Written Assignment #2

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1) (From 7.4 in the book) For each of the following statements, prove if it is true or false.
   a) $(A \wedge B) \models (A \iff B)$

A	B	$A \wedge B$	$A \iff B$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	T

Since $(A \iff B)$ is true whenever $(A \wedge B)$ is true, then $(A \wedge B) \models (A \iff B)$ is therefore true.

   b) $(C \vee (\neg A \wedge \neg B)) \equiv ((A \rightarrow C) \wedge (B \rightarrow C))$ becomes $(C \vee (\neg A \wedge \neg B)) \equiv ((\neg A \vee C) \wedge (\neg B \vee C))$

A	B	C	$C \vee (\neg A \wedge \neg B)$	$(\neg A \vee C) \wedge (\neg B \vee C)$
T	T	T	T	T
T	T	F	F	F
T	F	F	F	F
T	F	T	T	T
F	T	T	T	T
F	T	F	F	F
F	F	T	T	T
F	F	F	T	T

Since $(C \vee (\neg A \wedge \neg B))$ has the same truth values as $((\neg A \vee C) \wedge (\neg B \vee C))$, then $(C \vee (\neg A \wedge \neg B)) \equiv ((\neg A \vee C) \wedge (\neg B \vee C))$ is therefore true.

   c) $(A \vee B) \wedge \neg(A \rightarrow B)$ becomes $(A \vee B) \wedge (A \wedge \neg B)$

A	B	$(A \vee B) \wedge (A \wedge \neg B)$
T	T	F
T	F	T
F	T	F
F	F	F

Since when A is true and B is false $(A \vee B) \wedge (A \wedge \neg B)$ is evaluated to true, then it is satisfiable.

2) (From 7.10 in the book) Decide whether each of the following sentences is valid, unsatisfiable or neither. Verify your decisions using truth tables or the equivalence rules of Figure 7.11.
   a) $Smoke \rightarrow Smoke$ becomes $\neg Smoke \vee Smoke$

$Smoke$	$\neg Smoke \vee Smoke$
T	T
F	T

This statement is satisfiable and valid as shown in the truth table for $Smoke$ above.

   b) $(Smoke \rightarrow Fire) \rightarrow (\neg Smoke \rightarrow \neg Fire)$ becomes $(Smoke \wedge \neg Fire) \vee (Smoke \vee \neg Fire)$

$Smoke$	$Fire$	$(Smoke \wedge \neg Fire) \vee (Smoke \vee \neg Fire)$
T	T	T
T	F	T
F	T	F
F	F	T

This statement is satisfiable and valid as shown in the truth table for $Smoke$ and $Fire$ above.

c) $(Smoke \rightarrow Fire) \rightarrow ((Smoke \wedge Heat) \rightarrow Fire)$ becomes $(Smoke \wedge \neg Fire) \vee ((\neg Smoke \vee \neg Heat) \vee Fire)$

$Smoke$	$Fire$	$Heat$	$(Smoke \wedge \neg Fire) \vee ((\neg Smoke \vee \neg Heat) \vee Fire)$
T	T	T	T
T	T	F	T
T	F	F	T
T	F	T	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

Since every possible value for $Smoke$, $Fire$, and $Heat$ results in a true statement for $(Smoke \wedge \neg Fire) \vee ((\neg Smoke \vee \neg Heat) \vee Fire)$ it is valid.

3) (Exercise 7.2 in the book which was adapted from Barwise and Etchemendy (1993)). If a unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

a) Knowledge Base:

$$Mythical \rightarrow Immortal \tag{1}$$
$$\neg Mythical \rightarrow (\neg Immortal \wedge Mammal) \tag{2}$$
$$(Immortal \vee Mammal) \rightarrow Horned \tag{3}$$
$$Horned \rightarrow Magical \tag{4}$$

$$\neg Mythical \vee Immortal \quad [1, logical\ equiv.] \tag{5}$$
$$Mythical \vee (\neg Immortal \wedge Mammal) \quad [2, logical\ equiv.] \tag{6}$$
$$(\neg Immortal \wedge \neg Mammal) \vee Horned \quad [3, logical\ equiv.] \tag{7}$$
$$\neg Horned \vee Magical \quad [4, logical\ equiv.] \tag{8}$$
$$(\neg Immortal \wedge \neg Mammal) \vee Magical \quad [7, 8, res.\ rule] \tag{9}$$
$$Immortal \vee (\neg Immortal \wedge Mammal) \quad [5, 6, res.\ rule] \tag{10}$$
$$Immortal \vee Mammal \quad [10, dist.] \tag{11}$$
$$Magical \quad [11, 9, res.\ rule] \tag{12}$$
$$Horned \quad [7, 11, res.\ rule] \tag{13}$$

b) Mythical? : No, Cannot be shown.
c) Magical? : Yes, Shown above.
d) Horned? : Yes, Shown above.