A computer-based educational tool for generating experimental design examples

# Abstract

In this article, challenges in designing example problems of multifactorial experiments for an academic purpose are identified, and an algorithm for generating multifactorial experiments is proposed. This algorithm has the following features: 1) It is implemented on a platform based on the program languages JavaScript, HTML and CSS; and 2) It allows undergraduate and graduate students to simulate the behavior of multifactorial experiments. Moreover, the algorithm employs a proposed mathematical function that mimic a physical system. The performance of the proposed algorithm has been verified in the learning of Response Surface Methodology.

¿Problemas de que?

1. Escribir en tercera persona, evitar authors, major en voz pasiva.
2. ED no es necesario
3. ¿Cuales metodos?
4. Resumen en un solo parrafo
5. Para mi no es necesario este texto: Experimental design subject deals with optimization, i.e., maximizing yield or minimizing cost by controlling various factors..
6. Es factors o parameters ?

# Keywords

Experimental design, educational tool, generating examples, problem-based learning

# Introduction

Almost all the fields involving experimentation use experimental design [1]–[4]. It is part of various undergraduate and graduate curriculum, ranging from engineering to biological sciences. The objective of experimental design is to minimize cost and time of experiments and maximize its yield. As an example, it can be used to find the values of factors such as pH, oxygen concentration, sugar concentration, with which the enzyme production is maximized. Different techniques can be employed to find the minimum number of experiments [5] . On the other hand, an improper design of experiment may lead to inaccurate or false conclusions, as well as a loss of money, material and time [6].

Solving many numerical examples helps students to learn statistics or mathematics in general [7], and to develop insight into the topics [8] . Moreover, students show good learning experience using visual examples and perform better with examples of experiments, which they can relate [9] . On the other hand, solving optimization problems and finding the most accurate mathematical model for a process in experimental design involves performing various experiments with different combinations of factors. Conducting experiments on a real system is not always feasible due to any of the following limitations.

1. Experiments on a real system can be costly.
2. A considerable amount of time may take each experiment.
3. The combination of factors associated with an optimum-response is constant for a physical system. Therefore, teachers may not provide a new problem. (No entendi este punto)
4. On the other hand, teachers may involve students in finding experiments to teach a specific topic [10]–[12] . However, it is teacher's task to generate examples for the classroom and the practice [13].
5. En mi opinion esto de verde se puede borrar porque no es algo util, ni se relaciona con algortimos de simulacion o con el tema del articulo, o con optimizacion.

Hence, a computer program generating responses for some specific inputs is an excellent alternative to mimic the physical systems. In this article, a methodology is presented to generate numerical examples, which simulate experiments. The objective is to generate a unique process that takes into account factor limits selected by the user, which outputs experimental data for a given combinations of parameters. Teachers may adopt this methodology in order to generate numerical examples, which highlight all the characteristics they want to present to students. Teachers can use the proposed methodology for evaluating the knowledge acquired by students during the course. Moreover, this technique allows teachers to implement problem-based learning. In this pedagogy, a student learns the topic while solving a problem given by the teacher.

So, it is teacher's responsibility to design the examples such that they help to develop all the skills that are intended for the student to learn.

Sometimes teachers may need to generate several problems to achieve it. Failing which a student may acquire incorrect intuition (or insight).

Este texto Amarillo creo que no es necesario porque no dice nada acerca de la tecnica propuesta, solo habla acerca de maestros y alumnus.

Note that a numerical example for an experimental design is a mathematical model representing a physical process. This model is a set of static functions, i.e., it does not have derivative or integral terms, which maps the factors to the responses. The response of a physical system may present more than one local maximum. However, most of the experimental design methods find the local maximum based on the initial base value. Hence, the proposed algorithm is designed to present only one peak.

Let a multi-response system given by

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

where ,  are responses, ,  are factors, ,  are nonlinear functions mapping the  factors to the  responses; moreover,,  are zero mean random noise.

All factorsare constrained by upper and lower limits. The numerical examples should produce unique optimal responses, $y\_j^M$, for a set of factors within its limits.

Como se prueba que los ejemplos numericos produciran solo una respuesta unica

The proposed algorithm presents the case of single response. A set of  single response systems can mimic a multi-response system.

The following section presents the mathematical functions which can simulate a real-life process with the following properties.

1. There exists a unique maximum value.

2. The functions are static. Time can be a factor but does not use the derivative and integral terms.

Creo que es conveniente eliminar las otras tres funciones porue no son utilizes, es decir no las empleas para mostrar a tu algoritmo. Un revisor te menciono precismanete eso. (ver punto 5 del revisor 2)

# Proposed mathematical function

Adding a nonlinear term to a Gaussian function gives the following proposed novel mathematical function.

|  |  |  |
| --- | --- | --- |
|  |  | (7) |

Figure 4 depicts  for a two variable case with . The gradient of  is

|  |  |  |
| --- | --- | --- |
|  |  | (8) |

where . Peaks, dips or saddle points are formed at

|  |  |  |
| --- | --- | --- |
|  | . | (9) |

The union of the solutions to the following three equations gives the solution to (9).

|  |  |  |
| --- | --- | --- |
|  |  | (10) |
|  |  | (11) |
|  |  | (12) |

The following procedure identifies the requirements to ensure the existence of one single peak by eliminating the occurrence of other peaks, dips or saddle points. Note that solution for (10) is , which can don't affect the solution ¿Cual?. On the other hand, the solution for (12) is

|  |  |  |
| --- | --- | --- |
|  | . | (13) |

Selecting a value $a$, such that  allows suppressing all real solutions of (13) .

Equation (11) can be rewriting as

|  |  |  |
| --- | --- | --- |
|  | , | (14) |

This expression implies that solution ¿Cua? lies at . Equation (14) is equivalent to

|  |  |  |
| --- | --- | --- |
|  | . | (15) |

Since  for all the values of  except for , the solution ¿Cuall? can be limited to only the point  provided that . By considering a positive value for , only one peak at origin is guaranteed when .

# Properties of the proposed mathematical function

The proposed mathematical function has the following properties:

1. Its gradient is not proportional to the distance from its optimum combination.
2. It has a unique maximum value at  i.e. at , provided that 
3. It is not a concave function.
4. The optimal value of an arbitrary factor is not constant throughout the factorial space.

In the next section, a method is presented to adapt the function  defined at (7) in order to generate random experiments.

# Adapting the proposed function

A scaled version of the proposed function  in (7) is

|  |  |  |
| --- | --- | --- |
|  |  | (16) |
|  | , | (17) |
|  | , | (18) |
|  | , | (19) |
|  | , | (20) |
|  | , | (21) |
|  | , | (22) |
|  | , | (23) |
|  | , | (24) |
|  |  | (25) |
|  |  | (26) |

With ,  is a random variable such that  and , where  denotes expected value. Moreover,  is a random variable with the properties  and . On the other hand,  is the function range,  is a noise factor,  is difficulty factor,  are the optimal combination of factors, where the function  reaches its maximum value. In addition, and  are lower and upper limits of the  factor,  and  are padding constants for limiting the maximum value of the function  and limiting the optimal combination within the desired region, respectively.

It is worth mentioning that function  shown in (16) preserves all the following mathematical properties of the function .

In the next section, an algorithm that implements the proposed procedure is presented.

# Algorithm

The objective of the algorithm is to generate experiment results by simulating a multi-factorial process. The teacher gives the constants required by the multi-factorial process, and students are allowed to perform experiments at different factors. The algorithm generates responses for the given  inputs using the equations and inequalities given from (16) to (26). The objective for the student is to minimize the number of experiments conducted to achieve the optimum value of the process.

Figure 5 shows the flowchart of the proposed algorithm. The teacher should define the values of  and , where and  should be less than 0.5. Based on our experience, It is recommended to use .  is the difficulty factor, the bigger the value is assigned, the harder it is to reach the optimum value. It is recommended to use . The noise factor  introduces noise into the system.

Esto creo que no es necesario, puesto que es evidente and  are lower and upper limits of the values generated in the experiments. Hence, a user should select the values such that .

This algorithm can be implemented in any programming language to distribute the students.

# Application

The triad of JavaScript (JS), Hypertext markup language (HTML) and Cascading Style Sheets (CSS) is used to create an application [16]. JS is used to implement the algorithm discussed in the previous section.

This application is executed with , , , , , , , , , and . The application calculated , , , , , , and . Figure 6 and 7 show, respectively, the contour plot and surface plot of the function  without the noise term i.e., considering . The RSM is applied to find the optimal values of  and , results of each iteration are also superimposed over the contours.

The master students of Biological Sciences Faculty, Universidad Autónoma de Coahuila, Torreón were instructed using this application. This tool became part of the classroom as well as the homework in the learning of Response Surface Methodology (RSM). The application does not give access to the optimum values. A distance tool is available in this application to provide the information how far is the optimum value from any set of given factors. This distance tool lets the students notice if they have arrived the optimum result.

This technique allowed the teacher to implement the problem-based learning. Also, individualize the problem, i.e., the teacher assigns a unique problem to each student. It helped the students to work in groups to discuss the technique at the same time every student has to work on themselves to solve his/her unique problem. The teacher also reported that the students became more interactive in the classroom.

# Conclusion

The Construction of a single response, unique peak multivariable mathematical function is proposed in this article. This article presents four mathematical functions (, , , and) which are suitable to mimic a real-life process for the experimental design purpose. The properties of these functions are studied to infer that certain functions are less challenging while compared to others. The functions are ordered in the order of difficulty with being the easiest and  being the hardest. The function, , is adapted to generate experimental data for a selected range of factors. This article also presents an algorithm to generate experimental results. This algorithm generates a random process based on the constants provided by the teacher. An application of the algorithm is designed in HTML, CSS, and JavaScript. It is used in the classroom to teach the topic of Response Surface Methodology (RSM). Teachers may use this application to teach other optimization techniques such as Taguchi methods. The function developed in this article has only one peak, i.e., the optimal value is a maximum value. Multiplying the function, , by a negative number transforms the single-peak function to a single-dip (opposite to a peak) function. It also requires to modify (17) and (18) such that the upper and lower limits are respected.

This work can further be extended to a multiple response case by generating  number of functions  where , which requires to generate  number of values for , i.e. the values  are replaced by  where  and .

A non-concave function gives an additional challenge in solving the optimization problem.

This property is not recommended because it trivializes the multi-factorial problems.

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