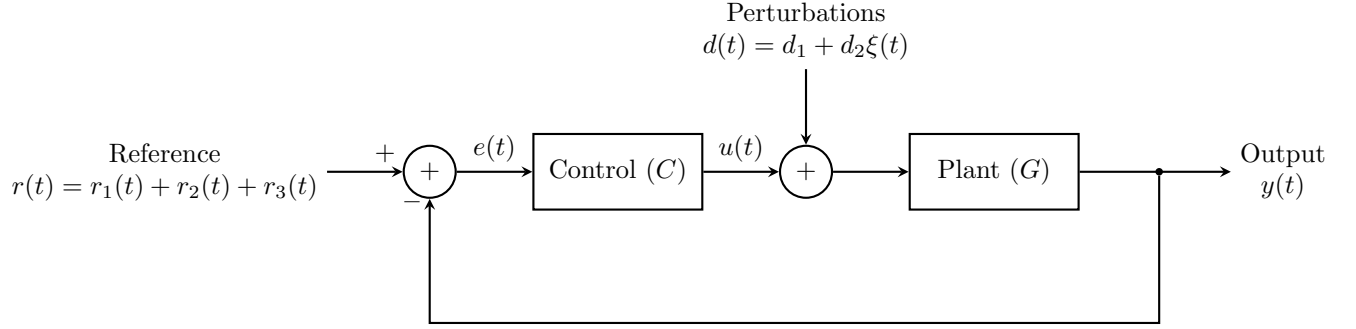


Figure 1: Open loop system

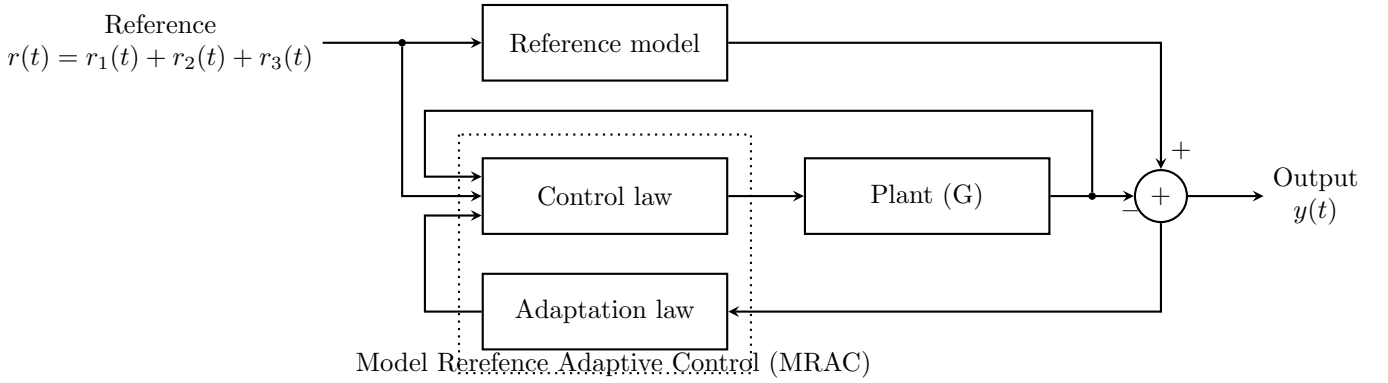


$$C(s) = \frac{U(s)}{E(s)} = K_P + \frac{K_I}{s} + K_D s$$

$$C(z) = \frac{U(z)}{E(z)} = K_P + K_I \frac{T_S(z+1)}{2(z-1)} + K_D \frac{z-1}{zT_S}$$

T_S is sampling time and $\xi(t)$ is white noise with zero mean power 1.

Figure 2: Closed loop system with a PID Controller



Reference model:

$$\dot{y}_m + a_m y_m = b_m r$$

Error:

$$e = y - y_m$$

Adaptation law:

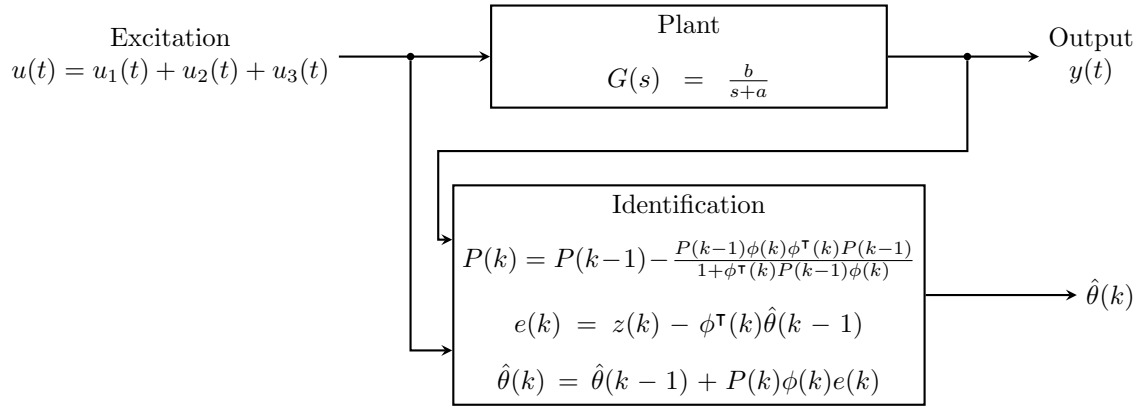
$$u = \hat{a}_r r + \hat{a}_y y$$

$$\dot{\hat{a}}_r = -\gamma e r$$

$$\dot{\hat{a}}_y = -\gamma e y$$

T_S is sampling time.

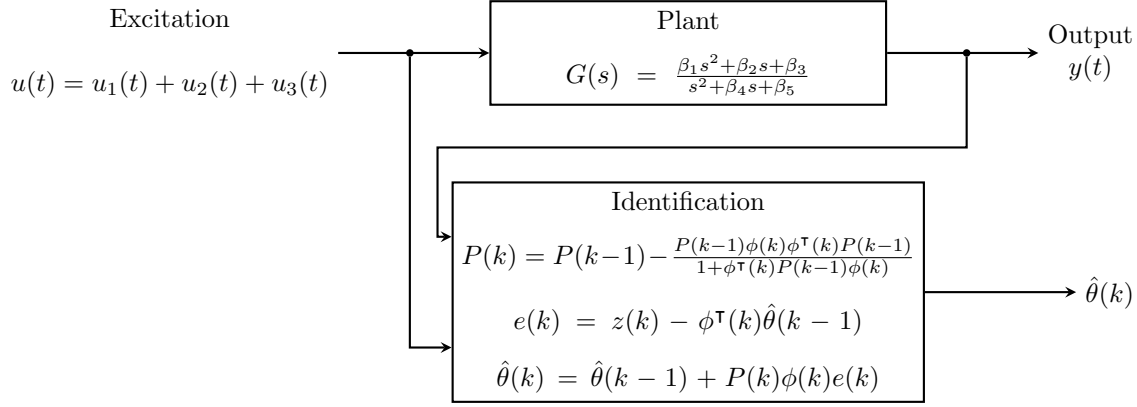
Figure 3: Adaptive control



$$P(0) = P_0 = P_0^T > 0, P_0 = \rho \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, z = y(k), \phi = [y(k-1) \quad u(k-1)]^T,$$

$$\theta = [\theta_1 \quad \theta_2]^T, \hat{a} = \frac{-\ln \hat{\theta}_1}{T_S}, \hat{b} = \frac{\hat{\theta}_2 \hat{a}}{1-\hat{\theta}_1}, \text{ and } T_S \text{ is sampling time.}$$

Figure 4: First order system identification



Discretized plant model: $G(z) = \frac{y(k)}{u(k)} = \frac{\alpha_1 z^2 + \alpha_2 z + \alpha_3}{z^2 + \alpha_4 z + \alpha_5}$, $z(k) = y(k)$,

T_S is sampling time, $P(0) = P_0 = P_0^T > 0$,

$$P_0 = \rho \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \phi = \begin{bmatrix} u(k) \\ u(k-1) \\ u(k-2) \\ -y(k-1) \\ -y(k-2) \end{bmatrix}, \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \end{bmatrix}.$$

Discretized plant model in state space:

$$x(k+1) = A_d x(k) + B_d u(k),$$

$$y(k+1) = C_d x(k) + D_d u(k),$$

where $A = \begin{bmatrix} 0 & 1 \\ -\alpha_5 & -\alpha_4 \end{bmatrix}$, $B_d = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C_d = [\theta_3 - \theta_5 \theta_1 \quad \theta_2 - \theta_4 \theta_1]$, and $D_d = [\theta_1]$.

Continuous plant model in state space:

$$\dot{x} = Ax + Bu,$$

$$y = Cx + Du,$$

where $A = \frac{2}{T_S} R(I - \frac{8}{21} R^2 - \frac{4}{105} R^4)(I - \frac{5}{7} R^2)^{-1}$, $B = A(A_d - I)^{-1} B_d$, $C = C_d$, $D = D_d$,

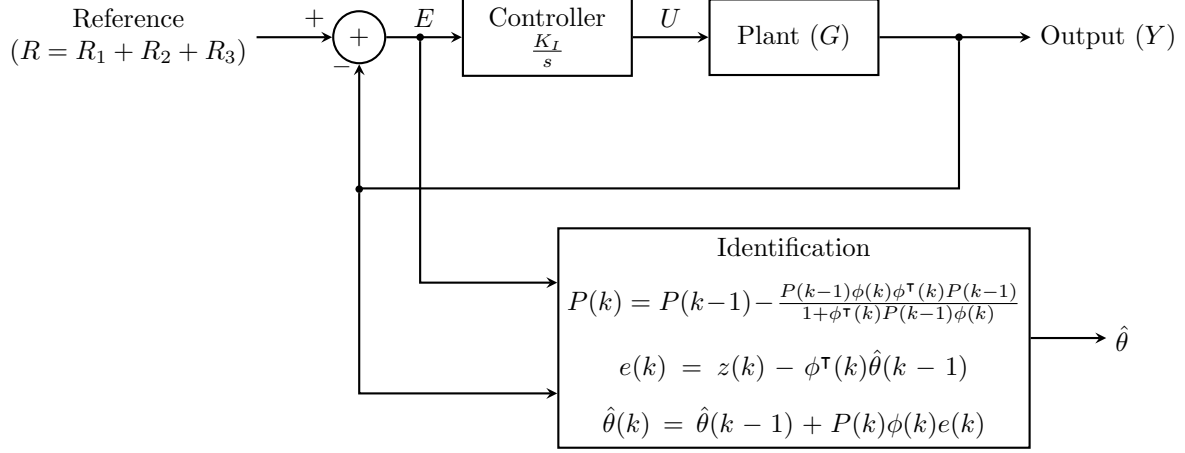
$$R = (A_d - I)(A_d + I)^{-1}, \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\beta_1 = D_{1,1}, \beta_2 = C_{1,1} B_{1,1} + C_{1,2} B_{2,1} - D_{1,1}(A_{1,1} + A_{2,2}),$$

$$\beta_3 = C_{1,1}(B_{2,1} A_{1,2} - B_{1,1} A_{2,2}) + C_{1,2}(B_{1,1} A_{2,1} - B_{2,1} A_{1,1}) + D_{1,1}(A_{1,1} A_{2,2} - A_{1,2} A_{2,1}),$$

$$\beta_4 = -(A_{1,1} + A_{2,2}), \text{ and } \beta_5 = A_{1,1} A_{2,2} - A_{1,2} A_{2,1}$$

Figure 5: Second order system identification



$$P(0) = P_0 = P_0^T > 0, P_0 = \rho \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, z = y(k) - y(k-1),$$

$$\phi = \begin{bmatrix} e(k-1) + e(k-2) \\ y(k-1) - y(k-2) \end{bmatrix}, \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}, \hat{a} = -\frac{\ln \hat{\theta}_2}{T_S}, \hat{b} = \frac{2\hat{\theta}_1 \hat{a}}{T_S(1-\hat{\theta}_2)K_I},$$

and T_S is sampling time.

Figure 6: Parameter estimation of a first order system with integral controller