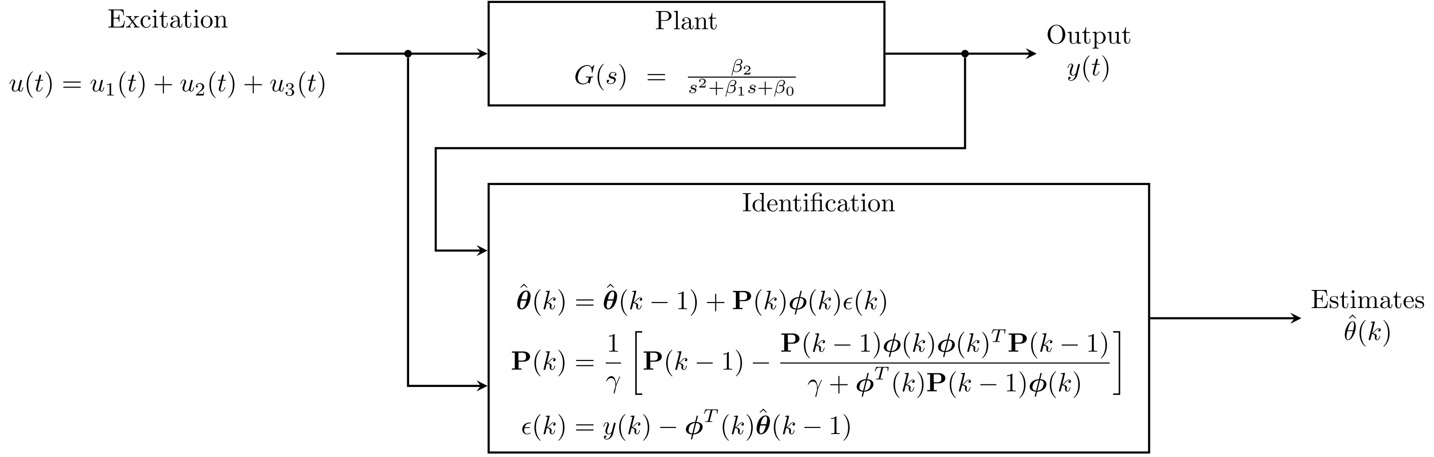

Discretized plant model: $y(k) = \theta_1 y(k-1) + \theta_2 y(k-2) + \theta_3 u(k-1) + \theta_4 u(k-2) = \boldsymbol{\phi}^T(k)\boldsymbol{\theta}$,
$\boldsymbol{\theta} = [\theta_1,\ \theta_2,\ \theta_3,\ \theta_4]^T$,
$\boldsymbol{\phi}(k) = [y(k-1),\ y(k-2),\ u(k-1),\ u(k-2)]^T$,
$\hat{\boldsymbol{\theta}}(k) = [\hat{\theta}_1(k),\ \hat{\theta}_2(k),\ \hat{\theta}_3(k),\ \hat{\theta}_4(k)]^T$,
T_S is sampling time,

$$\boldsymbol{P}(0) = \boldsymbol{P_0} = \boldsymbol{P_0}^\intercal > 0,\ \boldsymbol{P_0} = \rho \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Discretized plant model in state space:

$$\begin{aligned} \mathbf{x}(k+1) &= \hat{\mathbf{A}}_d \mathbf{x}(k) + \hat{\mathbf{B}}_d u(k) \\ y(k+1) &= \hat{\mathbf{C}}_d \mathbf{x}(k) \end{aligned}$$

where

$$\hat{\mathbf{A}}_d = \begin{bmatrix} 0 & 1 \\ -\hat{\theta}_2 & -\hat{\theta}_1 \end{bmatrix}, \qquad \hat{\mathbf{B}}_d = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad \hat{\mathbf{C}}_d = [\hat{\theta}_4,\ \hat{\theta}_3]$$

. Estimated continuous model is:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \widehat{\mathbf{A}}_c x(t) + \widehat{\mathbf{B}}_c u(t) \\ y(t) &= \widehat{\mathbf{C}}_c \mathbf{x}(t) \end{aligned}$$

where

$$\begin{aligned} \widehat{\mathbf{A}}_c &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \frac{2}{T}\mathbf{R}\left[\mathbf{I}_{2x2} - \frac{8}{21}\mathbf{R}^2 - \frac{4}{105}\mathbf{R}^4\right]\left[\mathbf{I}_{2x2} - \frac{5}{7}\mathbf{R}^2\right]^{-1} \\ \widehat{\mathbf{B}}_c &= \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} = \widehat{\mathbf{A}}_c\left[\widehat{\mathbf{A}}_d - \mathbf{I}_{2x2}\right]^{-1}\widehat{\mathbf{B}}_d, \qquad \widehat{\mathbf{C}}_c = [c_{11},\ c_{12}] = \widehat{\mathbf{C}}_d \\ \mathbf{R} &= \left[\widehat{\mathbf{A}}_d - \mathbf{I}_{2x2}\right]\left[\widehat{\mathbf{A}}_d + \mathbf{I}_{2x2}\right]^{-1} \end{aligned}$$

So, we calculate:

$$\hat{\beta}_0(k) = a_{11}a_{22} - a_{12}a_{21}, \quad \hat{\beta}_1 = -(a_{11} + a_{22}), \quad \hat{\beta}_2 = c_{11}(b_{21}a_{12} - b_{11}a_{22}) + c_{12}(b_{11}a_{21} - b_{21}a_{11})$$