Written Assignment #3

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CS331 - Intro to AI
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1) Probability Distributions

a) $\mathbf{P}(Toothache)$

$Toothache$	$\mathbf{P}(Toothache)$
False	$0.576 + 0.144 + 0.008 + 0.072 = \mathbf{0.8}$
True	$0.064 + 0.016 + 0.012 + 0.108 = \mathbf{0.2}$

b) $\mathbf{P}(Cavity)$

$Toothache$	$\mathbf{P}(Toothache)$
False	$0.576 + 0.144 + 0.064 + 0.016 = \mathbf{0.8}$
True	$0.008 + 0.072 + 0.012 + 0.108 = \mathbf{0.2}$

c) $\mathbf{P}(Toothache|Cavity)$

$Toothache$	$Cavity$	$\mathbf{P}(Toothache\|Cavity)$
True	True	$(0.012 + 0.108)/0.2 = \mathbf{0.6}$
True	False	$(0.064 + 0.016)/0.8 = \mathbf{0.1}$
False	True	$(0.008 + 0.072)/0.2 = \mathbf{0.4}$
False	False	$(0.576 + 0.144)/0.8 = \mathbf{0.9}$

2) Show Equivalence

$$
\begin{aligned}
P(X,Y) &= P(X)P(Y): \\
&= P(X|Y)P(Y) && [Chainrule] \\
&= (\frac{P(X \wedge Y)}{P(Y)})P(Y) && [Kolmogorov\ def] \\
&= (\frac{P(X)P(Y)}{P(Y)})P(Y) && [mult.\ rule\ when\ indep.] \\
&= P(X)P(Y) && [division] \quad \square
\end{aligned}
$$

$$
\begin{aligned}
P(X|Y) &= P(X): \\
P(X|Y) &= \frac{P(X \wedge Y)}{P(Y)} && [Kolmogorov\ def] \\
&= \frac{P(X)P(Y)}{P(Y)} && [mult.\ rule\ when\ indep.] \\
&= P(X) && [division] \quad \square
\end{aligned}
$$

$$
\begin{aligned}
P(Y|X) &= P(Y): \\
P(Y|X) &= \frac{P(Y \wedge X)}{P(X)} && [Kolmogorov\ def] \\
&= \frac{P(Y)P(X)}{P(X)} && [mult.\ rule\ when\ indep.] \\
&= P(Y) && [division] \quad \square
\end{aligned}
$$

3) Suppose you are given a coin that lands heads with probability x and tails with probability (1-x).
   a) *Are the outcomes of successive flips of the coin independent of each other given that you know the value of x?*

      Regardless of knowing probabilty x the outcomes of previous coin flips have no effect on the next.
   b) *Are the outcomes of successive flips of the coin independent of each other if you do not know the value of x?*

      Like part a, whether or not you know the probability of the coin flip's outcome the successive flips would have no effect on any other. Knowing or not knowing doesn't change the probability of an event that is observed.

4) After your yearly checkup, the doctor has bad news and good news.

   It's good news, because even though the test is highly accurate the probability of having the disease is quite low. The actual probability of having the disease given testing positive is:

$$\begin{aligned} P(Have \mid Pos) &= \frac{P(Pos \mid Have)P(Have)}{P(Pos)} \\ &= \frac{P(Pos \mid Have)P(Have)}{P(Pos \mid Have)P(Have) + P(Pos \mid Doesn't\ Have)P(Doesn't\ Have)} \\ &= \frac{(0.99 * 0.0001)}{(0.99 * 0.0001) + (0.01 * 0.9999)} \\ &= \frac{0.000099}{0.000099 + 0.009999} \\ &\approx 0.0098 \\ &\approx 0.98\% \end{aligned}$$

5) Suppose you are witness to a nighttime hit-and-run accident involving a taxi in Athens.
   a) It is not possible to calculate the most likely color of the taxi without more information. In order to get this probability we'd need to know the number of blue and green taxis in the city.
   b) Now that we know 9 out of 10 taxis in Athens are green we can calculate the most likely color.

$$\begin{aligned} P(Blue \mid Saw\ Blue) &= \frac{P(Saw\ Blue \mid Blue)P(Blue)}{P(Saw\ Blue)} \\ &= \frac{P(Saw\ Blue \mid Blue)P(Blue)}{P(Saw\ Blue \mid Blue)P(Blue) + P(Saw\ Blue \mid Green)P(Green)} \\ &= \frac{0.75 * 0.1}{(0.75 * 0.1) + (0.25 * 0.9)} \\ &= 0.25 \end{aligned}$$

      Since the probability you saw blue and the car was actually blue is only 0.25, it's most likely that you saw blue and the car was actually green!