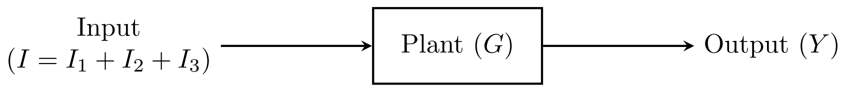
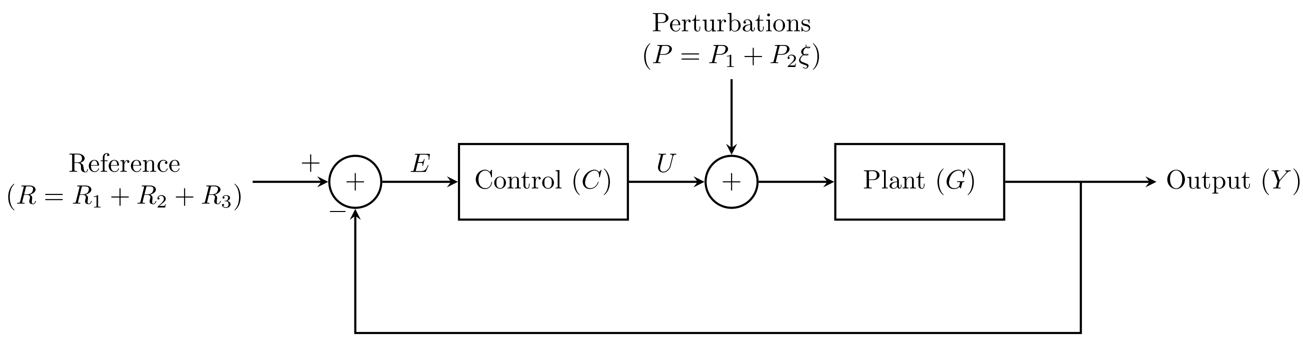
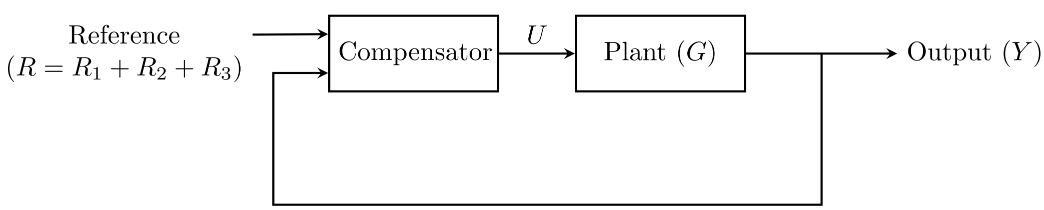



Figure 1: Open loop system



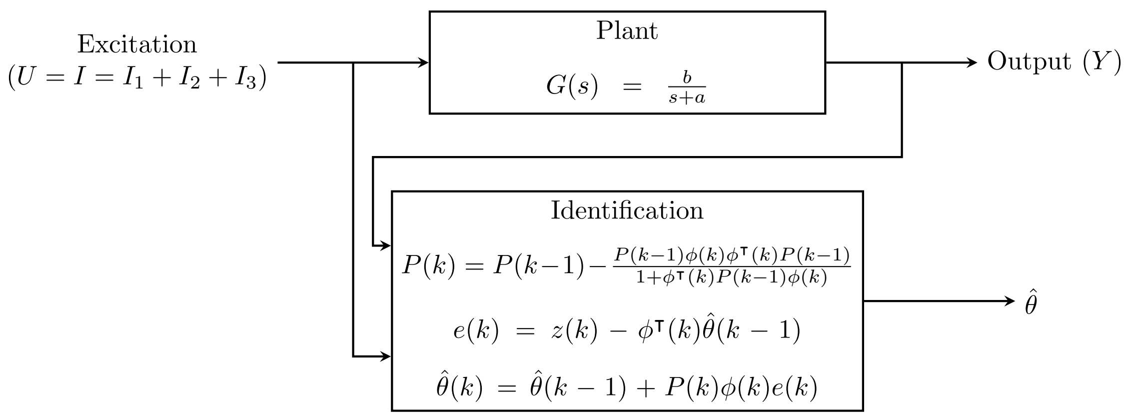

$C(s) = \frac{U(s)}{E(s)} = K_P + \frac{K_I}{s} + K_D s$
$C(z) = \frac{U(z)}{E(z)} = K_P + \frac{K_I T_S (z+1)}{(z-1)} + \frac{K_D (z-1)}{z T_S}$
$T_S$ is sampling time and $\xi$ is white noise with zero mean power 1.

Figure 2: Closed loop system with a PID Controller



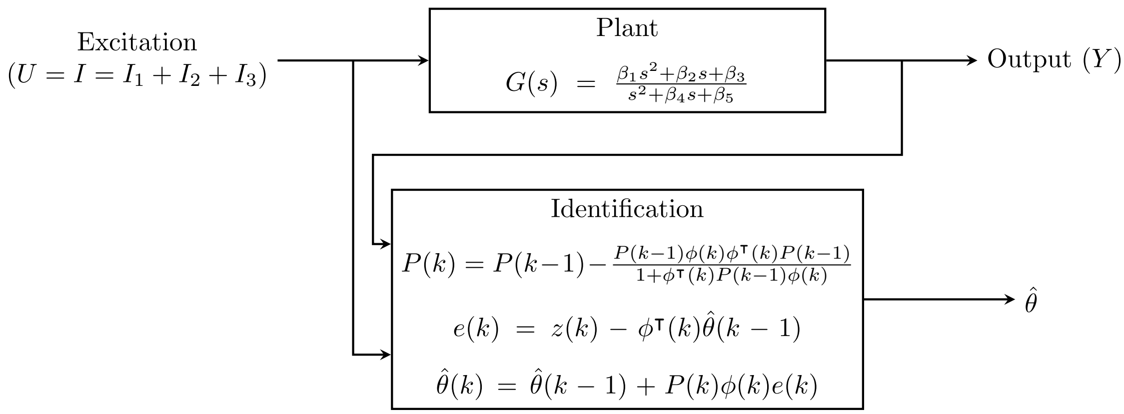

Reference model:
$\dot{y}_m + a_m y_m = b_m r$
Error:
$e = y - y_m$
Adaptive control algorithm:
$u = \hat{a}_r r + \hat{a}_y y$
$\dot{\hat{a}}_r = -\gamma e r$
$\dot{\hat{a}}_y = -\gamma e y$
$T_S$ is sampling time.

Figure 3: Closed loop system with an Adaptive controller




$P(0) = P_0 = P_0^\intercal > 0$, $P_0 = \rho \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $z = y(k)$, $\phi = \begin{bmatrix} y(k-1) & u(k-1) \end{bmatrix}^\intercal$, $\theta = \begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix}^\intercal$, $\hat{a} = \frac{-\ln \hat{\theta}_1}{T_S}$, $\hat{b} = \frac{\hat{\theta}_2 \hat{a}}{1-\hat{\theta}_1}$, and $T_S$ is sampling time.

Figure 4: First order system identification

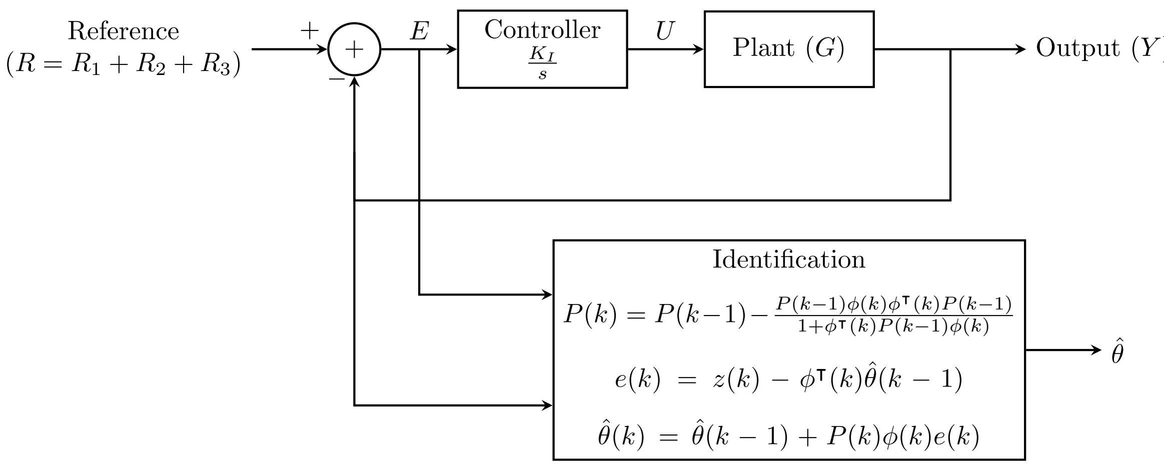



Discretized plant model: $G(z) = \frac{y(k)}{u(k)} = \frac{\alpha_1 z^2+\alpha_2 z+\alpha_3}{z^2+\alpha_4 z+\alpha_5}$, $z(k) = y(k)$,
$T_S$ is sampling time, $P(0) = P_0 = P_0^\intercal > 0$,

$$P_0 = \rho \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \; \phi = \begin{bmatrix} u(k) \\ u(k-1) \\ u(k-2) \\ -y(k-1) \\ -y(k-2) \end{bmatrix}, \; \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \end{bmatrix}.$$

Discretized plant model in state space:
$x(k+1) = A_d x(k) + B_d u(k)$,
$y(k+1) = C_d x(k) + D_d u(k)$,
where $A = \begin{bmatrix} 0 & 1 \\ -\alpha_5 & -\alpha_4 \end{bmatrix}$, $B_d = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C_d = \begin{bmatrix} \theta_3 - \theta_5\theta_1 & \theta_2 - \theta_4\theta_1 \end{bmatrix}$, and $D_d = \begin{bmatrix} \theta_1 \end{bmatrix}$.
Continous plan model in state space:
$\dot{x} = Ax + Bu$,
$y = Cx + Du$,
where $A = \frac{2}{T_S} R(I - \frac{8}{21}R^2 - \frac{4}{105}R^4)(I - \frac{5}{7}R^2)^{-1}$, $B = A(A_d - I)^{-1}B_d$, $C = C_d$, $D = D_d$,
$R = (A_d - I)(A_d + I)^{-1}$, and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$\beta_1 = D_{1,1}$, $\beta_2 = C_{1,1}B_{1,1} + C_{1,2}B_{2,1} - D_{1,1}(A_{1,1} + A_{2,2})$,
$\beta_3 = C_{1,1}(B_{2,1}A_{1,2} - B_{1,1}A_{2,2}) + C_{1,2}(B_{1,1}A_{2,1} - B_{2,1}A_{1,1}) + D_{1,1}(A_{1,1}A_{2,2} - A_{1,2}A_{2,1})$,
$\beta_4 = -(A_{1,1} + A_{2,2})$, and $\beta_5 = A_{1,1}A_{2,2} - A_{1,2}A_{2,1}$

Figure 5: Second order system identification




$P(0) = P_0 = P_0^{\intercal} > 0$, $P_0 = \rho \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $z = y(k) - y(k-1)$,
$\phi = \begin{bmatrix} e(k-1) + e(k-2) \\ y(k-1) - y(k-2) \end{bmatrix}$, $\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$, $\hat{a} = -\frac{\ln \hat{\theta_2}}{T_S}$, $\hat{b} = \frac{2\hat{\theta_1}\hat{a}}{T_S(1-\hat{\theta_2})K_I}$,
and $T_S$ is sampling time.

Figure 6: Parameter estimation of a first order system with integral controller