Computer Science 350

Assignment One

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1. is no output alphabet (d)
From class, a DFA is a five-tuple $M = (Q, \Sigma, \delta, s, F)$
Where:

1. Q is the finite set of machine states
2. Σ is the finite input alphabet
3. δ is a transition function from $Qx\Sigma$ to Q
4. $s \in Q$ is the start state
5. $F \subseteq Q$ is the accepting state

There is no mention of an "Output alphabet" in the five-tuple DFA
...

2. The answer is 7. With the Kleene star on the alphabet $(0, 1)$ and with the string length being at most 2, we have:
$L = \{\epsilon, 0, 1, 00, 01, 10, 11\}$
$|L| = 7$
...

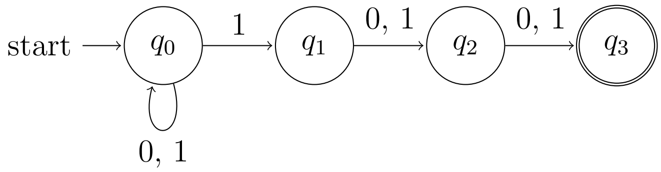
3. (A) $input \in \Sigma^*$ Because of "input" being lowercase "i"

4. The set $(A^* \cap B) \cup (B^* \cap A)$ is equal to $\emptyset$
for $A^* \cap B = \emptyset$
$= \{\epsilon, Hello, World, HelloWorld, HelloHello, \dots\} \cap \{Input, Output\} = \emptyset$
for $B^* \cap A$
$= \{\epsilon, Input, Output, InputOutput, InputInput, \dots\} \cap \{Hello, World\} = \emptyset$
$\emptyset \cup \emptyset = \emptyset$ Therefore the set is the empty set

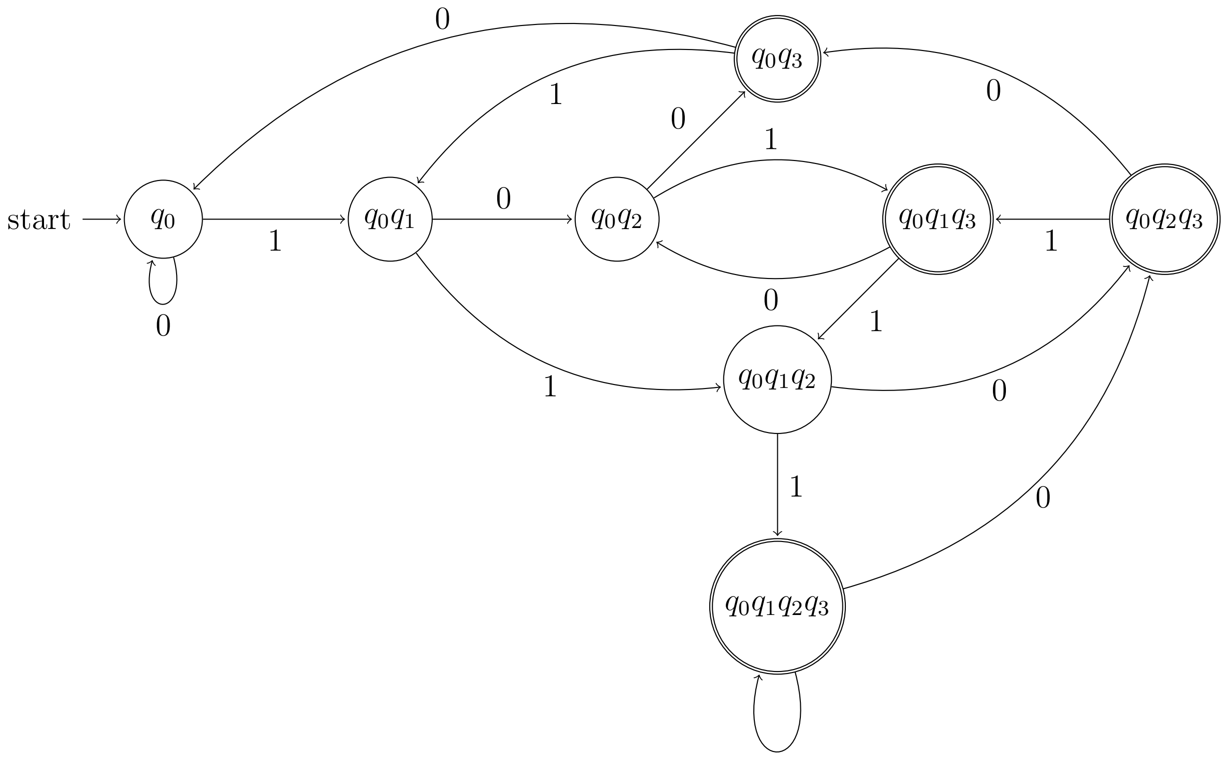
. . .

5.

NFA M


start
q_0
q_1
q_2
q_3
1
0, 1
0, 1
0, 1

DFA N

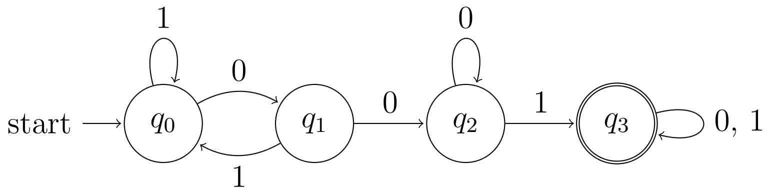

start
q_0
q_0q_1
q_0q_2
q_0q_3
$q_0q_1q_3$
$q_0q_2q_3$
$q_0q_1q_2$
$q_0q_1q_2q_3$
0
1
0
1
0
1
0
1
0
1
1
0
1
0
1
0

6.

a) $L = \{w \in \{0,1\} \mid w \textit{ contains the substring } 001\}$

b) We notice that L contains infinitely many strings, for example $\{(001)^n | n > 1\} \in L$. we also notice that there are infinitely many strings not in L, for example $\{(01)^n | n > 1\} \notin L$. These are both infinitely large sets, however, one is in L and the other is not.

c) DFA M



First we take $u \in L, u = \sigma_1 \ldots \sigma_n, \sigma_{n+1}, \sigma_{n+2}, \ldots \sigma_k$ where $\sigma_n, \sigma_{n+1}, \sigma_{n+2} =$ 001. We take the sequence $r_0 \ldots r_n, r_{n+1}, r_{n+2}, \ldots, r_k$ where $r_i = q_0$ for $0 \geq i \geq n$, and $r_{n+2} = q_3$. Now we will show that this sequence of states is indeed an accepting trace of u on DFA M. Observe that $r_0 = q_0 \in S$ and $r_{n+2} = q_3 \in F$. Further, note that $r_n = q_1 \in \delta(r_{n-1}, \sigma_n) = \delta(q_0, 0)$, $r_{n+1} = q_2 \in \delta(r_n, \sigma_{n+1}) = \delta(q_1, 0)$, $r_{n+2} = q_3 \in \delta(r_{n+1}, \sigma_{n+2}) = \delta(q_2, 1)$. Notice this is the only trace to get to the accepting state, hence, u is accepting by the DFA M.

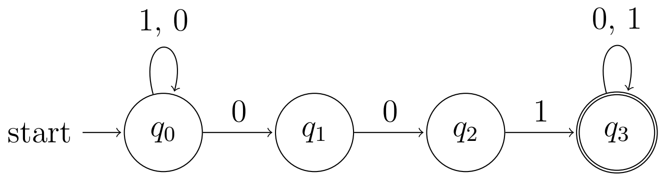
Now we take $u \in L(M), u = \sigma_1 \ldots \sigma_n, \sigma_{n+1}, \sigma_{n+2}, \ldots \sigma_k$ where $\sigma_n, \sigma_{n+1}, \sigma_{n+2} =$ 001. Note that there is some trace $r_0 \ldots r_n, r_{n+1}, r_{n+2}, \ldots, r_k$ where $r_0 = q_0$, $r_n = q_1$, $r_{n+1} = q_2$ and $r_{n+2} = q_3$. Note that:
$r_{n+2} = q_3 \in \delta(q_i, x)$ iff $q_i = q_2$ and $x = 1 = \sigma_{n+2}$
and either
$r_{n+1} = q_2 \in \delta(q_i, x)$ iff $q_i = q_2$ and $x = 0 = \sigma_{n+1}$
$r_n = q_2 \in \delta(q_i, x)$ iff $q_i = q_2$ and $x = 0 = \sigma_n$
Or
$r_{n+1} = q_2 \in \delta(q_i, x)$ iff $q_i = q_2$ and $x = 0 = \sigma_{n+1}$
$r_n = q_1 \in \delta(q_i, x)$ iff $q_i = q_1$ and $x = 0 = \sigma_n$

So, $run() = q_{0_{r_1}} \ldots q_{0_{r_{n-1}}} q_1 q_2 q_3 \ldots {q_3}^k$ so it must be the case that $u = (0,1)^j 001 (0,1)^k \in L$

d) $L = (1+0)^* \cdot (001) \cdot (0+1)^*$

e) So, in this case $P = 001$ and $A(P) = \{u \mid u\ \textit{contains the pattern}\ 001\}$ and even further, $A(P) = \{u001v \mid u, v \in \{1, 0\}^*\}$.

f) NFA N



g) The number of states of the DFA and NFA are both the same however, the NFA has two less transition functions than the DFA. Mainly $\delta(q_2, 0) = q_2$ and $\delta(q_0, 0) = q_1$. This means that the NFA is simpler than the DFA.