SESSION 3 OF

STATISTICS FOR BUSINESS

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TODAY'S TOPIC

INTRODUCTION
PROBABILITY

COUNTING PRINCIPLE/RULE

If there are m ways of doing activity-1 and n ways of doing activity-2, then there are $m \times n$ ways of doing both activities 1 and 2.

EXAMPLES

1. Martin has 2 shirts and 3 pants, then he has ______ possible outfit choices.
2. You have a deck of 52 cards, a die with 6 sides, and a coin with 2 sides. If you are asked to select one card from the deck, one side of die, and one side of the coin, then how many options do you have?
3. John received a new credit card. The card came with a default PIN which is a 4 digit number. The bank manager advised him to change the PIN as early as possible. How many options does John has to for the new PIN?

SOLUTIONS

1. Martin has $2 \times 3 = 6$ options for his outfit
2. You have $52 \times 6 \times 2 = 624$
3. Number of possible PINs are $10 \times 10 \times 10 \times 10 = 10000$. And John has $10000 - 1 = 9999$ options for his new PIN.

PERMUTATIONS

Permutation is selecting n elements from N elements in a specific order. The number of possible permutations are

$$_{N}P_{n} = P_{n}^{N} = \frac{N!}{(N-n)!},$$

where,

$$N! = 1 \times 2 \times 3 \times \cdots \times (N-1) \times N$$

EXAMPLE

You are organizer of a prestigious 4 day conference. Each day of the conference should start with a key-note speaker. Your team identified 10 possible candidates for the key-note speakers. How many options do you have?

SOLUTION

You are organizer of a prestigious 4 day conference. Each day of the conference should start with a key-note speaker. Your team identified 10 possible candidates for the key-note speakers. How many options do you have?

Number of possible options are

$$\frac{10!}{(10-4)!} = \frac{10!}{6!} = \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10}{1 \times 2 \times 3 \times 4 \times 5 \times 6} = 7 \times 8 \times 9 \times 10$$
$$= 5040$$

COMBINATIONS

Combination is selecting n elements out of N elements without worrying about the order. The number of possible combinations are

$$_{N}C_{n} = C_{n}^{N} = \binom{N}{n} = \frac{N!}{n!\,(N-n)!}$$

EXAMPLE

As a portfolio manager in a small hedge fund, you've decided to create a new fund that will attract risk-taking investors. The fund will include five stocks of fast-growing companies that offer high growth potential. Your team of analysts identified 20 companies that suit the profile. How many possible combinations are there for your new fund?

SOLUTION

As a portfolio manager in a small hedge fund, you've decided to create a new fund that will attract risk-taking investors. The fund will include five stocks of fast-growing companies that offer high growth potential. Your team of analysts identified 20 companies that suit the profile. How many possible combinations are there for your new fund?

Solution

$$\binom{20}{5} = \frac{20!}{5!(20-5)!} = \frac{20!}{5!\times 15!} = \frac{16\times 17\times 18\times 19\times 20}{1\times 2\times 3\times 4\times 5} = 16 \times 17 \times 3 \times 19 = 15504$$

PROBABILITY

- Examples
  - How likely is it going to rain on a random day?
  - How likely is it going to rain on a cloudy day?
- Representation $P(A)$
- Values between 0 and 1

TERMINOLOGY

- Experiment
- Random experiment
- Sample space
- Outcome
- Event
- Trail

SAMPLE SPACE REPRESENTATION

- List of the possible outcomes
- Tree diagram
- Venn diagram

EXAMPLE
LIST OF POSSIBLE OUTCOMES

		Die 2					
Die 1		1	2	3	4	5	6
	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

EXAMPLE
LIST OF POSSIBLE OUTCOMES

A restaurant offers a set menu with a choice of three starters - soup (S), prawn cocktail (P) or bruschetta (B), and three main courses - lamb (L), hake (H) or chicken (C).

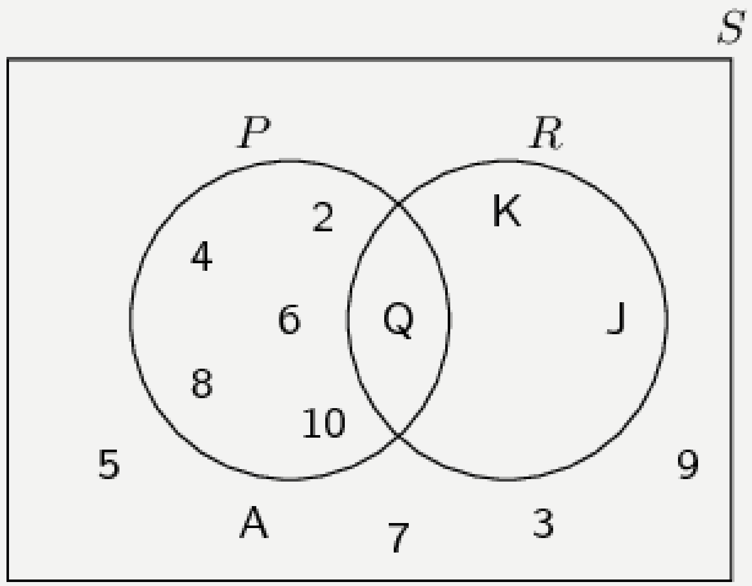
	Soup	**Prawn cocktail**	**Bruschetta**
Lamb	L, S	L, P	L, B
Hake	H, S	H, P	H, B
Chicken	C, S	C, P	C, B

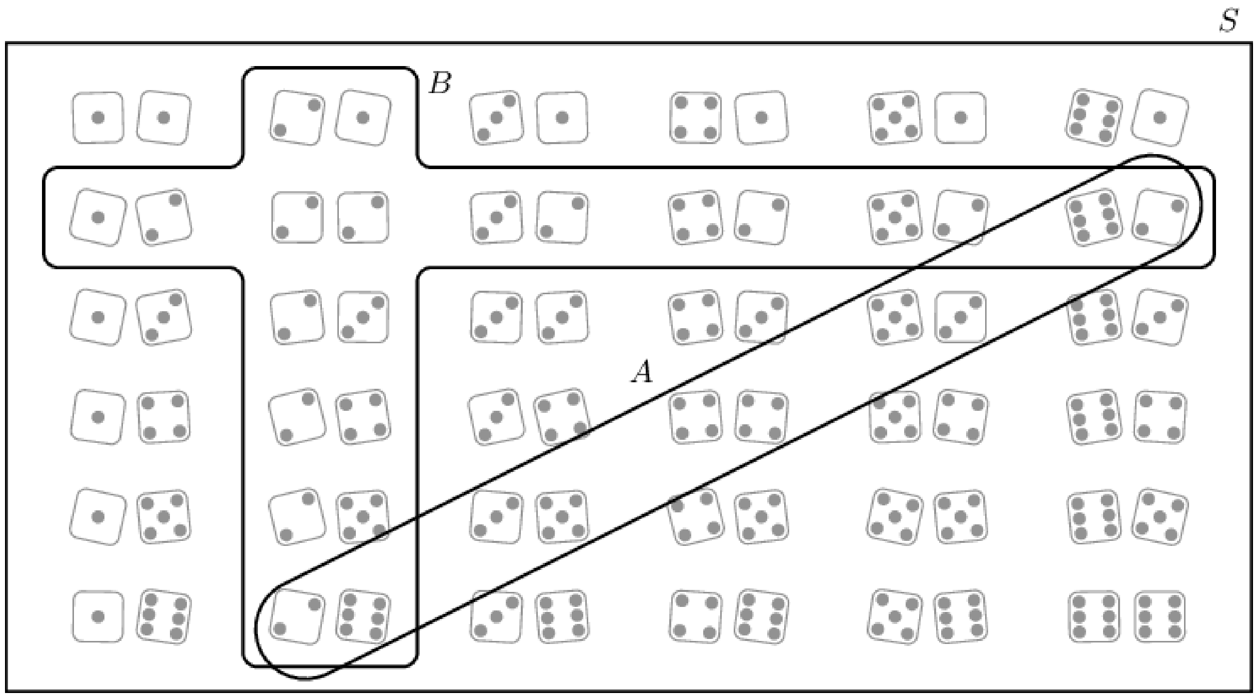
EXAMPLE
VENN DIAGRAMS

Consider the following two events from a set of diamonds removed from a deck of cards.

- P: An even diamond is chosen
- R: A royal diamond is chosen

Represent the sample space S and events P and R using a Venn diagram.





EXAMPLE
VENN DIAGRAM

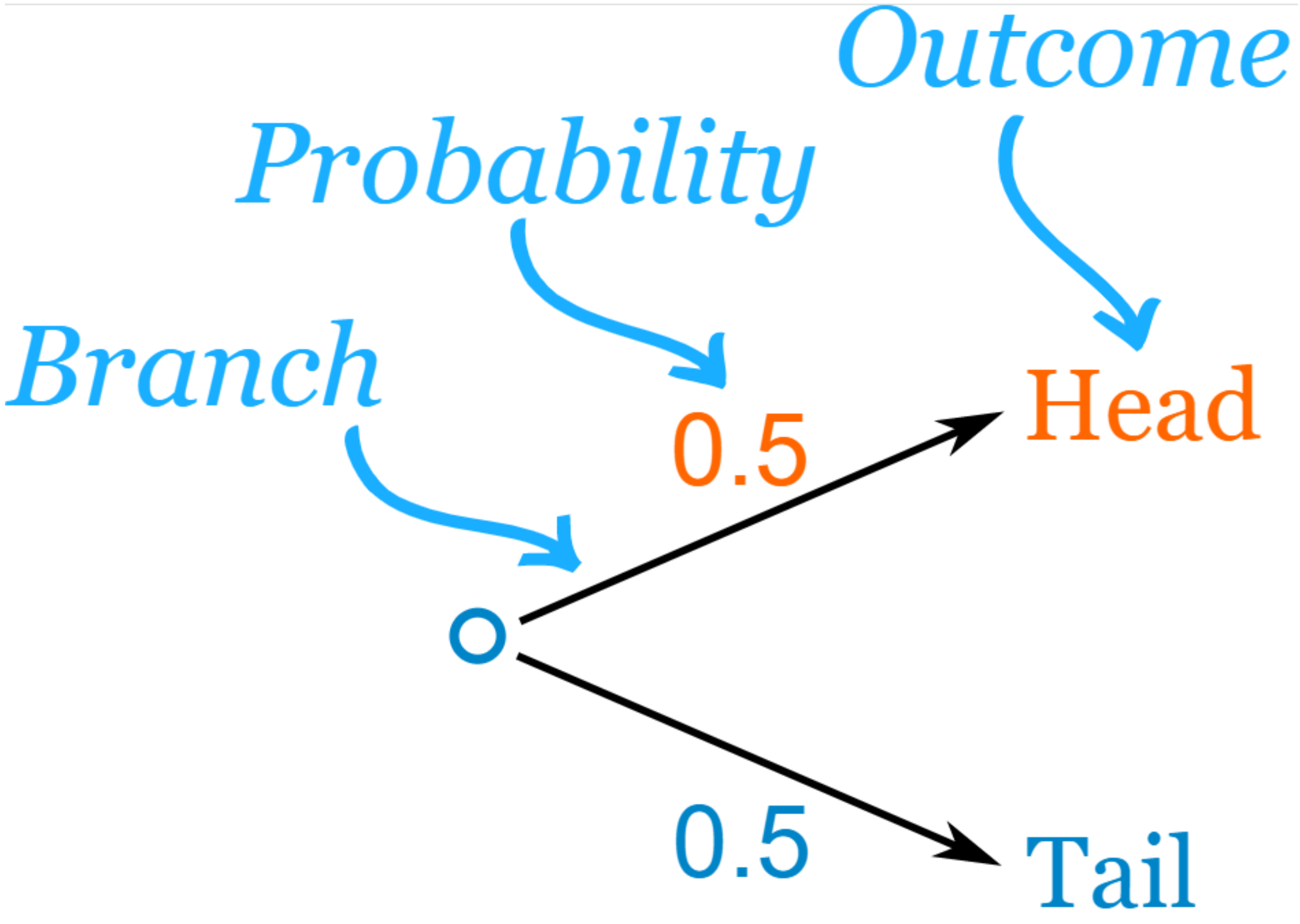
Represent the sample space of two rolled dice and the following two events using a Venn diagram:

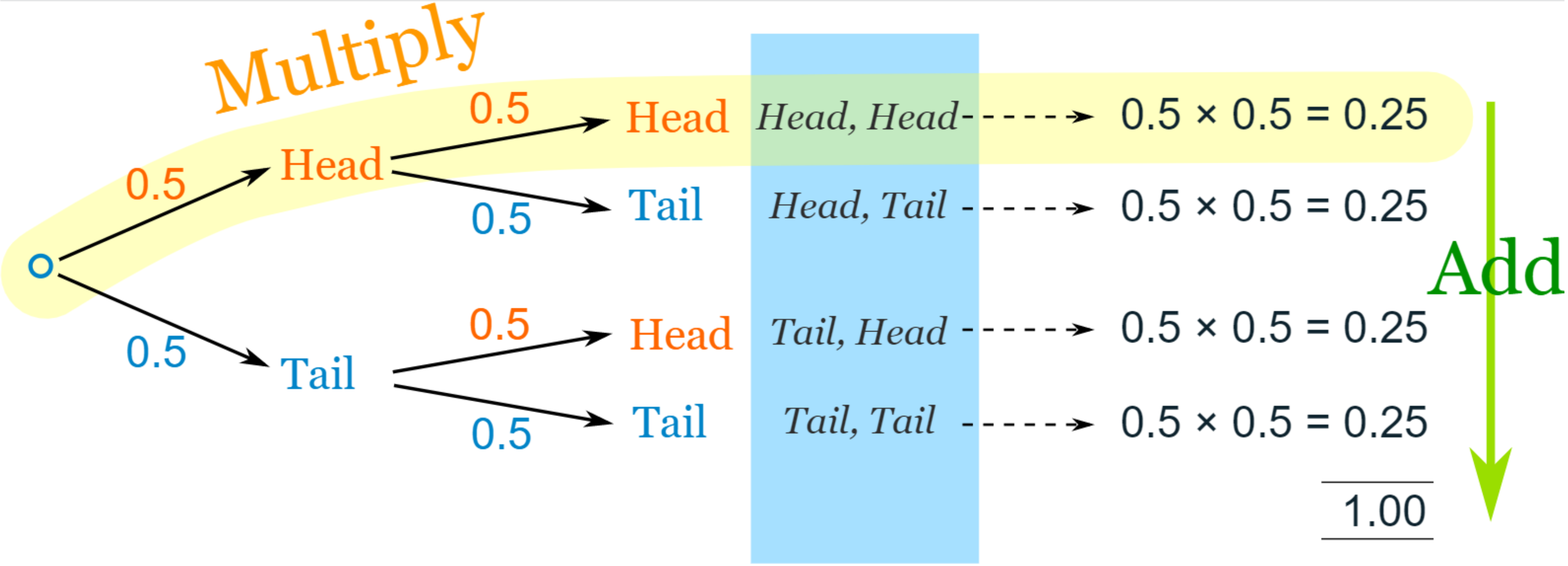
Event A: the sum of the dice equals 8

Event B: at least one of the dice shows 2

TREE DIAGRAM

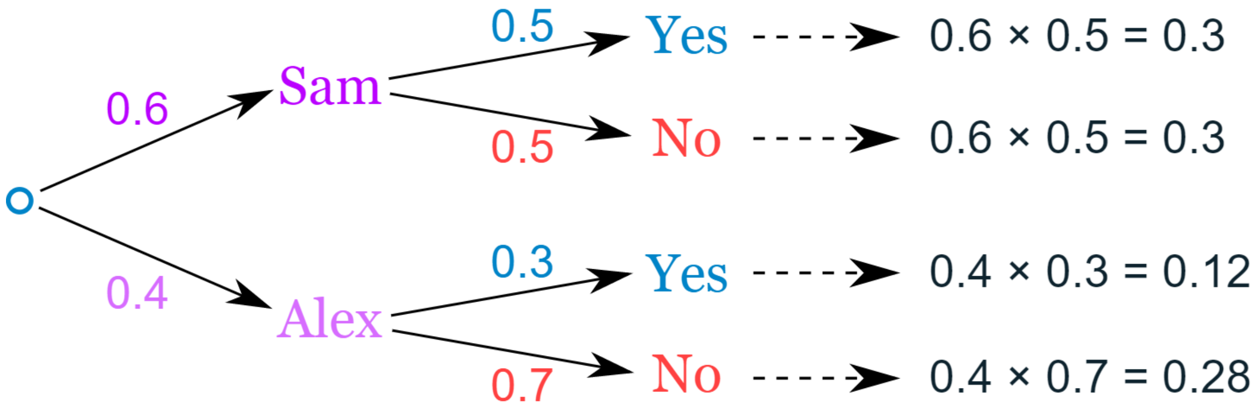
HOW TO PREPARE IT?





TREE DIAGRAM

HOW DO WE USE IT?



TREE DIAGRAM EXAMPLE

You are off to soccer, and love being the Goalkeeper, but that depends who is the Coach today:

with Coach Sam the probability of being Goalkeeper is 0.5

with Coach Alex the probability of being Goalkeeper is 0.3

Sam is Coach more often ... about 6 out of every 10 games (a probability of 0.6).

EXAMPLE
RELATIVE FREQUENCY

The following table displays causes of death in the workplace in the US. If we select of death from this population

- What is the probability that this death was from a fall?
- What is the probability that this death was from a transportation accident?

Cause	Number of deaths
Transportation	1705
Violent events	837
Equipment accidents	741
Falls	645
Chemical	404
Fires and explosions	113

Cause	Number of deaths	Relative frequency	% Relative frequency
Transportation (T)	1705	0.3836	38.36
Violent events (V)	837	0.1883	18.83
Equipment accidents (E)	741	0.1667	16.67
Falls (F)	645	0.1451	14.51
Chemical (C)	404	0.0909	9.09
Fires and explosions (X)	113	0.0254	2.54
Total	**4445**	**1.0000**	**100.00**

SOLUTION
RELATIVE FREQUENCY

What is the probability that this death was from a fall?

$$P(F) = 0.1451$$

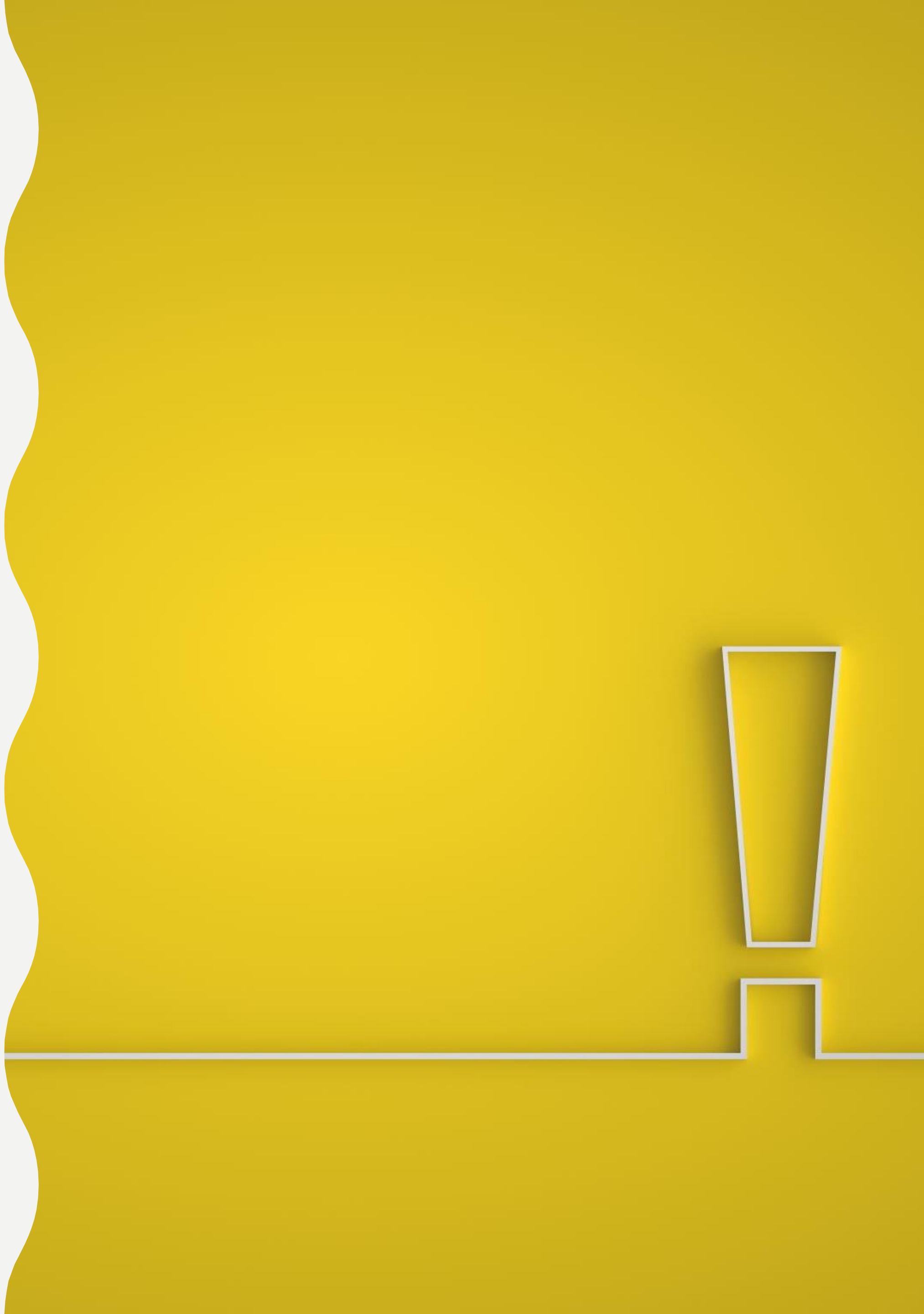
What is the probability that this death was from a transportation accident?

$$P(T) = 0.3836$$

COMPLIMENT

Given an event A, the complement of A, denoted as A^c or $\bar{A}$ contains all the outcomes that are NOT in A. Union of A and $\bar{A}$ occupies the whole sample space. So, we can say

$$P(A) + P(\bar{A}) = 1$$



EXAMPLE COMPLIMENT

If the sample space (S) of rolling dice experiment is $S = \{1, 2, 3, 4, 5, 6\}$. If event $A = \{1, 2, 3, 4\}$, then find the probability of $\bar{A}$.

SOLUTION COMPLIMENT

If the sample space (S) of rolling dice experiment is $S = \{1, 2, 3, 4, 5, 6\}$. If event $A = \{1, 2, 3, 4\}$, then find the probability of $\bar{A}$.

$A = \{1, 2, 3, 4\}$

$\bar{A} = \{5, 6\}$

$P(A) = \frac{4}{6} = 0.6667$

$P(\bar{A}) = \frac{2}{6} = 0.3333$

Verify the results:

$P(A) + P(\bar{A}) = 1$

EXAMPLE COMPLIMENT

In our previous example, we determined that the probability of dying from a fall $P(F)$ at the workplace was $0.1451.$ What is the probability of dying from all other causes?

SOLUTION COMPLIMENT

In our previous example, we determined that the probability of dying from a fall $P(F)$ at the workplace was $0.1451.$ What is the probability of dying from all other causes?

We know that:

$P(F) + P(\bar{F}) = 1$

Hence probability of dying from all other causes is:

$P(\bar{F}) = 1 - P(F) = 1 - 0.1451 = 0.8549$

UNION

An outcome is in the event $A \cup B$ (A union B) if the outcome is in A or is in B or is in both A and B.

Example:

$A = \{1, 2, 3, 4, 5\}$

$B = \{4, 5, 6\}$

$A \cup B = \{1,2,3,4,5,6\}$

INTERSECTION

An outcome is in the event $A \cap B$ (A intersection B) if the outcome is in both A and B at the same time. Using the same sets A and B

Example:
$A = \{1, 2, 3, 4, 5\}$
$B = \{4, 5, 6\}$
$A \cap B = \{4,5\}$

ADDITION LAW OF PROBABILITY

Addition law gives the probability of union of two events A and B as

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

EXAMPLE

Imagine we are studying complaints from a service provided. Out of 100 services, we found out that 10 out of the 100 had complaints regarding delay in the service, 15 out of the 100 had complaints regarding the quality of the service, and 5 out of 100 had complaints regarding delays and quality of service. Based on this we decided to assign warnings to employees providing services that have delays or with low quality. What is the probability that we assign a warning to one of our employees?

SOLUTION

Imagine we are studying complaints from a service provided. Out of 100 services, we found out that 10 out of the 100 had complaints regarding delay in the service, 15 out of the 100 had complaints regarding the quality of the service, and 5 out of 100 had complaints regarding delays and quality of service. Based on this we decided to assign warnings to employees providing services that have delays or with low quality. What is the probability that we assign a warning to one of our employees?

A: Delays $P(A) = 0.1$

B: Bad quality $P(B) = 0.15$

$A \cap B$: Delays and Bad quality $P(A \cap B) = 0.05$

$A \cup B$: Delays or Bad quality $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.1 + 0.15 - 0.05 = 0.2$

MUTUALLY EXCLUSIVE EVENTS

We say that two events are mutually exclusive if when one occurs, the other one can't occur (at the same time). Therefore, for A and B to be mutually exclusive we see that there can't be elements in the intersection of the two events.

So,

$$P(A \cap B) = 0$$

And hence addition law is simplified to

$$P(A \cup B) = P(A) + P(B)$$

CONDITIONAL PROBABILITY

It is probability of an event when another event is guaranteed. Conditional probability of event A when event B is guaranteed is given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

EXAMPLE

What is probability of obtaining 2 as outcome on a die roll if it is know that the outcome is an even number?

SOLUTION

What is probability of obtaining 1 or 2 as outcome on a die roll if it is know that the outcome is an even number?

Sample space $S = \{1, 2, 3, 4, 5, 6\}$

A: Outcome is 2: $A = \{1, 2\}$

B: Outcome is even: $\mathrm{B} = \{2, 4, 6\} \Rightarrow P(B) = \frac{3}{6} = 0.5$

$$A \cap B = \{2\} \Rightarrow P(A \cap B) = \frac{1}{6}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/6}{0.5} = \frac{1}{3}$$

EXAMPLE

Supposed we have classified 161 employees according to two criteria as shown in the table. If we select a person at random and we know this person is female, what is the probability she is under 30?

	Male (M)	Female (F)	Total
Under 30 (U)	54	47	101
Above 30 (A)	28	32	60
Total	82	79	161

SOLUTION

Supposed we have classified 161 employees according to two criteria as shown in the table. If we select a person at random and we know this person is female, what is the probability she is under 30?

$$P(U|F) = \frac{P(U \cap F)}{P(F)} = \frac{47/161}{79/161} = \frac{47}{79} = 0.5949$$

	Male $(M = \overline{\boldsymbol{F}})$	**Female** $(F = \overline{\boldsymbol{M}})$	**Total**
Under 30 $(U = \bar{A})$	54	47	101
Above 30 $(A = \overline{U})$	28	32	60
Total	82	79	161

INDEPENDENT EVENTS

If outcome of event A is not dependent on outcome of event B, we call them independent events.

Mathematically,

$$P(A|B) = P(A)$$

and

$$P(B|A) = P(B)$$

MULTIPLICATION LAW

Rewriting the conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

gives the multiplication law:

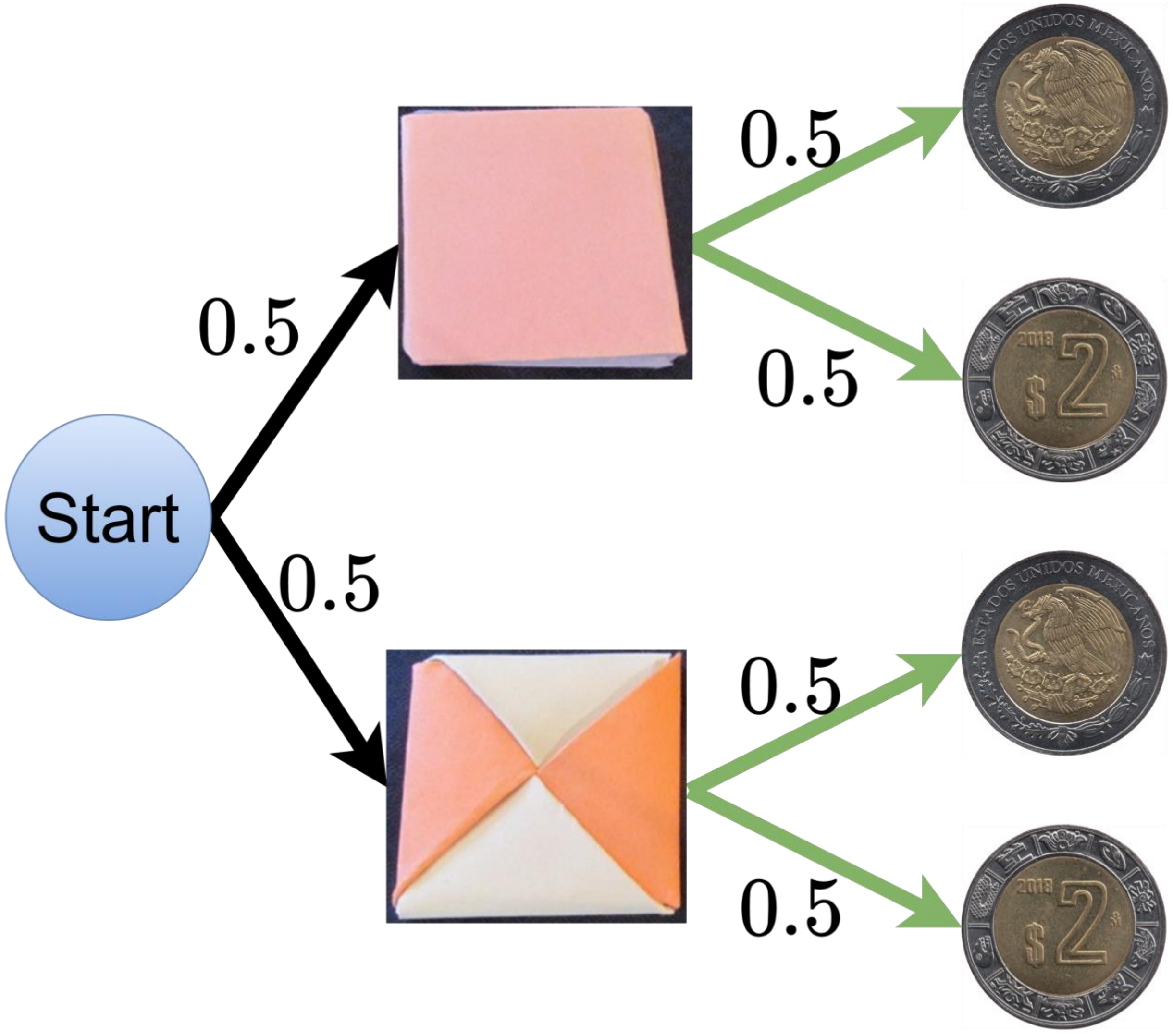
$$P(A \cap B) = P(A|B)P(B)$$

If the events are independent, we get

$$P(A \cap B) = P(A)P(B)$$

BAYES THEOREM

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)}$$



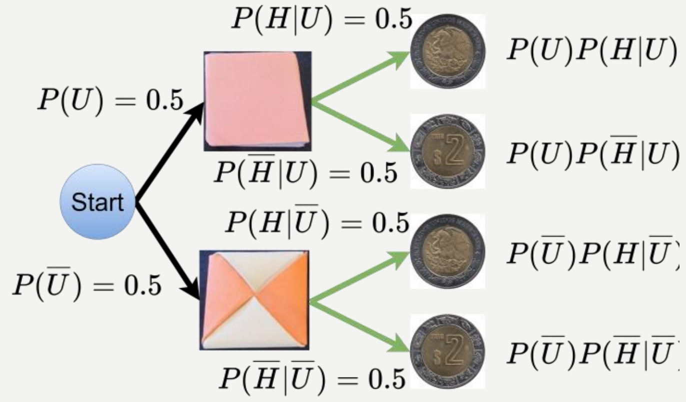
EXAMPLE

Calculate probability of falling heads in the game where first we flip a Ddakji and then we toss a coin. Assume that the coin and the Ddakji are fair.

SOLUTION

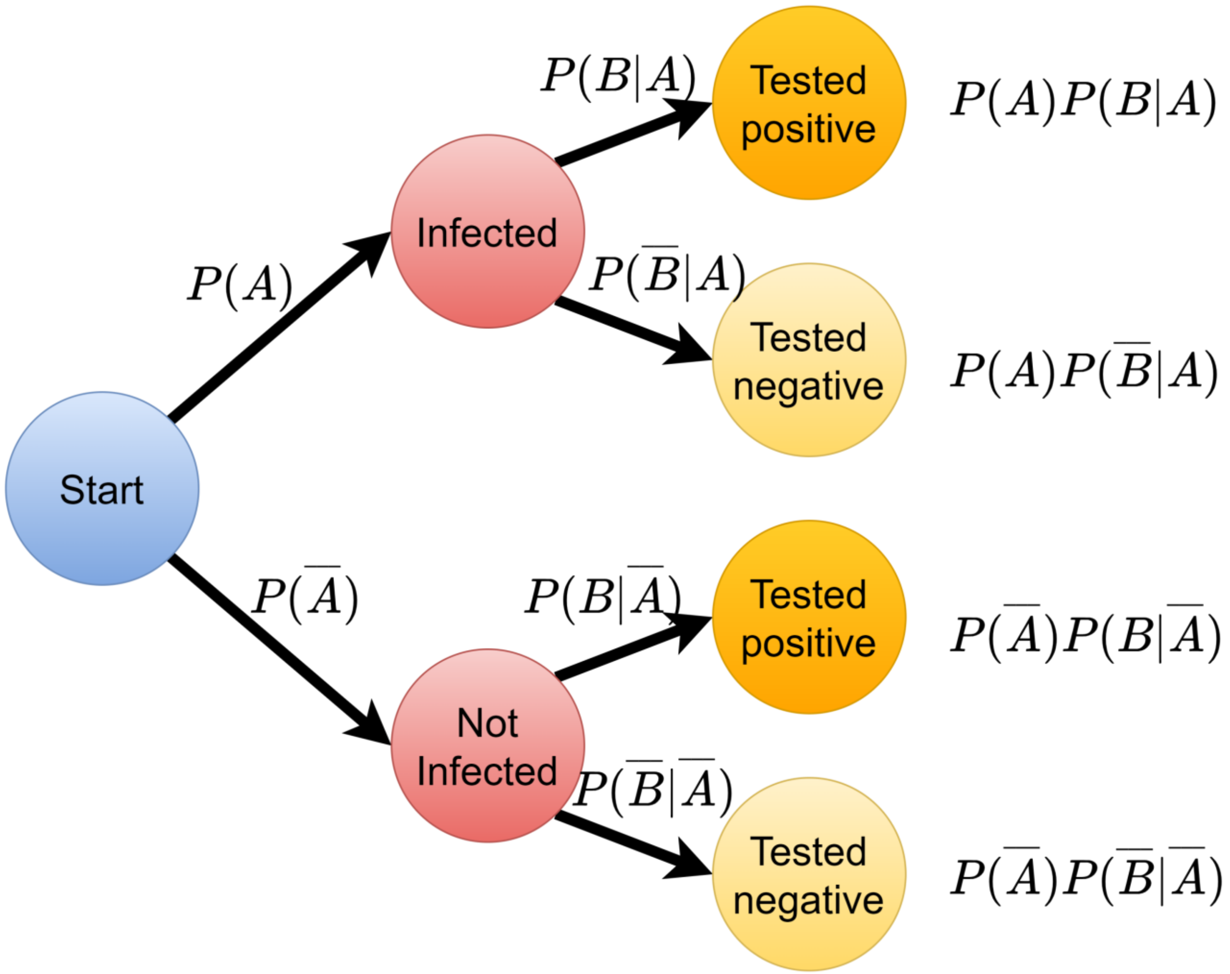
Calculate probability of falling heads in the game where first we flip a Ddakji and then we toss a coin. Assume that the coin and the Ddakji are fair.

$$P(H) = P(U)P(H|U) + P(\bar{U})P(H|\bar{U})$$
$$= 0.25 + 0.25 = 0.5$$



EXAMPLE

1. John visited New York city (of population is 8.5 million) to celebrate new year in the year 2020. Unfortunately, corona virus broke-out in the city and infected 85 people. Worried John, had himself tested with Polymerase chain reaction (PCR) and received a positive test result. What is the probability that John is infected with Corona virus if we asssume
   - PCR has gives correct positive 98% and
   - PCR gives false positive 1%.
2. John is not convinced with the test and took test from another diagnostic center and found positive again. What is the probability now with the latest information?
3. He repeated the test again and found positive yet again, what is the probability now?



TREE DIAGRAM

A: John is infected with Corona virus

B: John is tested positive in a test

SOLUTION

So, we can say

$$P(A) = \frac{85}{8500000} = \frac{85}{85 \times 10^5} = 10^{-5}$$
$$P(B|A) = 0.98 \Rightarrow P(A)P(B|A) = 9.8 \times 10^{-6}$$
$$P(B|\bar{A}) = 0.01 \Rightarrow P(\bar{A})P(B|\bar{A}) = (1 - 10^{-5})0.01 = 9.9999 \times 10^{-3}$$

We are interested to know $P(A|B)$, which is obtained by using Bayes Theorem

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)} = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(\bar{A})P(B|\bar{A})} = \frac{10^{-5} \times 0.98}{9.8 \times 10^{-6} + 9.9999 \times 10^{-3}}$$
$$= 9.790503 \times 10^{-4}$$

Test number	$P(A)$	$P(\bar{A})$	$P(B\|A)$	$P(B\|\bar{A})$	$P(A)P(B\|A)$	$P(\bar{A})P(B\|\bar{A})$	$P(B)$	$P(A\|B)$
1	0.00001	0.99999	0.98	0.01	0.0000098	0.0099999	0.0100097	0.00097905
2	0.00097905	0.99902095	0.98	0.01	0.000959469	0.009990209	0.010949679	0.087625339
3	0.087625339	0.912374661	0.98	0.01	0.085872832	0.009123747	0.094996579	0.903957103

SOLUTION

TANK YOU

FOR YOUR ATTENTION