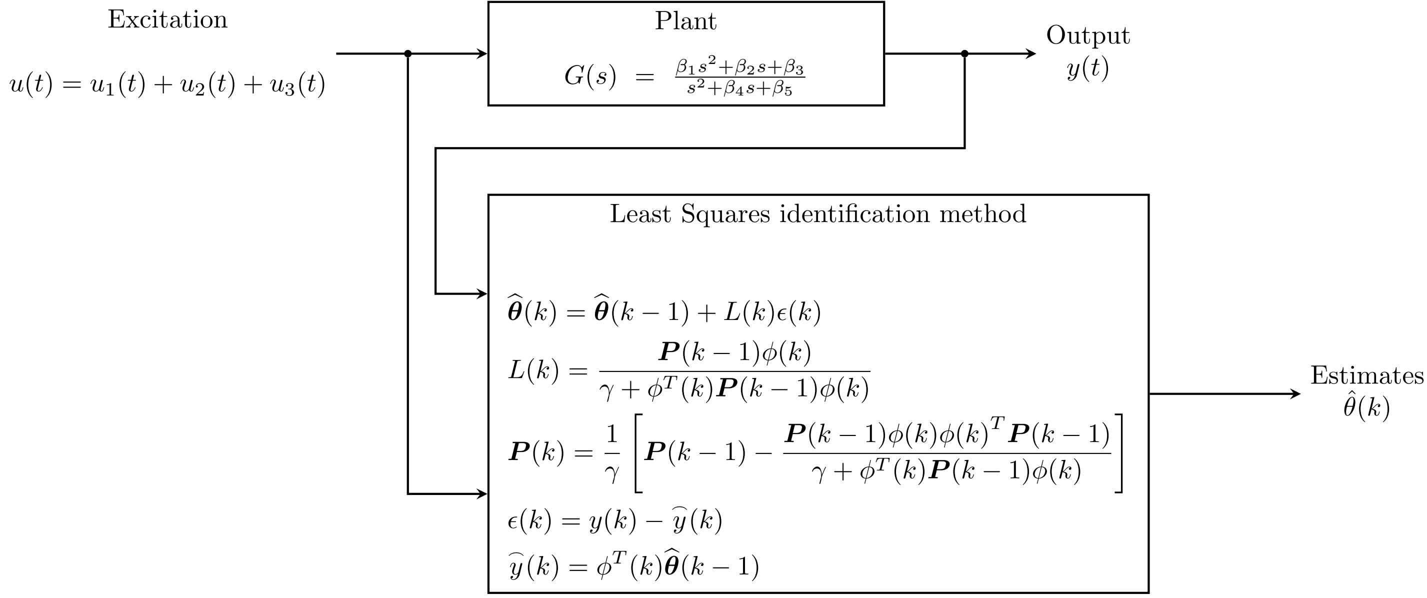

Discretized plant model:

$$y(k) = \theta_1 u(k) + \theta_2 u(k-1) + \theta_3 u(k-2) - \theta_4 y(k-1) - \theta_5 y(k-2) = \phi^T(k)\boldsymbol{\theta}$$
$$\boldsymbol{\theta} = [\theta_1,\ \theta_2,\ \theta_3,\ \theta_4, \theta_5]^T, \qquad \phi(k) = [u(k), u(k-1),\ u(k-2), -y(k-1),\ -y(k-2)]^T$$

$\hat{\boldsymbol{\theta}}(k) = [\hat{\theta}_1(k),\ \hat{\theta}_2(k),\ \hat{\theta}_3(k),\ \hat{\theta}_4(k),\ \hat{\theta}_5(k)]^T$,
T_S is sampling time,

$$\boldsymbol{P}(0) = \boldsymbol{P_0} = \boldsymbol{P_0}^{\intercal} > 0,\ \boldsymbol{P_0} = \rho\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}.$$

Discretized plant model in state space:

$$\boldsymbol{x}(k+1) = \hat{\boldsymbol{A}}_d\boldsymbol{x}(k) + \hat{\boldsymbol{B}}_d u(k)$$
$$y(k+1) = \hat{\boldsymbol{C}}_d\boldsymbol{x}(k) + \hat{D}_d u(k)$$

where

$$\hat{\boldsymbol{A}}_d = \begin{bmatrix} 0 & 1 \\ -\hat{\theta}_5 & -\hat{\theta}_4 \end{bmatrix}, \qquad \hat{\boldsymbol{B}}_d = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad \hat{\boldsymbol{C}}_d = [\hat{\theta}_3 - \hat{\theta}_5\hat{\theta}_1,\ \hat{\theta}_2 - \hat{\theta}_4\hat{\theta}_1], \qquad \widehat{\mathrm{D}}_d = \hat{\theta}_1$$

. Estimated continuous model is:

$$\dot{\boldsymbol{x}}(t) = \widehat{\boldsymbol{A}}_c x(t) + \widehat{\boldsymbol{B}}_c u(t)$$
$$y(t) = \widehat{\boldsymbol{C}}_c\boldsymbol{x}(t) + \widehat{D}_c u(t)$$

where

$$\widehat{\mathbf{A}}_c = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \frac{2}{T_s}\mathbf{R}\left[\mathbf{I}_{2x2} - \frac{8}{21}\mathbf{R}^2 - \frac{4}{105}\mathbf{R}^4\right]\left[\mathbf{I}_{2x2} - \frac{5}{7}\mathbf{R}^2\right]^{-1}$$
$$\widehat{\mathbf{B}}_c = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} = \widehat{\mathbf{A}}_c\left[\widehat{\mathbf{A}}_d - \mathbf{I}_{2x2}\right]^{-1}\widehat{\mathbf{B}}_d, \qquad \widehat{\mathbf{C}}_c = [c_{11},\ c_{12}] = \widehat{\mathbf{C}}_d, \qquad \widehat{\mathrm{D}}_c = \widehat{\mathrm{D}}_d$$
$$\mathbf{R} = \left[\widehat{\mathbf{A}}_d - \mathbf{I}_{2x2}\right]\left[\widehat{\mathbf{A}}_d + \mathbf{I}_{2x2}\right]^{-1}$$

So, we calculate:

$\hat{\beta}_1 = \widehat{\mathrm{D}}_c, \quad \hat{\beta}_2 = c_{11}b_{11} + c_{12}b_{21} - \widehat{\mathrm{D}}_c\left[a_{11} + a_{22}\right],$
$\hat{\beta}_3 = c_{11}(b_{21}a_{12} - b_{11}a_{22}) + c_{12}(b_{11}a_{21} - b_{21}a_{11}) + \hat{\beta}_5\widehat{\mathrm{D}}_c, \quad \hat{\beta}_4 = -(a_{11} + a_{22}), \quad \hat{\beta}_5 = a_{11}a_{22} - a_{12}a_{21}$