Title

Cristy
F bio, Universidad Autónoma de Coahuila,
Torreón, México.

Nagamani
F bio WWW, Universidad Autónoma de Coahuila

Raji
Department of ZZZ, University of WWW

S. K. Gadi*
FIME, Universidad Autónoma de Coahuila,
Torreón, México.

May 11, 2017

Abstract

One of the challenges in teaching the subject Design of Experiments is to come up with a proper numerical example. In this article, authors present a methodology to generate a numerical example for multifactorial experiments. Also, it presents a simple algorithm, which can be implemented in any programming language to generate unique models.

Index terms— Experimental design; educational tool; generating examples

1 Introduction

The subject experimental design is part of various undergraduate and graduate curriculum, ranging from the engineering to the biological sciences. Learning statistics or mathematics in general is effective by solving a number of numerical examples. It is teacher's task to generate them (Ball, Thames, & Phelps, 2008).

Teachers spend a lot of time in generating an appropriate examples, which meet all the characteristics they want to highlight. In this article we present a methodology to generate such example for the subject experimental design. The algorithm is described in

*Author to whom all correspondence should be addressed.

the section ((???)). Readers interested only in the implementation of algorithm may skip the mathematical construction presented in the section ((???)).

One of the objectives in the experimental design is to find the factors which optimize the responses of a physical process. Solving problems in this subject involves performing various experiments with a different combinations of factors. Conducting experiments on a real system for the classroom purpose is not always feasible due to any the following limitations.

1. The cost of conducting experiments on a real system is not always negligible.

2. A considerable amount of time may take for each experiment.

3. The combination of factor associated for optimum response is constant for a physical system. Therefore, teachers may not provide a fresh problem.

A computer program generating responses is a good alternative to mimic the physical systems. Hence, a numerical example for an experimental design is mathematical model representing a physical process. This model is a set of static functions (i.e. it does not have derivative or integral terms) which maps the factors to the responses. A multi-response system can be represented as

$$y_i = f_i(x_1, x_2, x_3, \ldots, x_n) + \xi_i \tag{1}$$

where y_i, $i \in \{1, 2, 3, \ldots, m\}$ are the responses, x_j, $j \in \{1, 2, 3, \ldots, n\}$ are the factors, f_i, $i \in \{1, 2, 3, \ldots, m\}$ are the nonlinear functions mapping the n factors to the m responses and ξ_i, $i \in \{1, 2, 3, \ldots, m\}$ are the noise.

All the factors, x_j, are constrained by upper and lower limits. The numerical examples should produce an unique optimal responses, y_i, for a set of factors within its limits. Also, it should be guaranteed that there exists no other peaks in the surface. A convex function meets our requirements. Construction of a one such mathematical function is presented in the next section.

2 Construction of an convex function to suit our requirements

2.1 Quadratic convex function

A second order polynomial function, such as

$$y_i = -\sum_{i=1}^{i=n} x_i^2 \tag{2}$$

is a convex function, which serve the purpose of providing a unique optimal point. Figure 2.1 depicts (2) for the two variables case. However, it doesn't represent a real physical system due to the following limitations.

1. Response surface methodology uses a second order fit algorithm. Hence, the process of reaching optimal solution becomes trivial.

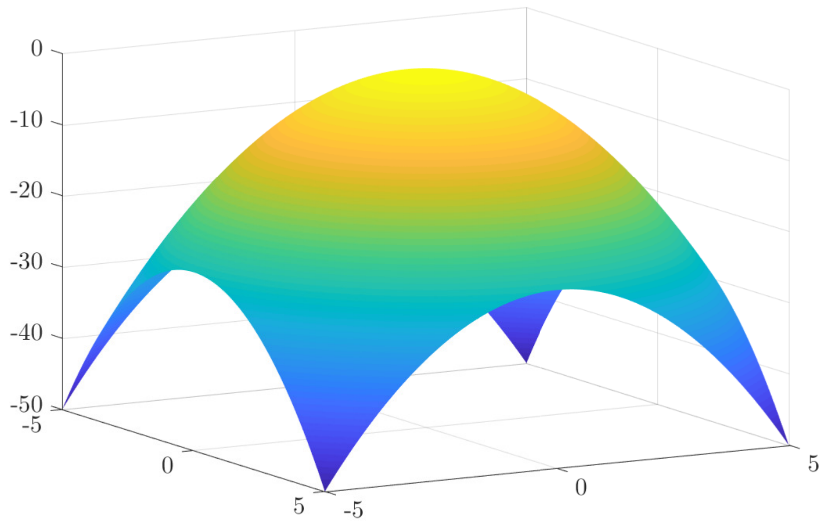

Figure 1: A second order polynomial convex function

2. A quadratic function is having a property that its slope increases as it moves far from the optimal point. This property trivializes the process of selecting a new base value.

2.2 Sigmoid convex function

Keeping above limitation in mind, a sigmoid based convex function is proposed. One variable sigmoid function is

$$S(x) = \frac{1}{1 - e^{-x}} \tag{3}$$

and its derivative is

$$S_d(x) = S'(x) = \frac{e^x}{(e^x + 1)^2} = S(x)(1 - S(x)). \tag{4}$$

The function S_d addresses limitations of mentioned in the previous sub-section. The function S_d is plotted in Figure 2.2. The function S_d can be extended to a n variables case as

$$S_d(x_1, x_2, x_3, ..., x_n) = \sum_{i=1}^{i=n} \frac{e_i^x}{(e^{x_i} + 1)^2}. \tag{5}$$

Figure 2.2 shows sigmoid based function for two variables. Hessian matrix for $S_d(x_1, x_2, x_3, ..., x_n)$ is

$$H_{i,j} = \begin{cases} \frac{\partial^2 S_d}{\partial x_i^2}, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases} = asflkasjflaksjflksfj \tag{6}$$

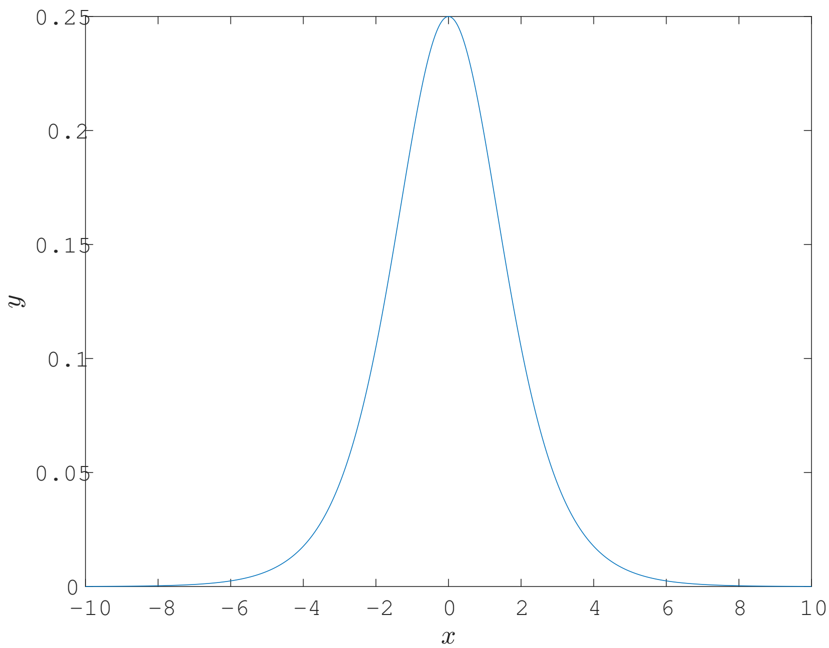

Figure 2: One variable convex function

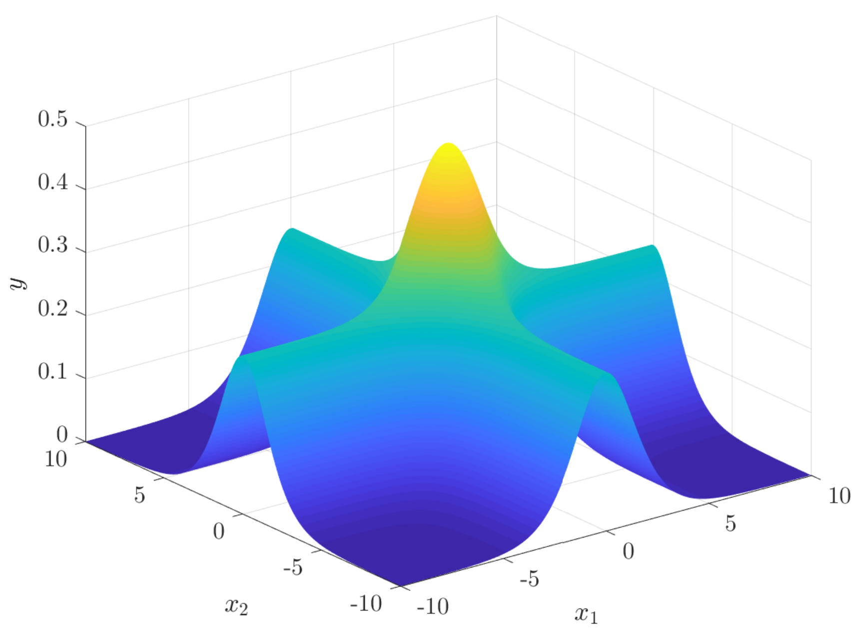

Figure 3: Two variable convex function

The Hessian matrix, $H_{i,j}$ is having a positive definate because it is a diagonal matrix and the diagnal values are positive. Hence, S_d is a convex function, i.e. has a unique maximum value with no other peaks. In the next section a method is presented to adapt S_d to generate random experiments within given limits.

3 Adapting convex function

The convex function, S_d, proposed in the previous section has a maximum value at $x_i = 0, \forall i \in \{1, 2, 3, \ldots, n\}$ and a value close to zero for $x_i > 5, \forall i \in \{1, 2, 3, \ldots, n\}$

References

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching. *Journal of Teacher Education*, *59*(5), 389-407. Retrieved from `http://dx.doi.org/10.1177/0022487108324554` doi: 10.1177/0022487108324554