

Figure 1: Background for the app

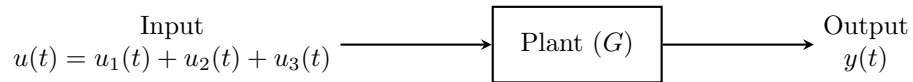
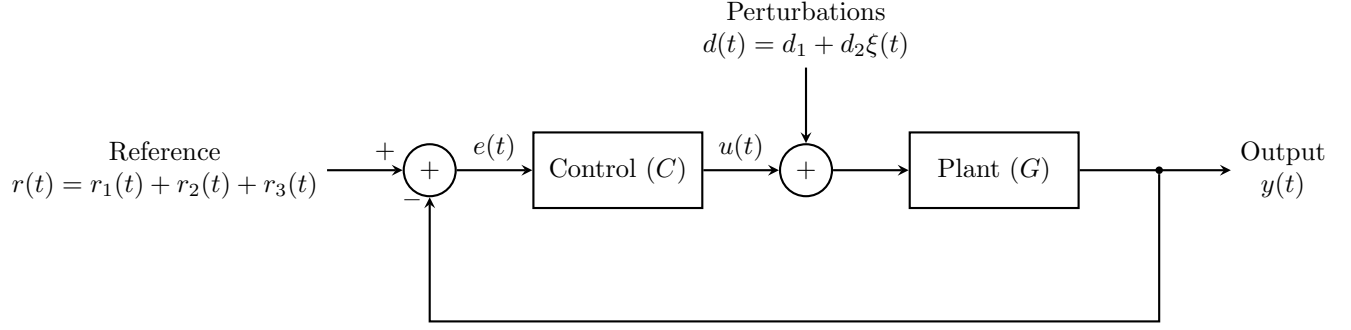


Figure 2: Open loop system

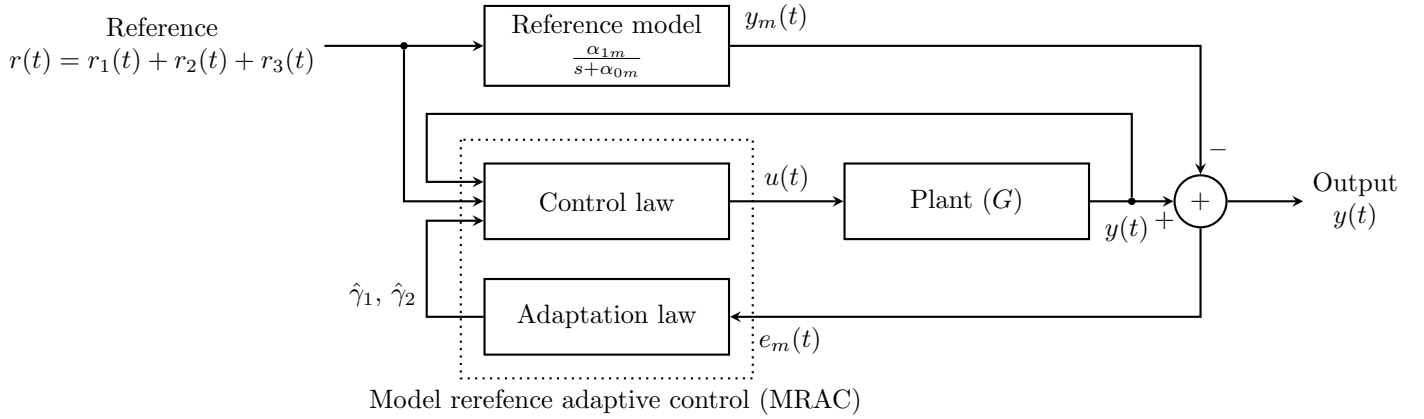


$$C(s) = \frac{U(s)}{E(s)} = K_P + \frac{K_I}{s} + K_D s$$

$$C(z) = \frac{U(z)}{E(z)} = K_P + K_I \frac{T_S(z+1)}{2(z-1)} + K_D \frac{z-1}{zT_S}$$

T_S is sampling time and $\xi(t)$ is white noise with zero mean power 1.

Figure 3: Closed loop system with a PID Controller



Control law:

$$u(t) = \hat{\gamma}_1(t)r(t) + \hat{\gamma}_2(t)y(t)$$

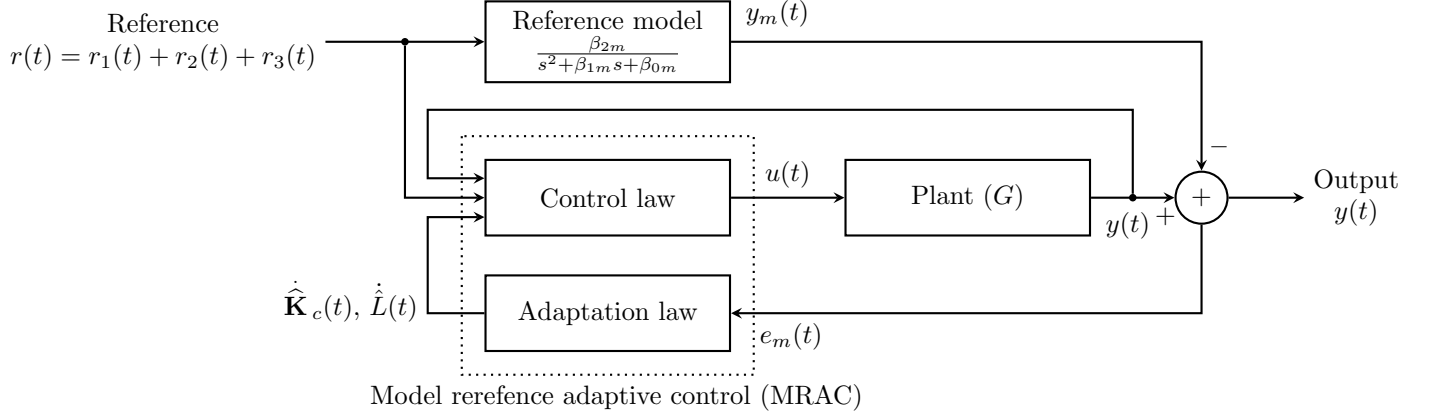
Adaptation law:

$$\dot{\hat{\gamma}}_1(t) = -\varrho e_m(t)r(t)$$

$$\dot{\hat{\gamma}}_2(t) = -\varrho e_m(t)y(t)$$

where $e_m = y - y_m$, $\varrho > 0$ is called adaptation gain and T_S is sampling time.

Figure 4: Adaptive control



Model:

$$\dot{\mathbf{x}}_m(t) = \mathbf{A}_{mc}\mathbf{x}_m(t) + \mathbf{B}_{mc}r(t)$$

$$\mathbf{A}_{mc} = \begin{bmatrix} 0 & 1 \\ -\beta_{0m} & -\beta_{1m} \end{bmatrix}, \quad \mathbf{B}_{mc} = \begin{bmatrix} 0 \\ \beta_{2m} \end{bmatrix}, \quad \mathbf{x}_m(t) = [y_m(t), \dot{y}_m(t)]^T$$

Control law:

$$u(t) = \hat{\mathbf{K}}_c(t)\mathbf{x}(t) + \hat{L}(t)r(t)$$

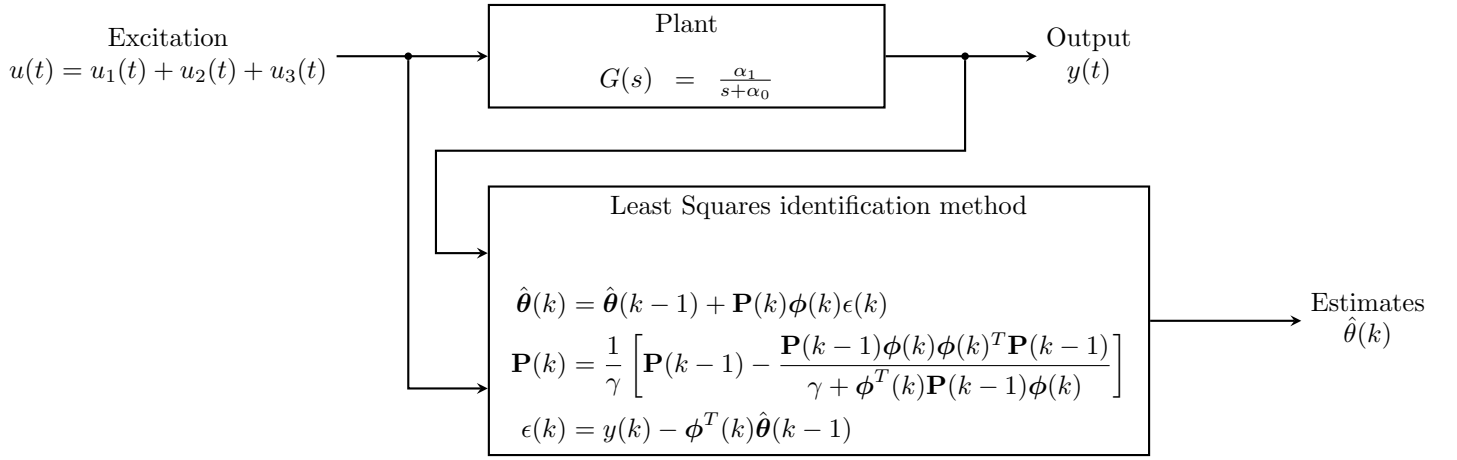
Adaptation law:

$$\begin{aligned} \dot{\hat{\mathbf{K}}}_c(t) &= \gamma \mathbf{B}_{mc}^T \mathcal{P} \mathbf{E}(t) \mathbf{x}^T(t) \\ \dot{\hat{L}}(t) &= \gamma \mathbf{B}_{mc}^T \mathcal{P} \mathbf{E}(t) r(t) \end{aligned}$$

where T_S is sampling time, $\gamma > 0$ is the adaptation gain, $\mathbf{x}(t) = [y(t), \dot{y}(t)]^T$, $\mathbf{E}_m(t) = [e_m(t), \dot{e}_m(t)]^T$ and matrix $\mathcal{P} = \mathcal{P}^T$ which satisfies:

$$\mathbf{A}_{mc}^T \mathcal{P} + \mathcal{P} \mathbf{A}_{mc} = -\mathbf{I}_{2 \times 2}$$

Figure 5: Adaptive control

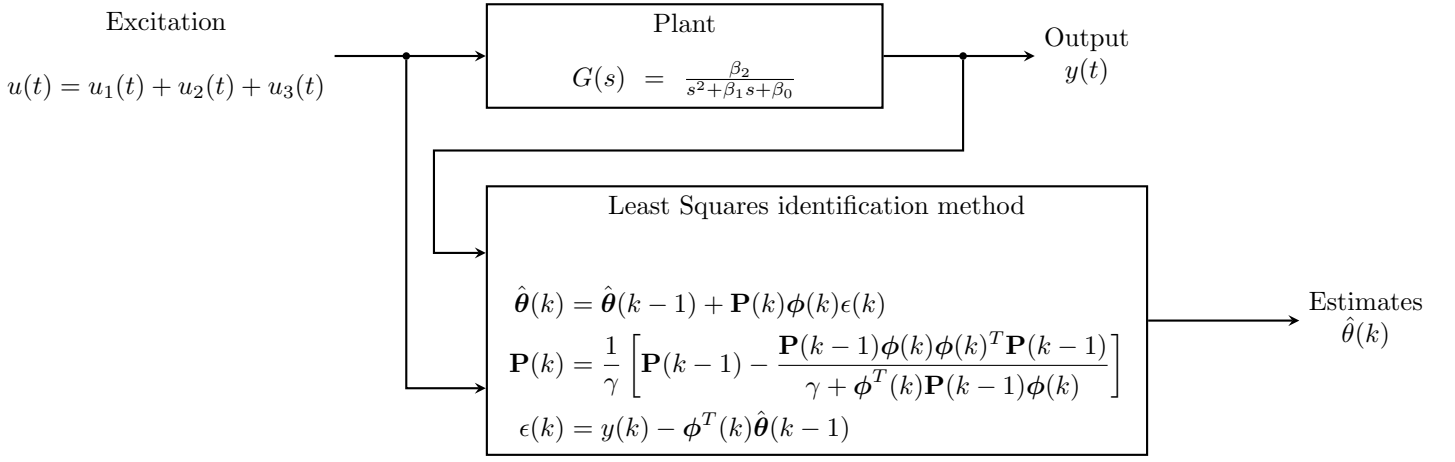


$$\mathbf{P}(0) = \mathbf{P}_0 = \mathbf{P}_0^T > 0, \mathbf{P}_0 = \rho \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \boldsymbol{\phi} = [y(k-1) \quad u(k-1)]^T,$$

$$\boldsymbol{\theta} = [\theta_1 \quad \theta_2]^T, \hat{\boldsymbol{\theta}} = [\hat{\theta}_1 \quad \hat{\theta}_2]^T, \hat{\alpha}_0 = \frac{-\ln \hat{\theta}_1}{T_S}, \hat{\alpha}_1 = \frac{\hat{\alpha}_0 \hat{\theta}_2}{1 - \hat{\theta}_1}, \text{ and}$$

T_S is sampling time.

Figure 6: First order system identification



Discretized plant model: $y(k) = \theta_1 y(k-1) + \theta_2 y(k-2) + \theta_3 u(k-1) + \theta_4 u(k-2) = \phi^T(k)\theta$,
 $\theta = [\theta_1, \theta_2, \theta_3, \theta_4]^T$,
 $\phi(k) = [y(k-1), y(k-2), u(k-1), u(k-2)]^T$,
 $\hat{\theta}(k) = [\hat{\theta}_1(k), \hat{\theta}_2(k), \hat{\theta}_3(k), \hat{\theta}_4(k)]^T$,
 T_S is sampling time,

$$P(0) = P_0 = P_0^T > 0, P_0 = \rho \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Discretized plant model in state space:

$$\begin{aligned} \mathbf{x}(k+1) &= \hat{\mathbf{A}}_d \mathbf{x}(k) + \hat{\mathbf{B}}_d u(k) \\ y(k+1) &= \hat{\mathbf{C}}_d \mathbf{x}(k) \end{aligned}$$

where

$$\hat{\mathbf{A}}_d = \begin{bmatrix} 0 & 1 \\ -\hat{\theta}_2 & -\hat{\theta}_1 \end{bmatrix}, \quad \hat{\mathbf{B}}_d = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \hat{\mathbf{C}}_d = [\hat{\theta}_4, \hat{\theta}_3]$$

. Estimated continuous model is:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \hat{\mathbf{A}}_c \mathbf{x}(t) + \hat{\mathbf{B}}_c u(t) \\ y(t) &= \hat{\mathbf{C}}_c \mathbf{x}(t) \end{aligned}$$

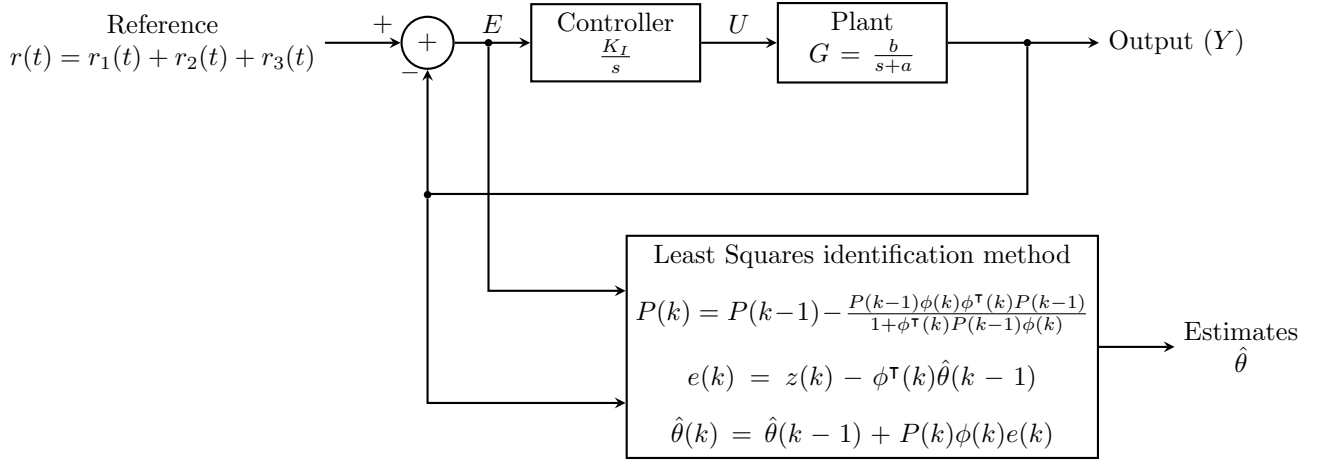
where

$$\begin{aligned} \hat{\mathbf{A}}_c &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \frac{2}{T} \mathbf{R} \left[\mathbf{I}_{2 \times 2} - \frac{8}{21} \mathbf{R}^2 - \frac{4}{105} \mathbf{R}^4 \right] \left[\mathbf{I}_{2 \times 2} - \frac{5}{7} \mathbf{R}^2 \right]^{-1} \\ \hat{\mathbf{B}}_c &= \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} = \hat{\mathbf{A}}_c \left[\hat{\mathbf{A}}_d - \mathbf{I}_{2 \times 2} \right]^{-1} \hat{\mathbf{B}}_d, \quad \hat{\mathbf{C}}_c = [c_{11}, c_{12}] = \hat{\mathbf{C}}_d \\ \mathbf{R} &= \left[\hat{\mathbf{A}}_d - \mathbf{I}_{2 \times 2} \right] \left[\hat{\mathbf{A}}_d + \mathbf{I}_{2 \times 2} \right]^{-1} \end{aligned}$$

So, we calculate:

$$\hat{\beta}_0(k) = a_{11}a_{22} - a_{12}a_{21}, \quad \hat{\beta}_1 = -(a_{11} + a_{22}), \quad \hat{\beta}_2 = c_{11}(b_{21}a_{12} - b_{11}a_{22}) + c_{12}(b_{11}a_{21} - b_{21}a_{11})$$

Figure 7: Second order system identification



$$P(0) = P_0 = P_0^T > 0, P_0 = \rho \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, z = y(k) - y(k-1),$$

$$\phi = \begin{bmatrix} e(k-1) + e(k-2) \\ y(k-1) - y(k-2) \end{bmatrix}, \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}, \hat{a} = -\frac{\ln \theta_2}{T_S}, \hat{b} = \frac{2\theta_1 \hat{a}}{T_S(1-\theta_2)K_I},$$

and T_S is sampling time.

Figure 8: Parameter estimation of a first order system with integral controller