# Gravity Modelling–Gravity & Crustal Structure

**After completing this practical you should be able to:**

The aim of this practical is to advance our understanding of the concept of **forward modelling** as a tool for interpreting geophysical anomalies. We will do this by implementing the semi-infinite slab model (using Excel) which predicts the Free Air and Bouguer gravity anomaly across a mountain belt. We will use **standard statistical tests** to quantify how well, or not, a particular model fits a set of observed data. In this case you will have to process the raw gravity observations (i.e. calculate and apply appropriate Free Air and Bouguer corrections).

After completing this practical you should be able to:

- Use Excel to perform more complex calculations multiple times using formulae.
- Use the semi-infinite slab model to predict the Free Air and Bouguer gravity anomaly measured at the surface across a simple model of a mountain belt and passive margin.
- Be able to use a simple forward model to estimate the physical and geological parameters of an object consistent with the measured gravity anomaly across that object.
- Use standard statistics such as the mean, standard deviation and root mean square deviation to assess the “goodness of fit” between model predictions and observations.

**Task 1.**
Implement a model which predicts the Free Air and Bouguer gravity anomalies measured at the surface along a transect across a mountain range (*please review pages 248-250 and 256-260 in Chapter 8 of Lillie before attempting these tasks*).

The relevant equation we need to use is (see page 250, Chapter 8, Lillie);

$$gz(x) = 13.34\, \Delta\rho\, \Delta h\, \left[\left(\frac{\pi}{2} + \arctan\left(\frac{x}{z}\right)\right)\right]$$

where gz(x) is the predicted gravity effect/contribution at position x due to a semi-infinite slab with the finite edge at x=0. The x position is the distance along the traverse measured from the zero point which is located above the slab edge (see Fig. 8.30, page 249 Lillie, Chapter 8). To calculate this gravity contribution at any position x we also need to define a value for $\Delta\rho$, the density contrast, $\Delta$h , the thickness of the slab and z, the depth to the centre of the slab (measured from the surface of the model to the mid-point of the slab (see Fig. 8.30, page 249, Lillie, Chapter 8). So as we move from one position to the next, along the transect, the x position changes (and so the x variable is incremented/changed) but the variables for density contrast, thickness and depth do not change (because we want to calculate gz at all positions for each of the model slabs).

To begin with let us set up a semi-infinite slab model for a simple mountain range in Airy isostatic equilibrium as described step-by step on pages 256-259 in Chapter 8, Lillie. Here we will only model one side of the range (the model is symmetrical) from -250km to +250km (recall that the 0 distance is located at the slab edge, or mountain front here). Use an interval of 5km. So to begin with we need to set up a column in Excel to store our x-position values. Make sure you use the correct units for each parameter (see below).

**Note:** This equation is scaled for use with $\Delta\rho$ expressed in units of **g.cm$^{-3}$** and distances **h, x** and **z** in **kilometres.**

**x position**
(use cell reference for cell in Excel column of x-positions)

$$g_z(x) = 13.34\,\Delta\rho\,\Delta h\,\left[(\pi/2 + \arctan(x/z)\right]$$

**delrho** (density contrast)

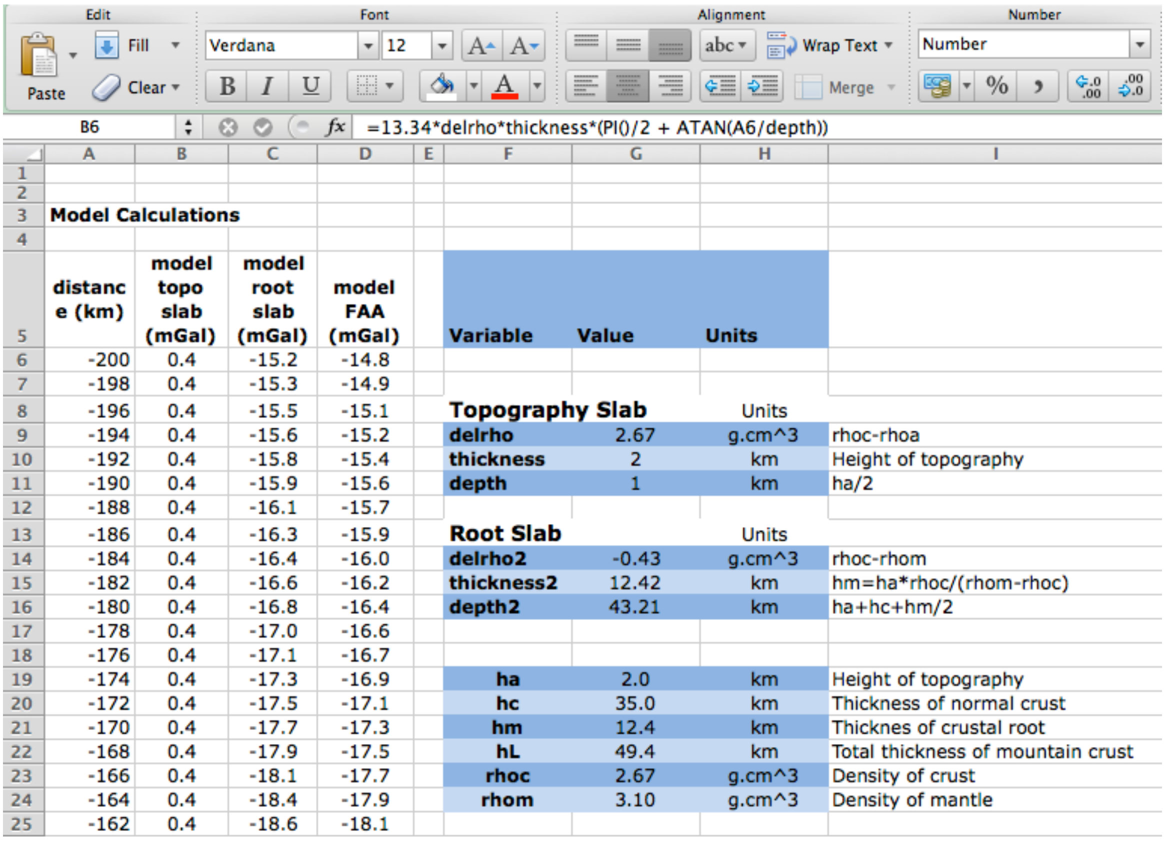
**thickness of slab**

**depth to centre of slab**

And in Excel this equation can be implemented as;

=13.34*delrho*thickness*(PI()/2 + ATAN(A6/depth))

with reference to row 6 in this case, using the named variables as shown in the example template/layout illustrated by the screen image below.

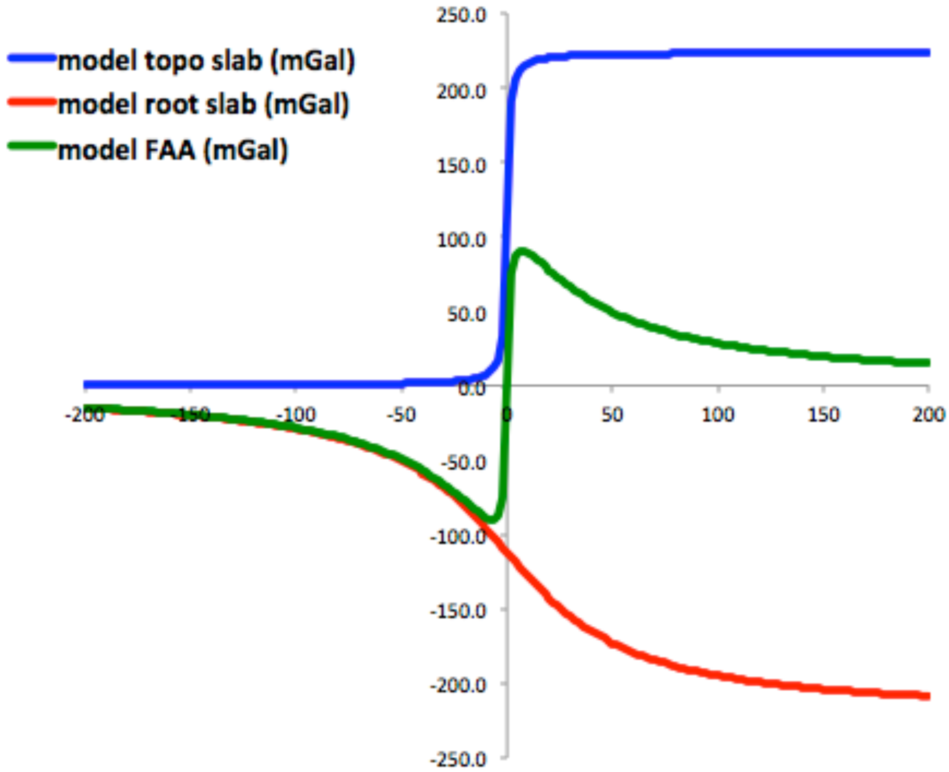



| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | |
| 2 | | | | | | | | | |
| 3 | **Model Calculations** | | | | | | | | |
| 4 | | | | | | | | | |
| 5 | **distance (km)** | **model topo slab (mGal)** | **model root slab (mGal)** | **model FAA (mGal)** | | **Variable** | **Value** | **Units** | |
| 6 | -200 | 0.4 | -15.2 | -14.8 | | | | | |
| 7 | -198 | 0.4 | -15.3 | -14.9 | | | | | |
| 8 | -196 | 0.4 | -15.5 | -15.1 | | **Topography Slab** | | Units | |
| 9 | -194 | 0.4 | -15.6 | -15.2 | | **delrho** | 2.67 | g.cm^3 | rhoc-rhoa |
| 10 | -192 | 0.4 | -15.8 | -15.4 | | **thickness** | 2 | km | Height of topography |
| 11 | -190 | 0.4 | -15.9 | -15.6 | | **depth** | 1 | km | ha/2 |
| 12 | -188 | 0.4 | -16.1 | -15.7 | | | | | |
| 13 | -186 | 0.4 | -16.3 | -15.9 | | **Root Slab** | | Units | |
| 14 | -184 | 0.4 | -16.4 | -16.0 | | **delrho2** | -0.43 | g.cm^3 | rhoc-rhom |
| 15 | -182 | 0.4 | -16.6 | -16.2 | | **thickness2** | 12.42 | km | hm=ha*rhoc/(rhom-rhoc) |
| 16 | -180 | 0.4 | -16.8 | -16.4 | | **depth2** | 43.21 | km | ha+hc+hm/2 |
| 17 | -178 | 0.4 | -17.0 | -16.6 | | | | | |
| 18 | -176 | 0.4 | -17.1 | -16.7 | | | | | |
| 19 | -174 | 0.4 | -17.3 | -16.9 | | **ha** | 2.0 | km | Height of topography |
| 20 | -172 | 0.4 | -17.5 | -17.1 | | **hc** | 35.0 | km | Thickness of normal crust |
| 21 | -170 | 0.4 | -17.7 | -17.3 | | **hm** | 12.4 | km | Thicknes of crustal root |
| 22 | -168 | 0.4 | -17.9 | -17.5 | | **hL** | 49.4 | km | Total thickness of mountain crust |
| 23 | -166 | 0.4 | -18.1 | -17.7 | | **rhoc** | 2.67 | g.cm^3 | Density of crust |
| 24 | -164 | 0.4 | -18.4 | -17.9 | | **rhom** | 3.10 | g.cm^3 | Density of mantle |
| 25 | -162 | 0.4 | -18.6 | -18.1 | | | | | |

Once we have a column of x position values we then need to calculate the gravity contributions for the topography and the crustal root separately at each of the x positions. The sum of these two contributions then gives us the model Free Air Anomaly (FAA), and the contribution from the root alone gives us the model Bouguer anomaly (BA). So you need to set-up two model columns, one for the topography slab and one for the root slab (see Fig. 8.37, page 258, Chapter 8, Lillie). The x-position value in the equations is taken directly from our x-position column using the appropriate cell reference, while you'll need to define the other three variables (for each slab) as named variables (See example layout shown in the screen image above).

Note you need to calculate the appropriate value for the thickness of the crustal root, $h_m$, using the expression derived by simultaneous solution of the two isostatic equations, and standard densities of $\rho_c$=2.67 g.cm$^{-3}$ and $\rho_m$=3.1 g.cm$^{-3}$. See below for details for how this is done. Make sure you understand this step, and that you could derive the expression yourself, as you will need to do carry out this procedure for the Tibet Plateau exercise.

Plot graphs of the gravity contributions for the topography and crustal root and the model FAA prediction (gravity values on the y-axis) against x-distance (distance on the x-axis) and compare your model output to the graphs shown in Fig. 8.37a, b and c on page 258, Chapter 8, Lillie. Your model predictions should look like those plotted below.




Please make sure you understand what each curve is representing, and how your curves/columns relate to those shown in Lillie, Chapter 8 (Fig. 8.37). Please also spend some time to read the discussion provided by Lillie, Chapter 8 (pages 258-260) about these model results, and how they relate to/indicate the state of isostasy of the model mountain.

**Task 2.**

**a)** Down load the synthetic mountain gravity data from the Moodle site, or the G4G Lab 4 web site. The simplest way to achieve this is to select and download the Excel formatted file, or if you use the ascii text format then open the link, then select and copy the data (make sure you select ALL the columns and headings) and then paste this into an empty Excel workbook page (use *Paste Special-> text*). Calculate the Free Air anomaly and the Bouguer anomaly for each of the station locations. This will require calculation of the appropriate theoretical gravity value at each station (use the 1967 standard formula, page 226, Chapter 8, Lillie as we did for Lab 1) and then calculating and applying the appropriate Free Air correction (i.e. FAC = 0.308*h mGal) and Bouguer Correction (BC = 0.112*h mGal) as required.

**b)** Use a suitable semi-infinite slab model to test whether the mountain range (from where the data in *a* above were obtained) is in Airy isostatic equilibrium or not. To do this adapt/copy your semi-infinite slab model from Task 1, and predict a model value for BOTH the Free Air anomaly and the Bouguer anomaly at all stations where there are observed data using a simple 2-slab mountain model (as described in the example shown in Chapter 8, page 256-260 in Lillie and implemented in Task 1 above).

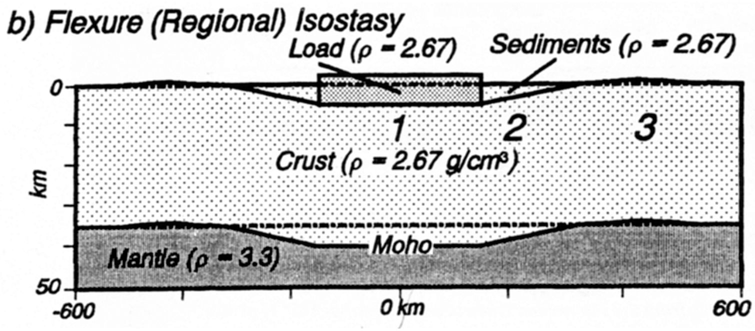
A simple visual inspection of the observed anomalies should give you a good idea of the answer. But, to produce a quantitative and robust answer, you will need to show that the observed gravity anomalies are accurately predicted by a simple model that is in Airy isostatic equilibrium.

To calculate the thickness of the mountain root $h_m$, assuming it is in Airy isostatic equilibrium, you should use the formula derived from simultaneous solution of the Airy isostatic equations (see below and also page 257, Chapter 8, Lillie) after inspecting the observed topography to determine a sensible value for $h_a$. Use standard densities of $\rho_c$=2.67 g.cm$^{-3}$ and $\rho_m$=3.1 g.cm$^{-3}$.

Make sure that you make a **valid statistical assessment** of how well your model fits the observations (see notes at end of manual for Lab Session 2). This will require calculating the RMSD between your model Free Air anomaly and the observed Free Air anomaly, and the model Bouguer anomaly and the observed Bouguer anomaly. Then you can use the mean of these two RMSD values to test/measure the “goodness of fit” of your model to both the FAA and the BA. You should also examine the distribution of the residuals to assess whether there is structure to the way they are distributed. Ideally we want these to be randomly distributed about the zero value, and to be less than (or similar to) the uncertaint on our measured values (typically we use the standard deviation on the measured data as a measure of the uncertainty on the observations).

**Task 3.**

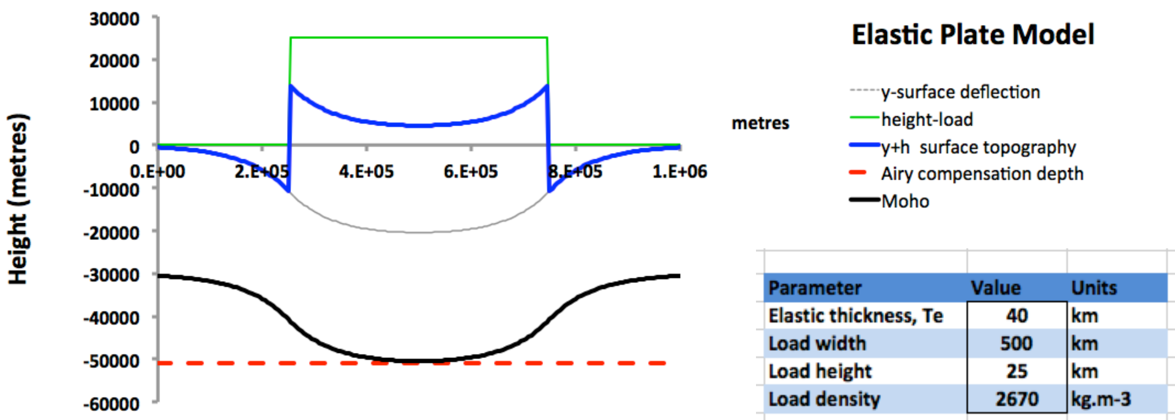
Download the Visco-Elastic Plate Model example Excel spread sheet file (Moodle Lab 4 page or link top right of G$G Lab 4 web page). Use this model of regional isostatic compensation to investigate how the lithosphere responds to different surface loads (i.e. topography).




The strength of the lithosphere, and specifically how this effects the deflection of the Moho caused by surface loading and the isostatic compensation of the load, can be approximated using a model describing the behaviour of an infinite elastic plate. If the elastic modulus (elastic strength) of the plate weakens over time as in viscous behaviour we can approximate this response by adding time dependent term to the elastic modulus. The resulting visco-elastic plate model provides a tool forof quantifying and understanding regional flexural isostatic compensation of surface loads, and how this may change over geological time scales.

In the extreme case where the elastic plate is very weak (i.e. very thin, or negligible, effective elastic thickness or the loading time is very long, i.e. t is much longer than tau) the elastic plate (or flexural) model is equivalent to the Airy local compensation model.

Experiment with systematically increasing the width of a load (keeping Te fixed) and then also by systematically varying the elastic thickness (Te) while keeping the width of the load fixed. You will see that for a given width and Te the lithosphere will appear weaker for increasing load times. The Maxwell time constant, tau, controls the time over which the full viscous relaxation occurs.




| Parameter | Value | Units |
|---|---|---|
| Elastic thickness, Te | 40 | km |
| Load width | 500 | km |
| Load height | 25 | km |
| Load density | 2670 | kg.m-3 |

For more details and the equations used in the model see; Watts, A. B. (2001). Isostasy and Flexure of the Lithosphere. Cambridge University Press. Chapter 3. P. 91-102.

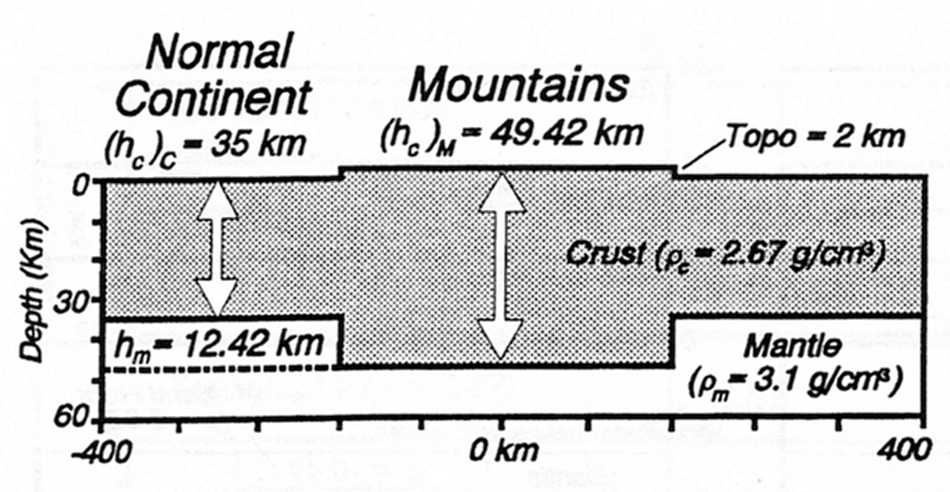



***Fig. 8.36 (Lillie, Chpt 8, p. 257)*** *Airy isostatic model for a 2km high mountain range. See Lillie p. 257 for further details and discussion.*

To calculate the thickness of the crustal root, $h_m$, needed to support the 2km high topography assuming local Airy-type isostatic compensation we solve the two Airy equations simultaneously to obtain an expression for $h_m$.

$h_M = h_a + h_c + h_m$ Equation 1: Equal thickness

$\rho_c h_M g = \rho_a h_a g + \rho_c h_c g + \rho_m h_m g$ Equation 2: Equal pressure

Two unknowns, $h_M$ and $h_m$ with two equations, so substitute for unknown, $h_M$ in equation 2 using equation 1;

Divide both sides of equation 2 by $g$ to give;

$$\rho_c h_M = \rho_a h_a + \rho_c h_c + \rho_m h_m$$

Then replace $h_M$ on the left hand side using equation 1 to give;

$$\rho_c(h_a + h_c + h_m) = \rho_a h_a + \rho_c h_c + \rho_m h_m$$

Multiply out and cancel like terms on both sides, noting that density of air is negligible here (zero) to get;

$$\rho_c h_a + \rho_c h_m = \rho_m h_m$$

Collect terms containing $h_m$ on one side;

$$\rho_c h_a = \rho_m h_m - \rho_c h_m$$

Factorise (common factor of $h_m$);

$$\rho_c h_a = h_m(\rho_m - \rho_c)$$

Divide both sides by $(\rho_m - \rho_c)$, giving;

$$h_m = h_a \ \rho_c/(\rho_m - \rho_c)$$