Measuring Stellar Elemental Abundances

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1 Introduction

Understanding the composition of stars is critical to understanding the formation, evolution, and resulting make-up of exoplanets. Finding the relative abundance of certain elements in solar systems beyond our own can give us insight into how close their planets chemically resemble Earth.

Stars can be analyzed by looking at their spectrograph; each element can be represented by a wavelength, and their respective abundances can be defined by the strength of their absorption lines. Atomically, this means looking at different states of atoms, their number density, and comparing that to the abundance of a star's most common element: hydrogen (H).

In this paper, we will be examining the absorption line and the resulting relative abundance of sodium (Na) in the Sun. This, in effect, will show that the methodology here can be extended to other stars and other elements. In doing so, we will be laying the basis for analyzing stellar composition and therefore examining the potential structure and formation of exoplanets.

2 Methodology

To analyze the absorption line of Na, we look at the doublet of the 3p - 3s transition, which correspond to Na 5890 and 5896.

First, using high resolution solar spectrum data from the BASS2000 Solar Survey Archive, we measure the equivalent width of Na in the Sun's absorption spectrum (Figure 1). The equivalent width (EW) is the width of a rectangle such that its area is equal to the area within the spectral line. We find that the EW of the Na absorption line is 0.83 Å.

Then, we find the correlating location on the curve of growth [1] for the Sun (Figure 2). To start, we calculate the corresponding value on the Y-axis:

$$\log(\frac{EW}{\lambda}) = -3.85 \tag{1}$$

We can see that this corresponds to a value of 14.8 on the x-axis:

$$\log(Nf\frac{\lambda}{5000\mathring{A}}) = 14.8 \tag{2}$$

Here f, the oscillator strength is assumed to be 0.65 and λ is the emission wavelength of 5890 Å. Finally, we use this to calculate the number density N for sodium:

$$N = \frac{10^{14.8}}{f\lambda} * 5000\mathring{A} = 8.24 * 10^{14} \text{ atoms/cm}^3 \tag{3}$$

Next, we use the Boltzmann equation to find the ratio of Na atoms in the ground state to Na atoms in excited states.

$$\frac{N_2}{N_1} = \frac{g_2}{g_1}\exp(-\frac{E_2 - E_1}{kT}) \tag{4}$$

Here, N_2 and N_1 correspond to the number density of Na atoms in the 3p and 3s states, respectively. E_2 and E_1 correspond to the energy at those states. Lastly, g_2 and g_1 are the number of unique states at that energy - for our case these are $g_2 = 6$ and $g_1 = 2$.

Next, we utilize the Saha equation to find the ratio of neutral Na atoms to ionized Na atoms:

$$\frac{Na_{II}}{Na_I} = \frac{2kT}{P_e}\frac{Z_{II}}{Z_I}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \exp(-\frac{\chi}{kT}) \tag{5}$$

Here, $P_e = 1.0$ Nm^{-2} is the electron pressure, $\chi = 5.1$ eV is the ionization energy, and $Z_{II} = 1.0$, $Z_I = 2.4$ is the partition function. So, we find:

Now we finally use the number density N we derived earlier to compute the total column density of Na atoms:

$$N_1(1 + \frac{N_2}{N_1})(1 + \frac{Na_{II}}{Na_I}) \tag{6}$$

From here, we can express the abundance by the three major methods: the mole ratio between Na and H, the log of the mole ratio between Na and H (offset by 12), and the log of the mole ratio compared to the known mole ratio of the Sun.

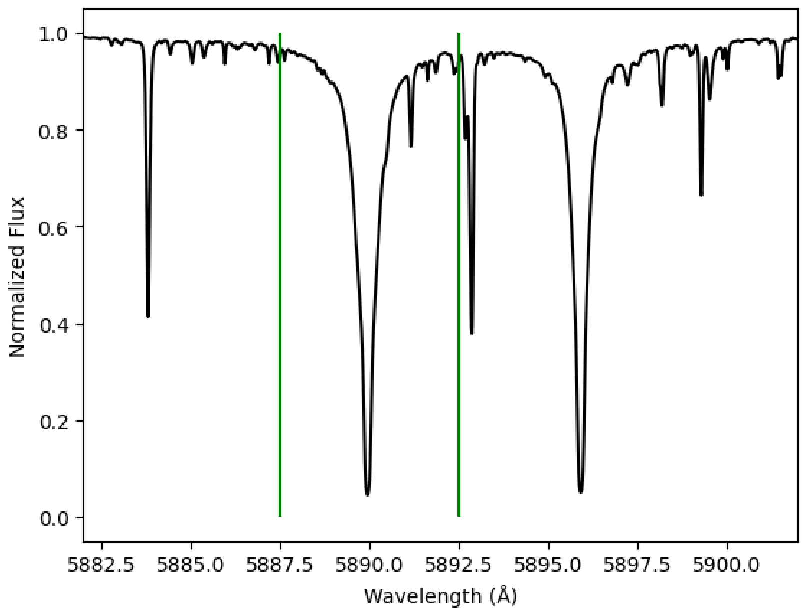

Fig. 1. The strongest absorption line of Na in the Sun is shown, the red lines highlighting the chosen line.

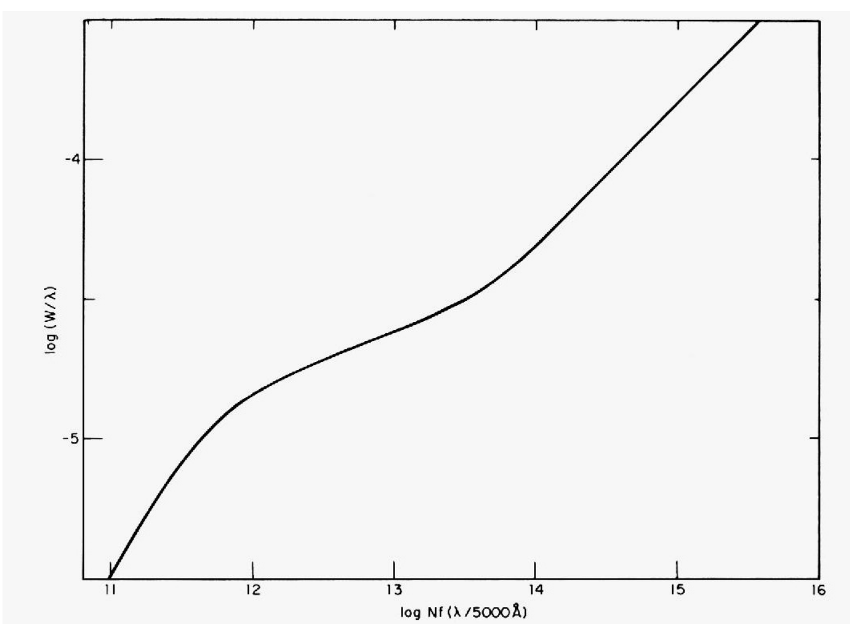

Fig. 2. The curve of growth for the Sun from L. H. Aller.

3 Results

From the equations outlined in the Methodology section, we find the following:

The Boltzmann ratio is:

$$\frac{N_2}{N_1} = \frac{6}{2}\exp(-\frac{2.107\text{ MeV}}{k * 5772\text{ K}}) = 0.0434 \tag{7}$$

The ion ratio is:

$$\frac{Na_{II}}{Na_I} = \frac{(2k)5772\text{ K}}{1.0\text{ Nm}^{-2}}\frac{1.0}{2.4}\left(\frac{(2\pi m_e k)5772\text{ K}}{h^2}\right)^{3/2}\exp(-\frac{5.1}{(k)5772\text{ K}}) = 2477 \tag{8}$$

The total column density is:

$$N_1(1+\frac{N_2}{N_1})(1+\frac{Na_{II}}{Na_I}) = 2.13 * 10^{18}\text{ atoms/m}^2 \tag{9}$$

From here, the abundance of Na in the Sun was computed in the three major ways to describe elemental abundance (Table 1).

Table 1. Calculated abundance of Na in the Sun.

Physics	Galactic Physics	Stellar Physics
$N_{\text{Na}}/N_{\text{H}}$	$12 + \log(\text{Na/H})$	$\log(\frac{N_{\text{Na}}/N_{\text{H}}}{(N_{\text{Na}}/N_{\text{H}})_\odot})$
3.23×10^{-6}	6.51	0.209

4 Conclusion

We find our measurement and calculations for the abundance of Na to be consistent with known values. The obvious number to look at is the mole ratio of Na to H compared to the actual value of the mole ratio for the Sun; here, our value of 0.209 is close to the ideal value of 0.0. Additionally, comparing to Palme et al. [2], we find that our value for the log abundance of 6.51 is comparable to their value of 6.30.

Table 2. Actual Values vs Calculations.

	Ideal/Known Value	Calculated Value
$N_{\mathrm{Na}}/N_{\mathrm{H}}$	2×10^{-6}	3.23×10^{-6}
$12 + \log(\mathrm{Na/H})$	6.30	6.51
$\log(\frac{N_{\mathrm{Na}}/N_{\mathrm{H}}}{(N_{\mathrm{Na}}/N_{\mathrm{H}})_{\odot}})$	0.0	0.209

As for why our values seem to suggest a higher abundance of Na in the Sun, this error could be due to a combination of many factors. For example, there could be slight errors in our assumption of oscillator strength, our estimate for the corresponding $\log(Nf\frac{\lambda}{5000\mathring{A}})$ value in the curve of growth, or our calculation of the equivalent width.

References

1. Aller, L.H.: Atoms, Stars, and Nebulae. Cambridge University Press, Cambridge, Massachusetts (1971)
2. Palme, H., Lodders, K., Jones, A.: Solar System Abundances of the Elements. Treatise on Geochemistry **2**, 15–36 (2014).