

Written Assignment #2

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CS331 - INTRO TO AI
SPRING TERM

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1) (From 7.4 in the book) For each of the following statements, prove if it is true or false.

a) $(A \wedge B) \models (A \iff B)$

A	B	$A \wedge B$	$A \iff B$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	T

Since $(A \iff B)$ is true whenever $(A \wedge B)$ is true, then $(A \wedge B) \models (A \iff B)$ is therefore true.

b) $(C \vee (\neg A \wedge \neg B)) \equiv ((A \rightarrow C) \wedge (B \rightarrow C))$ becomes $(C \vee (\neg A \wedge \neg B)) \equiv ((\neg A \vee C) \wedge (\neg B \vee C))$

A	B	C	$C \vee (\neg A \wedge \neg B)$	$(\neg A \vee C) \wedge (\neg B \vee C)$
T	T	T	T	T
T	T	F	F	F
T	F	F	F	F
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	F	F
F	F	T	T	T
F	F	F	T	T

Since $(C \vee (\neg A \wedge \neg B))$ has the same truth values as $((\neg A \vee C) \wedge (\neg B \vee C))$, then $(C \vee (\neg A \wedge \neg B)) \equiv ((\neg A \vee C) \wedge (\neg B \vee C))$ is therefore true.

c) $(A \vee B) \wedge \neg(A \rightarrow B)$ becomes $(A \vee B) \wedge (A \wedge \neg B)$

A	B	$(A \vee B) \wedge (A \wedge \neg B)$
T	T	F
T	F	T
F	T	F
F	F	F

Since when A is true and B is false $(A \vee B) \wedge (A \wedge \neg B)$ is evaluated to true, then it is satisfiable.

2) (From 7.10 in the book) Decide whether each of the following sentences is valid, unsatisfiable or neither. Verify your decisions using truth tables or the equivalence rules of Figure 7.11.

a) $\text{Smoke} \rightarrow \text{Smoke}$ becomes $\neg \text{Smoke} \vee \text{Smoke}$

Smoke	$\neg \text{Smoke} \vee \text{Smoke}$
T	T
F	T

This statement is satisfiable and valid as shown in the truth table for *Smoke* above.

b) $(\text{Smoke} \rightarrow \text{Fire}) \rightarrow (\neg \text{Smoke} \rightarrow \neg \text{Fire})$ becomes $(\text{Smoke} \wedge \neg \text{Fire}) \vee (\text{Smoke} \vee \neg \text{Fire})$

Smoke	Fire	$(\text{Smoke} \wedge \neg \text{Fire}) \vee (\text{Smoke} \vee \neg \text{Fire})$
T	T	T
T	F	T
F	T	F
F	F	T

This statement is satisfiable and valid as shown in the truth table for *Smoke* and *Fire* above.

- c) $(Smoke \rightarrow Fire) \rightarrow ((Smoke \wedge Heat) \rightarrow Fire)$ becomes $(Smoke \wedge \neg Fire) \vee ((\neg Smoke \vee \neg Heat) \vee Fire)$

<i>Smoke</i>	<i>Fire</i>	<i>Heat</i>	$(Smoke \wedge \neg Fire) \vee ((\neg Smoke \vee \neg Heat) \vee Fire)$
T	T	T	T
T	T	F	T
T	F	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

Since every possible value for *Smoke*, *Fire*, and *Heat* results in a true statement for $(Smoke \wedge \neg Fire) \vee ((\neg Smoke \vee \neg Heat) \vee Fire)$ it is valid.

- 3) (Exercise 7.2 in the book which was adapted from Barwise and Etchemendy (1993)). If a unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

a) Knowledge Base:

$$Mythical \rightarrow Immortal \quad (1)$$

$$\neg Mythical \rightarrow (\neg Immortal \wedge Mammal) \quad (2)$$

$$(Immortal \vee Mammal) \rightarrow Horned \quad (3)$$

$$Horned \rightarrow Magical \quad (4)$$

$$\neg Mythical \vee Immortal \quad [1, \text{logical equiv.}] \quad (5)$$

$$Mythical \vee (\neg Immortal \wedge Mammal) \quad [2, \text{logical equiv.}] \quad (6)$$

$$(\neg Immortal \wedge \neg Mammal) \vee Horned \quad [3, \text{logical equiv.}] \quad (7)$$

$$\neg Horned \vee Magical \quad [4, \text{logical equiv.}] \quad (8)$$

$$(\neg Immortal \wedge \neg Mammal) \vee Magical \quad [7, 8, \text{res. rule}] \quad (9)$$

$$Immortal \vee (\neg Immortal \wedge Mammal) \quad [5, 6, \text{res. rule}] \quad (10)$$

$$Immortal \vee Mammal \quad [10, \text{dist.}] \quad (11)$$

$$Magical \quad [11, 9, \text{res. rule}] \quad (12)$$

$$Horned \quad [7, 11, \text{res. rule}] \quad (13)$$

b) Mythical? : No, Cannot be shown.

c) Magical? : Yes, Shown above.

d) Horned? : Yes, Shown above.