# EGME 308 Group Project
# Design of a telescopic radio antenna

## Checkpoint #3:
## Design for vibration (and Final Report)

California State University, Fullerton

Fall 2021

Due: Wednesday, December 1, 2021

## Project overview

An electronics manufacturer is developing a custom portable AM/FM radio for one of its clients. Your team is one of several teams working on the telescopic antenna, which will serve as the FM signal receiver. Figure 1 shows a schematic diagram of the antenna, as well as an enlarged view of the cross-section. The antenna is a quarter-wave whip, which consists of several interconnected metal segments. The segments are hollow circular tubes that fit inside each other when the antenna is fully retracted. In general, let the length of the antenna be $L$ when fully extended, let $N$ be the number of segments (Figure 1 shows $N = 4$ segments, but $N$ can in principle be any positive integer), and let the tube wall thickness of each segment be $h$. For the sake of simplicity, let us agree that each segment will have the same length $(L/N)$ and the same thickness $h$, and that the segments will fit together snugly, as shown in Figure 1. We will ignore the hollow part of the very last segment, which is negligible.

### Competition

Your supervisor wants each team to work independently and generate a different design. The team that delivers the best final design (the lightest and least expensive design satisfying the design requirements) will receive a substantial raise, and their design will be used. To simulate this, at the end of the semester, all teams will pitch their designs during the Group Presentation, after the Final Report is due. The one team out of the entire class that delivers the single best final design (again, the lightest and least expensive design satisfying the design requirements) will receive a +10 point bonus on top of their final score in the course.

NOTE: The likelihood of two teams independently submitting identical designs is nil. It is in your team's best interest to keep your work confidential. You do NOT want another team to steal your design, improve on it, and get the bonus instead of you!

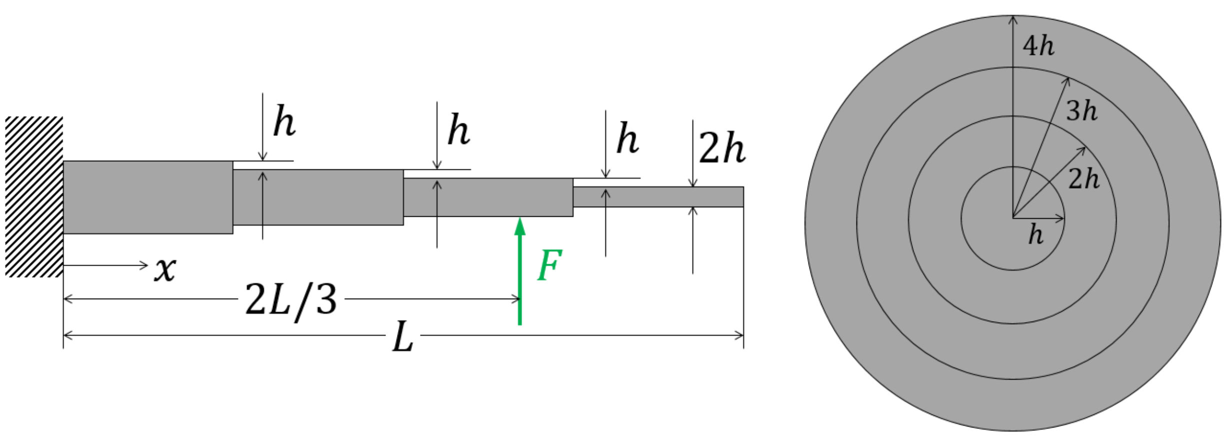


Figure 1: Schematic diagram of the antenna.

## Checkpoint #3: Design for vibration

The final step in the process—and your goal for Checkpoint #3—is to ensure that the antenna is not too flimsy, i.e., that it does not whip around too much under ordinary conditions. In particular, the antenna must satisfy the following design requirements:

- When fully extended, the antenna must be long enough to pick up FM radio signals.
- When completely retracted, the length of the antenna must not exceed 15 cm.
- The maximum diameter of the antenna must not exceed 1 cm.
- When fully extended, the antenna must be able to withstand a static, concentrated test force $F$, equal to 10 times the antenna's weight, applied transversely at a distance of two-thirds the fully extended length ($2L/3$) from the fixed base, as shown in Figure 1. Under said loading conditions, the following requirement(s) apply:
  - The maximum normal stress within the segments must not exceed the yield stress of the material, with a safety factor of at least 1.2.
  - The antenna must not deflect by more than $0.10L$ anywhere along the length of the antenna.
- **When fully extended, the antenna's fundamental frequency of vibration must be greater than 5 Hz.** NOTE: In this context, "frequency" refers to the frequency of the antenna's oscillatory whipping motion; it has nothing to do with the frequency of FM radio signals, which you used in Checkpoint #1. Do not confuse the two.
- The antenna should be as light and inexpensive as possible, while meeting all of the above requirements.

### Tasks

The following tasks are required for Checkpoint #3:

(a) Before you begin to work on Checkpoint #3, first revise your work for Checkpoint #2 based on the instructor's feedback.

(b) To find the fundamental frequency of the antenna, in general you would need to solve the dynamic Euler-Bernoulli beam equation[1],

$$\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 u}{\partial x^2}\right) = -\rho A(x)\frac{\partial^2 u}{\partial t^2}, \tag{1}$$

subject to appropriate boundary conditions, along with the initial conditions

$$u(x,0) = u_0(x), \quad \dot{u}(x,0) = 0. \tag{2}$$

[1] The dynamic beam equation enforces Newton's second law, $\mathbf{F} = m\mathbf{a}$, at every point within a beam. If you are curious, you can find the derivation in any vibrations textbook (you can also see the derivation in EGME 402, EGME 430, or EGME 431 with me). You are not required to include the derivation of the dynamic beam equation in this assignment.

Here $E$ is the elastic modulus of the material, $I(x)$ is the second moment of area of the cross-section (which you used in Checkpoints #1 and #2), $\rho$ is the volumetric mass density of the material, $A(x)$ is the cross-sectional area, $u_0(x)$ is the initial shape of the beam, and $t$ is time. Taken together, the governing partial differential equation, boundary conditions, and initial conditions constitute an initial-boundary-value problem for the time-dependent deflection curve $u(x, t)$. In this case, $I(x)$ and $A(x)$ are piecewise functions, and an analytical solution for $u(x, t)$, while possible, is *extraordinarily* tedious and time-consuming. In practice, this problem would be solved numerically. However, the numerical solution of partial differential equations is beyond the scope of this course, so instead of doing that, we will make a simplification.

We expect that the first segment (the segment on the fixed end, with the largest diameter) will influence the fundamental frequency more than the other segments. Accordingly, we might approximate the antenna's fundamental frequency with the fundamental frequency of just the first segment, by itself. The fewer segments there are, the more accurate this approximation will be (in fact, the approximation is exact when $N = 1$). Conversely, the more segments there are, the less accurate this approximation will be. Still, as long as $N$ is not too large, we expect this simplification to provide a reasonable estimate for the fundamental frequency. Therefore, for the purpose of estimating the antenna's fundamental frequency—and for this purpose *only*—we will treat the antenna as **a cantilever beam consisting of just the first segment by itself**, as shown in Figure 2.

Explain why the first segment would influence the fundamental frequency more than the other segments. Think of multiple reasons (not just one).

(c) Show that, under this simplification, the dynamic beam equation reduces to

$$\frac{\partial^4 u}{\partial x^4} = -\frac{1}{k^2}\frac{\partial^2 u}{\partial t^2}, \quad k = \sqrt{\frac{EI_0}{\rho A_0}}, \tag{3}$$

where $I_0 = I(0)$ and $A_0 = A(0)$.

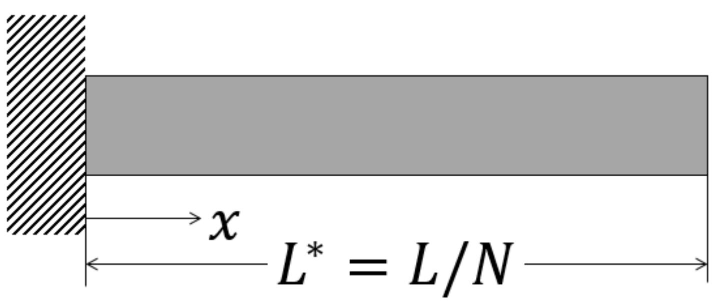


Figure 2: Schematic diagram of just the first segment of the antenna. The fundamental frequency of the antenna will be approximated by the fundamental frequency of the first segment by itself.

(d) Using the separation of variables technique, solve (3) on the interval $x \in (0, L^*)$, where $L^* = L/N$ is the length of the first segment. Seek a separable solution of the form

$$u(x,t) = X(x)T(t). \tag{4}$$

You should find that

$$\frac{X''''(x)}{X(x)} = -\frac{1}{k^2}\frac{\ddot{T}(t)}{T(t)} = \lambda. \tag{5}$$

Show that the separation constant must be positive. Writing $\lambda = \beta^4$, show that the general solution for $X(x)$ is given by

$$X(x) = A^* \cos(\beta x) + B^* \sin(\beta x) + C^* \cosh(\beta x) + D^* \sinh(\beta x). \tag{6}$$

By enforcing the boundary conditions for a cantilever beam of length $L^*$, show that $\beta$ must satisfy

$$\cos(\beta L^*) \cosh(\beta L^*) = -1. \tag{7}$$

There are infinitely many values of $\beta L^*$ satisfying (7). Compute the first three positive, nonzero values. Note that (7) is a transcendental equation that cannot be solved analytically for $\beta L^*$: it must be solved numerically using software or computer code. Based on all of the above, estimate the first three natural frequencies of the antenna ($\omega_1$, $\omega_2$, and $\omega_3$) in terms of $E$, $I_0$, $\rho$, $A_0$, and $L^*$.

(e) Compute the fundamental frequency ($\omega_1$) of the antenna you designed for Checkpoint #2. Remember that $\omega$ has units of rad/s, so you will need to convert to Hz (cycles/s) using the conversion

$$1 \text{ cycle} = 2\pi \text{ rad}. \tag{8}$$

Does the fundamental frequency exceed the value specified in the design requirements?

(f) If necessary, modify your design until it satisfies all of the design requirements. This may require many iterations, which is why it is better to perform your analysis and write your code for arbitrary system parameters rather than specific parameter values. If you have written the code well, modifying your design should be as simple as changing the parameter values at the very beginning of the code and running it again.

(g) Once you have arrived at a final design that satisfies all of the requirements, estimate the cost of the raw material required to construct your antenna.

(h) Complete the Design Specifications Form (located on Canvas in Word format).

(i) After you write the report (see below), complete the Authorship Declaration Form (located on Canvas in Word format).

(j) Each member of your group must complete a Peer Evaluation Form (located on Canvas in Word format). You will submit the Peer Evaluation Forms individually and confidentially, separate from the report. Missing Peer Evaluation Forms will be treated as reporting the lowest amount of participation across the board, for all team members.

(k) Document your work in the Final Report, with the following sections:

1. *Title page* A title page with the title of your report, your team number, and a list of all team members in alphabetical order by last name.
2. *Abstract* A concise summary of the entire report in a single paragraph, written for a layperson (non-engineer). A layperson should be able to read the Abstract and glean all of the important information from the report without any technical details. Important information includes everything on the Design Specifications Form. You should not start writing the Abstract until all of the other sections have been written. The Abstract will seem redundant—it is supposed to be.
3. *Introduction* A discussion of the problem, and a review of the relevant background required for subsequent sections. Include a discussion of the frequency/wavelength range for FM radio signals, citing the (reputable) sources you found.
4. *Design* A detailed description of your design. This section should contain all the information needed for the manufacturing company to create your antenna, including all of the geometric and material parameters listed on the Design Specifications Form. Also include a detailed CAD drawing of your antenna, with all dimensions labeled.
5. *Static stress analysis* A step-by-step derivation of your stress analysis, presented in a logical and coherent manner. Include exact free-body, shear-force, and bending-moment diagrams for your antenna, your symbolic derivation of $M(x)$ using the Laplace transform technique, exact plots of $c(x)$, $I(x)$, and $\sigma(x)$ versus $x$, and exact numerical values for the maximum normal stress and the associated safety factor for your antenna.
6. *Static deflection analysis* A step-by-step derivation of your static deflection analysis, presented in a logical and coherent manner. Include an explanation of the explicit Euler method, the exact increment $\Delta x$ that you used, an exact plot of $u(x)$ versus $x$, and the exact value of the maximum deflection for your antenna.
7. ***Modal analysis*** **A step-by-step derivation of your analysis for the fundamental frequency of the antenna, presented in a logical and coherent manner. Include a valid explanation for why the first segment has the greatest influence on the fundamental frequency, your solution for the time-dependent deflection curve $u(x,t)$, symbolic expressions for the first three natural frequencies of the antenna ($\omega_1$, $\omega_2$, and $\omega_3$) in terms of $E$, $I_0$, $\rho$, $A_0$, and $L^*$, and the numerical value of the fundamental frequency ($\omega_1$) for the antenna you designed.**

   While these last three sections will include plenty of equations, they are NOT simply a sequence of equations: you must incorporate the equations into a narrative. A good rule of thumb is to write as if the reader were not familiar with the material, but you should also avoid being pedantic with the reader.
8. *Cost* An estimate of the cost of the raw material required to construct the antenna. This should include the exact value of the total mass of the antenna.

9. *Conclusion* A concise summary of the report, highlighting the bottom line: your design, its performance, and its cost. Include all information contained in the Design Specifications Form. Also include a discussion of the merits of your design, as well as possible problems, issues, and areas for improvement (consider the inherent assumptions in your analysis and in choosing your design). The Conclusion is different from the Abstract in that the Conclusion is written for someone who has just read the entire report, while the Abstract is written for someone who does not have the time or expertise to read the entire report.
10. *References* You must cite all external references you consulted during the course of your work, in IEEE format. References should be cited appropriately throughout the text.
11. *Appendix A* Include your Design Specifications Form as Appendix A.
12. *Appendix B* Include your Authorship Declaration Form as Appendix B.
13. *Appendix C* Include the code used to generate results as Appendix C. If there is more than one code, give each code a separate appendix, continuing with Appendix D, Appendix E, etc.

There is no length requirement for the report: it should be as long as it needs to be to convey all of the necessary information, and no longer. Tables and figures should be numbered, captioned, and referenced appropriately (table captions go *above* the corresponding tables, while figure captions go *below* the corresponding figures). Equations should be numbered sequentially, with the labels right-justified in parentheses. See the "Helpful Hints for Technical Writing" guide for more tips. Reports written in LaTeXwill receive a 5% bonus.

## Grading

As stated in the course syllabus, your report will be scored on a standards-based grading scale. The standards-based grading scale corresponds to the following levels of competency:

| | | |
|---|---|---|
| 4 | A | Mastery of the material needed for subsequent courses and engineering practice. |
| 3 | B | Proficiency in the material needed for subsequent courses and engineering practice. |
| 2 | C | Bare minimum proficiency needed for subsequent courses. |
| 1 | D | Insufficient proficiency for engineering practice. |
| 0 | F | Insufficient proficiency for subsequent courses or engineering practice. |

Each group will be assigned a baseline score of 0, 1, 2, 3, or 4. Each individual team member will receive the baseline score scaled by a participation factor $0 \leq p \leq 1$, computed from the average of that member's scores on the Peer Evaluation Form.

*Example*: Suppose your team's baseline score is 4, and you receive the following scores on the Peer Evaluation Forms: 0, 2, 2, 2, and 4 (you gave yourself the 4 rating). Your average Peer Evaluation Score is 2.0/4.0, which translates to a participation factor of $p = 0.50$. Your individual score for this checkpoint would be $0.50 \times 4 = 2.00$ (C-range).

*Example*: Suppose your team's baseline score is 4, and no one turns in a Peer Evaluation Form. Each missing Peer Evaluation Form is treated as reporting 0 for all team members. Your average Peer Evaluation Score is 0.0/4.0, which translates to a participation factor of $p = 0$. Your individual score for this checkpoint would be $0 \times 4 = 0.00$ (F-range).

Meeting all of the following conditions is sufficient to receive a baseline score of 4:

- A typeset report with all of the requested information in each section.
- All text and figures created using appropriate software; nothing drawn or written by hand.
- A Design Specifications Form with design parameters that do, in fact, satisfy the design requirements.
- All information (including text and figures) is accurate. No false statements, contradictions, logical inconsistencies, logical fallacies, or factual errors anywhere.
- No mathematical or computational errors anywhere.
- No dimensional inconsistencies or naked numbers anywhere.
- No spelling or grammar mistakes anywhere.
- No formatting mistakes anywhere.
- Shear-force and bending-moment diagrams plotted from a symbolic formula using Matlab or Microsoft Excel (no "beam calculators" used).
- Appropriate and reputable references cited for every claim asserted in the report. No websites cited, except possibly for pricing materials.
- No missing information. NOTE: If your report is missing crucial information necessary to assess your work (such as the Design Specifications Form or the Authorship Declaration Form), you will receive a score of 0.
- No academic dishonesty of any kind (including plagiarism, fabrication of results, or any other form of academic dishonesty). NOTE: It is in your team's best interest to monitor each other's work and ensure that no academic dishonesty takes place. If any member of your team commits an act of academic dishonesty, your entire team will receive an automatic F in the course and will be referred to the Office of Student Conduct.

Anything less than the above may result in a baseline score less than 4.

# Appendix A: Design Specifications Form

## Geometry

| | | |
|---|---|---|
| *Number of segments:* | | |
| *Length when fully extended:* | | cm |
| *Length when fully retracted:* | | cm |
| *Tube wall thickness:* | | cm |
| *Maximum diameter:* | | cm |

## Material

| | | |
|---|---|---|
| *Material name:* | | |
| *Density:* | | $\mathrm{kg/m^3}$ |
| *Elastic modulus:* | | GPa |
| *Yield strength:* | | MPa |

## Performance

| | | |
|---|---|---|
| *Maximum normal stress under given loading:* | | MPa |
| *Safety factor under given loading:* | | |
| *Maximum deflection under given loading:* | | cm |
| *Fundamental frequency of vibration:* | | Hz |

## Cost

| | | |
|---|---|---|
| *Mass of antenna:* | | kg |
| *Cost of raw materials:* | | USD |

## Appendix B: Authorship Declaration Form

| Section | Contributors | Typeset by | Checked by |
|---|---|---|---|
| Abstract | | | |
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| Introduction | | | |
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| Design | | | |
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| Static stress analysis | | | |
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| Static deflection analysis | | | |
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| Modal analysis | | | |
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| Cost | | | |
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| Conclusion | | | |
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| References | | | |
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# Peer Evaluation Form

*Team Number:*

Please type the names of all of your group members, including yourself, and rate the degree to which each member fulfilled his or her responsibilities in completing the group project on the following scale:

| 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| *Did little or no work* | | | | *Did their fair share of work* |

These ratings should reflect each individual's level of participation, effort, and sense of responsibility, not their academic ability.

| Name | Rating |
|---|---|
| | |
| | |
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| | |

Comments: