A Sensorless Optimal Control System for an Automotive Electric Power Assist Steering System

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Abstract—This paper considers the analysis and design of a double-pinion-type electric power assist steering (EPAS) control system. A simplified model of the augmented steering assembly-electric motor system is developed using Lagrangian dynamics, and an optimal controller structure for the model is proposed. Three main advances to the state of the art are presented in this paper. First, a state-space design model is used rather than an input-output model. A state-space formulation for a system model that incorporates motor electrical dynamics is obtained with the assist motor angular position as the output. Second, linear quadratic regulator (LQR) and Kalman filter techniques are employed to arrive at an optimal controller for the EPAS system. The selection of weighting coefficients for the LQR cost function is discussed. Finally, the authors present a control strategy that eliminates the steering column torque sensor, a critical component in existing EPAS controller designs. The proposed control strategy presents an opportunity to improve EPAS system performance and also reduce system cost and complexity.

Index Terms—Electric power assist steering (EPAS), linear quadratic regulator (LQR), optimal control.

I. INTRODUCTION

E LECTRIC power assist steering (EPAS) systems offer distinct advantages over conventional hydraulic assist steering systems in terms of fuel efficiency, modularity, tunability of steering feel, and environmental friendliness. Four main configuration types in EPAS systems have been proposed by EPAS system manufacturers [1]. Three of these are based on a rack-and-pinion steering assembly, and all four share three basic components: the control unit, an assist motor, and a torque sensor mounted on the steering column. The four arrangements differ mainly in the placement of the motor with respect to the steering rack and steering column assembly, which offers certain application advantages. Single-pinion or column-assist-type EPAS systems find applications in light vehicles [2], while the double-pinion configuration presents specific advantages for application in heavy vehicles. This paper addresses the double-pinion arrangement (Fig. 1), in which driver input acts on the steering rack through one pinion and a motor mounted in a separate housing drives the rack through a second pinion-gear arrangement.

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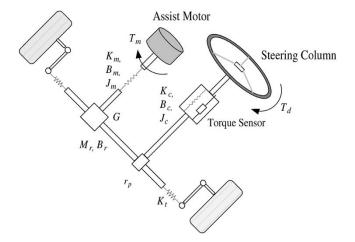


Fig. 1. Double-pinion-type EPAS system model.

A column-assist-type EPAS system has been addressed in [3] with respect to its modeling and the performance of a control system which utilizes the output of the torque sensor. Attempts have been made by authors [4] to improve the steering feel as perceived by the driver and ensure adequate assist levels using \mathcal{H}_{∞} control methods. Control and steering feel issues in the development of an EPAS system have been addressed in [5]. The authors have developed a method for fixed-structure optimal controller synthesis based on a nonlinear constrained optimization procedure. An EPAS control system performance has been analyzed with emphasis on system stability and reduction in the vibration of the steering column at high assist levels in [6]. Other issues that have been considered are perceived driver steering feel and control performance during static or low-speed conditions. An attempt has been made to reduce oscillations around the 30-Hz range using an observer for estimating motor position in [7].

A common feature in existing control strategies is their reliance on the steering column torque sensor. Popular approaches in systems on the road, e.g., the Honda Insight, employ classical proportional—integral—derivative (PID) controllers that interpret data from the torque sensor to generate current inputs for the assist motor. The torque sensors employed also incorporate an angular position sensor that measures the steering column deflection. The torque sensors are specialized instruments developed specifically for EPAS applications and are considerably expensive. Furthermore, all known torque sensor technologies introduce an element of compliance in the steering column, which affects steering feel and system stability. Elimination of the torque sensor would increase the stiffness of the steering column with associated mechanical benefits.

This paper presents three advances to the state of the art. First, a state-space formulation that incorporates electrical dynamics is devised for a dynamic model for a double-pinion-type EPAS system developed using Lagrangian dynamics. This is in contrast to the input—output approach common in current applications in systems on the road. Second, an optimal controller structure is proposed. A Kalman filter is used to estimate the system state variables and a linear quadratic regulator (LQR) is used to obtain an optimal controller. Finally, it is established that the system is observable if the assist motor position were considered to be an output and a controller is designed that uses only the motor position information. A useful feature of the design is that the controller does not use steering column torque feedback. Another advantage is that a variety of closed-loop system characteristics may be obtained through very few tuning parameters.

II. DYNAMIC MODEL OF AN EPAS SYSTEM

An exhaustive model of EPAS system dynamics would consist of all the masses and moments of inertia considered in relation to their interaction with various spring and damping elements that appear in the system. Considering that the fundamental behavior of the system is dominated by low-frequency modes and highly stiff elements that connect masses contribute high-frequency modes, a simplified model may be considered by neglecting these elements.

A simplified model of a steering mechanism equipped with a double-pinion-type EPAS system (Fig. 1) is composed of three basic elements: a steering rack, a steering column coupled to the steering rack through a pinion gear, and the assist motor on a separate column, coupled to the steering rack through a second torque-amplifying pinion gear. Tie-rods connect the steering rack to the tires. The model neglects the masses of the tie-rods and tires, tire motion, friction, etc., and the inertias of power transforming elements such as gears. It also combines the two tie-rod spring constants.

In Fig. 1, T_d and T_m represent applied driver torque and the torque applied by the motor, respectively. K_c and B_c represent the steering column and intermediate shaft torsional stiffness and steering column damping, respectively. M_r and B_r represent rack and wheel assembly mass and rack damping respectively, K_t is the tire or rack centering spring rate and K_m and B_m are the motor and gearbox torsional stiffness and damping respectively. The motor gearbox gear ratio is represented by G.

The equations of motion for the double-pinion-type EPAS system may be obtained using Lagrangian Dynamics. The general form of Lagrange's equation takes the form

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i, \qquad i = 1, 2, \dots, n \tag{1}$$

where q_i are the motion coordinates associated with the system, Q_i are the generalized external forces corresponding to q_i , and n is the number of degrees of freedom.

For the EPAS system, θ_c , the angular position of the steering column in the direction of T_d , θ_m , the angular position of the motor shaft in the direction of T_m , and the rack position, p are chosen as the motion coordinates. The external forces acting on θ_c and θ_m are the applied driver torque T_d and the motor

torque, T_m . The tires are assumed to be stationary, so no external force acts on the steering rack. The kinetic energy of the mechanical elements of the system is due to the motion of the steering column, the motor column and the steering rack and may be expressed as

$$T_{M} = T_{\text{steering column}} + T_{\text{motor column}} + T_{\text{steering rack}}$$

$$= \frac{1}{2} J_{c} \dot{\theta}_{c}^{2} + \frac{1}{2} J_{m} \dot{\theta}_{m}^{2} + \frac{1}{2} M_{r} \dot{p}^{2}$$
(2)

where J_c is the steering wheel and steering column moment of inertia, J_m is the motor column and gear-box moment of inertia, and p is the steering rack displacement. The potential energy of the mechanical elements of the system is the energy stored in the system due to the compliance in individual mechanical elements, and may be expressed as:

$$\begin{split} V_{M} &= V_{\text{steering column}} + V_{\text{motor column}} + V_{\text{steering rack}} \\ &= \frac{1}{2} K_{c} \left(\theta_{c} - \frac{p}{r_{p}} \right)^{2} + \frac{1}{2} K_{m} \left(\theta_{m} - \frac{p G}{r_{p}} \right)^{2} + \frac{1}{2} K_{t} p^{2} \end{split} \tag{3}$$

where r_p is the radius of the pinion (assumed to be equal for both the steering column and the motor column).

The Lagrangian for the mechanical system is found as

$$L_{M} = T_{M} - V_{M}$$

$$= \frac{1}{2} J_{c} \dot{\theta}_{c}^{2} + \frac{1}{2} J_{m} \dot{\theta}_{m}^{2} + \frac{1}{2} M_{r} \dot{p}^{2} - \frac{1}{2} K_{t} p^{2}$$

$$- \frac{1}{2} K_{c} \left(\theta_{c} - \frac{p}{r_{p}} \right)^{2} - \frac{1}{2} K_{m} \left(\theta_{m} - \frac{p G}{r_{p}} \right)^{2}.$$
(4)

Application of Lagrange's equation (1) for the three motion variables yields the following equations of motion that describe the dynamics of the mechanical elements in the double-pinion-type EPAS system:

$$J_{c} \ddot{\theta}_{c} + B_{c} \dot{\theta}_{c} - K_{c} \left(\theta_{c} - \frac{p}{r_{p}} \right) = \tau_{d}$$

$$J_{m} \ddot{\theta}_{m} + B_{m} \dot{\theta}_{m} - K_{m} \left(\theta_{m} - \frac{pG}{r_{p}} \right) = k i$$

$$M_{r} \ddot{p} + B_{r} \dot{p} + K_{t} p = \frac{K_{c}}{r_{p}} \left(\theta_{c} - \frac{p}{r_{p}} \right)$$

$$+ \frac{K_{m}G}{r_{p}} \left(\theta_{m} - \frac{pG}{r_{p}} \right).$$

$$(5)$$

Equation (5) describes steering column dynamics. The left-hand side represents the effect of the moment of inertia of the steering column and steering column damping. The right-hand side term is the torque applied by the driver and the torsional coupling to the steering rack. Equation (6) models the motor mechanical dynamics, which are similar in form to steering column dynamics. The right-hand side term in the equation is the output torque of the motor (which is proportional to the motor current). The motor torque constant is represented by k. The left-hand side terms represent the motor column moment of inertia, damping, and torsional coupling with the steering rack. Steering rack dynamics are described by (7). The

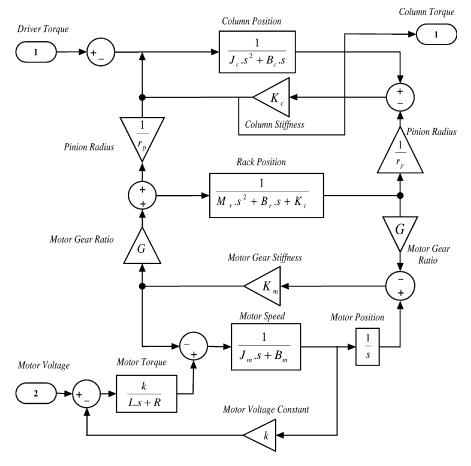


Fig. 2. Open-loop EPAS system model.

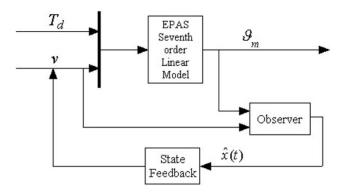


Fig. 3. Proposed controller structure.

left-hand side terms represent the effects of rack mass, damping and rack centering stiffness, respectively. The terms on the right-hand side describe torsional coupling with the steering column and the assist motor through the motor gearbox, respectively. Equations ((5)–(7)) are found to be consistent with the double-pinion-type system model presented in [6].

The motor contributes another degree of freedom to the system. Typically, the motor used in EPAS applications is a dc motor. The electrical dynamics of the motor are given by

$$L\dot{i} + Ri + k\dot{\theta}_m = v \tag{8}$$

where L is the inductance associated with the stator winding, R is the winding resistance, and v is the motor terminal voltage.

TABLE I NOMENCLATURE AND PARAMETER VALUES (COURTESY: VISTEON CORPORATION, MI, USA)

D	C11	V-1	Units
Parameter	Symbol	Value	Units
Steering wheel (driver) torque	T_d		N-m
Motor torque	T_m		N-m
Steering wheel angular position	θ_c		rad
Rack displacement	p		m
Motor column angular position	θ_m		rad
Motor current	i		A
Motor terminal voltage (control)	v		V
Steering wheel moment of inertia	J_c	0.04	kg-m ²
Steering column torsional stiffness	K_c	172	N-m/rad
Steering column damping	B_c	0.0225	N-m/(rad/s)
Rack and wheel assembly mass	M_r	32	kg
Rack damping	B_r	3920	N/(m/s)
Tire or rack centering spring rate	K_t	23900	N/m
Pinion radius	r_p	0.0071	m
Motor gear ratio	r_p G	0.4686	
Motor moment of inertia	J_m	4.52×10^{-4}	kg-m ²
Motor and gearbox torsional stiffness	K_m	625	N-m/(rad/s)
Motor and gearbox damping	B_m	3.339×10^{-3}	N/(m/s)
Motor torque and voltage constant	k	0.0345	N-m/A
Motor inductance	L	9.06×10^{-5}	Henry
Motor resistance	R	0.035	Ohm

Equations ((5)–(8)) together define the electromechanical dynamics of the double-pinion-type EPAS system. A block diagram representation of the open-loop EPAS system is shown in Fig. 2. The open-loop system has two inputs in the form of driver torque and assist motor voltage and one output in the form of column torque. The block labeled *Column Position* is derived from (5). The output of *Column position* is the steering column

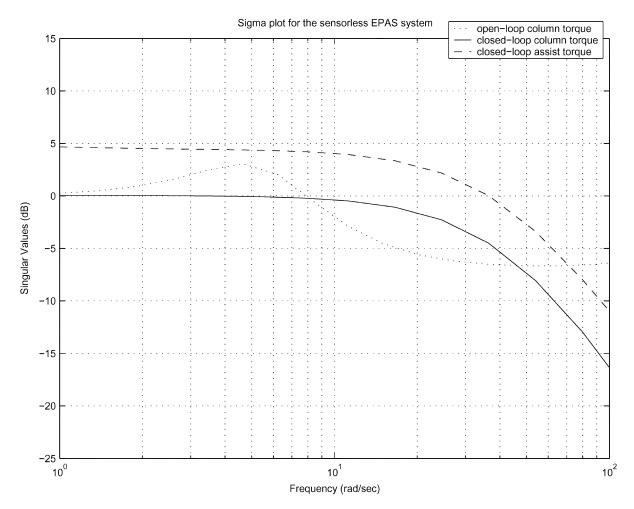


Fig. 4. Frequency response of sensorless EPAS system for $Q=aQ_2$. The y axis depicts the singular values of the transfer function matrices for steering column torque for the open-loop system (dotted), steering column torque for the closed-loop system (solid), and motor torque for the closed-loop system (dashed).

angular position θ_c . The block *Motor Speed* is derived from (8) and has the motor position θ_m as its output. The *Motor Speed* and *Column Position* blocks are coupled to the *Rack Position* block (derived from (7)) through the gains *Motor Gear Stiffness* and *Column Stiffness*, respectively. The *Motor Torque* block represents the electrical dynamics of the motor. It becomes apparent that the dynamics of the EPAS system are linear and of the seventh order.

III. STATE-SPACE FORMULATION

The linear EPAS system may be expressed in the state-space form

$$\dot{x} = Ax + Bu;
y = Cx + n(t)$$
(9)

where n(t) is the random measurement noise term. The state assignment for the EPAS model is $x = \begin{bmatrix} \theta_c & \dot{\theta}_c & \theta_m & \dot{\theta}_m & p & \dot{p} & i \end{bmatrix}^T$. The applied driver torque and the motor terminal voltage are considered as the inputs of the system such that, $u = \begin{bmatrix} T_d & v \end{bmatrix}^T$.

The motor column angular position, θ_m , as measured by a high-resolution position sensor on the motor shaft is considered as the output of the multi-input, single-output system. The

system matrix A, the input matrix B, the output matrix C, and the feedthrough matrix D are shown by (10), at the bottom of the next page.

IV. THE DESIGN APPROACH

All power assist systems must be able to provide a wide range of assist levels and the prime objective of the EPAS control system is to ensure closed-loop system stability at high levels of assist gain. The state-space approach provides a methodical solution to the problem of control of high-order systems and is thus employed to obtain a suitable controller for the double-pinion-type EPAS system. A state-variable feedback control system is proposed. Optimal closed-loop pole locations are computed using an LQR approach. The LQR approach is chosen as it has good stability and robustness properties, and leads to good gain and phase margins [8], [9].

A practical limitation of the LQR is that all the state variables of the system are required for feedback. For reasons of economy and certain physical constraints, it is not possible to measure all the state variables in EPAS systems. Therefore, the Kalman filter is used as an estimator of the system state.

The structure of the proposed controller is shown in Fig. 3. The EPAS system is modeled by the seventh-order linear two-input single-output model developed in Sections II and III. The

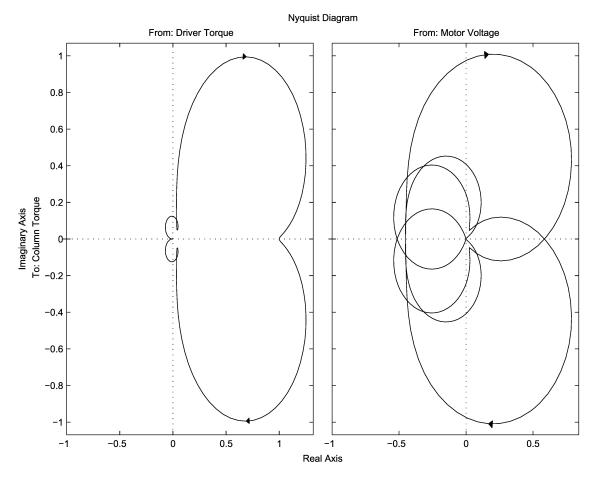


Fig. 5. Nyquist plots for uncompensated EPAS system.

driver torque T_d and the motor voltage v are the two inputs to the system. The motor column angular position, θ_m , is the output for the sensorless EPAS system. The system is considered sensorless in the sense that the usual column torque sensor is not used.

It can be shown that for the state-space model described in (10), the pair (A,B) is completely controllable and the pair (A,C) is completely observable. A state-variable feedback control scheme may thus be employed. The output and the motor terminal voltage v are used as inputs into an observer to gen-

erate \hat{x} , an estimate of the system state. The state estimate, \hat{x} is used to compute the state-feedback gain matrix K_{lqc} .

A Kalman filter is designed to obtain the required observer dynamics in the presence of stochastic disturbances. The state equation for the Kalman estimator is

$$\dot{\hat{x}} = A\,\hat{x} + B\,u + K_o\left(y - C\,\hat{x}\right) \tag{11}$$

where K_o is the estimator gain matrix. The stochastic disturbances in the EPAS model are assumed to be Gaussian and are due to the noise in the sensor measurements and the disturbances that are transmitted from the road. The noise covariances have to be specified for Kalman filter design. The variances will vary according to road conditions and vehicle type. In this treatment, the noise variances are chosen by selecting desirable time- or frequency-domain responses for the control loop.

V. SELECTION OF WEIGHTING MATRICES

The selection of the LQR weighting matrices Q and R is considered in this section. A study of the physical characteristics of the model ((5)–(7)) reveals that two quantities define the effort applied by the driver.

1) *The steering column torque:* The steering column torque is proportional to the deflection in the torsion bar and is given by

$$T_c = K_c \left(\theta_c - \frac{p}{r_p} \right). \tag{12}$$

A low steering column torque for a given amount of rack displacement implies that most of the effort is being produced by the motor rather than the driver.

2) The power expended in displacing the rack: The power expended in displacing the steering rack through a distance p is defined by the relation

$$K_t p \dot{p} = T_d \dot{\theta}_c + T_m \dot{\theta}_m. \tag{13}$$

The terms on the right-hand side are the products of driver torque and motor torque and the respective column angular velocities. The term on the left-hand side is the product of the force and the linear velocity of the steering rack. A low value for

$$T_d \,\dot{\theta}_c = K_t \, p \,\dot{p} - T_m \,\dot{\theta}_m \tag{14}$$

implies that most of the power expended in displacing the rack is applied by the electric motor and not the driver.

Equations (12) and (14) may be incorporated in the matrix Q. A reasonable choice for the Q matrix is found to be

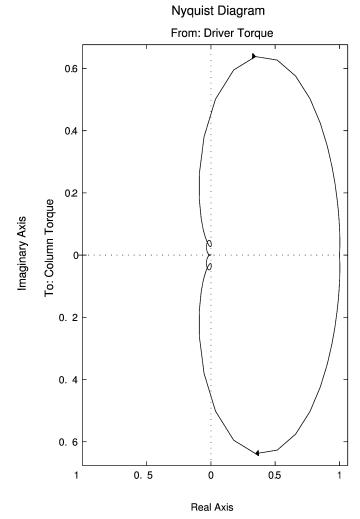


Fig. 6. Nyquist plot for compensated EPAS system.

where a is a tuning parameter that may be varied to modify the closed-loop characteristics of the system. The R matrix is chosen as

$$R = bI \tag{16}$$

where I is the identity matrix and b is a weight on the input.

It is observed that a finer control over closed-loop dynamics may be achieved by increasing the number of tuning parameters. Another choice for ${\cal Q}$ is the matrix

where a_3 is a weight on the motor column angular position, a_4 is a weight on the motor column angular velocity, and a_7 is a weight on the motor armature current. An appropriate selection of a, a_3 , a_4 , and a_7 ensures that an adequate level of motor assist

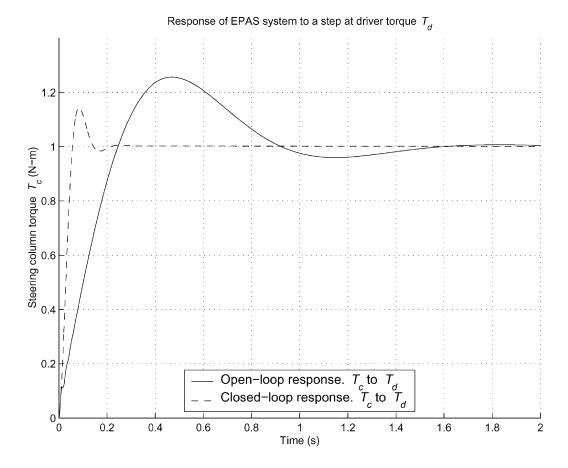


Fig. 7. Step response of EPAS system to driver torque.

exists and the motor current does not assume inordinately high values.

VI. RESULTS AND DISCUSSION

It is desirable in an EPAS system that the amount of assist provided by the motor at operating frequencies be adequately high and produces no oscillations in the system. A good driver is capable of producing inputs at 3–5 Hz [3] at the steering wheel. The assist levels in this frequency range should be higher than that at the higher frequencies. This enhances the ability of the controller to reject external disturbances (e.g., gearbox torque ripple).

The weighting matrices for the EPAS control system described in Section IV are selected as per the criteria detailed in Section V. The resulting control system is simulated in MATLAB and the characteristics of the closed-loop system are compared with the open-loop system.

A. Frequency Response and Robustness

Parameter values from Table I are used for the purpose of simulation. Fig. 4 displays the frequency response of the sensorless EPAS system in terms of its sigma plot. The weighting matrices are Q_2 and R. The values of the tuning parameters are $a=1\times10^6, a_3=1\times10^3, a_4=5\times10^6, a_7=100,$ and b=10.

The assist gain is quantified in terms of the motor column torque described by the expression

$$T_m = K_m \left(\theta_m - \frac{p G}{r_p} \right) \tag{18}$$

and the assist gain (between the driver torque and motor torque) is represented by the dashed line in the frequency response. It is observed that the closed-loop system has a flat response for low frequencies and a high rate of roll-off. This is a desirable characteristic since high gains for high-frequency driver inputs (generally unintended) are undesirable. The level of assist gain is also adequate. The assist gain may be reduced or increased by decreasing or increasing the value of a. Also, increasing the values of a_3 and a_4 increases the weights on motor column position and velocity, and decreases the level of gain.

The corresponding Nyquist plots for the open- and closed-loop systems are shown in Figs. 5 and 6, respectively.

B. Time Response

The response of the open-loop and the closed-loop system to a step at the driver torque (T_d) for the controller parameter values in Section VI-A are shown in Fig. 7. It is observed that the settling time of the closed-loop system is considerably lower than the open-loop system. Also, the system settling time

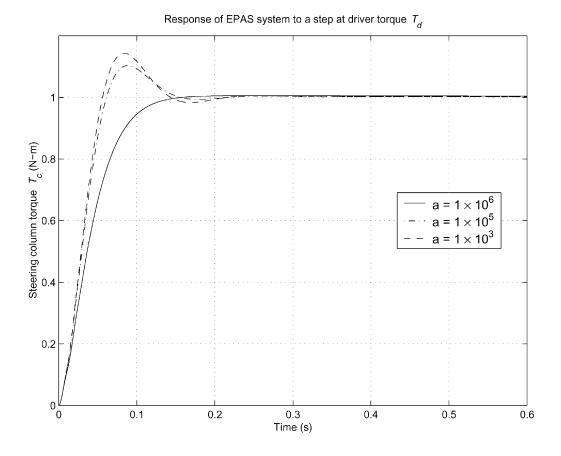


Fig. 8. Step response of EPAS system to driver torque for varying values of a. The parameters a_3 , a_5 , and a_7 remain the same as before.

may be varied by varying the value of a, where $Q=a\,Q_2$ (Fig. 8).

VII. CONCLUSION

The torque sensors on the steering column in all current EPAS systems introduce an element of compliance in the steering column. Elimination of the torque sensor increases the stiffness of the steering column and improves steering feel and system stability. In this paper, the authors proposed an EPAS control system that does not use steering column torque information. The dynamical equations of motion for a simplified model of a double-pinion-type EPAS system are derived and a state-space formulation is obtained for the model. An optimal-stochastic controller using LQR/Kalman filter techniques is designed for the system model. Notable advances offered by the proposed controller are: 1) stable closed-loop performance is attained for high levels of assist gain; 2) the controller is successful in attenuating oscillations at low-frequency ranges that are inherent in the open-loop system; and 3) closed-loop characteristics may be varied by a few tuning parameters. Furthermore, the controller does not

use the steering column torque sensor and may be termed as sensorless. This offers advantages in terms of both cost and mechanical performance.

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