

Discretized plant model:

$$q(k) = \theta_1 u(k) + \theta_2 u(k-1) + \theta_3 u(k-2) - \theta_4 q(k-1) - \theta_5 q(k-2) = \phi^T(k) \boldsymbol{\theta}$$

$$\boldsymbol{\theta} = [\theta_1, \ \theta_2, \ \theta_3, \ \theta_4, \theta_5]^T, \qquad \phi(k) = [u(k), u(k-1), \ u(k-2), -q(k-1), \ -q(k-2)]^T$$

$$\hat{\boldsymbol{\theta}}(k) = [\hat{\theta}_1(k), \ \hat{\theta}_2(k), \ \hat{\theta}_3(k), \ \hat{\theta}_4(k), \ \hat{\theta}_5(k)]^T,$$
 T_S is sampling time,

$$\boldsymbol{P}(0) = \boldsymbol{P_0} = \boldsymbol{P_0}^{\mathsf{T}} > 0, \ \boldsymbol{P_0} = \rho \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Discretized plant model in state space:

$$\mathbf{x}(k+1) = \hat{\mathbf{A}}_d \mathbf{x}(k) + \hat{\mathbf{B}}_d u(k)$$
$$y(k+1) = \hat{\mathbf{C}}_d \mathbf{x}(k) + \hat{\mathbf{D}}_d u(k)$$

where

$$\hat{\boldsymbol{A}}_d = \left[\begin{array}{cc} 0 & 1 \\ -\hat{\theta}_5 & -\hat{\theta}_4 \end{array} \right], \qquad \hat{\boldsymbol{B}}_d = \left[\begin{array}{c} 0 \\ 1 \end{array} \right], \qquad \hat{\boldsymbol{C}}_d = [\hat{\theta}_3 - \hat{\theta}_5 \hat{\theta}_1, \ \hat{\theta}_2 - \hat{\theta}_4 \hat{\theta}_1], \qquad \hat{\boldsymbol{D}}_d = \hat{\theta}_1$$

. Estimated continuous model is:

$$\dot{\boldsymbol{x}}(t) = \hat{\boldsymbol{A}}_c x(t) + \hat{\boldsymbol{B}}_c u(t)$$
$$y(t) = \hat{\boldsymbol{C}}_c \boldsymbol{x}(t) + \hat{\boldsymbol{D}}_c u(t)$$

where

$$\widehat{\mathbf{A}}_{c} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \frac{2}{T_{s}} \mathbf{R} \left[\mathbf{I}_{2 \times 2} - \frac{8}{21} \mathbf{R}^{2} - \frac{4}{105} \mathbf{R}^{4} \right] \left[\mathbf{I}_{2 \times 2} - \frac{5}{7} \mathbf{R}^{2} \right]^{-1}$$

$$\widehat{\mathbf{B}}_{c} = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} = \widehat{\mathbf{A}}_{c} \left[\widehat{\mathbf{A}}_{d} - \mathbf{I}_{2 \times 2} \right]^{-1} \widehat{\mathbf{B}}_{d}, \qquad \widehat{\mathbf{C}}_{c} = [c_{11}, c_{12}] = \widehat{\mathbf{C}}_{d}, \qquad \widehat{\mathbf{D}}_{c} = \widehat{\mathbf{D}}_{d}$$

$$\mathbf{R} = \left[\widehat{\mathbf{A}}_{d} - \mathbf{I}_{2 \times 2} \right] \left[\widehat{\mathbf{A}}_{d} + \mathbf{I}_{2 \times 2} \right]^{-1}$$

So, we calculate:

$$\hat{b} = c_{11}(b_{21}a_{12} - b_{11}a_{22}) + c_{12}(b_{11}a_{21} - b_{21}a_{11}), \quad \hat{a} = -(a_{11} + a_{22})$$