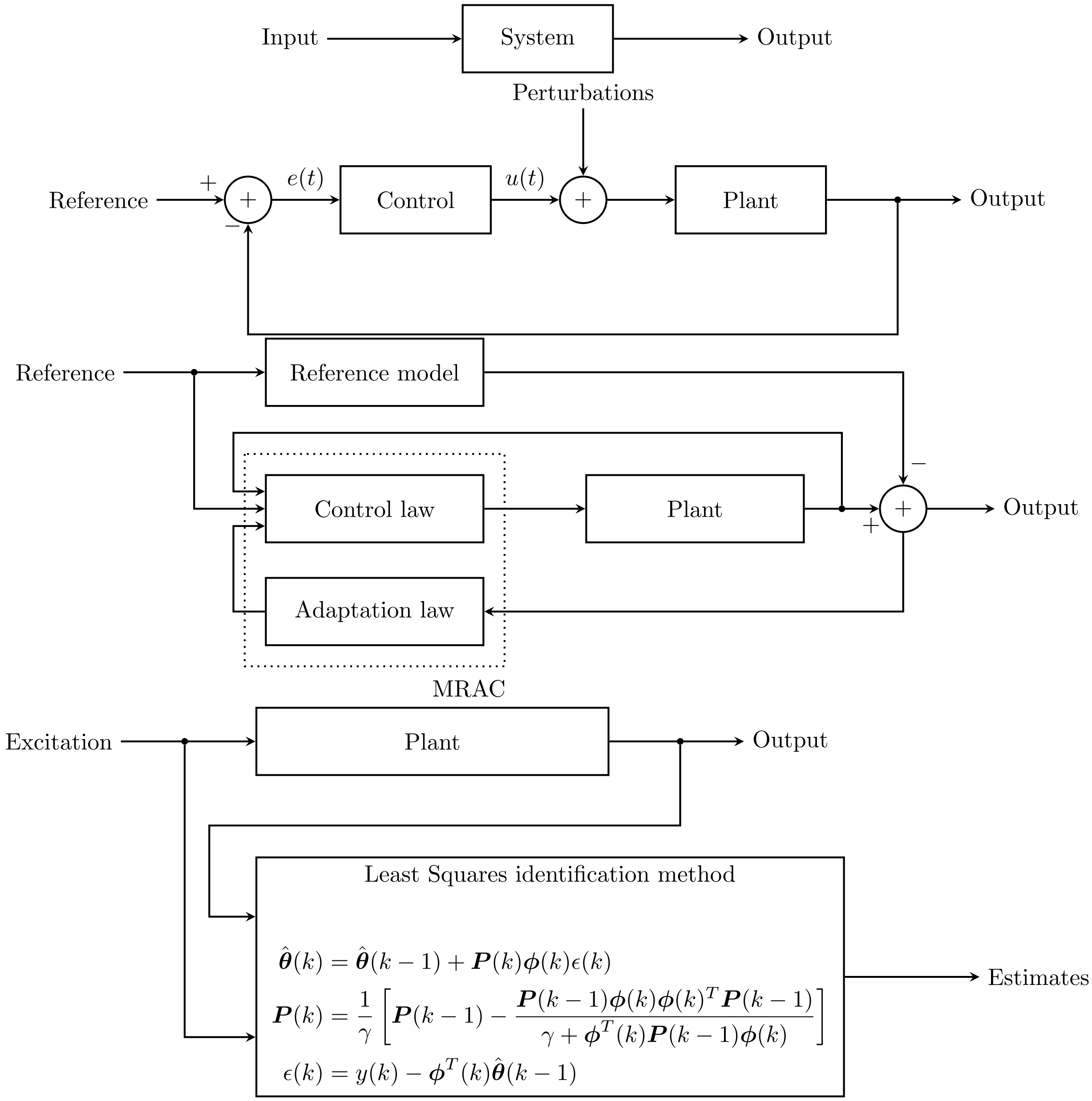


Figure 1: Background for the app

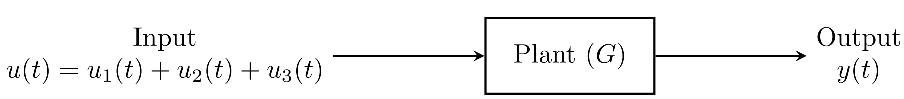
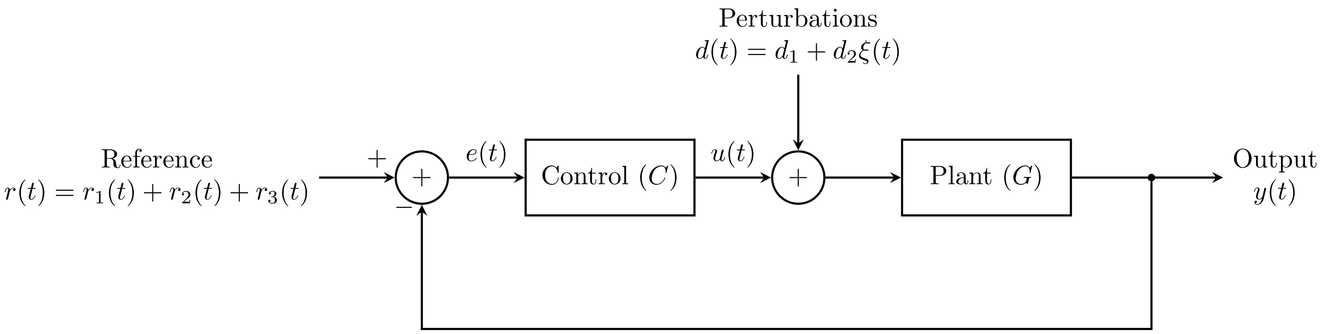
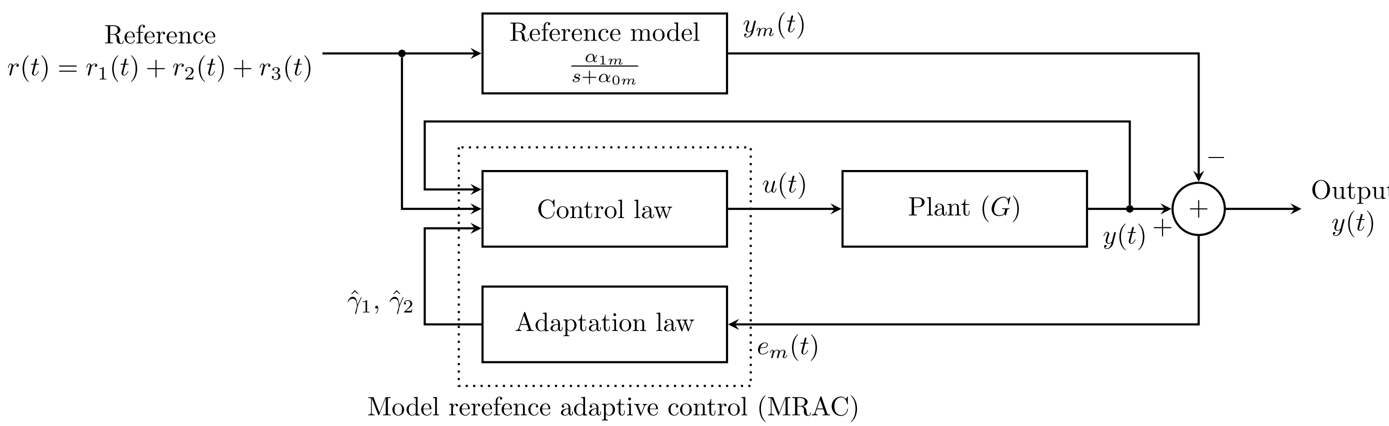


Figure 2: Open loop system



$C(s) = \frac{U(s)}{E(s)} = K_P + \frac{K_I}{s} + K_D s$
$C(z) = \frac{U(z)}{E(z)} = K_P + K_I \frac{T_S(z+1)}{2(z-1)} + K_D \frac{z-1}{zT_S}$
T_S is sampling time and $\xi(t)$ is white noise with zero mean power 1.

Figure 3: Closed loop system with a PID Controller



Control law:

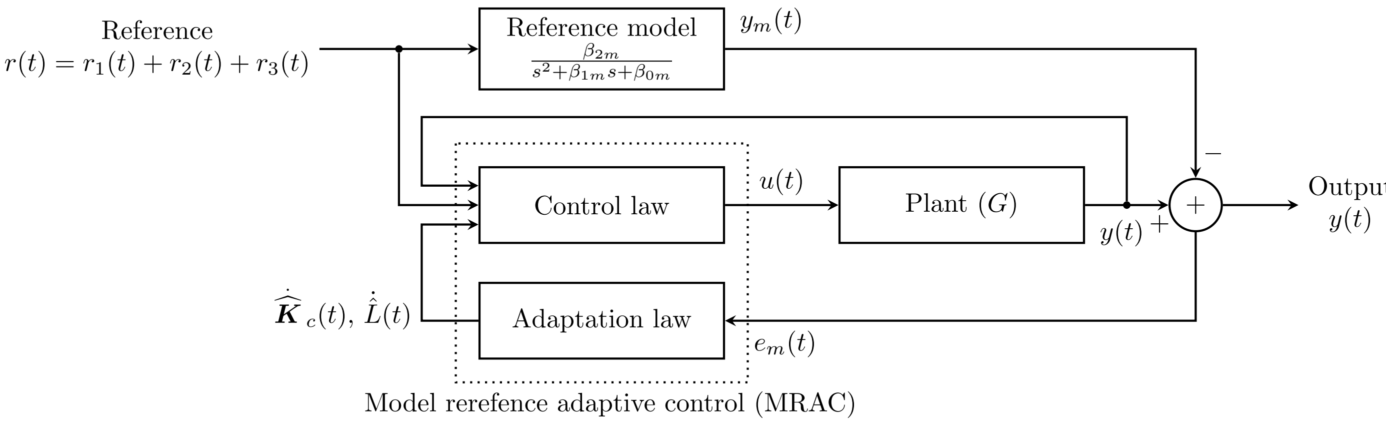
$$u(t) = \hat{\gamma}_1(t)r(t) + \hat{\gamma}_2(t)y(t)$$

Adaptation law:

$$\dot{\hat{\gamma}}_1(t) = -\varrho e_m(t)r(t)$$
$$\dot{\hat{\gamma}}_2(t) = -\varrho e_m(t)y(t)$$

where $e_m = y - y_m$, $\varrho > 0$ is called adaptation gain and T_S is sampling time.

Figure 4: Adaptive control



Model:

$$\dot{\boldsymbol{x}}_m(t) = \boldsymbol{A}_{mc}\boldsymbol{x}_m(t) + \boldsymbol{B}_{mc}r(t)$$

$$\boldsymbol{A}_{mc} = \begin{bmatrix} 0 & 1 \\ -\beta_{0m} & -\beta_{1m} \end{bmatrix}, \qquad \boldsymbol{B}_{mc} = \begin{bmatrix} 0 \\ \beta_{2m} \end{bmatrix}, \qquad \boldsymbol{x}_m(t) = [y_m(t),\ \dot{y}_m(t)]^T$$

Control law:

$$u(t) = \widehat{\boldsymbol{K}}_c(t)\boldsymbol{x}(t) + \hat{L}(t)r(t)$$

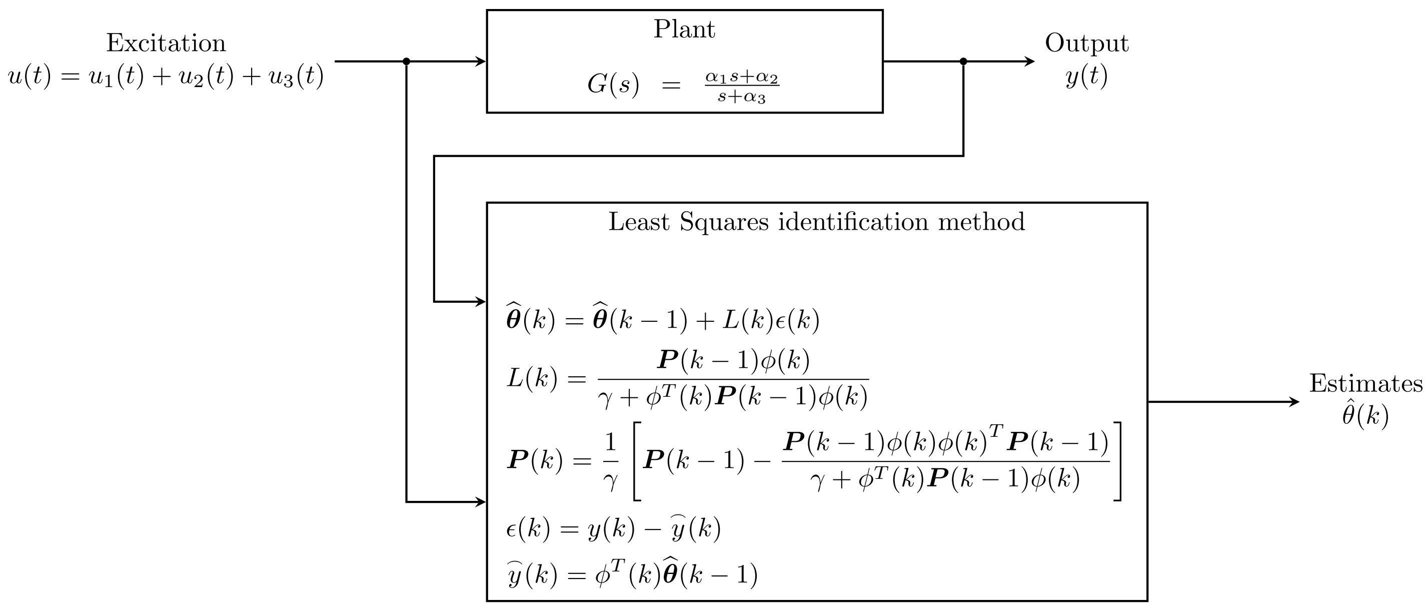
Adaptation law:

$$\dot{\widehat{\boldsymbol{K}}}_c(t) = \gamma\boldsymbol{B}_{mc}^T\mathcal{P}\boldsymbol{E}(t)\boldsymbol{x}^T(t)$$
$$\dot{\hat{L}}(t) = \gamma\boldsymbol{B}_{mc}^T\mathcal{P}\boldsymbol{E}(t)r(t)$$

where T_S is sampling time, $\gamma > 0$ is the adaptation gain, $\boldsymbol{x}(t) = [y(t),\ \dot{y}(t)]^T$, $\boldsymbol{E}_m(t) = [e_m(t),\ \dot{e}_m(t)]^T$, and matrix $\boldsymbol{\mathcal{P}} = \boldsymbol{\mathcal{P}}^T$ satisfies:

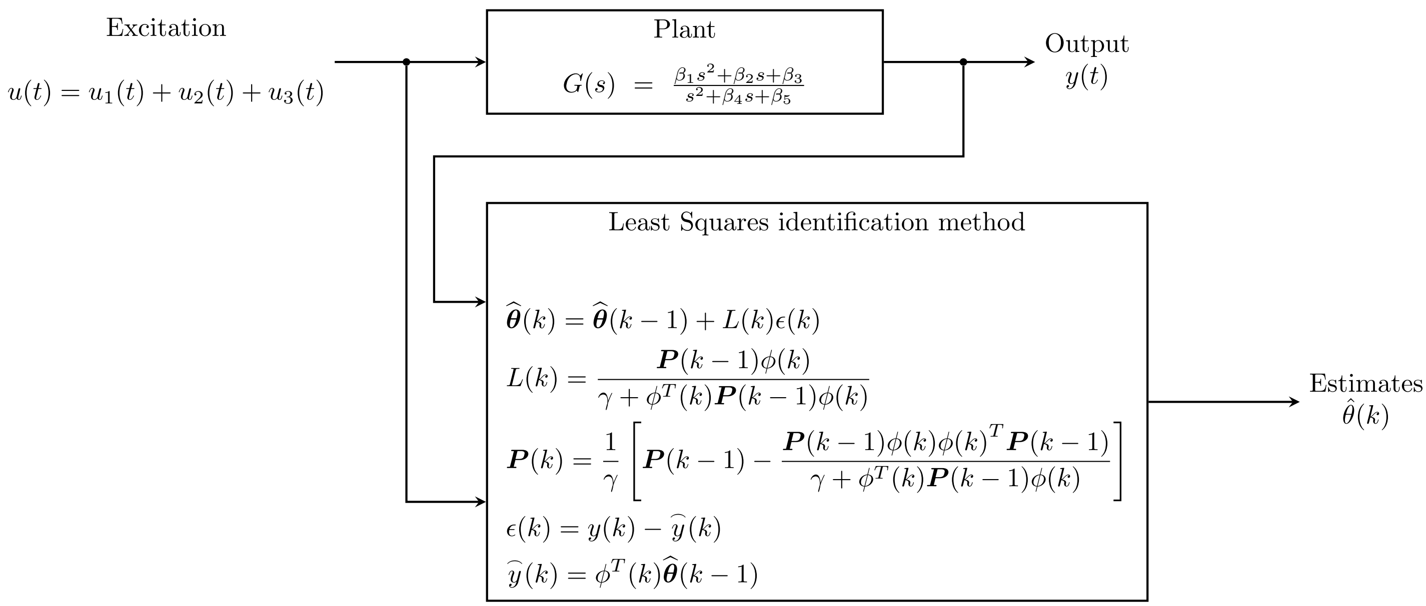
$$\boldsymbol{A}_{mc}^T\mathcal{P} + \mathcal{P}\boldsymbol{A}_{mc}^T = -\boldsymbol{I}_{2x2}$$

Figure 5: Adaptive control



$\boldsymbol{P}(0) = \boldsymbol{P_0} = \boldsymbol{P_0}^{\intercal} > 0$, $\boldsymbol{P_0} = \rho \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\boldsymbol{\phi} = \begin{bmatrix} u(k) & u(k-1) & y(k-1) \end{bmatrix}^{\intercal}$,
$\boldsymbol{\theta} = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 \end{bmatrix}^{\intercal}$, $\hat{\boldsymbol{\theta}} = \begin{bmatrix} \hat{\theta}_1 & \hat{\theta}_2 & \hat{\theta}_3 \end{bmatrix}^{\intercal}$,
$\hat{\alpha}_1(k) = \hat{\theta}_1(k)$, $\hat{\alpha}_2(k) = \hat{\alpha}_3(k)\left[\frac{\hat{\theta}_2(k) + \hat{\theta}_1(k)\hat{\theta}_3(k)}{1 - \hat{\theta}_3(k)} + \hat{\theta}_1(k)\right]$, $\hat{\alpha}_3(k) = -\frac{\ln(\hat{\theta}_{3*}(k))}{T_s}$,
$\hat{\theta}_{3*}(k) = \begin{cases} \hat{\theta}_3(k) & \text{if} \quad \hat{\theta}_3(k) > 0.01 \\ 0.01 & \text{if} \quad \hat{\theta}_{3*}(k) \leq 0.01 \end{cases}$ and T_S is sampling time.

Figure 6: First order system identification



Discretized plant model:

$$
\begin{aligned}
&y(k) = \theta_1 u(k) + \theta_2 u(k-1) + \theta_3 u(k-2) - \theta_4 y(k-1) - \theta_5 y(k-2) = \phi^T(k)\boldsymbol{\theta} \\
&\boldsymbol{\theta} = [\theta_1,\ \theta_2,\ \theta_3,\ \theta_4, \theta_5]^T, \qquad \phi(k) = [u(k), u(k-1),\ u(k-2), -y(k-1),\ -y(k-2)]^T
\end{aligned}
$$

$\hat{\boldsymbol{\theta}}(k) = [\hat{\theta}_1(k),\ \hat{\theta}_2(k),\ \hat{\theta}_3(k),\ \hat{\theta}_4(k),\ \hat{\theta}_5(k)]^T$,
T_S is sampling time,

$$
\boldsymbol{P}(0) = \boldsymbol{P_0} = \boldsymbol{P_0}^{\intercal} > 0,\ \boldsymbol{P_0} = \rho \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.
$$

Discretized plant model in state space:

$$
\begin{aligned}
\boldsymbol{x}(k+1) &= \hat{\boldsymbol{A}}_d \boldsymbol{x}(k) + \hat{\boldsymbol{B}}_d u(k) \\
y(k+1) &= \hat{\boldsymbol{C}}_d \boldsymbol{x}(k) + \hat{D}_d u(k)
\end{aligned}
$$

where

$$
\hat{\boldsymbol{A}}_d = \begin{bmatrix} 0 & 1 \\ -\hat{\theta}_5 & -\hat{\theta}_4 \end{bmatrix}, \qquad \hat{\boldsymbol{B}}_d = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad \hat{\boldsymbol{C}}_d = [\hat{\theta}_3 - \hat{\theta}_5\hat{\theta}_1,\ \hat{\theta}_2 - \hat{\theta}_4\hat{\theta}_1], \qquad \widehat{\mathrm{D}}_d = \hat{\theta}_1
$$

. Estimated continuous model is:

$$
\begin{aligned}
\dot{\boldsymbol{x}}\,(t) &= \widehat{\boldsymbol{A}}_c x(t) + \widehat{\boldsymbol{B}}_c u(t) \\
y(t) &= \widehat{\boldsymbol{C}}_c \boldsymbol{x}(t) + \widehat{D}_c u(t)
\end{aligned}
$$

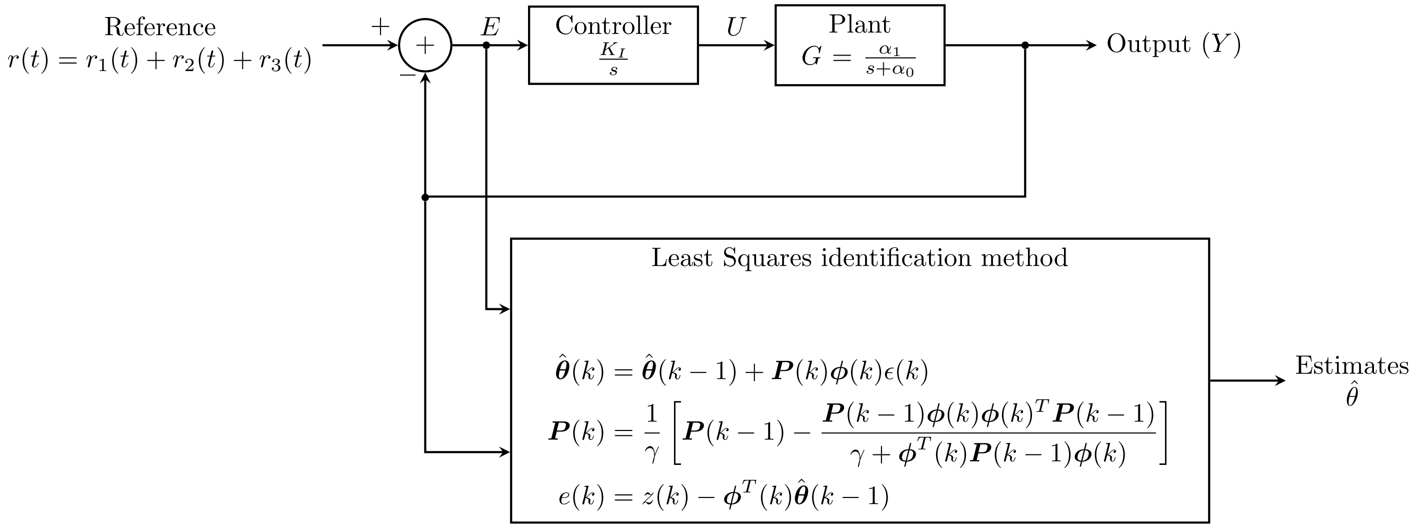
where

$$
\begin{aligned}
&\widehat{\mathbf{A}}_c = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \frac{2}{T_s}\mathbf{R}\left[\mathbf{I}_{2x2} - \frac{8}{21}\mathbf{R}^2 - \frac{4}{105}\mathbf{R}^4\right]\left[\mathbf{I}_{2x2} - \frac{5}{7}\mathbf{R}^2\right]^{-1} \\
&\widehat{\mathbf{B}}_c = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} = \widehat{\mathbf{A}}_c\left[\widehat{\mathbf{A}}_d - \mathbf{I}_{2x2}\right]^{-1}\widehat{\mathbf{B}}_d, \qquad \widehat{\mathbf{C}}_c = [c_{11},\ c_{12}] = \widehat{\mathbf{C}}_d, \qquad \widehat{\mathrm{D}}_c = \widehat{\mathrm{D}}_d \\
&\mathbf{R} = \left[\widehat{\mathbf{A}}_d - \mathbf{I}_{2x2}\right]\left[\widehat{\mathbf{A}}_d + \mathbf{I}_{2x2}\right]^{-1}
\end{aligned}
$$

So, we calculate:

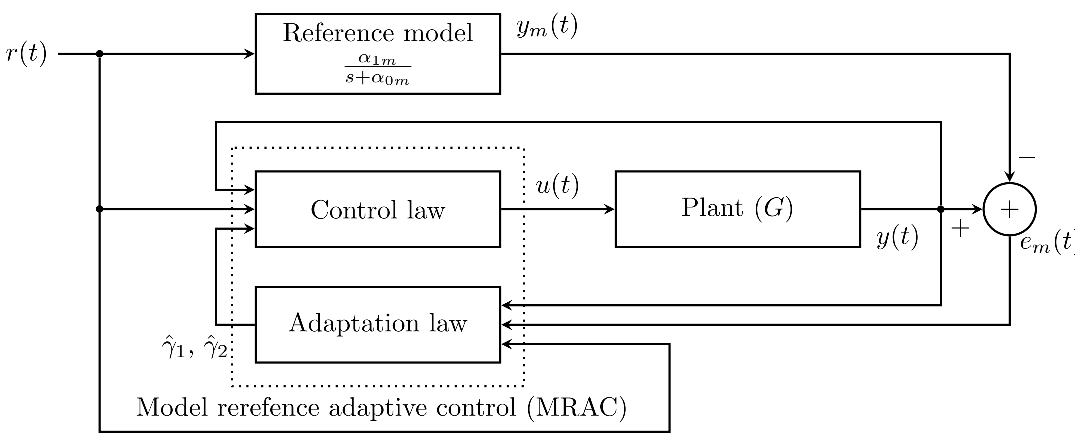
$\hat{\beta}_1 = \widehat{\mathrm{D}}_c, \quad \hat{\beta}_2 = c_{11}b_{11} + c_{12}b_{21} - \widehat{\mathrm{D}}_c\,[a_{11} + a_{22}]\,,$

$\hat{\beta}_3 = c_{11}(b_{21}a_{12} - b_{11}a_{22}) + c_{12}(b_{11}a_{21} - b_{21}a_{11}) + \hat{\beta}_5\widehat{\mathrm{D}}_c, \quad \hat{\beta}_4 = -(a_{11} + a_{22}), \quad \hat{\beta}_5 = a_{11}a_{22} - a_{12}a_{21}$



$\mathbf{P}(0) = \mathbf{P}_0 = \mathbf{P}_0^{\intercal} > 0$, $\mathbf{P}_0 = \rho \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $z = y(k) - y(k-1)$, $\boldsymbol{\phi} = \begin{bmatrix} e(k-1) + e(k-2) \\ y(k-1) - y(k-2) \end{bmatrix}$, $\boldsymbol{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$,
$\hat{\alpha_0} = -\frac{\ln \hat{\theta_2}}{T_S}$, $\hat{\alpha_1} = \frac{2\hat{\theta_1}\hat{\alpha_0}}{T_S(1-\hat{\theta_2})K_I}$, and T_S is sampling time.

Figure 8: Parameter estimation of a first order system with integral controller



Control law:

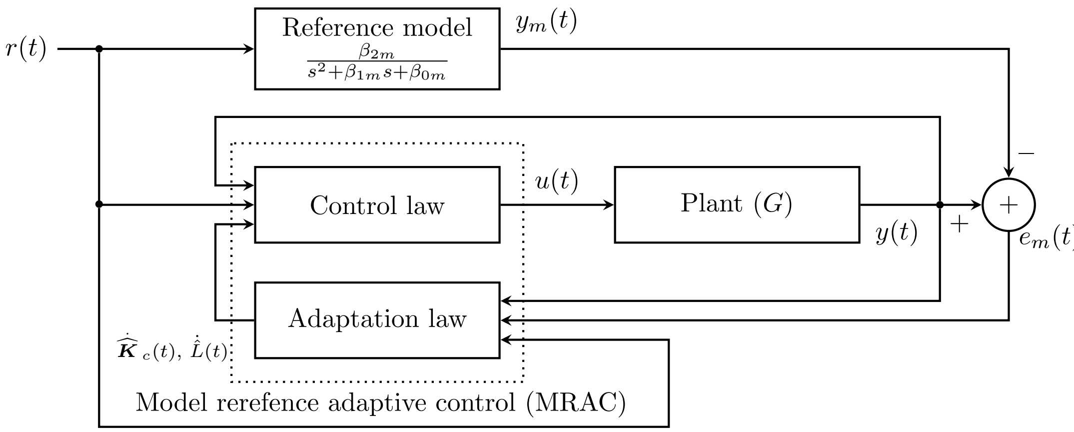
$$u(t) = \hat{\gamma}_1(t)r(t) + \hat{\gamma}_2(t)y(t)$$

Adaptation law:

$$\dot{\hat{\gamma}}_1(t) = -\varrho e_m(t)r(t)$$
$$\dot{\hat{\gamma}}_2(t) = -\varrho e_m(t)y(t)$$

where $e_m = y - y_m$, $\varrho > 0$ is called adaptation gain and T_S is sampling time.

Figure 9: Adaptive control



Model:

$$\dot{\boldsymbol{x}}_m(t) = \boldsymbol{A}_{mc}\boldsymbol{x}_m(t) + \boldsymbol{B}_{mc}r(t)$$

$$\boldsymbol{A}_{mc} = \begin{bmatrix} 0 & 1 \\ -\beta_{0m} & -\beta_{1m} \end{bmatrix}, \qquad \boldsymbol{B}_{mc} = \begin{bmatrix} 0 \\ \beta_{2m} \end{bmatrix}, \qquad \boldsymbol{x}_m(t) = [y_m(t),\ \dot{y}_m(t)]^T$$

Control law:

$$u(t) = \widehat{\boldsymbol{K}}_c(t)\boldsymbol{x}(t) + \hat{L}(t)r(t)$$

Adaptation law:

$$\dot{\widehat{\boldsymbol{K}}}_c(t) = \gamma \boldsymbol{B}_{mc}^T \mathcal{P}\boldsymbol{E}(t)\boldsymbol{x}^T(t)$$
$$\dot{\hat{L}}(t) = \gamma \boldsymbol{B}_{mc}^T \mathcal{P}\boldsymbol{E}(t)r(t)$$

where T_S is sampling time, $\gamma > 0$ is the adaptation gain, $\boldsymbol{x}(t) = [y(t),\ \dot{y}(t)]^T$, $\boldsymbol{E}_m(t) = [e_m(t),\ \dot{e}_m(t)]^T$, and matrix $\boldsymbol{\mathcal{P}} = \boldsymbol{\mathcal{P}}^T$ satisfies:

$$\boldsymbol{A}_{mc}^T\mathcal{P} + \mathcal{P}\boldsymbol{A}_{mc}^T = -\boldsymbol{I}_{2x2}$$

Figure 10: Adaptive control