Electric Charges & Fields

Electric discharge (examples)

- spark/crackle when taking off synthetic clothes/sweater in dry weather.
- lightning
- sensation of electric shock while opening a car door / holding the iron bar of a bus after sliding from our seat.

Reason :- discharge of electric charges through our body, which were accumulated due to rubbing of insulating surfaces.

Generation of static electricity.
doesnt move/change with time.

Electrostatics :- study of forces, fields & potential arising from static charges

Eletric Charges.

Electricity $\xrightarrow{\text{from}}$ Elektron [Greek]

↓

Amber.

Amber rubbed with wool/silk attracts light objects.

↓ discovered by

Thales of Miletus, Greek. mathemati

ironed white paper sticks get attracted to TV/monitor screens.

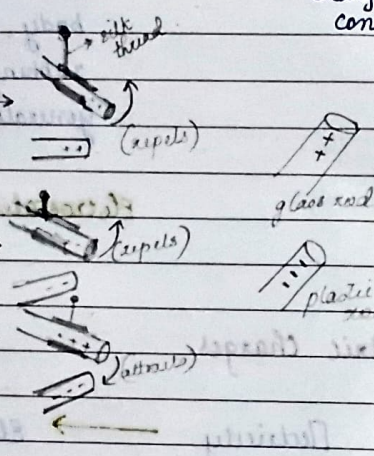
2 glass rods rubbed with 2 pieces of wool or silk cloth.

A B C D ABCD are electrified

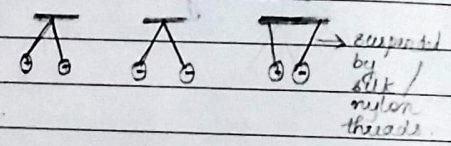
2 plastic rods rubbed with cat fur.

E F G E F G are electrified

Pith balls touched with glass rods (A/B) → H
Pith balls touched with plastic rods (E/F) → I
} Pith balls are electrified or charged on contact



A repels B
C repels D
A/B attracts C/D
E repels F
E/F attracts G
E/F attracts A/B
E/F repels C/D
A/B repels G.
H_1 repels H_2
I_1 repels I_2
H attracts I



Only 2 kinds of entities which is called, electric charge.

- like charges repels
- unlike charges attracts

The property that differentiates the 2 kinds of charges → **Polarity** of charge.

If A/B brought in **contact** with C/D, they:
→ no longer attracts each other
→ don't attract / repel other light objects as they did on being electrified.

Conclusion:- unlike charges acquired by the objects neutralise / **nullify** each other's effect.

↓

Charges named +ve & -ve. (sum of a +ve and -ve no. of same magnitude is zero.)

By American scientist Benjamin Franklin

By convention, A, B and G → +ve
C, D, E and F → -ve

If an obj. has charge : electrified / charged
no charge : electrically neutral.

Unification of Electricity & Magnetism.

- **Oersted** (1820 Danish scientist) → found compass needle is deflected by passing an electric current through a wire placed near the needed.
- **Ampere & Faraday** (supported Oersted by saying) → electric charges in motion produce magnetic field. & moving magnets generate electricity.
- **Maxwell** (Scotish physist) & **Lorentz** (Danish physist) (unification finally achieved) → put forward a theory showing interdependence of these 2 subjects.

This field is called : **Electromagnetism**.
one of the fundamental forces of nature.

Electromagnetism

most of the phenomena acting around us can be described by this example: friction, chemical force bet. atoms holding matter together, forces describing processes occuring in cells of living organisms.

Maxwell → 4 equations — same role in classical electromagnetism as Newton's equations of motion & gravitation law play in mechanics.

argued → claimed

- light is electromagnetic in nature & its speed can be found by making purely electrical and magnetic measurements.
- Science of optics is closely related to that of electricity and magnetism.

Foundation of modern technological civilisation.

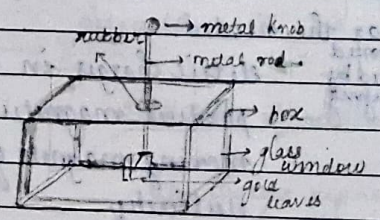
→ Electric power → TV & Radio
→ telecommunication → many other practical appliances.

Gold Leaf electroscope.

A vertical metal rod housed in a box with 2 thin gold leaves attached at its bottom end and a metal knob at top end.

When a charged body touches the knob, charge flows on to the leaves and they diverge.

↓ degree of divergence, indicate the amount of charge.



Making electroscope

- thin aluminium curtain rod. [20 cm]
- ball ends filled for hanging the curtain
- Cut out a piece of 20 cm length with the ball at one end and flatten the cut end.
- bottle with cork. A hole in the cork sufficient to hold the curtain rod snugly [5 cm of the rod's ball end projecting above the cork]
- 6 cm long thin Al. foil fold in at the centre and attach it to the flattened end using cellulose tape.
- A paper scale put inside the bottle: measures leaves separation.

Electric force is all pervasive & encompassed almost each & every field associated with our life.

All matter contain charges → exactly balanced so, electrically neutral (normally)

Arising from forces bet. charged particles :-

- force holding molecules together
- force holding together atoms in solids
- adhesive force of glue
- forces associated with surface tension.

Electrify neutral body

- add → excess
- remove (kind of charge) → deficit (of charge)

In solids, some electrons are less tightly bound. They r transferred from 1 body to another.

Glass rod rubbed with silk cloth.

+ve ← electrons flows → -ve

Conductors & Insulators

Substances

- readily allows electricity to pass through → conductors
- don't allows electricity to pass → Insulators

conductors — semiconductors (intermediate property) — Insulators

conductors: electrons, that are free to move inside them.

Insulators: non metals.

Charge transferred to:
- → conductors - readily distribute over the entire surface
- → Insulator - stays at the same place.

Earthing / Grounding

↓

Charged body in contact with earth, excess charge dissapears, causing a momentary current to pass through the body to the ground. This process of sharing charges with earth.

earthing ↓ safety measure for electrical circuit & appliances.

Thick metal plate buried deep into the earth and thick wires are drawn from it. → used in buildings for earthing near main supply.

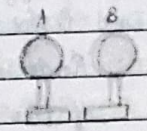
electric wiring in houses has 3 wires, live, neutral, earth.

- live: carry current from the power station
- earth: connected to the buried metal plate.

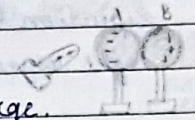
metallic bodies of appliances are connected to it. Thus if live wire touches the metallic body charge flows to earth without damaging the appliances or causing any injury to humans.

· charging by contact → charge
· charging by induction → charge

Charging by Induction

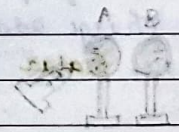


- Bring 2 metal sphires, supported by insulating stands.
- Bring a +ve ly charged rod near sphire A. (don't touch it). The free electrons in the sphires are attracted towards the rod. This leaves excess +ve charge on the rear surface of sphire B. Thus, left surface of A has an excess -ve charge & right surface of B has excess +ve charge.

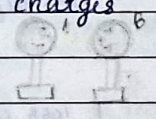


Not all electrons in sphires have accumulated on the left side of A. As -ve charge starts building, other electrons are repelled by these. In a short time equilibrium is reached under the action of force of attraction of the rod & the force of repulsion due to the accumulated charges. This process → induction of charge → happens almost instantly. The charges remain to the surface as long as the rod is held near. If rod is removed, charges redistribute - original position (neutral state).

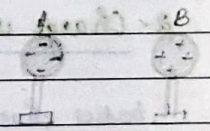
- Seperate sphire without removing the rod, A and B being oppositely charged attracts.



- Remove the rod, the charges rearranges.



- Seperate the sphires quite apart. The charges uniformly gets distributed over them.



Both sphires are oppositely charged. This is charging by induction. The rod doesn't loose any of it's charges contrary to the process of charging by contact.

Similar effect when electrified rods are brought near light objects.

Rods induce opp. charges on the near surface of the obj & similar charges move to the farther side of the obj. centres of the 2 types of charges are slightly separated. Magnitude of attractive/repulsive Force depends upon the distance between the 2 charges. Here attraction overweight repulsion (unlike charge is nearer/closer to the rod). Thus light objects are pulled towards the rod.

Basic Properties of Electric Charge

① Additivity of Charge
② Charge is conserved
③ Quantisation of charge.

Point charge. — All charge content of a body is concentrated at 1 point in space.

size of charged bodies is very small compared to the distance between them.

1 Additivity of Charge.

If a system has 2 point charges (q_1 & q_2) – Total charge of the system algebraic sum of q_1 and q_2.

If a system contains n charges: $q_1, q_2 \ldots q_n$, total charge in the system: $q_1 + q_2 + \ldots q_n$.

Thus charge adds up like real numbers

i.e, Scalar quantity → like mass.

One difference: Mass – always +ve
Charge – +ve / -ve.

2- Charge is conserved.

bodies charged by rubbing → transfer of electrons (↑ loss charge, ↑ gains charge.)

↓

no new charges are created nor destroyed within an isolated system with many charged bodies; charge may get redistributed - but,

The total charge of the isolated system is always CONSERVED.

conservation of charge – established experimentally,

not possible to create/destroy net charge carried by any ISOLATED SYSTEM

but charge carrying particles may be created/destroyed in the process

sometimes nature creates charged particles:
A neutron turns into a proton & an electron.
· equal & opp charges
· total charge is zero, bef & aft creation.

3- Quantisation of Charge.

All free charges are integral multiples of a basic unit charge denoted by e.] experimentally established

Charge q of a body is always given by

$$q = ne$$

n: any integer, +ve or -ve

e: charge an electron or proton carries. (-e, -ve ↑; +ve, +e ↓)

fact that electric charge is always an integral multiple of e. ↑

quantisation of charge

↓ Ist suggested by the experimental laws of electrolysis discovered by Faraday.

↓ demonstrated experimentally by Millikan 1912.

(In SI unit)

A unit of charge. → A Coulomb [C]

↓

It is defined in terms of the unit of electric current.

↓

1 coulomb is the charge flowing through a wire of in 1s if the current is 1A.

$e = 1.602 \times 10^{-19}$ C

thus,

6×10^{18} electrons to make → 1C.

↓

large magnitude for electrostatics

↓

smaller units → $1\mu C$ [10^{-6} C] (micro)

$1mC$ [10^{-3} C] (milli)

If protons & electrons are the only basic charges in the universe, all observable charges have to be integral multiples of e. Thus charges in any body is an integral multiple of e and can be increased/decreased in steps of e.

A body has n_1 elect. & n_2 protons.

Total amount of charge: $n_2 \times e + n_2 \times -e$

$\Rightarrow (n_2 - n_1)e$

Macroscopic level.

- step size is very small so we deal with a few μC.
- charge of body increasing/decreasing by steps of e is not visible. The granny nature of charge is lost, thus appear continous.
- At a larger scale [μC/mC], the fact that charge incre/decrea. in units of e is'nt very different from saying that charge can take continous values.

Thus at macroscopic level → quantisation of charge

↓

No practical consequence.

Compared with geometric concept of points & lines.

A dotted line looks continous from a distance as many points close together gives the impression of a cont. line.

Illy, many charges taken together appears continous.

1 electron/proton

$1e = 1.6 \times 10^{-19}$ C.

$1\mu C$ = around 10^{13} times the electronic charge

$\mu = 10^{-6}$

$m = 10^{-3}$

Microscopic level.

- charges - few tens / hundreds i.e, can be counted.
- Appear in discreate lumps
- Cannot be ignored.

Thus quantisation depends on the magnitude of scale involved.

Coulomb's Law

↓

Quantitave statement about the Force bet. 2 point charges.

Original experiments

↓

est. at macroscopic level, but est. down to subatomic level ($r \sim 10^{-10}$ m)

Coulomb → discovered this law without knowing the explicit magnitude of the charge. In fact it's the other way round: Coloumbs law can now be employed to furnish a definition for a unit of charge.

Coulomb measured F bet. 2 point charges and found:

→ varied inversely as the square of the distance bet. the charges

→ directly proportional to the magnitude of the 2 charges.

→ acted along the line joining the 2 charges

q_1, q_2 seperated by distance r in vacuum, magnitude of F → $F = k\frac{|q_1 q_2|}{r^2}$

↓

used a torsion balance to measure Force bet. 2 mytallic sphires (charges)

In his law k is arbitrary. We can choose any value for k.

The choice of k determines the size of the unit of charge.

↓

for a colomb, value of $k = 9 \times 10^9 \frac{Nm^2}{C^2}$. [SI unit]

Put this value for k in coulomb law for $q_1 = q_2 = 1C$ & $r = 1m$,

We get $F = 9 \times 10^9$ N.

Thus, 1 C is the charge when placed at a distance of 1m from another charge of the same magnitude in vacuum experiences an electric F of repulsion of magnitude 9×10^9 N.

(1 C → too big. 1μC / 1mC in electrostatics)

- when seperation bet. 2 sphires is much larger than the radius of each sphire, charged sphires regarded as point charges.
- charges were unknown. So, he took a met. sphire of charge q. he took an identical uncharged sphire and brough it in contact with the 1st. Charge spread over the 2 sphires. By symmetry each sphire has q/2 charge. similarly we get charges q/2, q/4 etc...
- Keeping charges constant, varied distance and measured Force.
- Keeping distance fixed, he varied charges in pairs

Thus comparing forces for diff. pairs of charges at diff. distances he arrived at his law.

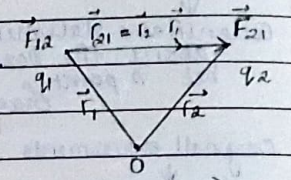
For later convenience constant K is usually put as $K = \frac{1}{4\pi\varepsilon_0}$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$$

Epsilon naught.
Permitivitty in free space (vaccum)

$$\varepsilon_0 = 8.854 \times 10^{-12}\ C^2/m^2N$$

F → Vector so need to write coulombs law in vector notation.

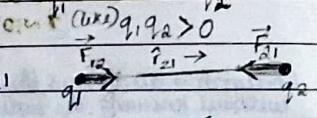


$\vec{r}_1, \vec{r}_2$ → position vectors of q_1 & q_2.
$\vec{F}_{12}$ → Force of^n q_1 by q_2
$\vec{F}_{21}$ → Force on q_2 by q_1
$\vec{r}_{12}$ → vector from q_2 to q_1
$\vec{r}_{21}$ → vector from q_1 to q_2.

$\vec{r}_{21} = r_2 - r_1$

$\vec{r}_{12} = r_1 - r_2 \Rightarrow -(r_2 - r_1) \Rightarrow -\vec{r}_{21}$

$\vec{r}_{12} = -\vec{r}_{21}$



(like) $q_1 q_2 > 0$

(unlike) $q_1 q_2 < 0$

$\vec{r}_{12}$ & $\vec{r}_{21}$ → vectors, r_{21}, r_{12} → magnitude

direction of a vector: specified by a unit vector along the vector

Thus, unit vectors (direction)

$$\hat{r}_{21} = \frac{\vec{r}_{21}}{r_{21}} \qquad \hat{r}_{12} = \frac{\vec{r}_{12}}{r_{12}}$$

[vector → dire × magn
dire (unit vect) → vector / magnitude]

Coulomb's force law bet. 2 point charges q_1 & q_2 located at $\vec{r}_{12}$ & $\vec{r}_{21}$:

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 \times q_2}{r_{21}} \times \hat{r}_{21}$$ → unit vector.

Remarks (based on this equation).

① F

- Valid for any sign of q_1 & q_2. (+ve / -ve). If q_1, q_2 → same sign, $\vec{F}_{21}$ along $\hat{r}_{21}$ → denotes repulsion. q_1, q_2 → opp. signs, $\vec{F}_{21}$ along $-\hat{r}_{21}$ ($\hat{r}_{12}$).

Thus → no need for 2 seperate equations for like & unlike charge (same equation will suffice)

② $\vec{F}_{12}$ on charge q_1 by q_2 obtained by interchanging 1 & 2. Thus

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} \times \hat{r}_{12}$$

Thus → Coulomb's law agrees with Newton's 3rd law

③ Coulombs law → F bet. q_1 & q_2 in vaccum.

If charges are placed in matter or space has matter situation complicated → charged constituents presence in matter.

Superposition principle

Force on any charge due to a no. of other charges is the vector sum of all the forces on that charge due to the other charges taken one at a time. The individual forces are unaffected by the presence of other charges.

Principle of superposition says that in a system of charges $q_1, q_2 \ldots q_n$, the force on q_1 due to q_2 is given by Coulomb's law i.e., it's unaffected by the presence of other charges $q_3, q_4 \ldots q_n$.

The total Force F_1 on q_1 → vector sum of Forces $F_{12}, F_{13} \ldots F_{1n}$.
↳ obtained: ||gram law of addition.

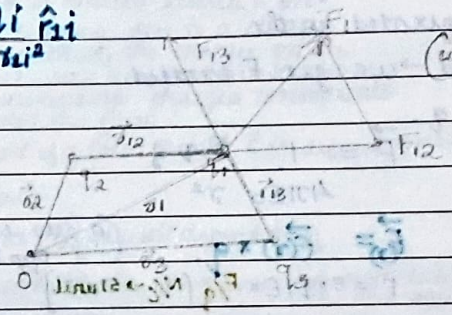
i.e,

$$F_1 = F_{12} + F_{13} + \ldots + F_{1n}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} + \frac{q_1 q_3}{r_{13}^2}\hat{r}_{13} + \ldots + \frac{q_1 q_n}{r_{1n}^2}\hat{r}_{1n}\right]$$

$$\frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{n}\frac{q_i}{r_{1i}^2}\hat{r}_{1i}$$

(when 3 charges present)



Electric field

Point Charge Q placed at O in vacuum produces an electric field everywhere in the surrounding. When another charge q is brought near it, the field act on it and produces force.

Consider → point charge Q in vacuum placed at → origin O. Place another point charge q placed at P, where OP = r, charge Q exerts a force on q (Coulomb's law).

what is left in the surrounding if q is removed? If there is nothing at point P, then how does a F act when we place q at P.

↓ (to explain)

introduce concept of field.

signifies how some distributed quantity varies with position.

The electric field produced by Q at a point P ⇒ $\vec{E}(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$ → unit vector from origin to point $\vec{r}$.

↓ Thus specified value of electric field for each value of position vector $\vec{r}$.

The effect of the charge (q) has been incorporated in the electric field → we get F exerted by Q on q.

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}\hat{r}$$

$$\vec{F}(r) = \vec{E}(r) \times q.$$ (q also exerts an equal and opp F on Q)

$F = E \times q$; $E = F/q$ [N/C → SI unit]

Electrostatic F bet. Q & q → interaction bet. q and Elect. field or Q and vice versa.

$$\vec{F}(r) = \vec{E}(r) \times q.$$

Remarks

① If q → unity, Electric field due to Q is numerically equal to the force exerted by it. [F = E×1 ; F = E] Thus, electric field due to a charge Q at a point in space may be defined as the force that a unit +ve charge would experience if placed at that point.

Q → produce electric field → Source Charge.

q → test effect of a source charge : Test charge.

must remain at it's orig. location. but → when q is brought near Q, Q experiences an electric F and tend to move.

make q negligibly small. thus F becomes negligibly small but ratio F/q is finite and defines electric field. ① ← (way out)

$$E = \lim_{q \to 0} = \left(\frac{F}{q}\right)$$

Practical way :- hold Q to it's location by unspecified forces. ②

eg : when we consider electric F on test charge q due to a charged planar sheet, the charges on the sheet are held to their location by unspecified charged constituents inside the sheet.

② $\vec{E}$ is independent of q (even though $\vec{E}$ is defined operationally in terms of q)
(electric field)

↓ (reason)

$F \propto q$ so F/q doesn't depend on q.

$\vec{F}$ on q by Q depend on the location of q → can take any value in the space around Q.

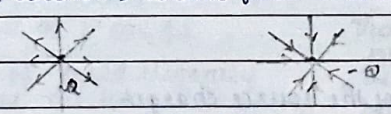
thus Q Elect. field E due to Q also depend on space coordinate $\vec{r}$.

(as F depent on r, E also depent on r)

different position of q → different value for $\vec{E}$.

Exist at every point in 3-D space. → Field.

③ +ve charge source → electric field → directed radially outwards
-ve " → radially inwards.



④ Electric field has spherical symmetry → magnitude of $\vec{E}$ due to a point charge same on a sphere with point charge at centre.

↓ equal distance from Q, magnitude of $\vec{E}$ same.

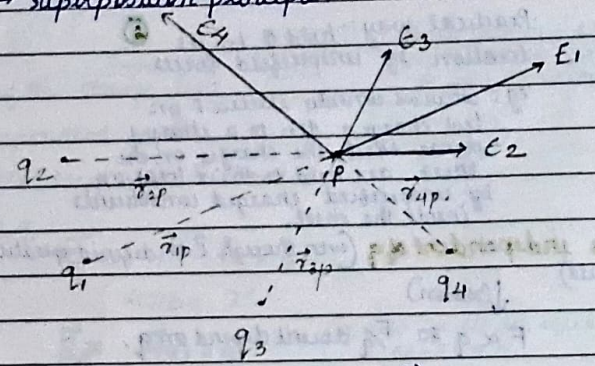
[F depend on r so magn. of E also dep. on r]

Electric field due to a system of charges

system of charges → $q_1, q_2 \ldots q_n$
Position vectors → $\vec{r}_1, \vec{r}_2 \ldots \vec{r}_n$. (relative to some origin O)

electric field at a point due to a system of charges → defined by
let us denote by position vector $\vec{r}$
force experienced by a unit test charge at that point. without disturbing the orig. position of charges $q_1, q_2 \ldots q_n$.

To determine this field at point P
↓
use → coulomb's law
→ superposition principle.



$$\vec{E}(r) = \vec{E}_1(r) + \vec{E}_2(r) + \ldots \vec{E}_n(r)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1p}^2}\hat{r}_{1p} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2p}^2}\hat{r}_{2p} + \ldots + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_{np}^2}\hat{r}_{np}$$

$$\vec{E}(r) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{ip}^2}\times\hat{r}_{ip}$$

E → vector quant.
determined by → position of the source charges.

Physical Significance of electric field.

Electric Field.
- Vector quantity
- Elegent way to charactrize the electrical environm. of a sys. of charges.
- convenient, but not necessary in electrostatics
- Elec. fiel. at a point tells U the force a unit +ve charge will exper. if placed at that point.
- Charectiristic of the sys. of charges → independent of test charge.
- field (physics) refers to a quant. that is defined at every point in space & may vary from point to point.

True phys signif. of electric field → dealing with time dependent electro-magnetic phenomena.

Consider distant charges $-q_1$ & q_2 in accelerated motion.
Greatest speed signal/infor. can go from 1 point to another → c (speed of light).

Thus, effect of q_1 on q_2 cannot arise instantaneously
↓
will be time delay bet. effect & cause
effect → F on q_2
cause → motion of q_1

concept of field → Ist intro. by Faraday

The accelerated motion of charge q_1 produces electromagnetic waves, which then propogate with the speed of c, reach q_2 and cause of F on q_2.
← This is wire notion of electric field is useful.

The notion of field elegantly accounts for the time delay.

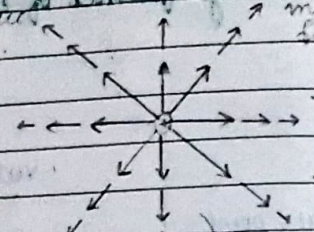
Thus, a source of time dependent electromagnetic fields, turned on for a short time interval and then switched off, leaves behind propogating electromagnetic fields transporting energy.

Thus, even though electric & magnetic fields can be detected only by their effects on charges → considered as physical entities. They can also transport energy. have their own independent dynamics.
↓
evolve according to laws of their own.

each arrow → indicates field strength experienced by a unit +ve charge placed at its tail end.

Electrical Field lines

Representing $\vec{E}$ pictorically

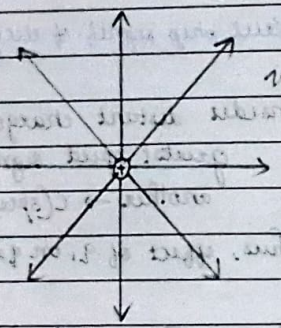


- place point charge at origin.
- Draw vectors along the direction of electric field.
- Length of line proportional to strength of elec field → Since magnitude of $\vec{E} \propto \frac{1}{r^2}$, the vector gets shorter & shorter as we move away from the origin.

info about field strength → density of field lines & length of arrows

If we want to estimate density of field lines (3-D)
↓
consider no. of lines per unit cross-sectional area
↓
perpendicular to the lines

connecting the arrows pointing in one direction gives you a field line



- electric field decreases → as the square of distance from a point charge
- area enclosing the charge increases as the square of the distance (sphere's area)

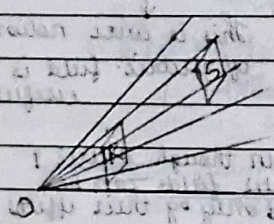
→ no. of field lines crossing the enclosing area remains constant, whatever maybe the distance of the area from the charge.

information about magnitude of field strength

density of field lines

denser near charge, as E stronger

no. of lines not imp (infinite lines can be drawn). relative density of lines → imp.



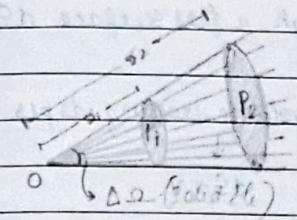
R, S → equal elements of area placed at 2 points R & S.

no of field lines ∝ strength of $\vec{E}$.

Thus $\vec{E}$ at R stronger.

Dependence of field lines on area / solid ∠le subtended by an area element.

For a given solid ∠le → no. of radial field lines same.



$P_1 = r_1^2 \Delta\Omega \quad (\Delta\Omega = \frac{\Delta S}{r^2})$

$P_2 = r_2^2 \Delta\Omega$

2-D → plane ∠le, $\Delta\theta = \frac{\Delta l}{r}$

3-D → solid ∠le, $\Delta\Omega = \frac{\Delta S}{r^2}$

no. of lines (n) cutting these area elements (P_1, P_2) are the same. ∴ $\frac{n}{r_1^2 \Delta\Omega} = \frac{n}{r_2^2 \Delta\Omega}$ ⇒ $\frac{1}{r^2}$ dependence.

Picture of field lines (invented by) → Faraday.

Faraday called them 'lines of force'. It is misleading. appropriate term → field lines.

↓ (reason for invention)

develop intuitive, non mathematical way to visualize electric field around charged configurations.

Electric field line: is a **curve** drawn in such a way that the **tangent** to it at each point is in the **direction of the net field** at that point.

→ arrow necessary (tangent has 2 possible directions)

space curve (curve in 3-D)

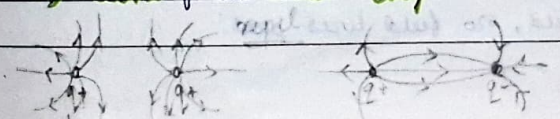
Field lines follow → imp general properties

i) **Start** at +ve charge, **end** at −ve charge. If only a single charge, start/end at infinity.

ii) Charge free region → field lines: **continuous curves** without breaks

iii) 2 field lines **never cross**. If they did, then point of intersection has → 2 directions (absurd)

iv) **don't form closed loops**. (Conservative nature of electric field)



+ve charges never cross
closed loops

Electric Flux

Intro → similarity to water flux.

liquid flow → velocity $\vec{v}$, through a flat surface dS (area)

$\vec{v}$ normal to dS.

rate of flow of liquid → volume crossing dS per unit time

↓

$dS \times v$

flux of liquid

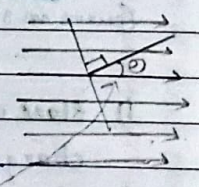
If normal to the surface isn't parallel to direction of $\vec{v}$, but makes $\angle$le θ with it, then projected area in a plane $\perp$ to $\vec{v}$ is $\delta dS\cos\theta$ ∴ flux going out of the surface is $\vec{v}\cdot\hat{n}dS$

no. of field lines crossing a unit area (perpendicular to the field) → measure of field strength

if we place a small planar element of area ΔS normal to $\vec{E}$ at a point,

no of field lines ∝ both $\vec{E}$ & ΔS [area increase more lines pass.]

∴ no of field lines ∝ $E\Delta S$



Now, tilt area element ΔS by an $\angle$le of θ.

Clearly no of field lines passing ΔS decreases.

Here, no of field lines ∝ $E\Delta S\cos\theta$

$\theta = 0°$, ΔS normal to the field, max field lines pass.

$E\Delta S\cos 0° = E\Delta S \times 1 = E\Delta S$

$\theta = 90°$, ΔS parallel to the field, no field lines pass.

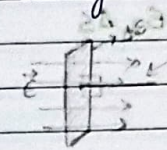
$E\Delta S\cos 90° = E\Delta S \times 0 = 0$

In a stream, amount of water entering a ring depends on how you hold it. Thus should be treated as a vector

An area element → vector : both direction & magnitude
(ΔS in this case)

↓

along its normal.



What if the area is a curved surface? how to associate a vector?

↓

divide the surface into a large no. of very small area elements.

↓

each area element treated as planar and vector associated to it.

direction of area element ← ambiguity (uncertainty / ambiguity)

↓

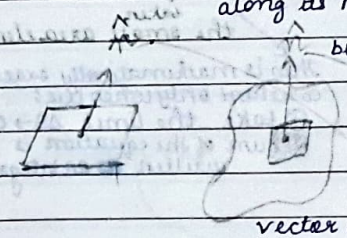
along its normal

↓

but normal can have 2 directions

↓

which direction to choose?



↓ resolved by some convention appropriate to context.

vector associated with every area element of a closed surface is taken to be in **direction of the outward normal**.

↓

$\hat{n}$

Thus area element vector $\Delta\vec{S}$ = $\Delta S \times \hat{n}$
at a point

direction. magni. direct. of outward normal at that point

definition

Electric flux $\Delta\phi$ through area element $\Delta\vec{S}$ is defined by

$$\Delta\phi = \vec{E}\cdot\Delta\vec{S} = E \times \Delta S \times \cos\theta$$ → $\angle$le bet. $\vec{E}$ & $\Delta\vec{S}$ (normal)

($\Delta\phi$ ∝ no of field lines cutting through ΔS)

SI unit of elec. flux = $\frac{N}{C} \times m^2$

Can look at expression $E \cdot \Delta S \cdot \cos\theta$ in 2 ways

- $E(\Delta S \cdot \cos\theta)$: E times the projection of area normal to E.

- $E_{\perp} \Delta S$: component of E along the normal to the area element TIMES the magnitude (mod) of area element.

Simple definition for electric flux → $\Delta\phi = \vec{E} \cdot \Delta\vec{S}$

In Principle, to calculate total flux → div the surface into small area elements, calcu flux at each elem. add them. thus

$$\phi \simeq \sum \vec{E} \cdot \Delta\vec{S}$$

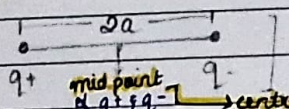
Total flux through surface S.

approx. sign, $\vec{E}$ is constant over taken the small area elem.

This is mathematically exact constant only when the:
① take the limit $\Delta S \to 0$
② Sum of the equation is written as an integral

Electric dipole

Electric dipole is a pair of equal and opp. point charges q & $-q$ seprated by a distance of $2a$.



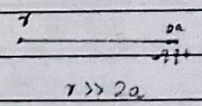
A dipole

direction of the line. By convention direction from $-q$ to q. (direction of a dipole)

net charge → 0 [$-q+q$]

electric field → not 0 [$-q$ and q seprated by some distance]

↳ but, when $r \gg 2a$ field due to q and $-q$ nearly cancel out.



at distances much large than seperation bet. 2 charges. (almost concentric)

thus, elect field due to dipole, fall off faster than field due to a single charge at large distance.

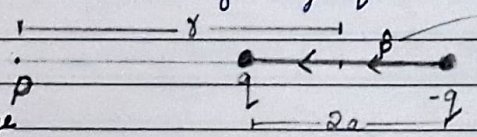
Field of an electric dipole

1. when dipole point is on dipole axis
2. point on equitorial plane of dipole. ↳ ⊥ to dipole axis through its centre

① For points on the axis

for point P at distance r from centre of dipole on the side of charge q.

Electric field at P, obtained by adding electric fields E_{-q} due to charge $-q$ and E_q due to charge q by ||gm laws of vectors.



$$E_{-q} = \frac{1}{4\pi\varepsilon_0}\frac{-q}{(r+a)^2}\vec{p}$$

$$E_{+q} = \frac{1}{4\pi\varepsilon_0}\frac{+q}{(r-a)^2}\vec{p}$$

Total field $E = E_{-q} + E_{+q} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\vec{p}$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+a)^2 - (r-a)^2}{(r-a)^2 \times (r+a)^2}\right]\vec{p}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r^2+a^2+2ar-r^2-a^2+2ar}{((r-a)(r+a))^2}\right]\vec{p}$$

$$E_{+q} + E_{-q} = \frac{q}{4\pi\varepsilon_0}\left[\frac{4ar}{(r^2-a^2)^2}\right]\vec{p}$$

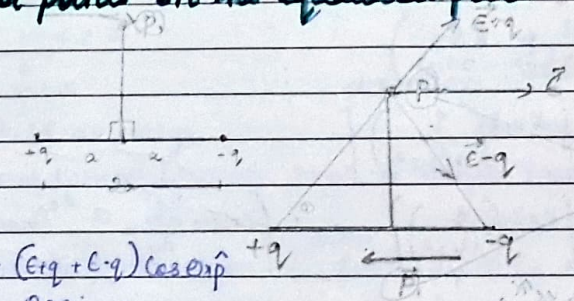
(electric field on p)

for $r \gg a$ [ignore a in denom. but not numer. (4 a.t, $a^2 = 0.01$ (small value so can ignore it but not a))]

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{4ar}{r^4}\right]\vec{p}$$

$$\vec{E} = \frac{4aq}{4\pi\varepsilon_0 r^3}\hat{p} \quad (r \gg a)$$

② For points on the equatorial plane.



$$E = -(E_{+q} + E_{-q})\cos\theta\,\hat{p}$$

$$= -\frac{2aq}{4\pi\varepsilon_0}\frac{1}{(r^2+a^2)^{3/2}}$$

at large distance $r \gg a$, $\vec{E} = \frac{-2aq}{4\pi\varepsilon_0 r^3}\hat{p}$

different top P → charge that experience the field

bottom P → dipole moment

$\vec{P}$ → dipole moment vector. direction → from -ve charge to +ve.

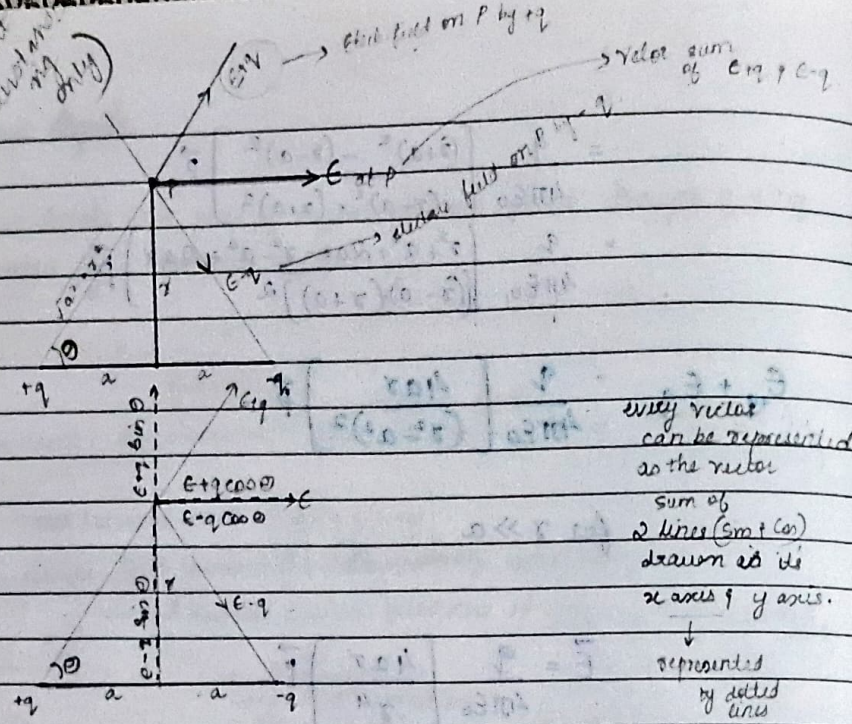
$P = 2aq$

$\vec{P} = 2aq \times \hat{p}$

along → $\vec{E} = \frac{KP}{r^3}$

on ⊥ → $\vec{E} = \frac{-KP}{r^3}$

This page is just for understanding only



every vector can be represented as the vector sum of 2 lines (sin + cos) drawn as the x axis + y axis.
↓
represented by dotted lines for understanding here

$E_{+q}\sin\theta$ + $E_{-q}\sin\theta$ cancels (equal and opp.)

- $E_{+q}\cos\theta$ + $E_{+q}\sin\theta$ → rep → E_{+q} line
- $E_{-q}\cos\theta$ + $E_{-q}\sin\theta$ → rep → E_{-q} line

$$E = E_{+q}\cos\theta + E_{-q}\cos\theta$$

$$E = \cos\theta\,(E_{+q} + E_{-q})$$

$$= \cos\theta\left(\frac{Kq}{r^2+a^2} + \frac{Kq}{r^2+a^2}\right)$$

$$= \cos\theta\left[K \times 2\left(\frac{q}{r^2+a^2}\right)\right]$$

$$= \frac{a}{\sqrt{a^2+r^2}}\left[K \times 2\left(\frac{q}{a^2+r^2}\right)\right] \qquad \left[\cos\theta = \frac{adj}{hy} = \frac{a}{\sqrt{a^2+r^2}} \text{ (from fig)}\right]$$

$$= \frac{a}{(a^2+r^2)^{1/2}} \times \frac{K2q}{(a^2+r^2)^1} \rightarrow \frac{2aqK}{(a^2+r^2)^{\frac{1}{2}+1}} \Rightarrow \frac{2aqK}{(a^2+r^2)^{3/2}}$$

when $r \gg a$ (ignore a^2 in denom.)

$$= \frac{2aqK}{r^{2\times 3/2}} \Rightarrow \frac{2aqK}{r^3} \quad \boxed{\frac{2aq}{4\pi\varepsilon_0 r^3}}$$

dipole moment

→ ability of a dipole to rotate in an external field

- electric f. on points on the axis → $\vec{E} = \frac{4aq}{4\pi\varepsilon_0 r^3}\hat{p}$
- elec f. on points on equi. plane → $\vec{E} = -\frac{2aq}{4\pi\varepsilon_0 r^3}\hat{p}$

In terms of $\vec{p}$, elec field of a dipole at large distance:

- At points on dipole axis

$$E = \frac{2\vec{p}}{4\pi\varepsilon_0 r^3} \quad (r \gg a)$$

- At points on equit. plane

$$E = \frac{-\vec{p}}{4\pi\varepsilon_0 r^3} \quad (r \gg a)$$

clear that dipole field at large distance does not involve q and a seperately. It depends on the product qa. → This suggests the definition of dipole momond. dipole moment vector $\vec{p}$ of an electric dipole is defined by,

$$\vec{p} = 2aq\,\hat{p}$$

(i.e. not magnitude, $p = 2aq$)

Thus for $\vec{p}$,
magnitude → charge q times the seperation 2a bet. q+ + q-.
direction → along the line for -q to q.

- dipole field at large distances → $\frac{1}{r^3}$ not $\frac{1}{r^2}$.
- magnitude + direction of dipole field depends not only on distance r but also on the ∠le bet. the position vector $\vec{r}$ and dipole moment $\vec{p}$.
- we can think of a limit when dipole size 2a approaches 0, the charge q approaches infinity such that product $p = q \times 2a$ remains finite. Such dipole → refered to as point dipole. For point dipole equations are exact for any r.

Physical Significance of Dipoles.

- Most molecules → centres of +ve & -ve charge lie at the same place (coincide)

∴ Dipole moment is zero.

But may develop a dipole moment when an electric field is applied

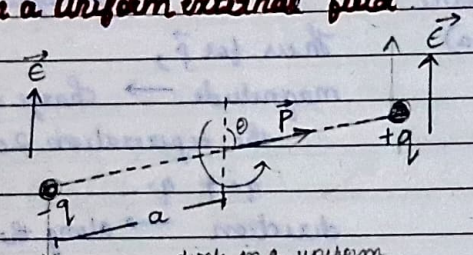
eg: CO_2, CH_4

- Some molecules → centres of +ve & -ve charge doesn't coincide

∴ have permanent electric dipole moment (even in the absence of electric field)

Polar molecules. eg: H_2O

Dipole in a Uniform external field



dipole in a uniform electric field

net induced by $\vec{E}$ exist independent of $\vec{E}$.

Consider a permanent dipole of dipole moment $\vec{p}$ in uniform electric field $\vec{E}$. → same magnitude throughout.

Force on q = $\vec{E} \times q$ $\left[F = E \times q \text{ i.e. } E = \frac{q}{4\pi\epsilon_0 r^2},\ F = \frac{Qq}{4\pi\epsilon_0 r^2}\right]$

Force on q⁻ = $\vec{E} \times -q$

net force on dipole = 0. [net force → force on q & q⁻ = $\vec{E} \times q + -\vec{E}q = 0$]

→ same value → since it's uniform

Since charges q & -q are separated, force act at different points resulting in a torque on the dipole.

When net force is zero, torque (couple) is independent of the origin.

$\vec{A} \cdot \vec{B} = AB\cos\theta$ $\boxed{\downarrow\uparrow \rightarrow \text{antiparallel}}$

$\vec{A} \times \vec{B} = AB\sin\theta$

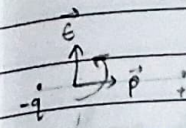
Magnitude of torque equals magnitude of each force multiplied by the arm of the couple (⊥ distance bet the 2 antiparallel forces)

Magnitude of torque = $qE \times 2a \sin\theta$.
= $2qaE\sin\theta$.

direction = normal to the paper - coming out of it → in this case.

To find direction, can use screw rule / right hand thumb rule.

Screw → when we rotate a screw in clockwise direction, it goes into the plane. Anticlockwise → then out of plane



In this case (fig) it's anticlockwise thus, out of the plane

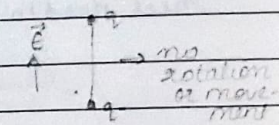
magnitude of torque = $2qaE\sin\theta$
= $pE\sin\theta$ (p = 2qa)

direction = normal to plane coming out of it.

Thus $\tau = \vec{p} \times \vec{E} = \underbrace{2a}_{\text{distance}}\ \underbrace{qE}_{\text{force}}$ (distance → m, force → N, so unit of torque → Nm)

Torque will tend to align the dipole with field $\vec{E}$.

When $\vec{p}$ aligned with $\vec{E}$, torque is zero.



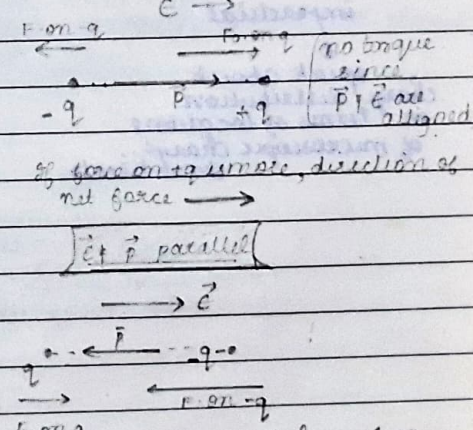
→ no rotation or movement

Dipole on a non-uniform field

net force won't be 0.

If $\vec{p}$ & $\vec{E}$ are ∥ or anti∥ then torque = 0.

→ This net force depends on the orientation of $\vec{p}$ with respect to $\vec{E}$.



no torque since $\vec{P}$ & $\vec{E}$ are aligned

If force on +q is more, direction of net force →

$\boxed{\vec{E} \text{ \& } \vec{P} \text{ parallel}}$

direc. of net force ←

$\boxed{\vec{E}, \vec{P} \text{ antiparallel}}$

This helps us understand usual paper pictures to combs.

PTO.

Reason. (comb & paper)
comb acquires charge through friction. This charged comb polarises the paper particle, induces a net dipole moment in the direction of the field. Further, the electric field due to the comb is non-uniform. This non-uniformity make a dipole to experience a net force on it. Thus paper moves in the direction of the comb.

Continous Charge distribution

So far dealt with charged configurations involving discrete charges $q_1, q_2 \dots q_n$ → but impractical.

(reason)
- mathematical treatment simpler
- does not involve calculus

Thus need to use continous charge distribution than discrete charge distri.

Surface of a charged conductor

impractical → speak about charge distribution in terms of locations of microscopic charge constituents.

To consider charge distribution
↓
feasible method,
↓
consider area element ΔS on the surface of the conduc. ← very small in macroscopic level but big enough to have a large no. of electrons
& specify charge ΔQ on the element.

Then, we define **surface charge density** as $\sigma = \frac{\Delta Q}{\Delta S}$ (C/m^2)

$\frac{\Delta Q}{\Delta S}$

we can do this at diff. points in the conductor and arrive at a **continous function** σ called surface charge density

σ → ignores
- quantisation of charge
- discontinuity in charge distribution at microscopic level.

σ → represent macroscopic surf. char. dens.
↑
smoothed out average of the microscopic char. dens. over an area element ΔS.

Line charge distribution

$\lambda = \frac{\Delta Q}{\Delta l}$ = **linear charge density** (C/m)

Δl → small line element of wire → macroscopic scale
ΔQ → charge contained in that line element.

Volume charge distribution

volume charge density (or simply charge density)

$\rho = \frac{\Delta Q}{\Delta V}$ (C/m^3)

ΔQ → charge included in macroscopically small vol. elem. ΔV

obtained same way as for discrete charges.

Field due to a Continuous Charge distribution.

Suppose, a continuous charge distribution in space
↓
charge density → ρ.

Choose a convinient Origin O.
(drawn from O)
Let position vector of any point in the charge distribution be $\vec{r}$.

ρ is a function of $\vec{r}$. (ρ varies with $\vec{r}$)

divide charge distribution into small volume element of size ΔV

The charge in $\Delta V = \rho \Delta V$ ($\Delta q = \rho \times \Delta V$)

Consider point P with position vector $\vec{R}$.

Electric field by charge $\Delta V \rho$ is given by:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \times \frac{\rho \Delta V}{r'^2} \times \hat{r}'$$

r' → distance bet. charge element & P.
$\hat{r}'$ → unit vector in direction from charge element to P.

Now, by superposition principle, total electric field due to the charge distribution obtained by: Summing over electric fields due to different volume elements:

$$\vec{E} \cong \frac{1}{4\pi\varepsilon_0} \sum_{\text{all } \Delta V} \frac{\rho \Delta V}{r'^2} \hat{r}'$$

$\rho, r', \hat{r}'$ → can vary from point to point.

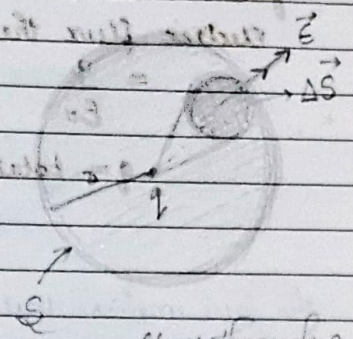
To find electric field → use Coulomb's law & Superposition principles
of · any charged dist.
· discrete / continuous
· part discrete / part continuous

In strict mathematical method let $\Delta V \to 0$, then sum becomes an integral. (but not used for simplicity)

→ significance: true in general, not only for simple cases.

Gauss's Law

Consider total flux through a sphere of radius r which encloses a point charge q at its centre.



div the sphere into small area elements ΔS.
The flux through area element ΔS is

$$\Delta\phi = \vec{E} \cdot \Delta\vec{S}$$
$$= \frac{q}{4\pi\varepsilon_0 r^2} \times \hat{r} \cdot \Delta\vec{S}$$

flux through a sphere enclosing a point q at the centre.

· Unit vector $\hat{r}$ is along the radius vector from the centre to area element.
· normal to a sphere at every point is the along the radius vector at that point
So, area element $\Delta\vec{S}$ & $\hat{r}$ have same direction.

$$\therefore \Delta\phi = \frac{q}{4\pi\varepsilon_0 r^2} \times \Delta S$$ (Since magnitude of a unit vector is 1)

flux through ΔS.

Total flux through the sphere → adding up flux through all the different area elements:

$$\phi = \sum_{\text{all } \Delta S} \frac{q}{4\pi\varepsilon_0 r^2} \times \Delta S$$

Since, each area element of sphere is at the same distance r from the charge,

$$\phi = \frac{q}{4\pi\varepsilon_0 r^2} \sum_{\text{all } \Delta S} \Delta S$$
$$= \frac{q}{4\pi\varepsilon_0 r^2} \times S$$

area of sphere $= 4\pi r^2$.

$$\phi = \frac{q}{4\pi\varepsilon_0 r^2} \times 4\pi r^2 \Rightarrow \frac{q}{\varepsilon_0}$$

→ is stated without proof

Gauss's law states:

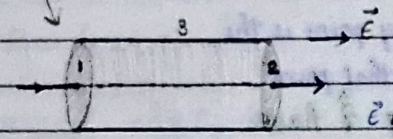
electric flux through a closed surface S

$$= \frac{q}{\epsilon_0}$$

q = total charge enclosed by S.

The law implies that - total electric flux through a closed surface is **zero if no charge is enclosed** by the surface.

(example)



$\vec{E}$ is uniform.

consider a closed cylindrical surface, its axis parallel to the uniform electric field $\vec{E}$.

Total flux $\phi = \phi_1 + \phi_2 + \phi_3$ + where ϕ_1 & ϕ_2 → flux through surface 1 & 2. ϕ → flux through the curved cylindrical part.

$\phi_3 = 0$. → normal to surface 3 at every point is ⊥ to $\vec{E}$, So $\cos 90° = 0$; $\phi_3 = 0$.

$\phi_1 = -ES_1$ [normal to 1 → opp to $\vec{E}$]

$\phi_2 = +ES_1$ [normal to 2 → along $\vec{E}$.]

$S_1 = S_2 = S$ → area of circular cross section

Thus, total flux = 0. $\left[\phi_1 + \phi_2 + \phi_3 = 0 \quad \underset{0}{} + ES_2 - ES_2\right]$

So, whenever net electric flux through a closed surface is zero, total charge contained in the closed surface is zero.

Imp points regarding gauss's law:

(i) Gauss's law is **true for any closed surface**, no matter what it's shape/size

(ii) The term q includes **the sum of all charges enclosed** by surface. The charges may be located anywhere inside the surface.

(iii) In the situation when the surface is so chosen that there are some charge inside and some outside then,
electric field → due to all charges (inside & outside S)
q → represent **only the total charge inside S.**

(iv) **Gaussian surface** can pass through a continuous charge distrib.
can't pass through any discrete charge

surface chosen for the application of gauss's law.

electric field due to a system of discrete charges is not well defined at the location of any charge.

(v) Gauss's law → **useful** → easier **calculation** of electrostatic field when system has some **symmetry**

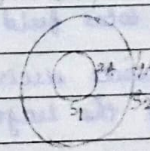
Facilitated by - choice of a suitable gaussian surface.

(vi) Gauss's law is **based on → inverse square dependence on distance** contained in the Coulomb's law.

any violation of Gauss's law - indicate departure from the inverse square law.

Example: Gaussian surface can pass through a cont. charge dist.

Question:



S_1 & S_2 → 2 spheres.

Find: ratio of electric flux $S_1 : S_2$

$\phi(S_1) = \frac{Q}{\epsilon_0} = 2/\epsilon_0$

$\phi(S_2) = 3/\epsilon_0 - 6/\epsilon_0$

ratio $\frac{2/\epsilon_0}{6/\epsilon_0} = 1:3$

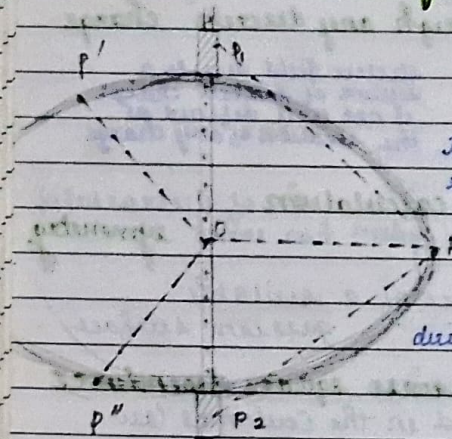
Applications of Gauss's law

$E \cong \frac{1}{4\pi\varepsilon_0} \sum_{\Delta V} \rho \Delta V / r'^2 \times \hat{r}'$ → electric field due to a general charge distribution.

Most cases → summation / integration involved in this equation cannot be carried out to give electric field at every point in space.

For some symmetric charge distributions configurations → possible to obtain the electric field in a simple way. Using Gauss's law. (3 cases)

· Field due to an infinitely long straight uniformly charged wire.



consider - infinitely long thin straight wire uniform linear char. dens. λ

wire is obviously an axis of symmetry.

Take radial vector from O to P and rotate it around the wire. point P, P', P'' are completely equivalent with respect to the charged wire. Thus it implies that at these points, electric field have same magnitude.

direction of electric field radial.

($\lambda > 0$ outward, $\lambda < 0$ inward)

consider a pair of line elements P_1 & P_2. The electric field produced by the 2 elements of the pair when summed give a resultant electric field which is radial. This is true for any such pair. hence, total field at any point P is radial.

Since, wire is infinite electric field don't depend on position of P along the length of the wire.

Electric field is everywhere radial in the plane cutting the wire normally and it's magnitude only depends on the radial distance r.

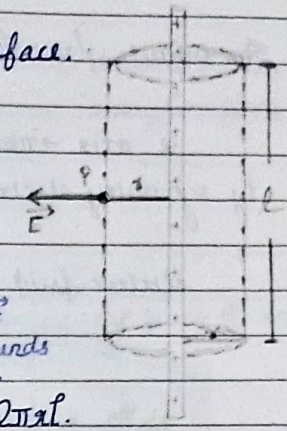
To calculate the field,

imagine a cylindrical gaussian surface.

Since $\vec{E}$ is radial, flux through the circular ends is zero.

At the cylindrical part of the surface, $\vec{E}$ is normal to the surface at every point & it magnitude is constant

↳ since magn of $\vec{E}$ in this case depends only on r.



Surface area of curved path = $2\pi r l$.

Flux through the gaussian surface

= flux through the curved cylindrical part of the surface

$= 2\pi r l \times E$

Surface includes charge = λl

$$\phi = \frac{q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

$$2\pi r l \times E = \frac{\lambda l}{\varepsilon_0}$$

$$E = \frac{\lambda l}{\varepsilon_0} \times \frac{1}{2\pi r l}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$ vectorically; $$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{n}$$

direction: inward: $\lambda(-ve)$, outward: $\lambda(+ve)$

radial unit vector in the plane normal to the wire passing through the point.

vector $\vec{A}$ is a scalar (A) multiplied by a unit vector $\hat{a}$. i.e → $\vec{A} = A \times \hat{a}$

A → algebraic no. (-ve / +ve).

when we want to restrict to non-neg. values, use symbol $|\vec{A}|$. (modulus of $\vec{A}$)

$|\vec{A}| \geq 0$.

direction of $\vec{A}$:-

same as $\hat{a}$ – $\vec{A} > 0$

opp as $\hat{a}$ – $\vec{A} < 0$

note :-

- though only charge enclosed by surface was included above, $\vec{E}$ is due to the charge of the entire wire.
- assumption that wire is infinitely long is crucial. without it, can't say $\vec{E}$ to be normal to the cylindrical gaussian surface. However, this equation is approximately true for electric field around the central portions of a long wire where end effects may be ignored.

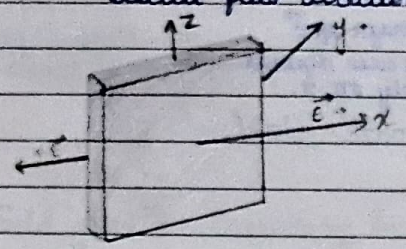
Field due to a uniformly charged infinite plane sheet

An infinite plane sheet → with uniform surface charge density $-\sigma$

x axis → normal to the plane

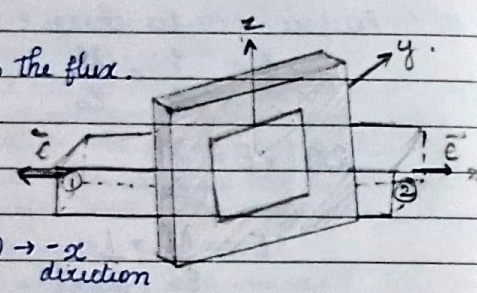
By symmetry electric field do not depend on y & z coordinates and its directions

electric field direction – always // to x axis.



Take Gaussian surface to be a rectangular parallelepiped of cross sectional area A.

Only 2 faces ① & ② contribute to the flux. electric field lines are // to the other faces so no flux.



Unit vector normal to surface ① → $-x$ direction

unit vector normal to surface ② → x direction

Flux $\vec{E}\cdot\Delta\vec{S}$ due to both surfaces add up. So, net flux through gaussian surface → $2EA$

charge enclosed by the surface → σA $\left(\sigma = \frac{\Delta q}{\Delta A}\right)$

By Gauss's law $\phi = \frac{q}{\varepsilon_0}$

$$\vec{E}\cdot\Delta\vec{S} = q/\varepsilon_0$$

$$2EA = \sigma A/\varepsilon_0 \Rightarrow E = \sigma A/\varepsilon_0 \times 2A$$

$$\therefore E = \frac{\sigma}{2\varepsilon_0}$$

vectorically, $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$. (unit vector normal to the plane going away from it)

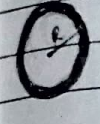
$\vec{E}$ → direction : toward plane, $\sigma(-ve)$
away from plane, $\sigma(+ve)$

from this equation

E is independent of x as well.

For finite planar sheet this equation – approx. true for middle regions away from the ends.

Field due to a uniformly charged spherical shell.



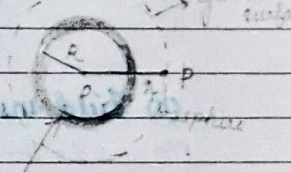
Take thin spherical shell of Radius R → uniform surface charge density σ.

electric field at any point P (inside / outside) depend only on r (radial distance from the centre of the shell to P) and P must be radial (i.e, along radius vector)

(1) Field outside the shell

point P outside the shell. Radius vector $\vec{r}$.

To calculate $\vec{E}$ at P take gaussian surface to be a sphere of radius r, with centre O passing through P



- All points on sphere – equivalent relative to the given charged configuration
- electric field at each point on the surface → has same magnitude E.
  → is along the radius vector at each point.

sur. char. dist. σ.

Thus $\vec{E}$ and $\Delta\vec{S}$ at every point are parallel and flux through each element is $E\Delta S$. Summing all ΔS, flux $= E\times S$

$\therefore \phi = E \times 4\pi r^2$ $(S = 4\pi r^2)$. charge enclosed $= \sigma \times 4\pi R^2$ $(\sigma = q/S)$

Q: A dipole is enclosed by a cube.
$\phi = 0$. (dipole moment $+q, -q$)
$\phi = \frac{q}{\varepsilon_0}, q + q + -q = 0$

thus by Gauss's law,

$$\phi = \frac{q}{\varepsilon_0} \quad (\phi = \sigma s)$$

$$E \times 4\pi r^2 = \frac{\sigma \times 4\pi R^2}{\varepsilon_0}$$

$$E = \frac{\sigma 4\pi R^2}{\varepsilon_0 \times 4\pi r^2}$$

$$E = \frac{\sigma R^2}{\varepsilon_0 r^2} \quad \left[\text{or } E = \frac{q}{4\pi\varepsilon_0 r^2} \right]$$

vectorially,

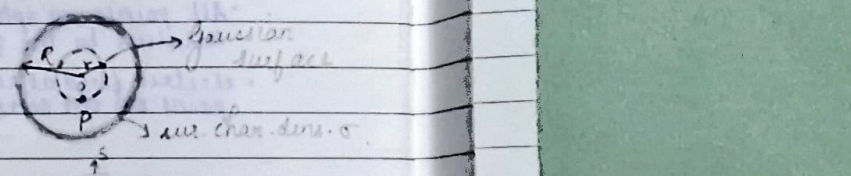
$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2} \times \hat{r}$$

directed,
outward $q > 0$
inward $q < 0$

Field produced by spherical shell is exactly the field produced by a charge q placed at centre O.
thus, for points outside the shell, field due to a uniformly charged shell is as if the entire charge of the shell is concentrated at the centre.

(ii) Field inside the shell

P → inside the shell.
Gaussian surface → sphere.
centred at O,
pass through P.



flux through gaussian surface $= E \times 4\pi r^2$
However, Gaussian surface encloses no charge. So, by Gauss's law, $E \times 4\pi r^2 = 0$

$$E = 0. \ (r < R)$$

(conclusion)

field due to a uniformly charged thin shell is zero at all points inside the shell.
(this result is very imp. because)
→ direct consequence of Gauss's law which follow Coloumb's law.

Experimental verification of this result confirms $1/r^2$ dependence of coloumb law.