



Discretized plant model:

$$y(k) = \theta_1 u(k) + \theta_2 u(k-1) + \theta_3 u(k-2) - \theta_4 y(k-1) - \theta_5 y(k-2) = \phi^T(k)\boldsymbol{\theta}$$

$$\boldsymbol{\theta} = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5]^T, \quad \phi(k) = [u(k), u(k-1), u(k-2), -y(k-1), -y(k-2)]^T$$

$$\hat{\boldsymbol{\theta}}(k) = [\hat{\theta}_1(k), \hat{\theta}_2(k), \hat{\theta}_3(k), \hat{\theta}_4(k), \hat{\theta}_5(k)]^T,$$

$T_S$  is sampling time,

$$\mathbf{P}(0) = \mathbf{P}_0 = \mathbf{P}_0^T > 0, \quad \mathbf{P}_0 = \rho \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Discretized plant model in state space:

$$\mathbf{x}(k+1) = \hat{\mathbf{A}}_d \mathbf{x}(k) + \hat{\mathbf{B}}_d u(k)$$

$$y(k+1) = \hat{\mathbf{C}}_d \mathbf{x}(k) + \hat{\mathbf{D}}_d u(k)$$

where

$$\hat{\mathbf{A}}_d = \begin{bmatrix} 0 & 1 \\ -\hat{\theta}_5 & -\hat{\theta}_4 \end{bmatrix}, \quad \hat{\mathbf{B}}_d = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \hat{\mathbf{C}}_d = [\hat{\theta}_3 - \hat{\theta}_5 \hat{\theta}_1, \hat{\theta}_2 - \hat{\theta}_4 \hat{\theta}_1], \quad \hat{\mathbf{D}}_d = \hat{\theta}_1$$

. Estimated continuous model is:

$$\dot{\mathbf{x}}(t) = \hat{\mathbf{A}}_c \mathbf{x}(t) + \hat{\mathbf{B}}_c u(t)$$

$$y(t) = \hat{\mathbf{C}}_c \mathbf{x}(t) + \hat{\mathbf{D}}_c u(t)$$

where

$$\begin{aligned} \hat{\mathbf{A}}_c &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \frac{2}{T_s} \mathbf{R} \left[ \mathbf{I}_{2 \times 2} - \frac{8}{21} \mathbf{R}^2 - \frac{4}{105} \mathbf{R}^4 \right] \left[ \mathbf{I}_{2 \times 2} - \frac{5}{7} \mathbf{R}^2 \right]^{-1} \\ \hat{\mathbf{B}}_c &= \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} = \hat{\mathbf{A}}_c \left[ \hat{\mathbf{A}}_d - \mathbf{I}_{2 \times 2} \right]^{-1} \hat{\mathbf{B}}_d, \quad \hat{\mathbf{C}}_c = [c_{11}, c_{12}] = \hat{\mathbf{C}}_d, \quad \hat{\mathbf{D}}_c = \hat{\mathbf{D}}_d \\ \mathbf{R} &= \left[ \hat{\mathbf{A}}_d - \mathbf{I}_{2 \times 2} \right] \left[ \hat{\mathbf{A}}_d + \mathbf{I}_{2 \times 2} \right]^{-1} \end{aligned}$$

So, we calculate:

$$\hat{\beta}_1 = \hat{\mathbf{D}}_c, \quad \hat{\beta}_2 = c_{11} b_{11} + c_{12} b_{21} - \hat{\mathbf{D}}_c [a_{11} + a_{22}],$$

$$\hat{\beta}_3 = c_{11}(b_{21} a_{12} - b_{11} a_{22}) + c_{12}(b_{11} a_{21} - b_{21} a_{11}) + \hat{\beta}_5 \hat{\mathbf{D}}_c, \quad \hat{\beta}_4 = -(a_{11} + a_{22}), \quad \hat{\beta}_5 = a_{11} a_{22} - a_{12} a_{21}$$