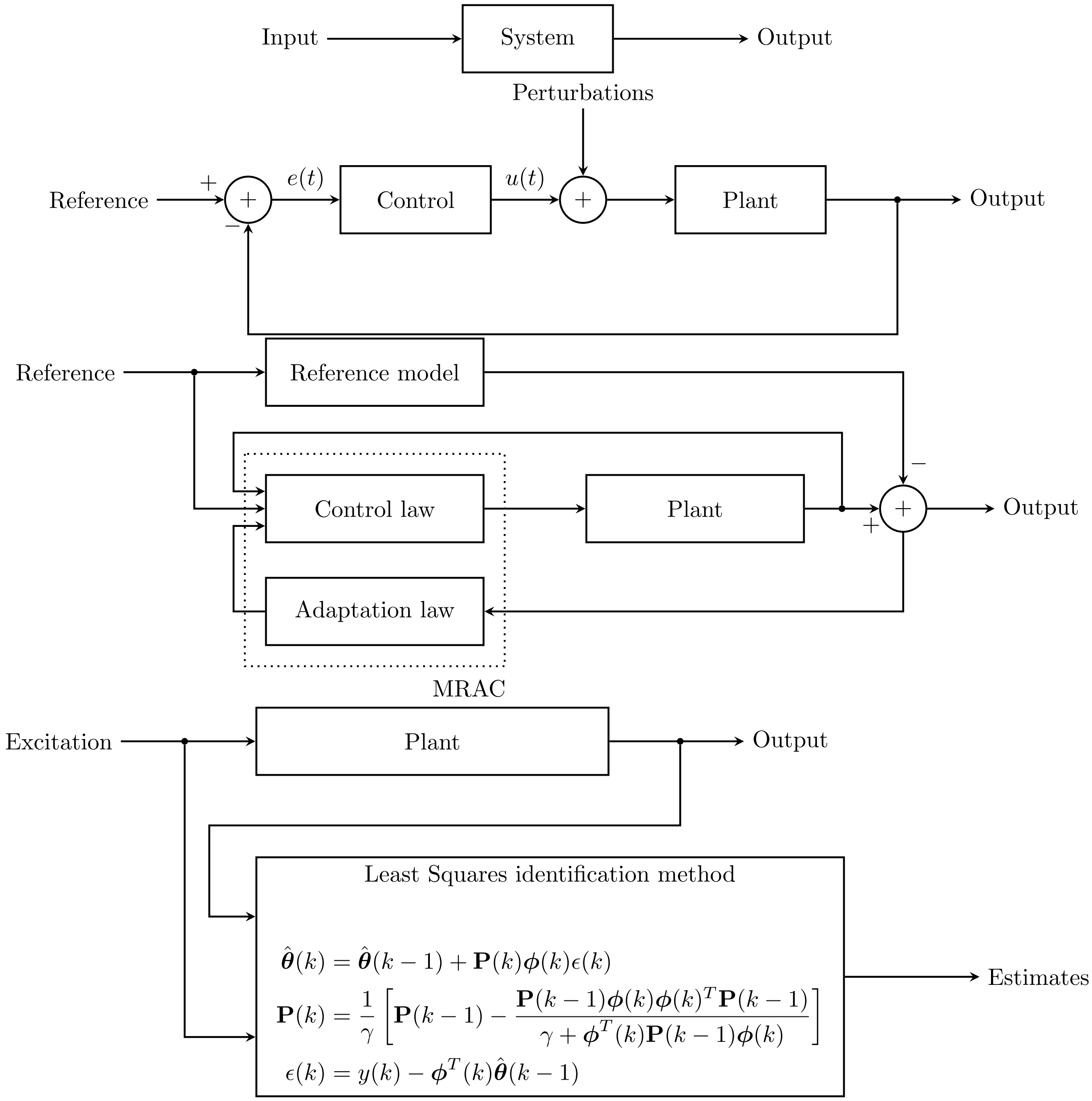


Figure 1: Background for the app

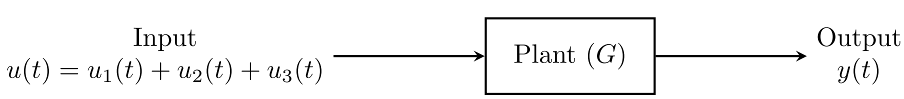
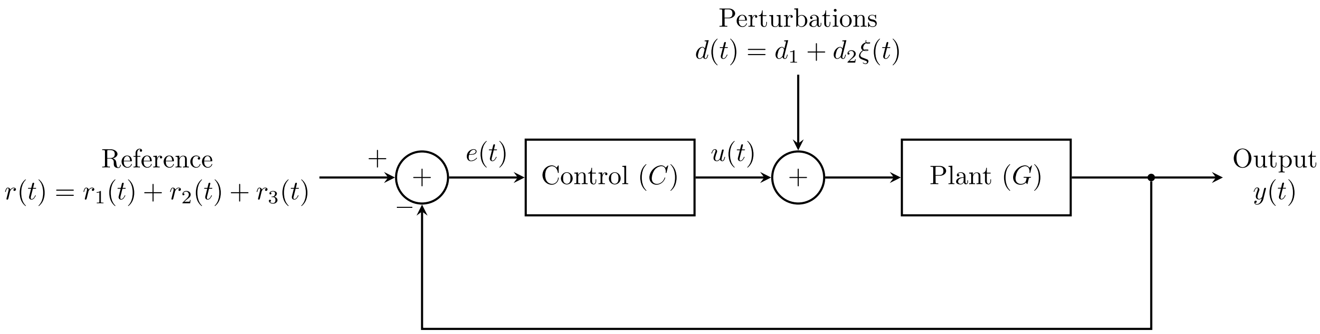
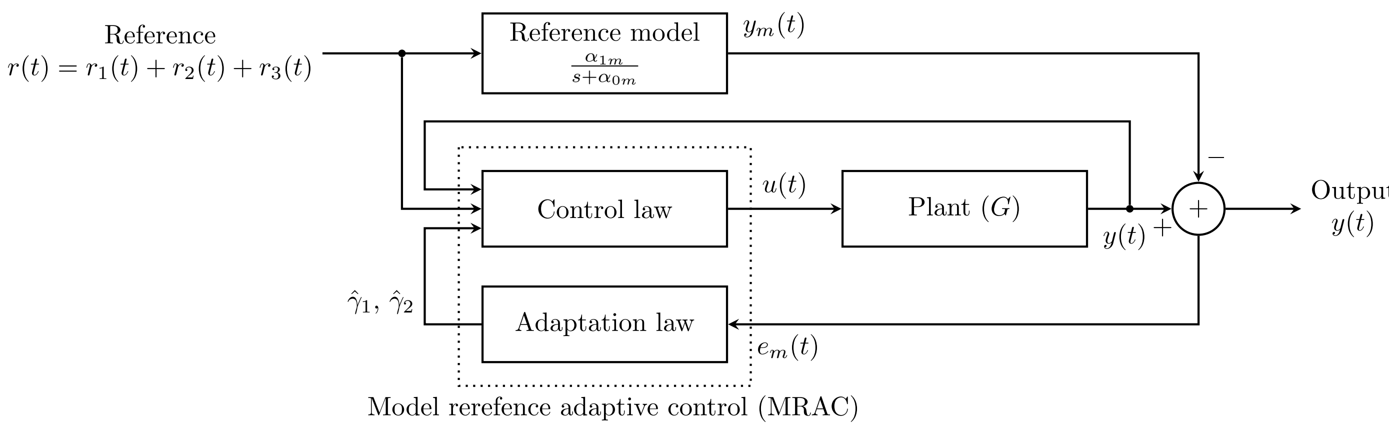


Figure 2: Open loop system



$C(s) = \frac{U(s)}{E(s)} = K_P + \frac{K_I}{s} + K_D s$
$C(z) = \frac{U(z)}{E(z)} = K_P + K_I \frac{T_S(z+1)}{2(z-1)} + K_D \frac{z-1}{zT_S}$
T_S is sampling time and $\xi(t)$ is white noise with zero mean power 1.

Figure 3: Closed loop system with a PID Controller



Control law:

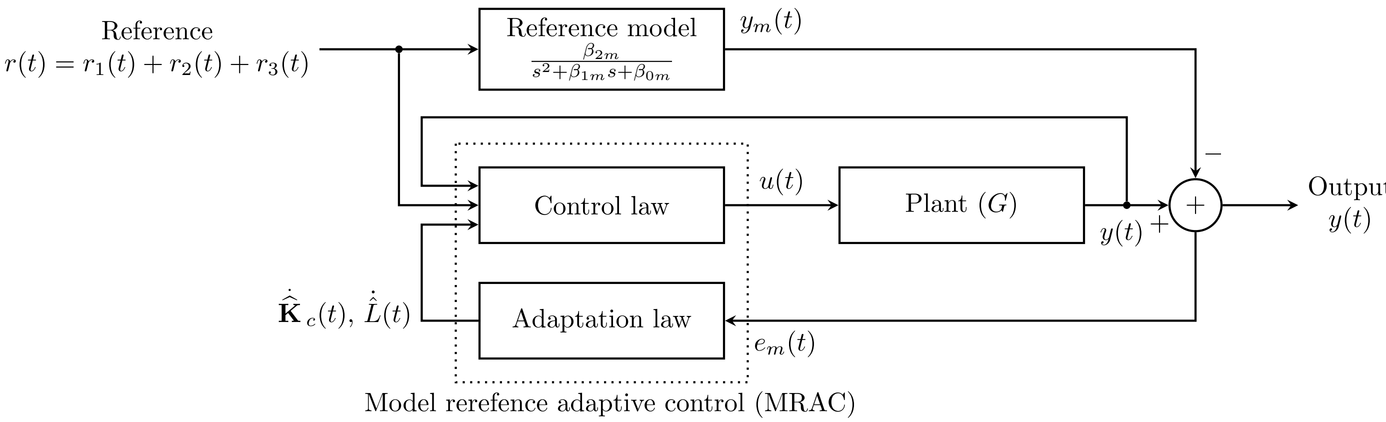
$$u(t) = \hat{\gamma}_1(t)r(t) + \hat{\gamma}_2(t)y(t)$$

Adaptation law:

$$\dot{\hat{\gamma}}_1(t) = -\varrho e_m(t)r(t)$$
$$\dot{\hat{\gamma}}_2(t) = -\varrho e_m(t)y(t)$$

where $e_m = y - y_m$, $\varrho > 0$ is called adaptation gain and T_S is sampling time.

Figure 4: Adaptive control



Model:

$$\dot{\mathbf{x}}_m(t) = \mathbf{A}_{mc}\mathbf{x}_m(t) + \mathbf{B}_{mc}r(t)$$

$$\mathbf{A}_{mc} = \begin{bmatrix} 0 & 1 \\ -\beta_{0m} & -\beta_{1m} \end{bmatrix}, \qquad \mathbf{B}_{mc} = \begin{bmatrix} 0 \\ \beta_{2m} \end{bmatrix}, \qquad \mathbf{x}_m(t) = [y_m(t),\ \dot{y}_m(t)]^T$$

Control law:

$$u(t) = \widehat{\mathbf{K}}_c(t)\mathbf{x}(t) + \hat{L}(t)r(t)$$

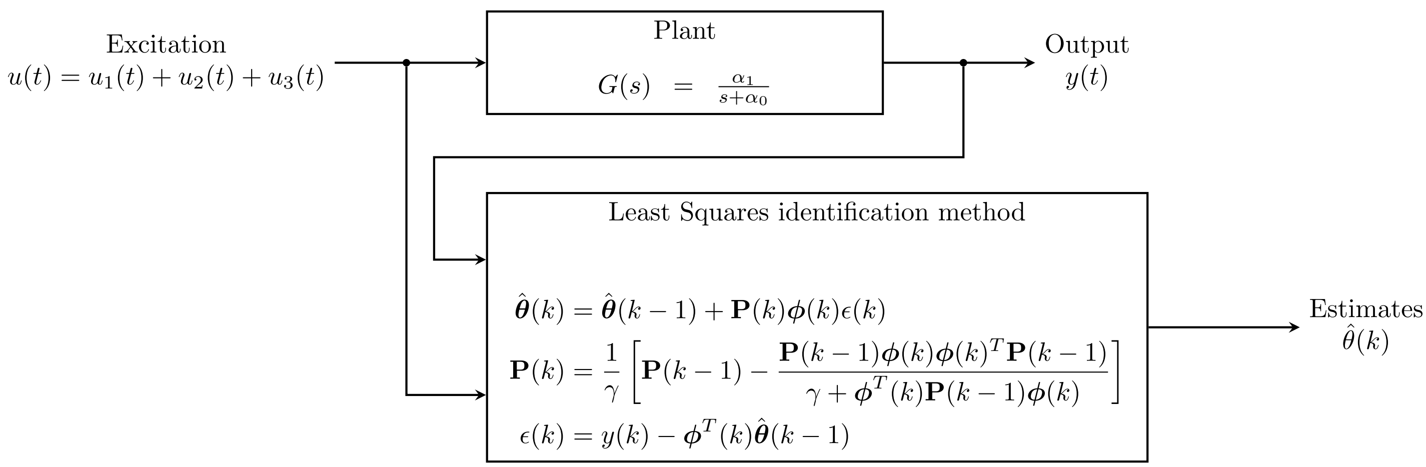
Adaptation law:

$$\dot{\widehat{\mathbf{K}}}_c(t) = \gamma \mathbf{B}_{mc}^T \mathcal{P}\mathbf{E}(t)\mathbf{x}^T(t)$$
$$\dot{\hat{L}}(t) = \gamma \mathbf{B}_{mc}^T \mathcal{P}\mathbf{E}(t)r(t)$$

where T_S is sampling time, $\gamma > 0$ is the adaptation gain, $\mathbf{x}(t) = [y(t),\ \dot{y}(t)]^T$, $\mathbf{E}_m(t) = [e_m(t),\ \dot{e}_m(t)]^T$, and matrix $\boldsymbol{\mathcal{P}} = \boldsymbol{\mathcal{P}}^T$ satisfies:

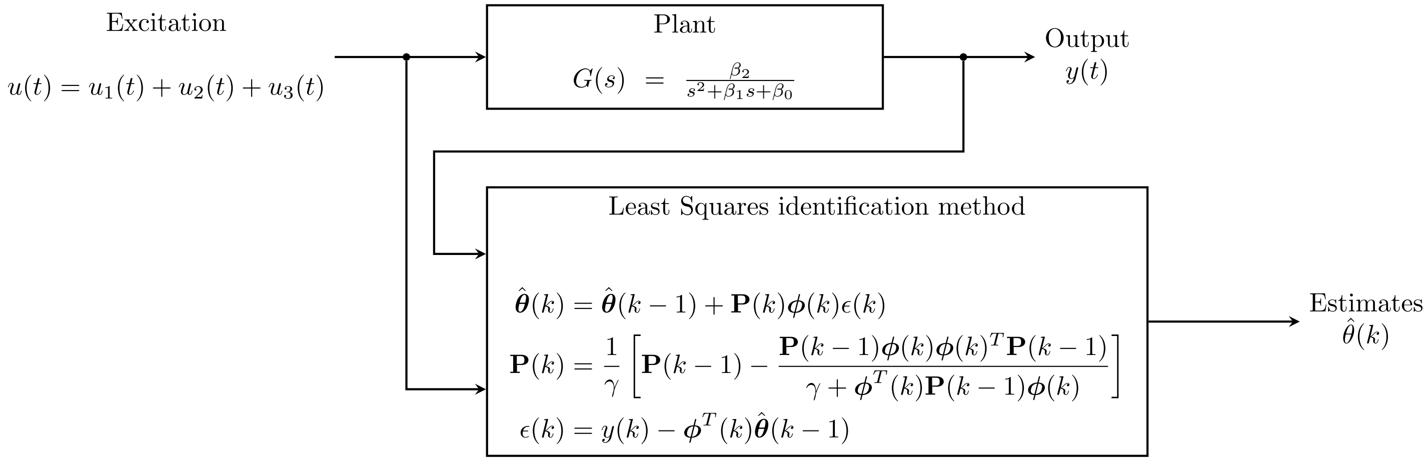
$$\mathbf{A}_{mc}^T\mathcal{P} + \mathcal{P}\mathbf{A}_{mc}^T = -\mathbf{I}_{2x2}$$

Figure 5: Adaptive control



$\mathbf{P}(0) = \mathbf{P_0} = \mathbf{P_0}^{\mathsf{T}} > 0$, $\mathbf{P_0} = \rho \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\boldsymbol{\phi} = \begin{bmatrix} y(k-1) & u(k-1) \end{bmatrix}^{\mathsf{T}}$,
$\boldsymbol{\theta} = \begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix}^{\mathsf{T}}$, $\hat{\boldsymbol{\theta}} = \begin{bmatrix} \hat{\theta}_1 & \hat{\theta}_2 \end{bmatrix}^{\mathsf{T}}$, $\hat{\alpha}_0 = \frac{-\ln \hat{\theta}_1}{T_S}$, $\hat{\alpha}_1 = \frac{\hat{\alpha}_0 \hat{\theta}_2}{1-\hat{\theta}_1}$, and
T_S is sampling time.

Figure 6: First order system identification



Discretized plant model: $y(k) = \theta_1 y(k-1) + \theta_2 y(k-2) + \theta_3 u(k-1) + \theta_4 u(k-2) = \boldsymbol{\phi}^T(k)\boldsymbol{\theta}$,
$\boldsymbol{\theta} = [\theta_1,\ \theta_2,\ \theta_3,\ \theta_4]^T$,
$\boldsymbol{\phi}(k) = [y(k-1),\ y(k-2),\ u(k-1),\ u(k-2)]^T$,
$\hat{\boldsymbol{\theta}}(k) = [\hat{\theta}_1(k),\ \hat{\theta}_2(k),\ \hat{\theta}_3(k),\ \hat{\theta}_4(k)]^T$,
T_S is sampling time,

$$\boldsymbol{P}(0) = \boldsymbol{P_0} = \boldsymbol{P_0}^{\mathsf{T}} > 0,\ \boldsymbol{P_0} = \rho \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Discretized plant model in state space:

$$\mathbf{x}(k+1) = \hat{\mathbf{A}}_d \mathbf{x}(k) + \hat{\mathbf{B}}_d u(k)$$
$$y(k+1) = \hat{\mathbf{C}}_d \mathbf{x}(k)$$

where

$$\hat{\mathbf{A}}_d = \begin{bmatrix} 0 & 1 \\ -\hat{\theta}_2 & -\hat{\theta}_1 \end{bmatrix}, \qquad \hat{\mathbf{B}}_d = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad \hat{\mathbf{C}}_d = [\hat{\theta}_4,\ \hat{\theta}_3]$$

. Estimated continuous model is:

$$\dot{\mathbf{x}}(t) = \widehat{\mathbf{A}}_c x(t) + \widehat{\mathbf{B}}_c u(t)$$
$$y(t) = \widehat{\mathbf{C}}_c \mathbf{x}(t)$$

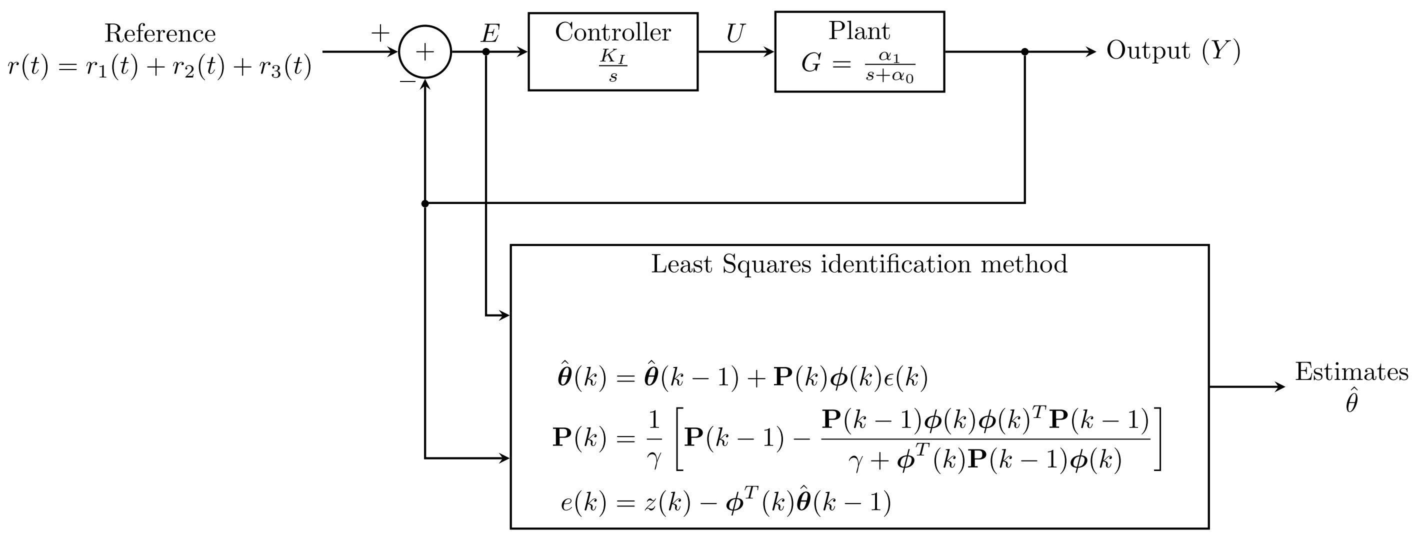
where

$$\widehat{\mathbf{A}}_c = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \frac{2}{T}\mathbf{R}\left[\mathbf{I}_{2x2} - \frac{8}{21}\mathbf{R}^2 - \frac{4}{105}\mathbf{R}^4\right]\left[\mathbf{I}_{2x2} - \frac{5}{7}\mathbf{R}^2\right]^{-1}$$
$$\widehat{\mathbf{B}}_c = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} = \widehat{\mathbf{A}}_c\left[\widehat{\mathbf{A}}_d - \mathbf{I}_{2x2}\right]^{-1}\widehat{\mathbf{B}}_d, \qquad \widehat{\mathbf{C}}_c = [c_{11},\ c_{12}] = \widehat{\mathbf{C}}_d$$
$$\mathbf{R} = \left[\widehat{\mathbf{A}}_d - \mathbf{I}_{2x2}\right]\left[\widehat{\mathbf{A}}_d + \mathbf{I}_{2x2}\right]^{-1}$$

So, we calculate:

$$\hat{\beta}_0(k) = a_{11}a_{22} - a_{12}a_{21}, \quad \hat{\beta}_1 = -(a_{11} + a_{22}), \quad \hat{\beta}_2 = c_{11}(b_{21}a_{12} - b_{11}a_{22}) + c_{12}(b_{11}a_{21} - b_{21}a_{11})$$

Figure 7: Second order system identification



$\mathbf{P}(0) = \mathbf{P}_0 = \mathbf{P}_0^\intercal > 0$, $\mathbf{P}_0 = \rho\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, $z = y(k) - y(k-1)$, $\boldsymbol{\phi} = \begin{bmatrix}e(k-1) + e(k-2)\\ y(k-1) - y(k-2)\end{bmatrix}$, $\boldsymbol{\theta} = \begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}$, $\hat{\alpha_0} = -\frac{\ln\hat{\theta_2}}{T_S}$, $\hat{\alpha_1} = \frac{2\hat{\theta_1}\hat{\alpha_0}}{T_S(1-\hat{\theta_2})K_I}$, and T_S is sampling time.

Figure 8: Parameter estimation of a first order system with integral controller