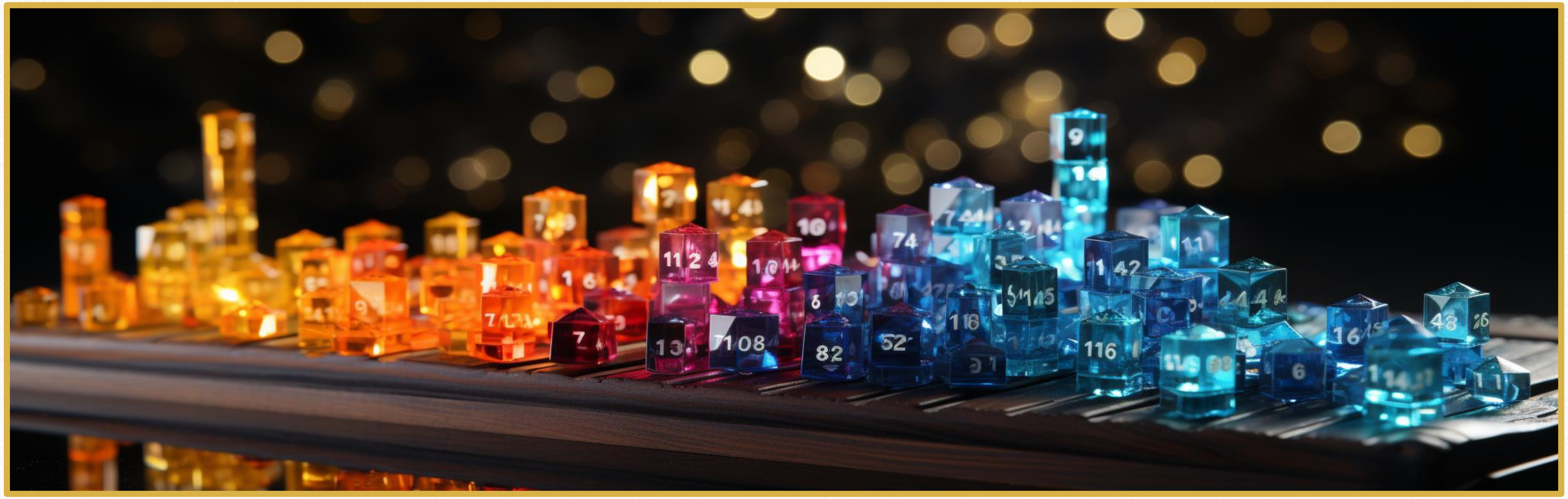
UXG1205 Lecture

11. Z-Scores and Standard Normal Distributions

LIN QINJIE

Outline

- Z-Scores and Standard Normal Distributions



Z-Scores and Standard Normal Distributions

- Calculate Z-scores
- Read Z-tables

- To understand Z-scores, you and Billy will go back to the mysteries of the post-alien invasion desert cacti. Remember, the ones that might be spy cacti are those that are over 11 kg.
- Recently, there are reports of spy cacti weighing just 10 kg. Better be careful and get rid of these too!

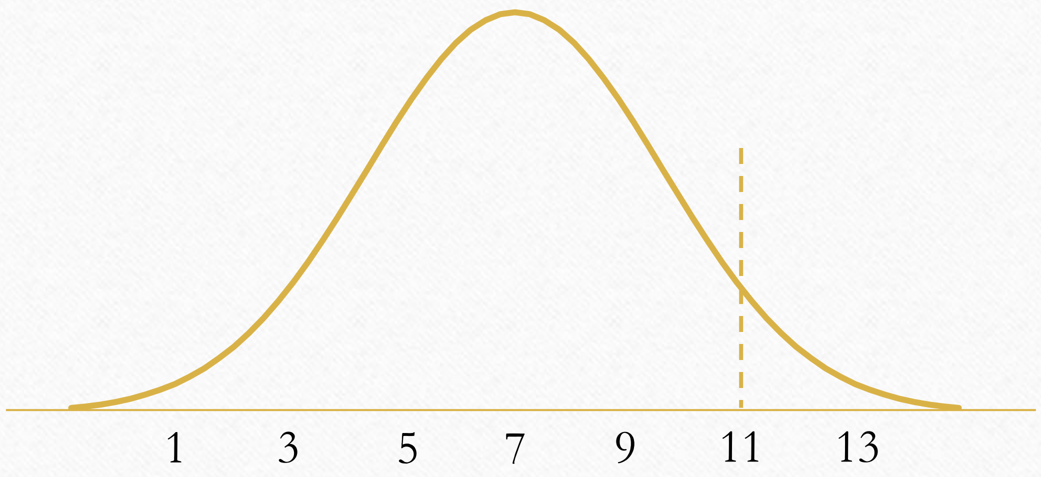
- The weights of desert cacti are distributed normally with a mean (μ) of 7 kg and a standard deviation (σ) of 2 kg.
- Now, you need to monitor any cactus weighing over 10 kg.
- To know how many you need to remove, you'll find the area of the curve to the right of 10 kg.

- The equation to calculate the area under the normal distribution to the right of 10 kg is given by:

$$\int_{10}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \times e^{-\frac{(x-\mu)^2}{2\times\sigma^2}} dx$$

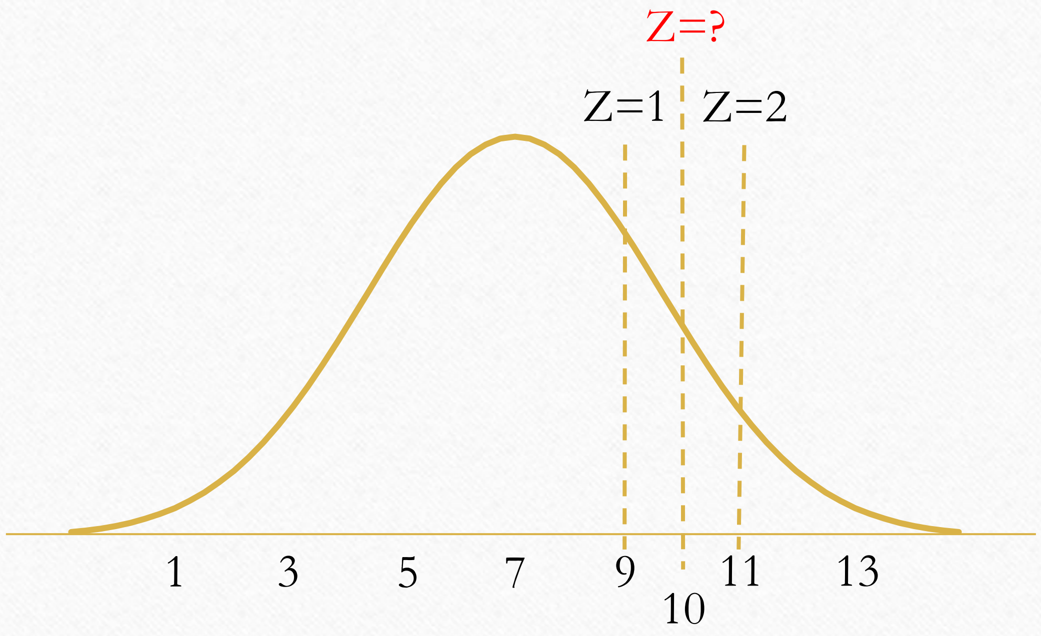
- If you know integral calculus, the above formula will give you an exact answer. (This is not tested.)

- If you don't know, fortunately, we have **Z-scores** to help us ☺



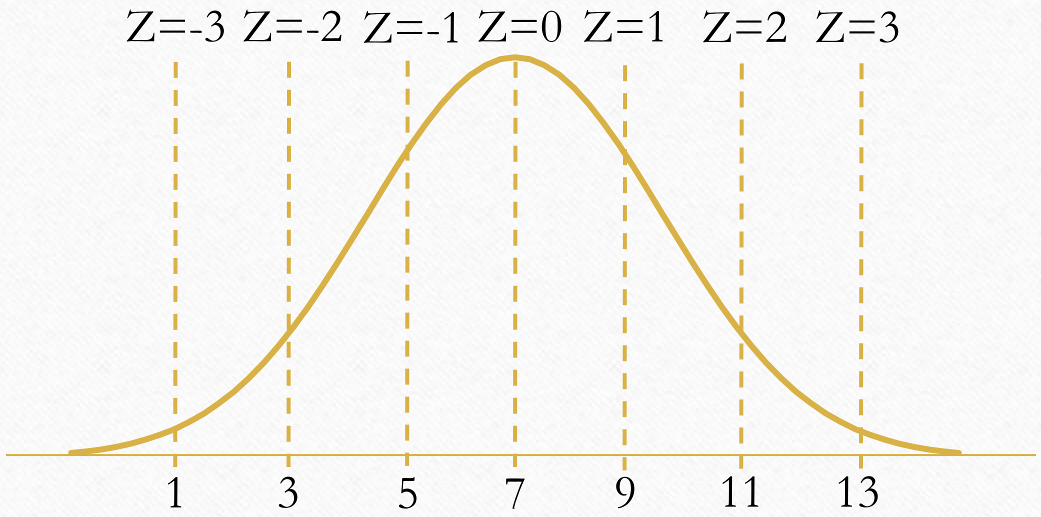
- **Z-score** indicates how many standard deviations a value is from the mean.
- Given our scenario of $\mu = 7$ and $\sigma = 2$, what's the Z-score for a 11-kg cactus?
  - Since 11 is 2 standard deviations more than the mean, we can conclude that 11 has a Z-score of 2. In other words, when $X = 11$, $Z = 2$.

- Let's try one more. For the same scenario, when X = 3, what is the Z-score?
  - A. Z = 1
  - B. Z = -2
  - C. Z = 2
  - D. Z = -4



- Now, 10 kg lies midway between Z = 1 and Z = 2. Based on your understanding, what do you infer about the Z-score for X = 10?

A. Z = 1
B. Z = 0.5
C. Z = 1.5
D. Z = 2

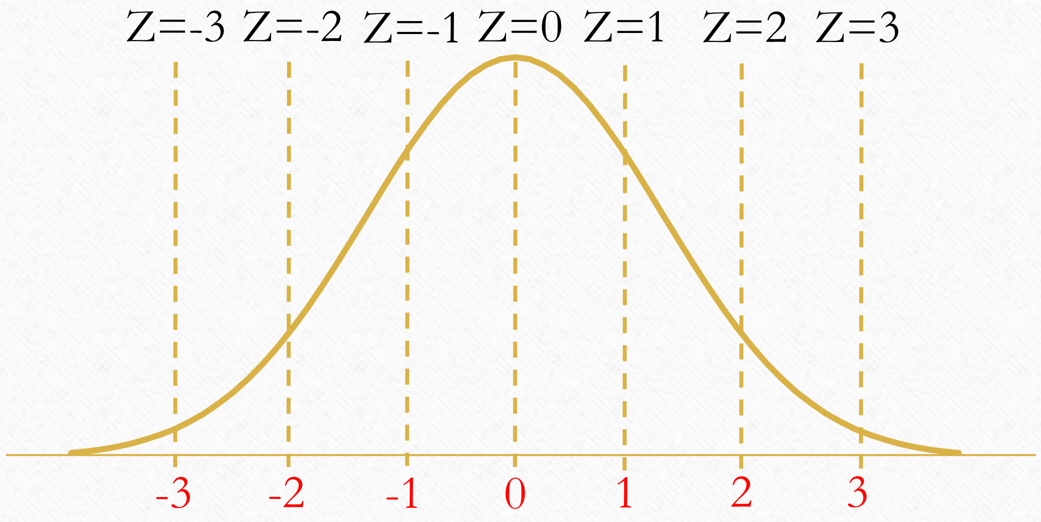


- You won't always have easily discernible values. For those instances, you can rely on the Z-score equation:

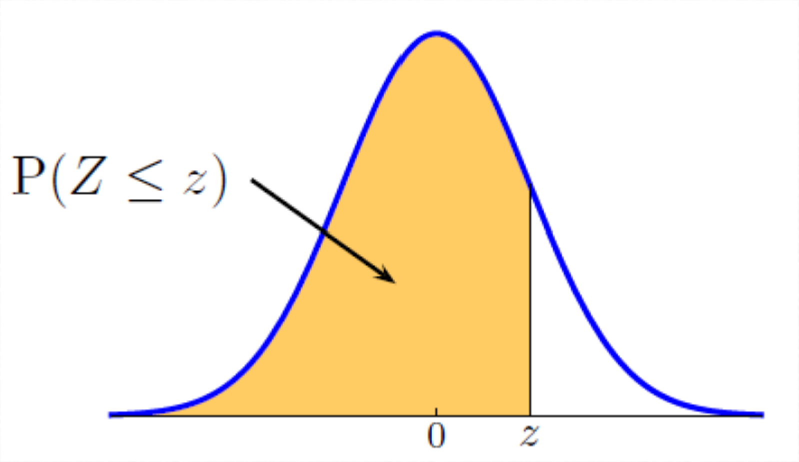
$$Z = \frac{X - \mu}{\sigma}$$

- Given that $\mu = 7$ and $\sigma = 2$, for a data point like $X = 2.5$:

$$Z = \frac{2.5 - 7}{2} = -2.25$$



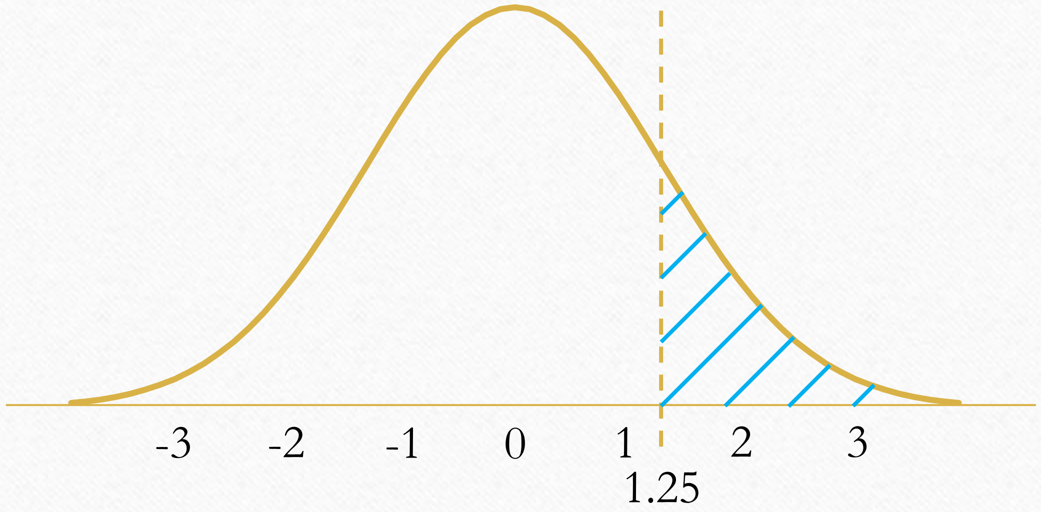
- Z-score essentially transforms our normal curve into the **standard normal distribution**, where $\mu = 0$ and $\sigma = 1$.
- For the **standard normal distribution**, almost all cumulative probabilities have been determined.
  - We can simply refer to a **Z-table** to find cumulative probabilities instead of using the complex integral equation!



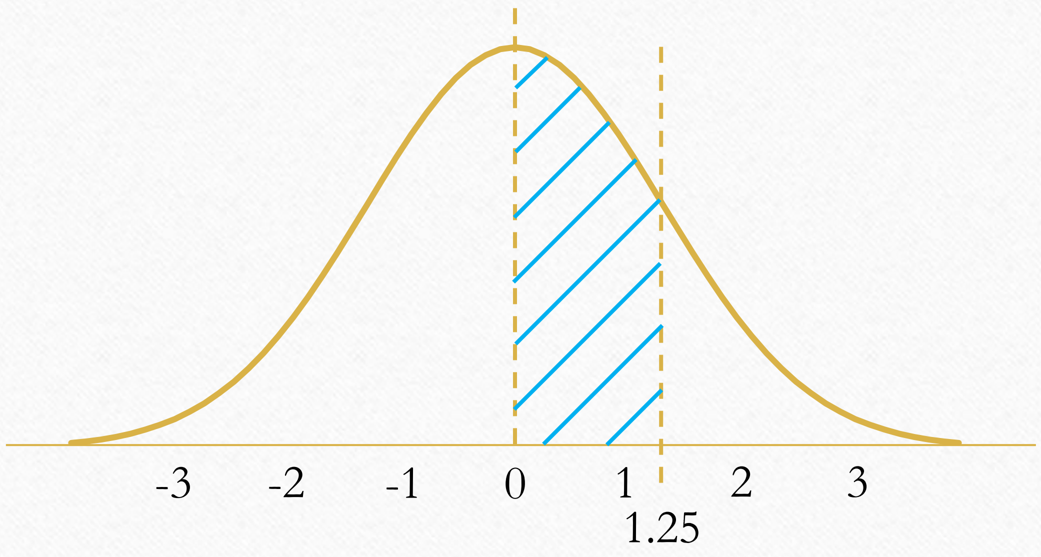
- There are a few types of Z-tables, but we will look at the most common one:
  - **Cumulative Z-table**
- You need to get a copy and refer to it.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084

- The initial column presents the first two digits of the Z-score, while the header row displays its second decimal place. To get the probability, find the correct value according to the Z-score.
- Using the Z-table provided,
  - Find $P(Z < -2.74)$?
  - Find $P(Z < 1.25)$?



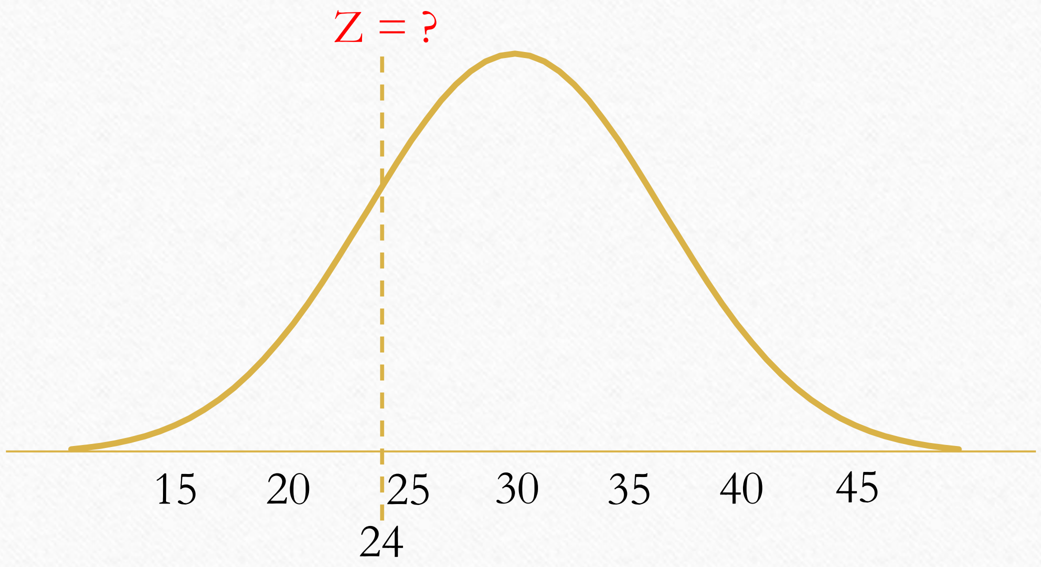
- If you need **complementary cumulative figures**, you can also use a cumulative Z-table. Just identify the complement of the required probability and then subtract it from 1.
- In other words, if you're trying to find $P(Z > 1.25)$, use the cumulative table to determine $1 - P(Z < 1.25)$.
- Since $P(Z < 1.25) = 0.8944$, $P(Z > 1.25) = 1 – 0.8944 = 0.1056$ or 10.56%



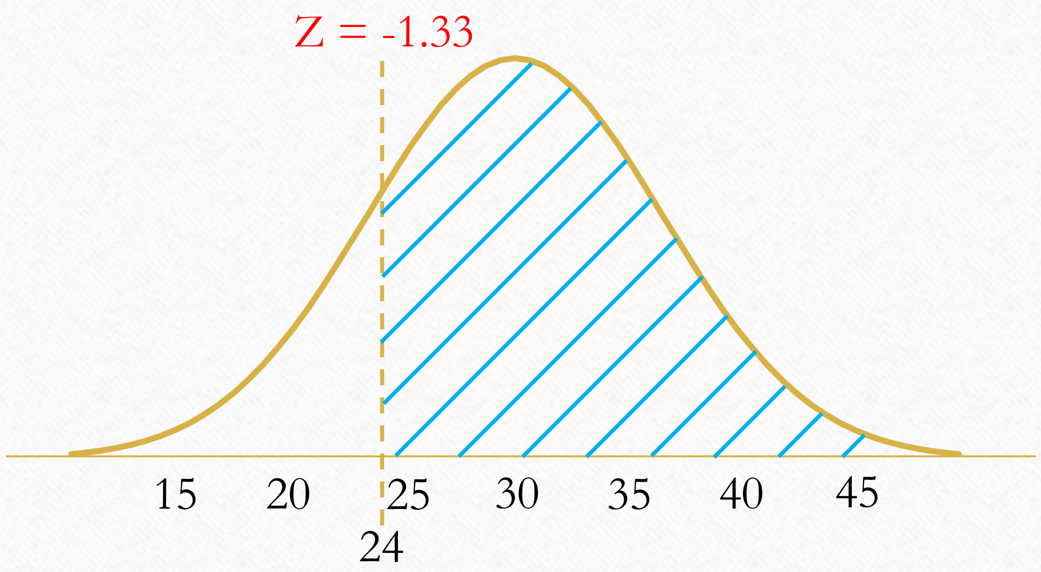
- Another practice.
- What is $P(0 < Z < 1.25)$?

- Back to those spy cacti. For 10-kg cactus, we know $Z = 1.5$. What's the percentage of cacti that you need to remove?
- In other words, we're finding $P(Z > 1.5)$.
- $P(Z > 1.5) = 1 – P(Z < 1.5) = 1 – 0.9332 = 0.0668 = 6.68\%$ (Ans)

- Also, remember Billy is hoping that there's a 90% chance that the batteries can survive at least 24 hours – i.e. $P(X > 24) = 0.9$. Can you now find out whether it's 90%?
- Empirical Rule couldn't help you with that. But now, **Z-scores** can!



- We were given some info previously: μ = 30 and σ = 4.5.
- First, we calculate the Z-score for X = 24.
- $Z = \frac{X-\mu}{\sigma} = \frac{24-30}{4.5} = -1.33$ (round to 2 decimal places)



- Next, to determine the likelihood that a radio will function beyond 24 hours, we should identify the area beneath the curve rightward of $X = 24$.
- For this, we can evaluate $1 - P(Z < -1.33)$ with a **cumulative Z-table**.
- $1 - P(Z < -1.33) = 1 - 0.0918 = 0.9082 = 90.82\%$

- This means you can now tell Billy that there's more than 90% chance that the radios can survive at least 24 hours. It's time to move out and get those aliens!

The End