29) a.) $\phi \in [0, \frac{\pi}{3}]$ $\rho \in [0, 4\cos\phi]$ $\theta \in [0, 2\pi]$

$$\int_0^{2\pi}\int_0^{\frac{\pi}{3}}\int_0^{4\cos\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= 2\pi \int_0^{\frac{\pi}{3}}\int_0^{4\cos\phi} \rho^2 \sin\phi \, d\rho \, d\phi$$

$$= 2\pi \int_0^{\frac{\pi}{3}} \left[\frac{1}{3} 64 \cos^3\phi \sin\phi \right] d\phi$$

$$= \frac{128\pi}{3} \int_0^{\frac{\pi}{3}} \cos^3\phi \sin\phi \, d\phi$$

$u = \cos\phi$
$du = -\sin\phi \, d\phi$
$-du = \sin\phi \, d\phi$

$$= -\frac{128\pi}{3} \int_1^{\frac{1}{2}} u^3 \, du$$

$$= \frac{128\pi}{3} \left[\frac{1}{4} - \frac{1}{64} \right] = \frac{128\pi}{3} \cdot \frac{15}{64} = \frac{30\pi}{3} = \boxed{10\pi}$$

b.) $\bar{x} = \frac{M_{yz}}{V}$ $\bar{y} = \frac{M_{xz}}{V}$ $\bar{z} = \frac{M_{xy}}{V}$

Due to symmetry $M_{yz} = M_{xz} = 0$

$z = \rho\cos\phi$

$$M_{xy} = \int_0^{2\pi}\int_0^{\frac{\pi}{3}}\int_0^{4\cos\phi} \rho^3 \cos\phi \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= 2\pi \int_0^{\frac{\pi}{3}}\int_0^{4\cos\phi} \rho^3 \cos\phi \sin\phi \, d\rho \, d\phi = 2\pi \int_0^{\frac{\pi}{3}} 64 \cos^5\phi \sin\phi \, d\phi$$

$$= 128\pi \left(-\frac{\cos^6\phi}{6} \Big|_0^{\frac{\pi}{3}} \right) = 21\pi \qquad z = \frac{21\pi}{10\pi} = 2.1\pi$$

$\boxed{\text{centroid: } (0, 0, 2.1)}$