\sqrt{2793745}\right)^{1/22}\left(-\frac{\frac{1}{8}+\frac{2551}{8}\text{Power}[\ll 2\gg]}{(13384+\text{Times}[\ll 2\gg])^{1/22}}+\frac{\frac{18403}{54}-\frac{11}{54}\text{Power}[\ll 2\gg]}{(13384+\text{Times}[\ll 2\gg])^{1/22}}\right)}-\frac{\left(\frac{1}{8}+\frac{343}{8\sqrt{2793745}}\right)\left(-\frac{1+\frac{2551}{\sqrt{2793745}}}{8\,(13384+8\,\text{Power}[\ll 2\gg])^{1/22}}+\frac{\frac{18403}{54}-\frac{11\sqrt{2793745}}{54}}{(13384+8\,\text{Power}[\ll 2\gg])^{1/22}}\right)}{\left(13384+8\sqrt{2793745}\right)^{1/22}\left(-\frac{\frac{1}{8}+\frac{2551}{8}\text{Power}[\ll 2\gg]}{(13384+\text{Times}[\ll 2\gg])^{1/22}}+\frac{\frac{18403}{54}-\frac{11}{54}\text{Power}[\ll 2\gg]}{(13384+\text{Times}[\ll 2\gg])^{1/22}}\right)}.$$

... General: Further output of GreaterEqual::meprec will be suppressed during this calculation.

In[83]:= `% // FullSimplify // Union`

Out[83]= {True}

In[84]:= `Map[Check134B, ToCheck134B];`

In[85]:= `% // FullSimplify // Union`

Out[85]= {True}

In[86]:=
```
(* -----------------------------------------------------------------------------
                                            ---------------------------------------
                                             ------- *)

(* The mathematica code above is sufficient to prove that the 62
 vectors have the desired property. Below we include a number of optional,
code-level 'sanity checks' to verify several of the preconditions used in the
  above analysis. When executed, all these statements should return true. *)
```

In[87]:=
```
(* Check that all the 62 vectors indeed have length 5. *)
```

In[88]:=
```
Union[Table[Length[v[i]] == 5, {i, 1, 62}]]
```

Out[88]= {True}

In[89]:=
```
(* All the original 62 vectors are non-negative *)

Union[Table[Part[v[i], 1] ≥ 0 && Part[v[i], 2] ≥ 0 &&
   Part[v[i], 3] ≥ 0 && Part[v[i], 4] ≥ 0 && Part[v[i], 5] ≥ 0, {i, 1, 62}]]

(* The first 39 vectors are decreasing on the first coordinate and non-
 decreasing on the second coordinate. *)

Union[Table[ Part[v[i], 1] > Part[v[i + 1], 1], {i, 1, 38}]]

Union[Table[Part[v[i], 2] ≤ Part[v[i + 1], 2], {i, 1, 38}]]
```

Out[89]= {True}

Out[90]= {True}

Out[91]= {True}

In[92]:=
```
(* The second group of vectors (v_1 and v_40... v_50) are decreasing on
  the first coordinate and non-decreasing on the second coordinate. *)

Part[v[1], 1] > Part[v[40], 1]

Part[v[1], 2] ≤ Part[v[40], 2]

Union[Table[ Part[v[i], 1] > Part[v[i + 1], 1], {i, 40, 49}]]

Union[Table[ Part[v[i], 2] ≤  Part[v[i + 1], 2], {i, 40, 49}]]

(* The third group of vectors (v_50 ... v_61) are decreasing on
  the first coordinate and non-decreasing on the second coordinate. *)
```

Out[92]= True

Out[93]= True

Out[94]= {True}

Out[95]= {True}

```
In[96]:= Union[Table[ Part[v[i], 1] > Part[v[i + 1], 1], {i, 50, 60}]]

         Union[Table[ Part[v[i], 2] ≤ Part[v[i + 1], 2], {i, 50, 60}]]

Out[96]= {True}

Out[97]= {True}
```