33) a) $\rho \in [0,a]$ $\theta \in [0, 2\pi]$ $\phi \in [0, \frac{\pi}{2}]$

$\bar{z} = \frac{M_{xy}}{M}$ $M = k\int dV$ $k =$ density constant

$M = k\int_0^{2\pi}\int_0^{\pi/2}\int_0^a \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta = 2\pi k \int_0^{\frac{\pi}{2}} \sin\phi \int_0^a \rho^2 d\rho$

$M = \frac{2\pi k a^3}{3}$

$M_{xy} = k\int_0^{2\pi}\int_0^{\pi/2}\int_0^a z\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta =$

$= k\int_0^{2\pi}\int_0^{\pi/2}\int_0^a \rho^3 \cos\phi \sin\phi \, d\rho \, d\phi \, d\theta = 2\pi k\left(\frac{1}{4}a^4\right)\int_0^{\frac{\pi}{2}} \frac{\sin 2\phi}{2} d\phi$

$= 2\pi k\left(\frac{a^4}{4}\right)\left(\frac{1}{2}\right) = \frac{k a^4 \pi}{4}$

$\bar{z} = \frac{\frac{\cancel{k} a^4 \cancel{\pi}}{4}}{\frac{2\pi k a^3}{3}} \cdot \frac{3}{2\pi k a^3} = \frac{3a}{8}$ $\boxed{\text{centroid: } (0, 0, \frac{3a}{8})}$

b) $I_x = \int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^a (x^2+y^2)\, k\, \rho(x,y)\, dV = k\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^a (\rho^2 \sin^2\phi) \cdot 1 \cdot \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

$= 2\pi k \cdot \int_0^{\frac{\pi}{2}}\int_0^a \rho^4 \sin^3\phi \, d\rho \, d\phi = 2\pi k\left(\frac{a^5}{5}\right)\int_0^{\frac{\pi}{2}} \sin^3\phi \, d\phi$

$= 2\pi k \frac{a^5}{5}\int_0^{\frac{\pi}{2}} (1 - \cos^2\phi)\sin\phi \, d\phi = 2\pi k \frac{a^5}{5}\int_0^1 1 - u^2 du$

$u = \cos\phi$
$du = -\sin\phi \, d\phi$
$-du = \sin\phi \, d\phi$

$= 2\pi k \frac{a^5}{5}\left(\frac{2}{3}\right)$

$\boxed{I_x = \frac{4\pi k a^5}{15}}$ $k =$ density constant