13.1 HW   3, 7, 11, 15, 19, 23, 27, 31, 41, 43, 45

(3) $\lim_{t\to 0}\left(e^{-3t}\, i + \frac{t^2}{\sin^2 t}\, j + \cos 2t\, k\right)$

$\lim_{t\to 0} e^{-3t}\, i + \lim_{t\to 0}\frac{t^2}{\sin^2 t}\, j + \lim_{t\to 0}\cos 2t\, k$

$\lim_{t\to 0}\frac{2t}{\sin 2t} = \frac{0}{0}$

$\lim_{t\to 0}\frac{\cancel{2}^{1}}{2\cos(2t)} = 1$

$e^{0}\, i + \lim_{t\to 0}\frac{2t}{2\sin t\cos t}\, j + \cos(0)\, k$

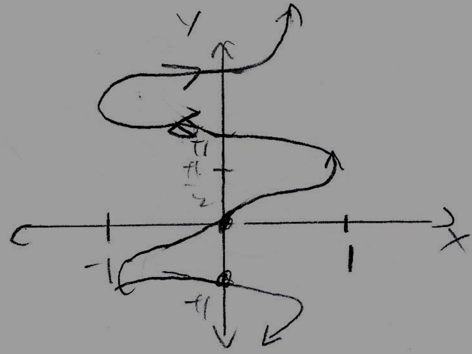
$\boxed{i + j + k}$

(7) $r(t) = \langle \sin t, t\rangle$

$x = \sin t \qquad y = t$

$t = \sin^{-1}(x) \qquad t = y$

$\langle \sin^{-1}(x), y\rangle$



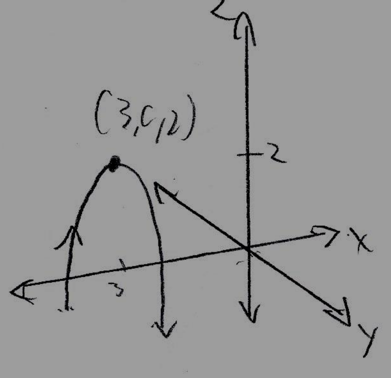
(11) $r(t) = \langle 3, t, 2 - t^2\rangle$

$x = 3 \qquad y = t \qquad z = 2 - t^2$

$t = \pm\sqrt{2 - z}$

$\langle 3, y, \sqrt{2 - z}\rangle$

(3, 0, 2) is peak.



(15) $r(t) = \langle t, \sin t, 2\cos t \rangle$

$x = t \quad y = \sin t \quad z = 2\cos t$

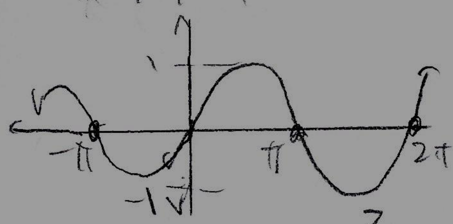
~~$t = x$~~ ~~$t = \sin^{-1}(y)$~~ ~~$t = \cos^{-1}\left(\frac{z}{2}\right)$~~

$\langle t, \sin t \rangle$

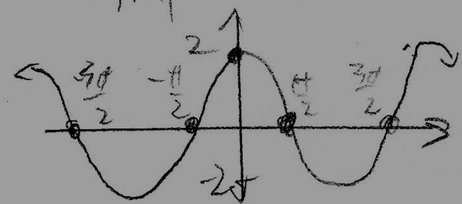
$\langle t, 2\cos t \rangle$

$\langle \sin t, 2\cos t \rangle$

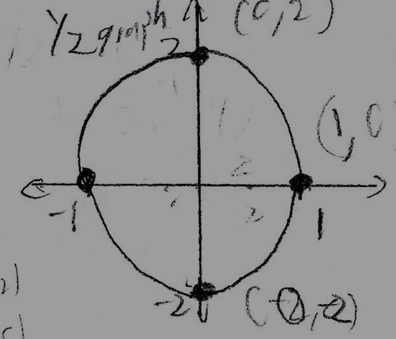
xy graph



xz graph



yz graph



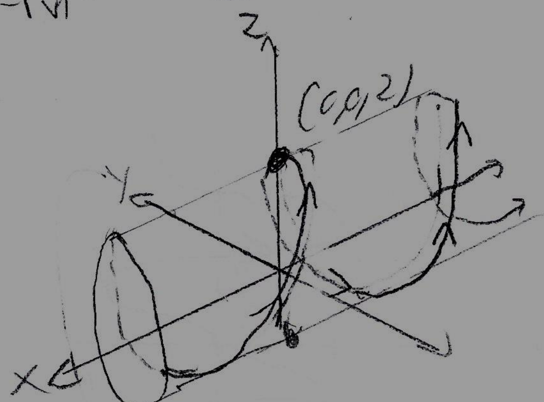
$t = 0$ a $(0,2)$

$t = \frac{\pi}{2}$ a $(1,0)$

$t = \pi$ a $(0,-2)$

$t = \frac{3\pi}{2}$ $(-1,0)$

$t = 2\pi$ a $(0,2)$



$r(0) = \langle 0, 0, 2 \rangle$

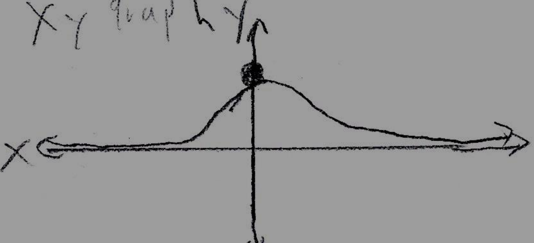
(19) $P(0,-1,1) \quad Q\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right) \qquad \vec{PQ} = \left\langle \frac{1}{2}, \frac{4}{3}, -\frac{3}{4} \right\rangle$

$r(t) = r_0 + t v = \langle 0, -1, 1 \rangle + t\left\langle \frac{1}{2}, \frac{4}{3}, -\frac{3}{4} \right\rangle$

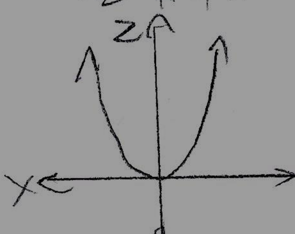
$$\boxed{\begin{array}{l} r(t) = \left\langle \frac{1}{2}t, \frac{4}{3}t - 1, -\frac{3}{4}t + 1 \right\rangle \quad t \in [0,1] \\ x = \frac{1}{2}t \quad y = \frac{4}{3}t - 1 \quad z = 1 - \frac{3}{4}t \quad t \in [0,1] \end{array}}$$

(23) $x = t \quad y = \frac{1}{1+t^2} \quad z = t^2$

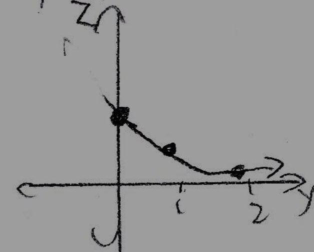
xy graph

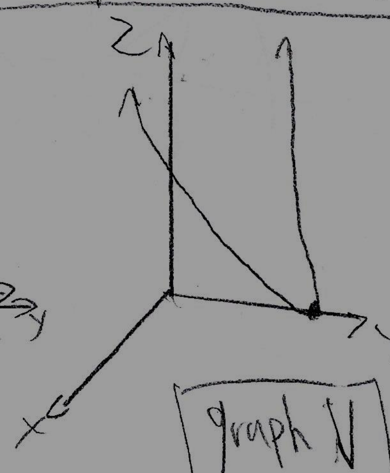


xz graph



yz graph



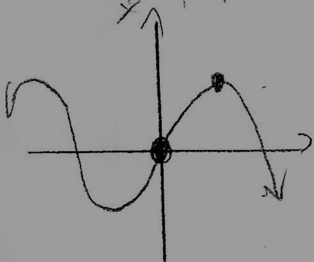


graph V

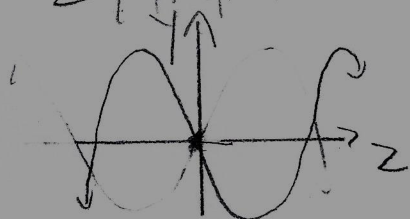
(27) $y = t\cos t$ $y = t\sin t$ $z = t$

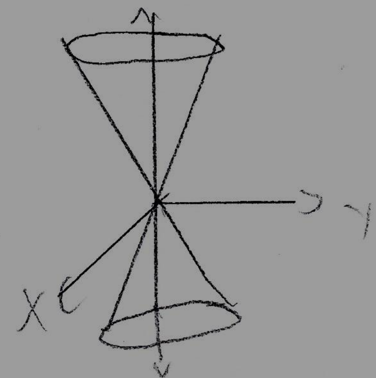
$z^2 = x^2 + y^2$

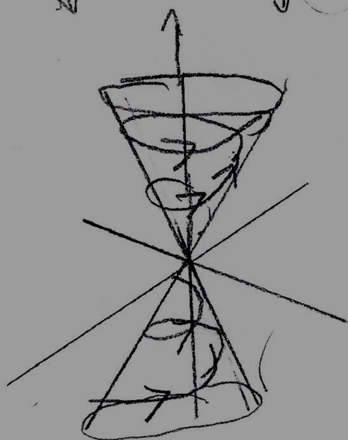
zx graph



zy graph







(31) $r(t) = t\,i + (2t - t^2 k)$ $z = x^2 + y^2$ $x^2 + y^2 - z = 0$

$r(t) = \langle t, 0, 2t - t^2 \rangle$

$x = t$ $y = 0$ $z = 2t - t^2$

$t^2 + 0 = 2t - t^2$

$2t^2 = \cancel{2}t$

$t = 1$ $t = 0$

$\langle 1, 0, 2(1) - 1 \rangle$

$=$

$\boxed{(1,0,1), (0,0,0)}$

$0 = 0^2 + 0^2$ $\langle 0, 0, 2(0) - 0 \rangle$

$(0,0,0)$ $(0,0,0)$

(41) $r(t) = \langle t^2, 1-3t, 1+t^2 \rangle$

$1 - 3t = 4$
$t = -1$

$1 - 3t = -8$ $t^2 = 4$
$t = -3$ $t = 2$ or $t = -2$

$r(-1) = \langle 1, 1+3, 1-1 \rangle = (1,4,0)$

$r(3) = \langle 9, 1-9, 1+27 \rangle = (9,-8,27)$

$r(2) = \langle 4, 1-6, 1+8 \rangle = (4,-5,9)$

$r(-2) = \langle 4, 1+6, 1-8 \rangle = (4,7,-7)$

$\boxed{(1,4,0), (9,-8,27) \text{ lie on } r(t)}$

$\boxed{(4,7,-6) \text{ doesn't lie on } r(t)}$

43. $Z = \sqrt{x^2+y^2}$ $\quad Z = 1+y$

$z^2 = x^2+y^2 \qquad z^2 = 1+2y+y^2$

$1+2y+\cancel{y^2} = x^2+\cancel{y^2}$

$1+2y = x^2$

$1 = x^2-2y$

$x = t \qquad y = \frac{x^2-1}{2}$

$= \frac{t^2-1}{2}$

$Z = 1+y$

$Z = 1+\frac{t^2-1}{2} = \frac{t^2+2-1}{2}$

$Z = \frac{t^2+1}{2}$

$$\boxed{r(t) = \left\langle t, \frac{t^2-1}{2}, \frac{t^2+1}{2} \right\rangle}$$

45. $z = x^2-y^2 \quad x^2+y^2=1$ $\qquad x = \cos t \quad y = \sin t \quad \sin^2 t+\cos^2 t = 1$

$z = \cos^2 t - \sin^2 t$

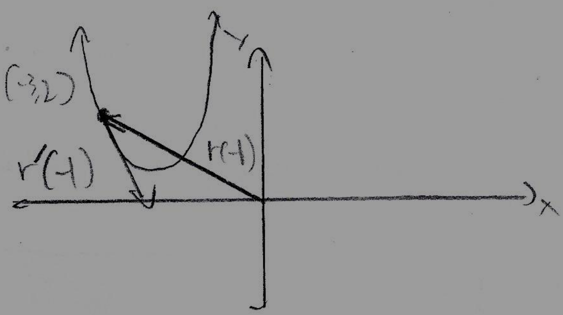
$z = \cos 2t$

$\langle \cos t, \sin t, \cos 2t \rangle$

$$\boxed{\begin{array}{l} r(t) = \cos t\, i + \sin t\, j + \cos 2t\, k \\ t \in [0, 2\pi] \end{array}}$$

13.2 Hw   3, 7, 11, 15, 19, 23, 27, 35, 39

(3) $r(t) = \langle t-2, t^2+1 \rangle \quad t = -1$

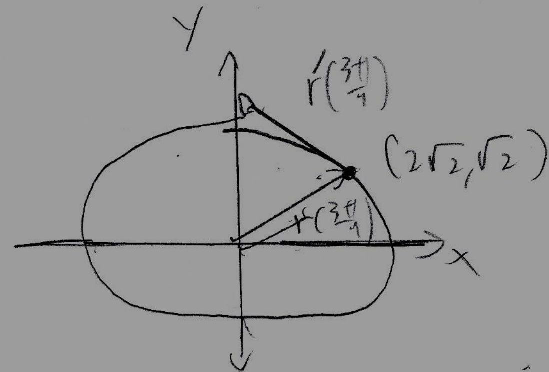


$r'(t) = \langle 1, 2t \rangle$

$r(-1) = \langle -3, 2 \rangle$

$r'(-1) = \langle 1, -2 \rangle$

(7) $r(t) = \langle 4\sin t, -2\cos t \rangle \quad t = \frac{3\pi}{4}$



$r'(t) = \langle 4\cos t, 2\sin t \rangle$

$r\left(\frac{3\pi}{4}\right) = \langle 2\sqrt{2}, \sqrt{2} \rangle$

$r'\left(\frac{3\pi}{4}\right) = \langle -2\sqrt{2}, \sqrt{2} \rangle$

(11) $r(t) = \langle t^2, \cos(t^2), \sin^2 t \rangle$

$r'(t) = \langle 2t, -2t\sin(t^2), 2\sin t\cos t \rangle$

$= \boxed{2t\,i - 2t\sin(t^2)\,j + \sin(2t)\,k}$

(15) $r(t) = a + tb + t^2c = \langle 1, t, t^2 \rangle$

$r'(t) = \langle 0, 1, 2t \rangle = \boxed{b + 2tc}$

19. $r(t) = \langle \cos t, 3t, 2\sin 2t \rangle \quad t=0$

$r'(t) = \langle -\sin t, 3, 4\cos 2t \rangle \qquad r'(0) = \langle 0, 3, 4 \rangle$

$|r'(t)| = \sqrt{\sin^2 t + 9 + 16\cos^2 2t} \qquad |r'(0)| = \sqrt{0+9+16} = \sqrt{25} = 5$

$T(t) = \dfrac{r'(t)}{|r'(t)|} = \quad T(0) = \dfrac{1}{5}\langle 0, 3, 4 \rangle = \boxed{\langle 0, \tfrac{3}{5}, \tfrac{4}{5} \rangle}$

23. $x = t^2+1 \quad y = 4\sqrt{t} \quad z = e^{t^2-t} \quad (2,4,1)$

$r(t) = \langle t^2+1, 4\sqrt{t}, e^{t^2-t} \rangle \qquad r(1) = \langle 1+1, 4\sqrt{1}, e^0 \rangle = (2,4,1)$
$t=1$

$r'(t) = \langle 2t, \dfrac{2}{\sqrt{t}}, (2t-1)e^{t^2-t} \rangle$

$r'(1) = \langle 2, 2, 1 \rangle$

$r = \langle 2+2t, 4+2t, 1+t \rangle$

$$\boxed{\begin{aligned} x &= 2+2t \\ y &= 4+2t \\ z &= 1+t \end{aligned}}$$

27. $x^2+y^2=25 \qquad y^2+z^2=20 \qquad (3,4,2)$

$y=t \quad x = \sqrt{25-y^2} = \sqrt{25-t^2} \qquad z = \sqrt{20-y^2} \qquad r(t) = \langle \sqrt{25-t^2}, t, \sqrt{20-t^2} \rangle$

$r'(t) = \langle \dfrac{-t}{\sqrt{25-t^2}}, 1, \dfrac{-t}{\sqrt{20-t^2}} \rangle$

$r'(4) = \langle \dfrac{-4}{3}, 1, -2 \rangle = \langle -4, 3, -6 \rangle$

$r(t) = \langle 3, 4, 2 \rangle + t\langle -4, 3, -6 \rangle = \langle 3-4t, 4+3t, 2-6t \rangle$

$\boxed{r(t) = \langle 3-4t, 4+3t, 2-6t \rangle}$

(35) $\int_0^2 \langle t, -t^3, 3t^5 \rangle \, dt$

$\langle \frac{1}{2}t^2, -\frac{1}{4}t^4, \frac{1}{2}t^6 \rangle \Big]_0^2$

$\langle \frac{1}{2}(4), -\frac{1}{4}(16), \frac{1}{2}(64) \rangle - \langle 0,0,0 \rangle = \boxed{\langle 2, -4, 32 \rangle}$

(39) $\int \sec^2 t \, i + t(t^2+1)^3 j + t^2 \ln t \, k \, dt$

$\int \sec^2(t) i \, dt + \int t(t^2+1)^3 j \, dt + \int t^2 \ln t \, k \, dt$

$a = t^2+1$
$da = 2t\,dt$
$\frac{1}{2} da = dt$

$\tan t \, i + \frac{1}{2} \int a^3 j \, da + \int t^2 \ln t \, k \, dt + C$

$\tan(t) i + \frac{1}{8}(t^2+1)^4 j + \int t^2 \ln t \, k \, dt + C$ $u = \ln t$
$dv =$

$\frac{1}{3}x^2$

$\int x^2 \ln x \, dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{3} x^3 \left(\frac{1}{x}\right) dx$

$u = \ln x \quad du = \frac{1}{x}dx$

$dv = x^2 \quad v = \frac{1}{3}x^3$

$\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C \quad x = t$

$$\boxed{\tan(t) i + \frac{1}{8}(t^2+1)^4 j + \left(\frac{1}{3}t^3 \ln t - \frac{1}{9}t^3\right) k + C}$$

13.3 homework 1, 5, 9, 13, 17, 21, 25, 27, 29, 31, 33, 43, 45, 49

(1) $r(t) = \langle t, 3\cos t, 3\sin t \rangle \quad t \in [-5, 5]$

$r'(t) = \langle 1, -3\sin t, 3\cos t \rangle$

$|r'(t)| = \sqrt{1 + 9\sin^2 t + 9\cos^2 t} = \sqrt{1 + 9(\sin^2 t + \cos^2 t)} = \sqrt{1+9} = \sqrt{10}$

$\int_{-5}^{5} \sqrt{10}\, dt = \sqrt{10}\, t \Big]_{-5}^{5} = 2\sqrt{10}(5) = \boxed{10\sqrt{10}}$

(5) $r(t) = \langle 1, t^2, t^3 \rangle \quad t \in [0, 1]$

$r'(t) = \langle 0, 2t, 3t^2 \rangle$

$|r'(t)| = \sqrt{4t^2 + 9t^4}$

$\int_0^1 t\sqrt{4 + 9t^2}\, dt$ $\quad u = 4 + 9t^2$, $du = 18t\, dt$, $\frac{1}{18} du = t\, dt$

$\frac{1}{18}\int_4^{13} \sqrt{u}\, du = \frac{1}{18} \cdot \frac{2}{3} u^{3/2} \Big|_4^{13} = \boxed{\frac{1}{27}\left(13^{3/2} - 8\right)}$

(9) $r(t) = \langle \cos \pi t, 2t, \sin 2\pi t \rangle$ $\quad (1, 0, 0)$ to $(1, 4, 0)$, $t = 0$, $t = 2$

$r'(t) = \langle -\pi \sin \pi t, 2, 2\pi \cos 2\pi t \rangle$

$|r'(t)| = \sqrt{(-\pi \sin \pi t)^2 + 4 + (2\pi \cos 2\pi t)^2}$

$\int_0^2 \sqrt{\pi^2 \sin^2 \pi t + 4 + 4\pi^2 \cos^2 2\pi t}\, dt \approx \boxed{10.3311}$

13. $r(t) = (5-t)i + (4t-3)j + 3tk \qquad P(4,1,3)$

$r'(t) = \langle -1, 4, 3 \rangle$

$|r'(t)| = \sqrt{1+16+9} = \sqrt{26}$

$\int_1^t \sqrt{26}\, dt \qquad \sqrt{26}\,t - \sqrt{26} \qquad t = \frac{s(t)}{\sqrt{26}} + 1$

$\boxed{s(t) = \sqrt{26}(t-1)}$

$$\boxed{\begin{aligned} r(t(s)) &= \left\langle 4 - \frac{s(t)}{\sqrt{26}},\ 1 + \frac{4s(t)}{\sqrt{26}},\ 3 + \frac{3s(t)}{\sqrt{26}} \right\rangle \\ r(t(4)) &= \left(4 - \frac{4}{\sqrt{26}},\ 1 + \frac{16}{\sqrt{26}},\ 3 + \frac{12}{\sqrt{26}} \right) \end{aligned}}$$

17. $r(t) = \langle t, 3\cos t, 3\sin t \rangle$

$r'(t) = \langle 1, -3\sin t, 3\cos t \rangle \qquad |r'(t)| = \sqrt{1 + 9(\sin^2 t + \cos^2 t)} = \sqrt{10}$

$T(t) = \frac{1}{\sqrt{10}} \langle 1, -3\sin t, 3\cos t \rangle = \boxed{\left\langle \frac{1}{\sqrt{10}}, \frac{-3}{\sqrt{10}}\sin t, \frac{3}{\sqrt{10}}\cos t \right\rangle}$

$T'(t) = \left\langle 0, \frac{-3}{\sqrt{10}}\cos t, \frac{-3}{\sqrt{10}}\sin t \right\rangle \qquad |T'(t)| = \sqrt{\frac{9}{10}(\cos^2 t + \sin^2 t)} = \frac{3}{\sqrt{10}}$

$N(t) = \frac{\sqrt{10}}{3} \left\langle 0, \frac{-3}{\sqrt{10}}\cos t, \frac{-3}{\sqrt{10}}\sin t \right\rangle = \boxed{\langle 0, -\cos t, -\sin t \rangle}$

$k(t) = \frac{|T'(t)|}{|r'(t)|} = \frac{\frac{3}{\sqrt{10}}}{\sqrt{10}} = \boxed{\frac{3}{10}}$

(21) $r(t) = \langle t^3, t^2 \rangle$

$r'(t) = \langle 0, 3t^2, 2t \rangle$ $\quad |r'(t)| = \sqrt{9t^4 + 4t^2}$ $\quad |r'(t)|^3 = (9t^4 + 4t^2)\sqrt{9t^4 + 4t^2}$

$r''(t) \langle 0, 6t, 2 \rangle$

$$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ 0 & 3t^2 & 2t \\ 0 & 6t & 2 \end{vmatrix} = \begin{vmatrix} 3t^2 & 2t \\ 6t & 2 \end{vmatrix} i - \begin{vmatrix} 0 & 2t \\ 0 & 2 \end{vmatrix} j + \begin{vmatrix} 0 & 3t^2 \\ 0 & 6t \end{vmatrix} k$$

$(6t^2 - 12t^2)i - 0j + 0k$

$\langle -6t^2, 0, 0 \rangle$

$|\langle -6t^2, 0, 0 \rangle| = \sqrt{36t^4} = 6t^2$

$$k(t) = \frac{6t^2}{(9t^4 + 4t^2)\sqrt{9t^4 + 4t^2}} = \boxed{\frac{6t^2}{(9t^4 + 4t^2)^{3/2}}}$$

(25) $r(t) = \langle t, t^2, t^3 \rangle$ $\quad (1,1,1)$ $\quad t = 1$

$r'(t) = \langle 1, 2t, 3t^2 \rangle$ $\quad |r'(t)| = \sqrt{1 + 4t^2 + 9t^4}$

$r''(t) = \langle 0, 2, 6t \rangle$

$$r'(t) \times r''(t) = \begin{vmatrix} 2t & 3t^2 \\ 2 & 6t \end{vmatrix} i - \begin{vmatrix} 1 & 3t^2 \\ 0 & 6t \end{vmatrix} j + \begin{vmatrix} 1 & 2t \\ 0 & 2 \end{vmatrix} k$$

$(12t^2 - 6t^2)i - 6tj + 2k$

$|\langle 6t^2, -6t, 2 \rangle| = \sqrt{36t^4 + 36t^2 + 4}$

$$k(t) = \frac{\sqrt{36t^4 + 36t^2 + 4}}{\left(\sqrt{1 + 4t^2 + 9t^4}\right)^{3/2}}$$

$$k(1) = \frac{\sqrt{36 + 36 + 4}}{\left(\sqrt{1 + 4 + 9}\right)^3} = \frac{\sqrt{76}}{(\sqrt{14})^3} = \boxed{\frac{1}{7}\sqrt{\frac{19}{14}}}$$

27) $y = x^4$

$$\frac{dy}{dx} = 4x^3$$

$$\frac{d^2y}{dx^2} = 12x^2$$

$$\boxed{k(x) = \frac{12x^2}{[1+16x^6]^{3/2}}}$$

29) $y = xe^x$

$$\frac{dy}{dx} = xe^x + e^x = e^x(x+1)$$

$$\frac{d^2y}{dx^2} = (x+1)e^x + e^x$$

$$\frac{(x+1)e^x + e^x}{[1 + (e^x(x+1))^2]^{3/2}}$$

$$= \boxed{\frac{e^x(x+2)}{[1+(xe^x+e^x)^2]^{3/2}}}$$

31) $y = e^x$

$y' = e^x$

$y'' = e^x$

$$k(x) = \frac{e^x}{(1+e^{2x})^{3/2}}$$

$$k'(x) = \frac{e^x(1+e^{2x})^{3/2} - e^x\left(\frac{3}{2}(1+e^{2x})^{1/2}(2e^{2x})\right)}{(1+e^{2x})^{3/2}}$$

$$0 = \cancel{e^x}\left((1+e^{2x})^{3/2} - \frac{3}{2}(1+e^{2x})^{1/2} 2e^{2x}\right)$$

$$\frac{3}{\cancel{2}}(1+e^{2x})^{1/2}e^{2x} = (1+e^{2x})^{\cancel{3/2}}$$

$$3e^{2x} = 1 + e^{2x}$$

$$2e^{2x} = 1$$

$$e^{2x} = \frac{1}{2}$$

$$2x = \ln\left(\frac{1}{2}\right)$$

$$x = -\frac{1}{2}\ln(2)$$

$$\boxed{\left(-\frac{1}{2}\ln(2), \frac{1}{\sqrt{2}}\right)}$$

$$\lim_{x\to\infty} k(x) = \frac{\lim_{x\to\infty} e^x}{\lim_{x\to\infty}(1+e^{2x})^{3/2}} = \frac{\infty}{\infty}$$

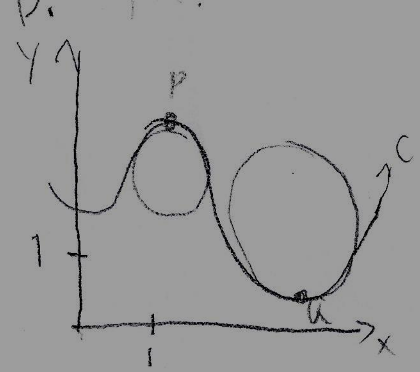
$$\frac{\lim_{x\to\infty} e^x}{\lim_{x\to\infty}\sqrt{e^{8x}+c}} \approx \frac{\lim e^x}{e^{4x}}$$

$$\sim \lim_{x\to\infty}\frac{1}{e^{3x}} = \frac{1}{\infty} = 0$$

$\boxed{\text{approaches } 0}$

(33) a. P. Since the arc length is less than Q, therefore the rate of change of unit tangent vector with respect to arc length is greater at P.

b.



assuming radius for P circle is 0.8 and radius for Q circle is 1.4

$$k_P = \frac{1}{0.8} = \boxed{1.3} \qquad k_Q = \frac{1}{1.4} = \boxed{0.7}$$

(43) $$k = \frac{|x'y'' - y'x''|}{[(x')^2 + (y')^2]^{3/2}}$$

$x = t^2 \qquad y = t^3$

$x' = 2t \qquad y' = 3t^2$

$x'' = 2 \qquad y'' = 6t$

$$k = \frac{|12t^2 - 6t^2|}{[4t^2 + 9t^4]^{3/2}} = \boxed{\frac{6t^2}{[4t^2 + 9t^4]^{3/2}}}$$

(45) $x = e^t\cos t \qquad y = e^t\sin t$

$x' = e^t\cos t - e^t\sin t \qquad y' = e^t\sin t + e^t\cos t$

$x'' = \cancel{e^t\cos t} - e^t\sin t - e^t\sin t - \cancel{e^t\cos t} \qquad y'' = e^t\sin t + e^t\cos t - \cancel{e^t\sin t} + e^t\cos t$

$= -2e^t\sin t \qquad = 2e^t\cos t$

$$k = \frac{|(e^t\cos t - e^t\sin t)(2e^t\cos t) - (e^t\sin t + e^t\cos t)(-2e^t\sin t)|}{[(e^t\cos t - e^t\sin t)^2 + (e^t\sin t + e^t\cos t)^2]^{3/2}}$$

45 cont)

$$\frac{2e^{2t}\cos^2 t - e^{2t}\sin 2t + 2e^{2t}\sin^2 t + e^{2t}\sin 2t}{\left[e^{2t}\cos^2 t - \cancel{e^{t}\sin 2t} + e^{2t}\sin^2 t + e^{2t}\sin^2 t + e^{t}\sin 2t + e^{t}\cos^2 t\right]^{3/2}}$$

$$\frac{2e^{2t}}{\left[2e^{2t}\right]^{3/2}} = \frac{2e^{2t}}{\sqrt{8e^{6t}}} = \frac{2e^{2t}}{2\sqrt{2}\,e^{3t}} = \boxed{\frac{1}{\sqrt{2}e^{t}}}$$

49) $x = \sin 2t \quad y = -\cos 2t \quad z = 4t \quad (0, 1, 2\pi) \quad t = \frac{2\pi}{4} = \frac{\pi}{2}$

$r(t) = \langle \sin 2t, -\cos 2t, 4t \rangle \quad r'(t) = \langle 2\cos 2t, 2\sin 2t, 4 \rangle$

$r'(\frac{\pi}{2}) = \langle -2, 0, 4 \rangle$

$-2(x-0) + 0(y-1) + 4(z-2\pi) = 0$

$-2x + 4z - 8\pi = 0$

$\boxed{x - 2z = 4\pi \quad = \text{Normal plane}}$

$T(t) = \frac{r'(t)}{|r'(t)|} = \frac{\langle 2\cos 2t, 2\sin 2t, 4 \rangle}{\sqrt{4\cos^2 2t + 4\sin^2 2t + 16}} = \frac{1}{\sqrt{5}}\langle \cos 2t, \sin 2t, 2 \rangle \quad T(\frac{\pi}{2}) = \frac{1}{\sqrt{5}}\langle -1, 0, 2 \rangle$

$T'(t) = \frac{1}{\sqrt{5}}\langle -2\sin 2t, 2\cos 2t, 0 \rangle \quad T'(\frac{\pi}{2}) = \frac{1}{\sqrt{5}}\langle 0, -2, 0 \rangle \quad |T'(\frac{\pi}{2})| = \frac{2}{\sqrt{5}}$

$N(\frac{\pi}{2}) = \frac{\frac{1}{\sqrt{5}}\langle 0, -2, 0 \rangle}{\frac{2}{\sqrt{5}}} = \langle 0, -1, 0 \rangle \quad B(\frac{\pi}{2}) = T(\frac{\pi}{2}) \times N(\frac{\pi}{2}) = \frac{1}{\sqrt{5}}\begin{vmatrix} i & j & k \\ -1 & 0 & 2 \\ 0 & -1 & 0 \end{vmatrix} = \frac{1}{\sqrt{5}}\langle 2, 0, 1 \rangle$

$\frac{2}{\sqrt{5}}(x-0) + 0(y-1) + \frac{1}{\sqrt{5}}(z-2\pi) = 0$

$\boxed{\text{oscillating}! \quad 2x + z = 2\pi}$

13.4 homework 3, 7, 11, 15, 19, 23, 27, 37, 39, 41

3. $r(t) = \langle -\frac{1}{2}t^2, t \rangle$ $t = 2$

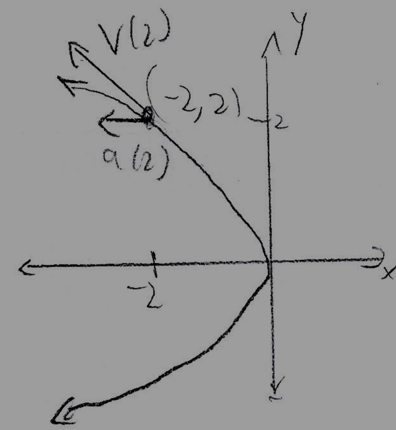
$v(t) = \langle -t, 1 \rangle$ $|v(t)| = \sqrt{t^2+1}$

$a(t) = \langle -1, 0 \rangle$

$v(2) = \langle -2, 1 \rangle$

$a(2) = \langle -1, 0 \rangle$

$|v(2)| = \sqrt{5}$

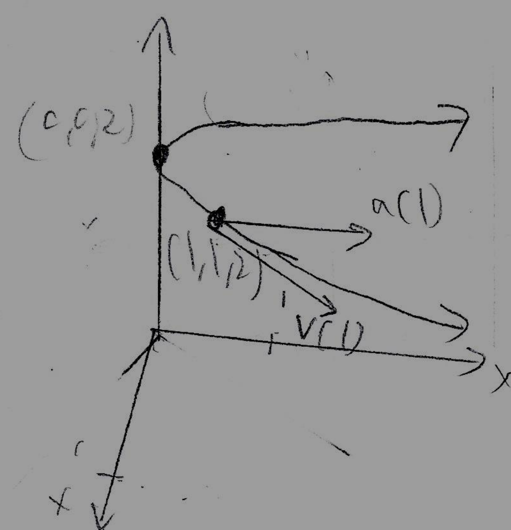


7. $r(t) = \langle t, t^2, 2 \rangle$ $t = 1$ $r(1) = (1,1,2)$

$v(t) = \langle 1, 2t, 0 \rangle$ $v(1) = \langle 1, 2, 0 \rangle$

$|v(t)| = \sqrt{1+4t^2}$ $|v(1)| = \sqrt{5}$

$a(t) = \langle 0, 2, 0 \rangle$ $a(1) = \langle 0, 2, 0 \rangle$



11. $r(t) = \langle \sqrt{2}t, e^t, e^{-t} \rangle$

$\boxed{v(t) = \langle \sqrt{2}, e^t, -e^{-t} \rangle}$

$|v(t)| = \sqrt{2 + e^{2t} + e^{-2t}} = \sqrt{(e^t + e^{-t})^2} = \boxed{e^t + e^{-t} = |v(t)|}$

$\boxed{a(t) = \langle 0, e^t, e^{-t} \rangle}$

15. $a(t) = 2i + 2tk$ $v(0) = 3i - j$ $r(0) = j + k$

$v(t) = \int 2i + 2tk \, dt = \langle 2t, 0, t^2 \rangle + c$

$c = \langle 3, -1, 0 \rangle$

$\boxed{v(t) = \langle 2t+3, -1, t^2 \rangle}$

$r(t) = \int \langle 2t+3, -1, t^2 \rangle dt = \langle t^2+t, -t, \frac{1}{3}t^2 \rangle + c$

$c = \langle 0, 1, 1 \rangle$

$\boxed{r(t) = \langle t^2+t, 1-t, \frac{1}{3}t^2 + 1 \rangle}$

19. $r(t) = \langle t^2, 5t, t^2 - 16t \rangle$

$v(t) = \langle 2t, 5, 2t - 16 \rangle$

$|v(t)| = \sqrt{4t^2 + 25 + 4t^2 - 64t + 16^2}$

$s(t) = \sqrt{8t^2 - 64t + 281}$

$s'(t) = \dfrac{1}{2\sqrt{8t^2 - 64t + 281}}(16t - 64)$

$0 = 16t - 64$

$\boxed{t = 4}$

23. $V_0 = 200$ m/s $\quad a = 60°$

$x = 200\cos(60)t \qquad y = 200\sin(60)t - \frac{1}{2}(9.8)t^2$

$r(t) = \langle 100t, 100\sqrt{3}t - 4.9t^2 \rangle$

$v(t) = \langle 100, 100\sqrt{3} - 9.8t \rangle$

$|v(t)| = \sqrt{100^2 + (100\sqrt{3} - 9.8t)^2}$

a. $100\sqrt{3}t - 4.9t^2 = 0$

$t(100\sqrt{3} - 4.9t) = 0$

$t = 0 \qquad t = \dfrac{100\sqrt{3}}{4.9} = 35.35$

$x = 100(35.35) = \boxed{3535\text{m}}$ range

b. $y' = 100\sqrt{3} - 9.8t \qquad t = \dfrac{100\sqrt{3}}{9.8} = 17.673$

$y = 200\sin(60)(17.673) - 4.9(17.673)^2 =$

$\boxed{1531\text{ m}} \Leftarrow 1530.61 \quad h_{max}$

c. $|v(35.35)| = \sqrt{100^2 + (100\sqrt{3} - 9.8(35.35))^2} = 200.017 \sim \boxed{200\text{ m/s}}$

27. $r_{max} = 1600$ $\alpha = 36°$ $g = 32$

$$1600 = v_0 \sin(36)t - \frac{1}{2}(32)t^2$$

$$0 = v_0 \sin(36) - 32t$$

$$t = \frac{v_0 \sin 36}{32}$$

$$1600 = \frac{(v_0 \sin 36)^2}{32} - \frac{1}{2}\frac{(v_0 \sin 36)^2}{32}$$

$$1600 = \frac{2(32)(v_0 \sin 36)^2 - 32(v_0 \sin 36)^2}{2(32)^2}$$

$$\frac{3200(32)^2}{32} = (v_0 \sin 36)^2$$

$$v_0 = 544.41 \approx 544 \text{ ft/s}$$

$$\boxed{v_0 = 544 \text{ ft/s}}$$

7. $r(t) = \langle t^2+1, t^3, 0 \rangle$

$r'(t) = \langle 2t, 3t^2, 0 \rangle$ $|r'(t)| = \sqrt{4t^2 + 9t^4}$

$r''(t) = \langle 2, 6t, 0 \rangle$

$$r'(t) \times r''(t) = \begin{vmatrix} 3t^2 & 0 \\ 6t & 0 \end{vmatrix} i - \begin{vmatrix} 2t & 0 \\ 2 & 0 \end{vmatrix} j + \begin{vmatrix} 2t & 3t^2 \\ 2 & 6t \end{vmatrix} k$$

$\langle 0, 0, 12t^2 - 6t^2 \rangle$

$\langle 0, 0, 6t^2 \rangle$

$$a_T = \frac{r'(t) \cdot r''(t)}{|r'(t)|} = \frac{4t + 18t^3}{t\sqrt{4+9t^2}} = \boxed{\frac{4+18t^2}{\sqrt{4+9t^2}} = a_T}$$

$$a_N = \frac{|r'(t) \times r''(t)|}{|r'(t)|} = \frac{|\langle 0, 0, 6t^2 \rangle|}{t\sqrt{4+9t^2}} = \frac{\sqrt{36t^4}}{t\sqrt{4+9t^2}} = \frac{6t^2}{t\sqrt{4+9t^2}}$$

$$\boxed{a_N = \frac{6t}{\sqrt{4+9t^2}}}$$

39. $r(t) = \langle \cos t, \sin t, t \rangle$

$r'(t) = \langle -\sin t, \cos t, 1 \rangle \qquad |r'(t)| = \sqrt{1 + \sin^2 t + \cos^2 t} = \sqrt{2}$

$r''(t) = \langle -\cos t, -\sin t, 0 \rangle$

$$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ -\sin t & \cos t & 1 \\ -\cos t & -\sin t & 0 \end{vmatrix} = \begin{vmatrix} \cos t & 1 \\ -\sin t & 0 \end{vmatrix} i - \begin{vmatrix} -\sin t & 1 \\ -\cos t & 0 \end{vmatrix} j + \begin{vmatrix} -\sin t & \cos t \\ -\cos t & -\sin t \end{vmatrix} k$$

$\langle \sin t, -\cos t, 1 \rangle$

$r'(t) \cdot r''(t) = \sin t \cos t - \sin t \cos t + 0 = 0$

$$a_T = \frac{r'(t) \cdot r''(t)}{|r'(t)|} = \frac{0}{\sqrt{2}} = \boxed{0}$$

$$a_N = \frac{|r'(t) \times r''(t)|}{|r'(t)|} = \frac{\sqrt{\sin^2 t + \cos^2 t + 1}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}} = \boxed{1}$$

41. $r(t) = \langle \ln t, t^2 + 3t, 4\sqrt{t} \rangle \quad (0, 4, 4)$

$\ln t = 0$

$t = 1$

$\langle \ln(1), 1^2 + 3, 4\sqrt{1} \rangle = (0, 4, 4)$

$r'(t) = \langle \frac{1}{t}, 2t + 3, \frac{2}{\sqrt{t}} \rangle$

$r''(t) = \langle \frac{-1}{t^2}, 2, \frac{-1}{\sqrt{t^3}} \rangle \qquad |r'(t)| = \sqrt{\frac{1}{t^2} + (2t+3)^2 + \frac{4}{t}}$

$r'(t) \cdot r''(t) = \frac{-1}{t^3} + 4t + 6 - \frac{2}{t^2}$

$$r'(t) \times r''(t) = \begin{vmatrix} 2t+3 & \frac{2}{\sqrt{t}} \\ 2 & \frac{-1}{\sqrt{t^3}} \end{vmatrix} i - \begin{vmatrix} \frac{1}{t} & \frac{2}{\sqrt{t}} \\ \frac{-1}{t^2} & \frac{-1}{\sqrt{t^3}} \end{vmatrix} j + \begin{vmatrix} \frac{1}{t} & 2t+3 \\ \frac{-1}{t^2} & 2 \end{vmatrix} k$$

$\langle \frac{-2t-3}{\sqrt{t^3}} - \frac{4}{\sqrt{t}}, \frac{-1}{\sqrt{t^5}} + \frac{2}{\sqrt{t^5}}, \frac{2}{t} + \frac{2t+3}{t^2} \rangle$

$r'(1) \times r''(1) = \langle -5 - 4, -1 + 2, 2 + 5 \rangle$

$\langle -9, 1, 7 \rangle$

$|\langle -9, 1, 7 \rangle| = \sqrt{81 + 1 + 49} = \sqrt{131}$

$$a_T = \frac{4t + 6 - \frac{1}{t^3} - \frac{2}{t^2}}{\sqrt{\frac{1}{t^2} + (2t+3)^2 + \frac{4}{t}}} = \frac{4 + 6 - 1 - 2}{\sqrt{1 + 25 + 4}} = \frac{7}{\sqrt{30}}$$

$$a_N = \frac{\sqrt{131}}{\sqrt{30}} = \sqrt{\frac{131}{30}}$$