March Madness Bracketology, Sports Ranking Methods

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Abstract

In March Madness we use Bracketology to predict outcomes of sports brackets. As with any sports, sports betting is a huge industry. Every year billions of dollars are bet on many major sports tournaments,including march madness. Bracketology offers two effective methods in predicting the outcomes of these brackets thus revealing important over/under gambling odds, which are an essential part in the sports betting world. More specifically bracketology uses two main methods. First it uses the Massey Method, based on the theory of least squares, which ranks teams by point differential. Secondly it uses the Colley Method based on Laplace rule of succession, which ranks teams on a modification of winning percentage. We apply these methods to help predict outcomes of March Madness brackets. We compare results of these methods with each other, and with real results from an actual bracket to analyze their efficacy and the differences between them. More generally, these proposed methods offer a flexible approach to conditional density forecasting for a broad class of applications.

1 Introduction

Sports betting is a massive industry in today's world. Every year billions of dollars are bet on different sports events. In this paper we will focus on the college basketball NCAA tournament, otherwise known as March Madness.Every year over 40 million people make predictions for March Madness Brackets filling out about 70 million brackets each year. Their objective is to successfully predict the winner of each game. The probability of successfully doing this is nearly zero, and although prior season performance of the teams is readily available from internet sources, a more challenging task is finding a plausible model capable of making credible predictions. Ideally, we would like a model that will predict for any particular pairing of teams the joint density of their final scores. From such a predictive density one can then design betting strategies based on point spreads, odds, the over/under, or other gambling opportunities. This is in essence what Bracketology is. In this paper we will look at 2 main methods in which we can sucessfully model these predictions and create these gambling odds.

The first method called the Massey Method was created by mathematics professor Kenneth Massey while an undergraduate student at Bluefield College in 1997, also referred to as the Point Spread Method, this method is based on the theory of least squares and dervies the ratings of each team by the sum of point differentials between each respective team. It is one of the methods currently used by the Bowl Championship Series and of coure the NCAA March Madness tournament.

The second Method called the Colley Method was developed by Wesley Colley. Colley's method of ranking, which was initially used in college football rankings prior to the change in the system, is a modification of the simplest method of ranking, the winning percentage. Colley's Method was shortly after, adopted for rankings in nearly all classes of sports . Colley's method gives us a rating for each team based on Laplace's Rule of succession, which we can use to find a ranking for the teams.

In this paper our goal is to understand the Massey and Colley Method examine their differences, and similarities, and then apply them to a real NCAA March Madness bracket. In order to do this we will first go over some definitions and formulas essential to develop and understanding of the Colley and Massey methods. Then we will go over the intuition behind these formulas and generalize these formulas for a sports competition with n teams, playing m games. Then we use both these methods to compute a simple example involving 4 teams, and compare the results from each method to each other. Finally we will use our two methods to compute a real March Madness bracket from 2013, and see how our methods did in comparison to what actually happened.

2 Background

In this section we start by defining some important terms so that we can better understand the processes behind the Massey and Colley methods. Let us start by defining terms needed to understand the Colley Method. Firstly, in both methods rankings and ratings come up alot, thus

Definition 1 (ranking)**.** *A ranking refers to a rank-ordered list of teams, where the first team is ranked the* $max\{|r_i|\}$*, where* r_i *is the* ***rating*** *of Team I.*

Definition 2 (rating)**.** *A rating (Colley Method) gives us a list of numerical scores (*$r_i \in \mathbb{Q}$*), where* r_i *is the rating of Team I, and* r_i *is computed by the* ***winning percentage*** *of each team.[3]*

Every rating gives us a ranking for the teams. These rankings will be important in use of our Colley Method. But how exactly do we come up with a rating of teams, in order to understand this let us define winning percentage.

Definition 3 (Winning Percentage)**.** *Let* t_i *be the number of times Team i has played and let* w_i *be the number of times Team i has won, then the winning percentage, thus the* ***rating*** *for Team I is given by*

$$r_i = w_i/t_i$$

[3]

Using this form of winning percentage is good but it has some key flaws, namely:

- Ties in rating often occur
- At the begining of the season ratings dont make sense (0/0)

Thus in order to correct for some these flaws the Colley Method uses Laplace's Rule

Definition 4 (Laplace's Rule)**.** *Laplace's Rule is the main idea behind Colley's method, starts with replacing the winning percentage by a slight modification of it called Laplace's rule of succession. The rating for Team i is then given by*

$$r_i = \frac{1 + w_i}{2 + t_i}$$

[3]

.Although here the addition of the 1 and the 2 may seem arbitrary, there is a precise reason for these numbers. Namely, we are equating the win/loss rating problem to the problem of locating a marker on a table by trial and error shots of dice. In other words, the expected place the die should land, is in the middle or at 1/2.[3] Comparatively, in the case for our example the expected value of a team that has no games played should be 1/2. Mathematically speaking, we are assuming a "flat" distribution, meaning that there is equal probability that the marker is anywhere on the table. When computing this explicitly, this is equivalent to finding the expected value (or weighted mean, or center of mass). If the probability distribution function of rating $\hat{r}$ is $f(\hat{r})$, then in the case of no games played (no dice thrown), $f(\hat{r}) = 1$, and the expectation value of $\hat{r}$ is

$$r = \hat{r} = \frac{\int_{r_0}^{r_1} \hat{r} \cdot f(\hat{r}) d\hat{r}}{\int_{r_0}^{r_1} f(\hat{r}) d\hat{r}} = \frac{\int_0^1 \hat{r} \cdot f(\hat{r}) d\hat{r}}{\int_0^1 f(\hat{r}) d\hat{r}} = 1/2$$

[4] The statistical expectation value of the location of the marker or the rating of your team for the one left throw (one win) case is therefore

$$r = \frac{\int_0^1 \hat{r}^2 d\hat{r}}{\int_0^1 \hat{r} d\hat{r}} = 2/3$$

The same is true for a loss except we repeat this in the opposite direction. More generally we have that for n_w wins and n_l losses

$$r = \frac{\int_0^1 (1-r)^{n_l} r^{n_w} r dr}{\int_0^1 (1-r)^{n_l} r^{n_w} dr} = \frac{1 + n_w}{2 + n_w + n_l}$$

[6] Hence we have derived our formula for Colley's Method. Now we need to define some terms necessary for the Massey Method.

The main idea proposed in the Massey Method is similar to that of the Colley Method, we still use **rating**, however in the Massey method it is defined differently.

Definition 5 (rating)**.** *A rating (Massey Method) gives us a list of numerical scores, computed by the sum of point differentials between each team.[2]*

Definition 6 (point differential)**.** *The point differential of two teams is the margin of score between the two teams* $s_j - s_k$ *, where* s_i *is the score of Team I.*

The main idea behind the Massey method is proposed in the equation

$$r_i - r_j = y_k$$

where r_i and r_j are the ratings of teams i and j respectively and y_k is the point differential or margin of victory in game k of Team I. Now, in order to generalize the Massey and Colley method to find the rating of n teams who play m games we need to set up a system of equations. But first lets show how we get there.

Since all teams begin with a rating r_k and the ratings are distributed around this initial number we have an approximation

$$\sum_{k \in O_i} r_k$$

where O_i denotes the set of teams that have played team i. Thus the ratings we get from the Colley method r_i approximately satisfy the system of n equations (n=number of teams)

$$r_i = \frac{1 + \frac{w_i - l_i}{2} + \sum_{k \in O_i} r_k}{2 + t_i}$$

Simplifying this equations on both sides we get

$$(2 + t_i)r_i - \sum_{k \in O_i} r_k = 1 + \frac{w_i - l_i}{2}$$

. Thus generalizing this for calculating ratings we have that for n teams playing m teams the system of equations for the Colley Method is

$$\begin{pmatrix} 2+t_i & -n_{12} & -n_{13} & \dots & -n_{1n} & \\ -n_{21} & 2+t_2 & -n_{23} & \dots & -n_{2n} & \\ :: & : & : & : & : & : \\ -n_{n1} & -n_{n2} & \dots & \dots & 2+t_n & \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ : \\ r_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ : \\ b_n \end{pmatrix}$$

where b_1 through b_n are each respective teams Laplace Rating in the Colley Method.[5]

For our derivation of the Massey Method we replicate the same steps but replace the **rating** from the Colley Method with the one from the Massey Method. Hence the generalized matrix for the Massey method would be

$$\begin{pmatrix} 2+t_i & -n_{12} & -n_{13} & \dots & -n_{1n} & \\ -n_{21} & 2+t_2 & -n_{23} & \dots & -n_{2n} & \\ :: & : & : & : & : & : \\ -n_{n1} & -n_{n2} & \dots & \dots & 2+t_n & \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ : \\ r_n \end{pmatrix} = \begin{pmatrix} p_1 \\ p_2 \\ : \\ p_n \end{pmatrix}$$

where p_i is the sum of all point differentials between the teams.

3 An Application of the Colley and Massey Method

In this section I will consider a simple example consisting of 4 teams, and utilize each method, to help us better understand how they work. Normally speaking these methods are used in NCAA March Madness Brackets, which contain over 20 teams, so for simplicity's sake we will start with an example with 4 teams.

$$\begin{tikzcd}
\langle A\rangle \arrow[r, dash] \arrow[d, "7"'] & \langle D\rangle \arrow[d, "10"] \arrow[dl, "3"] \\
\langle B\rangle \arrow[r, "17"'] & \langle C\rangle \arrow[ul, "3"]
\end{tikzcd}$$

Figure 1: Diagram of 4 Teams

3.1 Massey Method Applied

To better understand how this method works, we will begin with an example containing 4 Teams.Consider the four teams, Team A, Team B, Team C, and Team D seen in figure 1. Note that in Figure 1 the arrow denotes which team beat which team by how much.

Thus as we can see from Figure 1, Team A beat Team B by 7, Team C beat Team A by 3, Team D beat Team C by 10, and so on. Thus given this information we can form our Note that the point differential is the sum of each individual point differential. Hence we have the system

$$\begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ -24 \\ 13 \end{pmatrix}$$

where A,B,C and D are the ratings of each respective team.Notice that the nth row and nth column of the matrix is the number of games Team i played, and each Team j that Team i played has a -1 in their respective column. The sum of the point differentials of each Team is contained within the right most column vector. Solving for this system of equations we get

$$A = .125, B = 1.75, C = -6.00, D = 4.375$$

Hence our ranking of these teams using the Massey Method would be

1.Team D , 2. Team B ,3. Team A , 4. Team C.

3.2 Colley Method Applied

Now we will consider the same example from the previous section to demonstrate an understanding of the Colley Method. Consider the same figure from earlier with four teams: Team A, Team B, Team C and Team D.

$$\begin{tikzcd}
\langle A\rangle \arrow[r, dash] \arrow[d, "7"'] & \langle D\rangle \arrow[d, "10"] \arrow[dl, "3"] \\
\langle B\rangle \arrow[r, "17"'] & \langle C\rangle \arrow[ul, "3"]
\end{tikzcd}$$

Thus from here we can set up our matrix

$$\begin{pmatrix} 4 & -1 & -1 & 0 \\ -1 & 5 & -1 & -1 \\ -1 & -1 & 5 & -1 \\ 0 & -1 & -1 & 4 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1/3 \\ 1/3 \\ 1 \end{pmatrix}$$

Notice that from our derivation from earlier the 4 comes from the number of games played t_i plus 2. And the Negative 1's Denote the teams Team i has played. Now solving our system of equations we get our ratings and rankings for our teams where

$$A = .243, B = .236, C = .236, D = .368$$

Hence we have our rankings where Team C and Team B are tied for third.

1.Team D , 2.Team A, 3. Team C , Team B.

3.3 Massey and Colley Method Comparison

In this subsection we will compare the results of the Massey and Colley Method from our previous two sections. Notice that even with our simple example containing 4 teams, each method developed noticeably different ratings and rankings. This is because of the ranking systems, as stated before the Massey method uses point differential to obtain it's ratings,while the Colley Method uses a slight modification of winning percentage.

$$\begin{tikzcd} \langle A \rangle \arrow[r, dash] \arrow[d, "7"'] & \langle D \rangle \arrow[d, "10"] \arrow[dl, "3"] \\ \langle B \rangle \arrow[r, "17"'] & \langle C \rangle \arrow[ul, "3"'] \end{tikzcd}$$

Again referring back to our Figure 1, we note that the Colley Method doesn't take into account that Team C; although losing the same amount of games as Team B had a total point differential $r_c = -24$ while Team B had a total point differential $r_b = 7$. Hence it ranked them equally Team C.

Likewise for the Massey Method we note that Team B lost 2 out of 3 games, but Team A only lost 1 out of 2, Hence Team A had a better winning percentage. However Because Team A's point differential $r_a = 4$ was less than Team B's point differential $r_b = 7$ the Massey Method ranks Team B higher than Team A. Again intuitively we would say that Team A should be ranked higher than Team B, especially if the number of games played t_i increases.[1]

What does this imply? Judging on our results and comparison from this example we can conclude that both the Massey and Colley Method both have their flaws in calculating team ratings and rankings. However that doesn't mean that these methods are ineffective, in predicting brackets. As stated before the Massey and Colley Method are the top two methods used in predicting Sports Brackets and a huge part of the Sports Betting World.

4 The Massey and Colley Method Real World Example

Here we will use a real bracket taken from the 2013 NCAA tournament otherwise known as March Madness. We will compare the rankings from our methods to the actual bracket result. For simplicity's sake we will only consider the final 6 teams from the Bracket to predict which team will win. Here we must consider Consider the 6×6 matrix where each team has played games 1 against each other, but some teams have played more games due to more teams in their respective conference. Here in the figure we can see the results from the actual 2013 March Madness Bracket [7],

Our goal is to see if the predictions of our respective methods of the final 6 teams of each method will allign similarly to how the bracket actually turned out.

First we will compute the Massey Method where we compute the sum of point differentials in all the teams games.

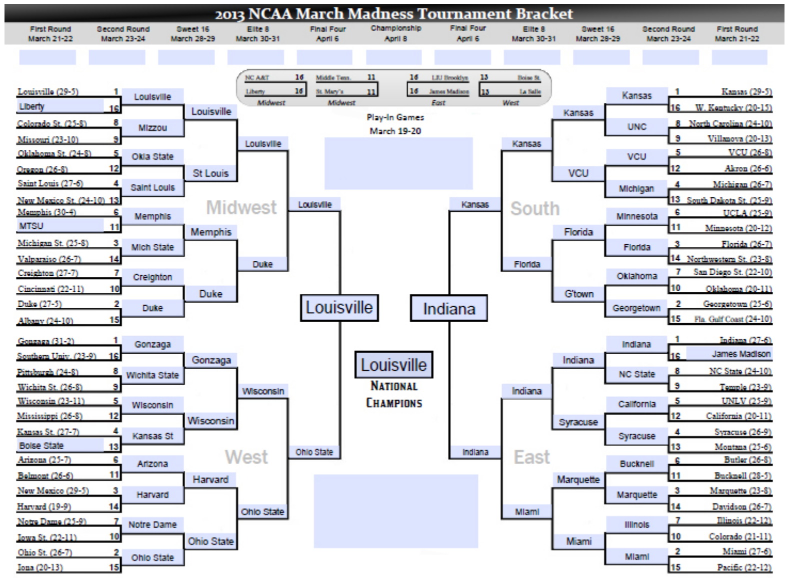

Figure 2: Results from the 2013 March Madness Bracket

$$\begin{pmatrix} 32 & -1 & -1 & -1 & -1 & -1 \\ -1 & 29 & -1 & -1 & -1 & -1 \\ -1 & -1 & 30 & -1 & -1 & -1 \\ 0 & -1 & -1 & 32 & -1 & -1 \\ -1 & -1 & -1 & -1 & 34 & -1 \\ -1 & -1 & -1 & -1 & -1 & 29 \end{pmatrix} \begin{pmatrix} Louisville \\ Kansas \\ Duke \\ OhioState \\ Florida \\ Indiana \end{pmatrix} = \begin{pmatrix} 93 \\ 82 \\ 18 \\ 13 \\ 23 \\ 44 \end{pmatrix}$$

Thus solving for this matrix we have our rankings for each team using the Massey Method:

1.Louisville, 2.Kansas, 3.Indiana, 4.Duke, 5.Ohio State, 6.Florida

Like wise setting up the matrix for the Colley Method we have

$$\begin{pmatrix} 34 & -1 & -1 & -1 & -1 & -1 \\ -1 & 31 & -1 & -1 & -1 & -1 \\ -1 & -1 & 32 & -1 & -1 & -1 \\ 0 & -1 & -1 & 34 & -1 & -1 \\ -1 & -1 & -1 & -1 & 36 & -1 \\ -1 & -1 & -1 & -1 & -1 & 31 \end{pmatrix} \begin{pmatrix} Louisville \\ Kansas \\ Duke \\ OhioState \\ Florida \\ Indiana \end{pmatrix} = \begin{pmatrix} 18 \\ 16.5 \\ 15.3 \\ 8 \\ 6.5 \\ 14.43 \end{pmatrix}$$

Now Calculating the ranking from the Colley Method we have :

1.Louisville, 2.Indiana, 3.Kansas, 4.Duke, 5.Ohio State ,6.Florida

As we can see from our rankings the Massey and Colley Methods ranked the respective teams very similarly, with Indiana being switched with Kansas in each respective ranking. However although each method ranked the teams slightly different, they both predicted Louisville to win the tournament. Now comparing this to the actual result of the March Madness Bracket seen in Figure 2 we can observe that Louisville did in fact win. Furthermore the Colley Method predicted the bracket results perfectly with each team ranked in accordance to the actual result of the bracket, while the Massey Method wrongly predicted Kansas over Indiana. However judging from our results from these two Methods on an actual bracket compared with the actual results we can see how well these methods do in ranking teams.

References

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