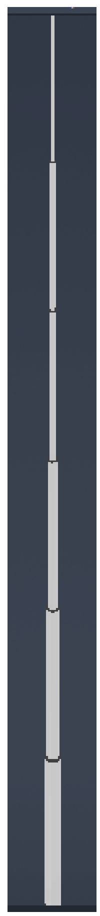
Telescopic Radio Antenna Design

Design For Stress Under Static Conditions

October 13th, 2021



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Reference Equations

- $\lambda f \rightarrow v$
  - The equation relating wavelength and frequency to the speed of light in a vacuum.

- $M''(x) \rightarrow w(x)$
  - The bending moment → $M(x)$
  - Transverse force per unit length applied to the antenna as a function of (x) → $w(x)$

- $\sigma(x) \rightarrow \frac{|M(x)|\, c(x)}{I(x)}$
  - Maximum normal stress → $\sigma(x)$
  - Absolute bending moment → $|M(x)|$
  - The maximum vertical distance from the neutral surface within the cross-sectional area at that point → $c(x)$
  - The second moment of the area at that point → $I(x)$

- $I \rightarrow \frac{\pi}{4}(r_0^4 - r_i^4)$
  - Annular cross-section of the inner radius → r_i
  - Annular cross-section of the outer radius → r_0

- $c \rightarrow r_0$
  - A solid circular cross-section can be considered an annular cross-section of the outer radius → r_0

- Safety Factor → $\frac{\sigma_y}{|\sigma|_{max}}$
  - Yield strength of material → σ_y

The single greatest normal stress of the material → σ_{max}

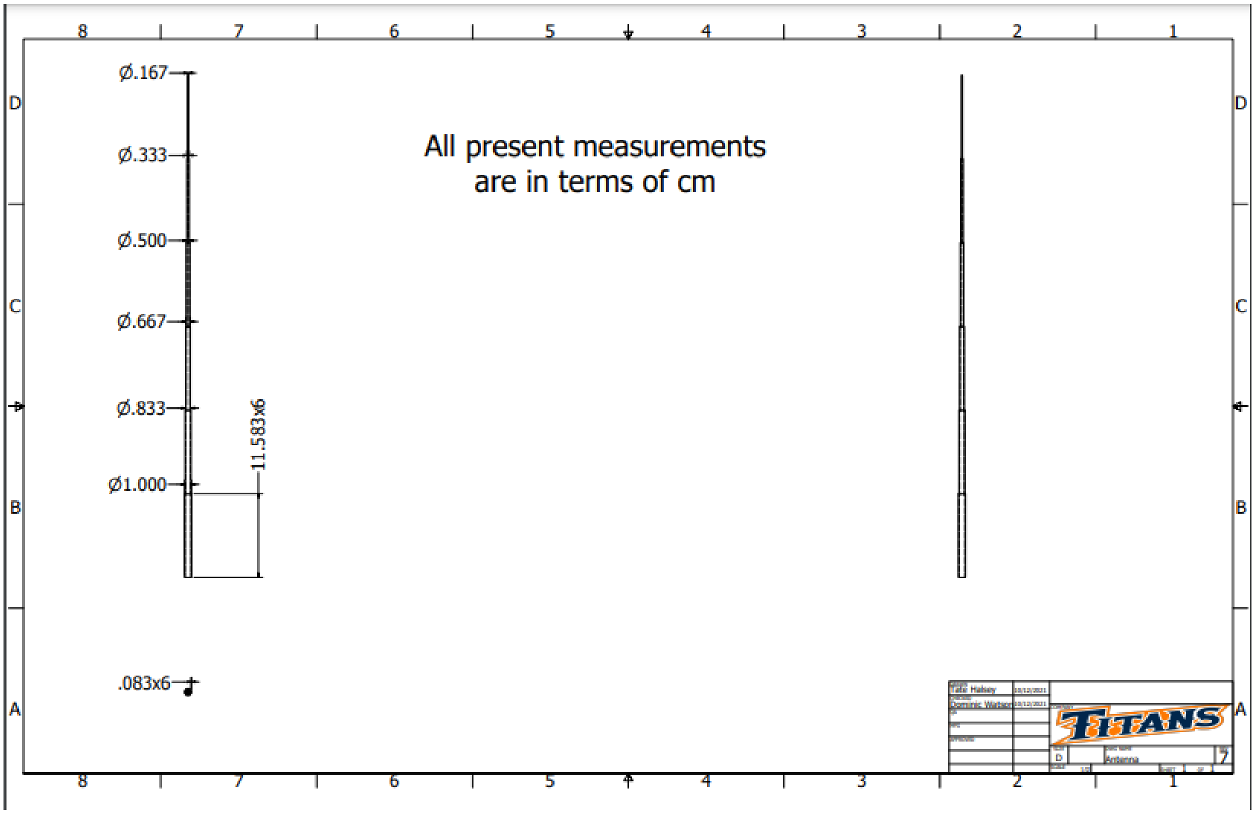
Abstract

Our team has been tasked with designing a telescopic radio antenna to pick up (FM) signals for a portable radio. The goal of this report is to ensure that the material and geometry of the antenna is functional without fail, whilst satisfying the manufacturer's design requirements. Approaching this specification with no restriction, the antenna is broken up into 6 segments. Upon full extension, the length of the antenna is 69.5 centimeters; fully retracted length measures out to approximately 11.5834 centimeters. Calculated tube wall thickness of the antenna is 0.0004167 centimeters, while the maximum diameter is 0.5 centimeter. Satisfying design requirements and functionality, the material chosen for this antenna was stainless steel 304. Density of the antenna comes out to 8000 kg/m^3. Calculating change in stress over change in strain, the elastic modulus of the antenna is 200 GPa. The yield strength is 207 MPa. Maximum normal stress for the antenna under a (given loading) is 167.96 MPa. The margin of security, or safety factor, for the antenna is 1.2324 (unitless). The total mass of the antenna comes out to 0.0182 kilograms. Total cost of raw materials needed to assemble the antenna was $0.08 USD.

Introduction

We have been tasked to design a telescopic antenna that would serve as an (FM) signal receiver for portable radios. This task requires a specific design geometry must be chosen including number of sections, length, and thickness, that meet the given requirements and can successfully pick up FM radio signals. In [1], the frequency for (FM) radio signals ranges from 87 to 108 (MHz). To find the wavelength related to the frequency, the range of frequency was divided by the speed of light in a vacuum. The calculated wavelength for an (FM) radio signal ranges from 2.78 to 3.45 meters. Understanding that the design is a quarter-wave antenna and can only pick up one-quarter of the wavelength of the intended (FM) radio signal, the admissible length (L) for the antenna ranges from 0.695 to 0.8625 meters.

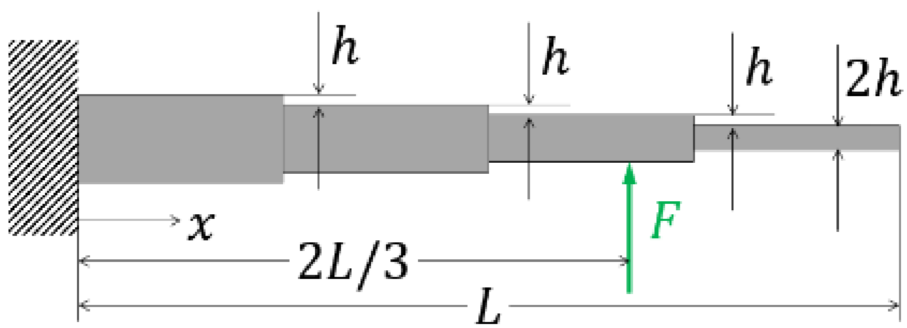
Design



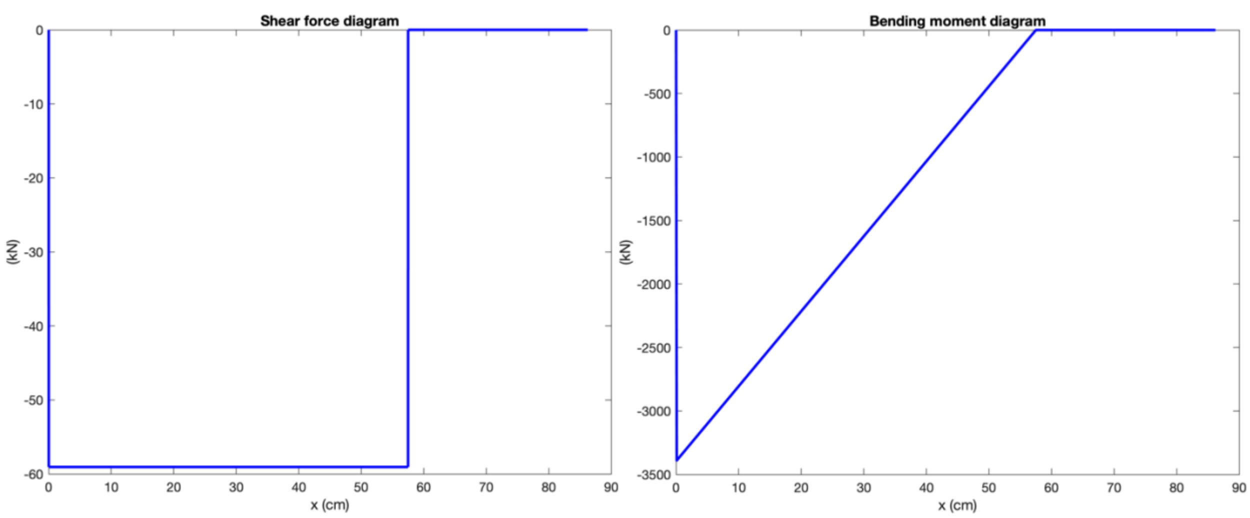
The material of this design is stainless steel 304. This material has a density of 8000 kg/m^3, an elastic modulus of 200 GPa, and a yield strength of 207 MPa. The antenna design has 6 segments that are all 11.583 centimeters in length, for a total length of 69.5 centimeters. The segments decrease in radius moving upwards by a factor of 0.04167. The diameter of the largest segment is 0.5 centimeter.

Static Stress Analysis

In the process of deriving our static stress analysis, there were a plethora of problems to be solved leading to our result. The given free-body diagram (Figure 1) helped create a shear-force (Figure 2) and bending-moment diagram (Figure 3).



(Figure 1): Free-Body Diagram



(Figures 2 & 3): Shear Force & Bending Moment Diagrams

This stage was simply recognizing that the arbitrary concentrated force F was the single external force acting on the beam, so the only internal reaction force R_y must be equal in magnitude but opposite in direction as to maintain static equilibrium. This concept stayed relevant in the bending moment diagram as well for the only moment was caused by the single concentrated force F at a distance 2L/3 therefore there must be a moment at the support equal in magnitude but opposite in direction to maintain static equilibrium. Now with this first step out of the way, the next step was to derive a symbolic equation for $M(x)$ using the Laplace transform technique. Originally, we were given equation (1) and had to decide on a form for $w(x)$ to take (2).

$$(1)\ \mathrm{M}''(\mathrm{x}) = \mathrm{w}(\mathrm{x})$$

$$(2)\ \mathrm{w}(\mathrm{x}) = \mathrm{Fd}(\mathrm{x} - \frac{2\mathrm{L}}{3})$$

Take the Laplace transformation of both sides (3) and, after inputting values gained from the bending moment graph, move appropriate values to the right side of the equation to end up with equation (4). At this point we are left with solving for the Laplace of the $w(x)$ function (5), before dividing by through by s^2 (6).

$$(3)\ \mathrm{s}^2\mathrm{L}[\mathrm{M}(\mathrm{x})] - \mathrm{sM}(0) - \mathrm{M}'(0) = \mathrm{L}[\mathrm{w}(\mathrm{x})]$$

$$(4)\ \mathrm{s}^2\mathrm{L}[\mathrm{M}(\mathrm{x})] = \mathrm{FL}\left[\mathrm{d}\left(\mathrm{x} - \frac{2\mathrm{L}}{3}\right)\right] - \frac{2\mathrm{FL}}{3}\mathrm{s} + \mathrm{F}$$

$$(5)\ s^2 L[M(x)] = Fe^{-\frac{2L}{3}} - \frac{2FL}{3}s + F$$

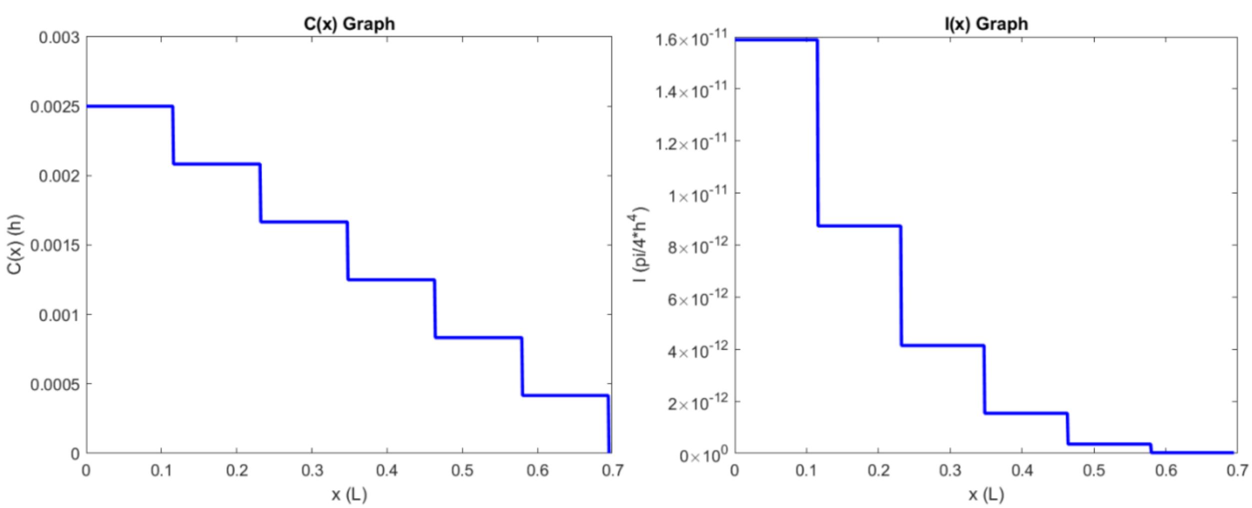
$$(6)\ L[M(x)] = \frac{Fe^{-\frac{2L}{3}\cdot s}}{s^2} - \frac{2FL}{3}\cdot\frac{1}{s} + F\cdot\frac{1}{s^2}$$

Take the inverse Laplace of both sides (7) and solve for M(x) (8).

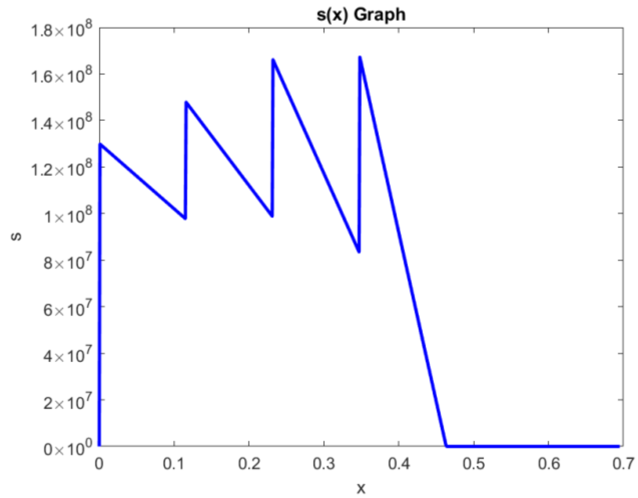
$$(7)\ M(x) = FL^{-1}\left(\frac{e^{-\frac{2L}{3}\cdot s}}{s^2}\right) - \frac{2FL}{3}\cdot L^{-1}\left(\frac{1}{s}\right) + FL^{-1}\left(\frac{1}{s^2}\right)$$

$$(8)\ M(x) = Fx - \frac{2F}{3L} + F\left(x - \frac{2L}{3}\right)\cdot H\left(x - \frac{2L}{3}\right)$$

Once the derivation was complete, it is onto plotting $c(x)$ (Figure 4), $I(x)$ (Figure 5), and $\sigma(x)$ versus x (Figure 6).



(Figures 4 & 5): Maximum Vertical Distance & Second Moment Area Diagrams



(Figure 6): Maximum Normal Stress Diagram

With these graphs available for inspection and their necessary code refined, important pieces of data can be found. The location of the maximum normal stress is 0.3475 meters, therefore the exact numerical value for normal stress is given by:

$$\rightarrow \frac{\left|F(0.3475) - \frac{2FL}{3} + F\left(0.3475 - \frac{2L}{3}\right) \cdot H\left(0.3475 - \frac{2L}{3}\right)\right| \cdot (N-3) \cdot h}{\frac{\pi}{4} h^4 \left(I_1 - d(3)\right)}$$

Then once inputted with appropriate constants, it resolves into:

$$\rightarrow \frac{\left|(7.1397)(0.3475) - \frac{2(7.1397)(0.695)}{3} + (7.1397)\left(0.3475 - \frac{2(0.695)}{3}\right) \cdot H\left(0.3475 - \frac{2(0.695}{3}\right)\right| \cdot (6-3) \cdot \frac{0.005}{6}}{\frac{\pi}{4}\left(\frac{0.005}{6}\right)^4 \left((6^4 - 5^4) - (-65)\right)}$$

$$\approx 1.6796 \cdot 10^8 \text{ Pascals}$$

Leading us to a safety factor of 1.2324.

Cost

The total mass of the antenna is 0.0182 kilograms, this was calculated by multiplying the density times the volume. The cost of stainless steel is approximately $0.23 per kilogram [4]. The cost of the stainless steel for the antenna design is $0.08.

Conclusion

For the antenna design the material that will be used is stainless steel 304. This material has a density of 8000 kg/m^3, an elastic modulus of 200 GPa and a yield strength of 207 MPa. Geometrically, this design has: 6 segments, when fully extended the length of the antenna is 69.5 cm, when fully retracted the length is 11.583 cm, the tube wall thickness is 0.04167 cm, and the maximum diameter is 0.5 cm. The cost of the raw material is $0.08 with the mass of are antenna being 0.0182 kg. The maximum normal stress is 167.96 MPa. With a concentrated force of 1.7849 N acting at 46.333 cm from the bottom of the antenna the factor of safety is 1.2324. While choosing the design, we wanted to minimize cost while maintaining a design that had a factor of safety of at least 1.2. When finding the geometry, the length was decreased to the lowest length that can still have an FM radio signal and determined h by dividing the radius by the number of segments. By inputting different numbers of segments, we were able to monitor the change in the factor of safety and come to a lower value that still meets design parameters. Problems that we encountered while working on the project were mostly mathematical, as well as issues incorporating that math into working code on MATLAB. Possible improvements include looking at a larger sample of materials and finding a cheaper cost that still satisfies design requirements.

References

[1] Y. Zhang, Z. Yang, L. Deng and S. Li, “Research on Wireless Positioning Technology Based on Digital FM Broadcasting”, *International Journal of Digital Multimedia Broadcasting,* vol. 2019, Article ID 1051386, 10 pages, 2019.

[2] H. M. Cobb, “History of Stainless Steel: Two New Classes of Stainless Steel.” 1st ed. Ohio: A S M International, 2010.

[3] Thyssenkrupp Materials LTD, C. Lane, C. Heath, West Midlands, “Stainless Steel 304 1.4301,” Jan. 2018. Accesses on: Oct. 11, 2021. [Online]. Available: https://www.thyssenkrupp-materials.co.uk/stainless-steel-304-14301.html

[4] “Stainless Steel: 304 SS,” [Online]. Available: https://www.scrapmonster.com/scrap/304-ss/48

Appendix A

Geometry

Number of segments:	6	
Length when fully extended:	69.5	cm
Length when fully retracted:	11.583	cm
Tube wall thickness:	0.0004167	cm
Maximum diameter:	0.5	cm

Material

Material name:	Stainless Steel 304	
Density:	8000	Kg/m^3
Elastic modulus:	200	GPa
Yield strength:	207	MPa

Performance

Maximum normal stress under given loading:	167.96	MPa
Safety factor under given loading:	1.2324	

Cost

Mass of antenna:	0.0182	kg
Cost of raw materials:	$0.08	USD

Appendix B

Section	Contributors	Typeset by	Checked by
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	Dominic Watson		Dominic Watson
	Serop Kelkelian		Shadan Amini
			Serop Kelkelian
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			Serop Kelkelian

Appendix C: Shear Force Script

```
% Shear Force
% Author: EGME 308-01 Group 3

% Clear previous data and command window
clear;
clc;

% Concentrated force (N)
F = 1.7849;
% Length (m)
L = .695;
% Matrix for x axis
a = 0:0.0001:L;

% Iterate through matrix a
for i = 1:numel(a)

  % Shear force
  % For values of a(i) that are less than 0, the values of s(i) are 0
  if a(i) < 0

    s(i) = 0;
  % For the values between the interval the value of s(i) equal to the
  % concentrated force
  elseif (a(i) > 0) && (a(i) < 2/3 * L)

    s(i) = -F;
  % When the values are not included in the interval (a(i) > 0) &&
  % (a(i) < 2/3*L) the s(i) are equal to zero
  else

    s(i) = 0;

  end

end

% Graph
% Plot all values of a versus all values of s
plot(a,s, 'blue', 'LineWidth', 2)
title('Shear force diagram')
xlabel('x (cm)')
ylabel('(kN)')

% Export Plot
ax = gca;
ax.YAxis.Exponent = 0;
exportgraphics(ax, 'Figures/curve_Shear_Force.png', 'Resolution', 300)
```

Appendix D: Bending Moment Script

```
% Bending Moment
% Author: EGME 308-01 Group 3

% Clear previous data and command window
clear;
clc;

% Concentrated force (N)
F = 1.7849;
% Length (m)
L = .695;
% Matrix for x axis
a = 0:0.0001:L;
% Formula for the bending moment
m = F;

% Iterate through matrix a
for i = 1:numel(a)

  % Bending moment
  % For values of a(i) that are less than 0, the values of s(i) are 0
  if a(i) < 0

    s(i) = 0;
  % For the valuse between the interval the value of s(i) is the (bending
  % moment formula * length) - (2 * force * length) / 3
  elseif (a(i) > 0) && (a(i) < 2/3*L)

    s(i) = m * a(i) - 2 * F * L/3;
  % When the values are not included in the interval (a(i) > 0) &&
  % (a(i) < 2/3*L) the s(i) are equal to zero
  else
    s(i) = 0;

  end

end

% Graph
% Plot all values of a versus all values of s
plot(a, s, 'blue', 'LineWidth', 2)
title('Bending moment diagram')
xlabel('x (cm)')
ylabel('(kN)')

% Export Plot
ax = gca;
ax.YAxis.Exponent = 0;
exportgraphics(ax, 'Figures/curve_Bending_Moment.png', 'Resolution', 300)
```

Appendix E: Absolute Bending Moment Script

```
% Absolute Bending Moment
% Author: EGME 308-01 Group 3

% Clear previous data and command window
clear;
clc;

% Concentrated force (N)
F = 1.7849;
% Length (m)
L = 0.695;
% Matrix of x
x = 0:0.001:L;

% Set range for values to be filled according to how many x values there are
for i=1:numel(x)

    % M(x)
    % For values of x(i) that are less than 0, the values of M(i) are 0
    if x(i) < 0

        M(i) = 0;

    % For the values between the interval
    elseif (x(i) > 0) && (x(i) < 2*L/3)

        % The value of M(i) is the ((force * x) - (2 * force * length) / 3)
        % + ((force * x) - (force * 2 * length) / 3) * heaviside of ( x -
        % (2 * length / 3))
        M(i) = F*x(i)-2*F*L/3+(F*x(i)-F*2*L/3)*heaviside(x(i)-2*L/3);

    % When the values are not included in the interval, the values of M(i) are 0
    else

        M(i) = 0;

    end

end

% Graph
% Plot all values of x versus all values of s
plot(x, M, 'blue', 'LineWidth', 2)
xlabel('x (L)')
ylabel('s')
title('M(x) Graph')

% Export Plot
ax = gca;
ax.YAxis.Exponent = 0;
exportgraphics(ax, 'Figures/Absolute_Bending_Moment.png', 'Resolution', 300)
```

Appendix F: Maximum Vertical Distance Script

```
% Maximum Vertical Distance
% Author: EGME 308-01 Group 3

% Clear previous data and command window
clear;
clc;

% Number of segments
N = 6;
% Length (m)
L = 0.695;
% Matrix for x axis
a = 0:0.001:L;
% Thickness of each segment on antenna
h = 0.000416666666666667;

% Iterate through matrix A
for i = 1:numel(a)

    % C(x)
    % Iterate from 0 to number of segments on antenna
    for n = 0:N

        % For values of a(i) that are between the interval, the values of C(i) are
(number of segments - iterable in sum) * thickness of segment
        if a(i) <= ((n + 1) * L / N) && a(i) >= ((n) * L / N)

            C(i) = (N - n) .* h;

        end

    end

end

% Graph
% Plot all values of a versus all values of C
plot(a, C, 'blue', 'LineWidth', 2)
xlabel('x (L)')
ylabel('C(x) (h)')
title('C(x) Graph')

% Export Plot
ax = gca;
ax.YAxis.Exponent = 0;
exportgraphics(ax, 'Figures/curve_C.png', 'Resolution', 300)
```

Appendix G: Second Moment Area

```
% Second Moment of Area
% Author: EGME 308-01 Group 3

% Clear previous data and command window
clear;
clc;

% Number of segments
N = 6;
% Length (m)
L = .695;
% Thickness of each segment on antenna (m)
h = 0.000416666666666667;
% Matrix for x axis
x = 0:0.001:L;
% Find first value of I, aka max I value, used later in loop for graph
I1 = pi/4*h^4*(N^4-(N-1)^4);
% Pre-allocate space
a = zeros(N-1,1);

% Set range for values to be filled according to how many x values there are, going
this route helps prevent errors regarding tryng to input an array starting at zero for
functions that only accept positive integers
for i = 1:numel(x)

    % I(x)
    % Set range for values according to how many sections are chosen, basically the
values for the summation
    for n = 1:N-1

        % Find and set the amount dropped per section in an array a
        a(n) = I1 - ((N-n).^4-(N-n-1).^4);

        % First section of plot, ranging from 0 to L/N (assumes n=1 as first n in
n*L/N)
        if (x(i)>=0) && (x(i) < L/N)

            % Value for I in first section of plot is maximum I value (I1 solved for
in lines previous)
            I(i)=I1;

        % Run through the following sections of plot according to n, ie
2*L/N,3*L/N,4*L/N, etc. until final range is reached
        elseif (x(i) >= n*L/N) && (x(i) <= (n+1)*L/N)

        % Find value for I in given section, set value in the array I
            I(i) = pi/4*h^4*(I1 - a(n));

        end

    end

end

% Graph
% Plot all values of x versus all values of I
plot(x, I, 'blue', 'LineWidth', 2)
xlabel('x (L)')
ylabel('I (pi/4*h^4)')
title('I(x) Graph')
```

```
% Export Plot
ax = gca;
ax.YAxis.Exponent = 0;
exportgraphics(ax, 'Figures/curve_I.png', 'Resolution', 300)
```

Appendix H: Maximum Normal Stress

```
% Maximum Normal Stress
% Author: EGME 308-01 Group 3

% mass calculated on line 26

% Clear previous data and command window
clear;
clc;

% Number of segments
N = 6;
% Length (m)
L_full = .695;
% Matrix for x axis
x = 0:0.001:L_full;
% Radius (m)
r = 0.0025;
% Thickness of each segment on antenna (m)
h = r/N;
% Density of metal (Kg/m^3)
D = 8000;
% Length of each antenna segment (m)
L = L_full/N;
% Volume of a cylinder (m^3)
V = pi * (r^2)* L;
% Formula of mass (mass calculation)
m = D * V;
% Concentrated force (N)
F = 10*m*9.81;
% Find I1
I1 = pi/4*h^4*(N^4-(N-1)^4);
% Yield Strength (Pa)
YS = 207000000;

% Pre-allocate space for drop matrix
d = zeros(N-1,1);

% Iterate through matrix X
for i = 1:numel(x)

    % I(x)
    % Set range for values according to how many sections are chosen, basically the
values for the summation
    for n = 1:N-1

        % Find and set the amount dropped per section in an array a
        d(n) = I1 - ((N-n).^4-(N-n-1).^4);

        % First section of plot, ranging from 0 to L/N (assumes n=1 as first n in
n*L/N)
        if (x(i)>=0) && (x(i) < L_full/N)

            % Value for I in first section of plot is maximum I value (I1 solved for
in lines previous)
            I(i)=I1;

        % Run through the following sections of plot according to n, ie
2*L/N,3*L/N,4*L/N, etc. until final range is reached
        elseif (x(i) >= n*L_full/N) && (x(i) <= (n+1)*L_full/N)
```

```
            % Find value for I in given section, set value in the array I
            I(i) = pi/4*h^4*(I1 - d(n));

        end

    end

    % C(x)
    % Iterate from 0 to number of segments on antenna
    for n = 0:N

        % For values of a(i) that are between the interval, the values of C(i) are
(number of segments - iterable in sum) * thickness of segment
        if x(i) <= ((n + 1) * L_full / N) && x(i) >= ((n) * L_full / N)

            C(i) = (N - n) .* h;

        end

    end

    % M(x)
    % For values of x(i) that are less than 0, the values of BM(i) are 0
    if x(i) < 0

        BM(i) = 0;

    % For the values between the interval
    elseif (x(i) > 0) && (x(i) < 2/3 * L_full)

        % The value of BM(i) is the (bending moment formula * length) - (2 * force *
length) / 3
        BM(i) = F*x(i)-2*F*L_full/3+(F*x(i)-F*2*L_full/3)*heaviside(x(i)-2*L_full/3);

    % When the values are not included in the interval, the values of BM(i) are 0
    else

        BM(i) = 0;

    end

    % s(x)
    % Calculate normal stress
    s(i) = (abs(BM(i)) * C(i))/(I(i));

end

    % Factor of saftey
    FOS = YS / max(s);

% Plotting
% Plot all values of x versus all values of I
plot(x, s, 'blue', 'LineWidth', 2)
xlabel('x')
ylabel('s')
title('s(x) Graph')

% Export Plot
ax = gca;
ax.YAxis.Exponent = 0;
exportgraphics(ax, 'Figures/curve_Normal_Stress.png', 'Resolution', 300)
```