③ $\oint_C xy\,dx + x^2y^3\,dy$ $(0,0)$ $(1,0)$ $(1,2)$

a.) $\int_{C_1} 0 + 0 = 0$

$$\int_{C_2} xy\,dx + x^2y^3\,dy = \int_{C_2} 1\cdot y(0) + y^3\,dy = \int_0^2 y^3\,dy = \frac{2^4}{4} = 4$$

$y = 2x$
$dy = 2dx$

$$\int_{C_3} xy\,dx + x^2y^3\,dy = \int_1^0 2x^2\,dx + x^2(8x^3)(2dx) = \int_0^0 2x^2 + 16x^5\,dx$$

$$= \left[\frac{2x^3}{3} + \frac{8x^6}{3}\right]_1^0 = -\frac{2}{3} - \frac{8}{3} = -\frac{16}{3}$$

$$4 - \frac{16}{3} = \boxed{\frac{2}{3}}$$

b.) $P = xy \quad Q = x^2y^3$

$Q_x = 2xy^3 \quad P_y = x$

$$\int_0^1 \int_0^{2x} 2xy^3 - x\,dy\,dx = \int_0^1 \left[\frac{x}{2}y^4 - xy\right]_0^{2x} dx = \int_0^1 8x^5 - 2x^2\,dx$$

$$= \left[\frac{4}{3}x^6 - \frac{2}{3}x^3\right]_0^1 = \frac{4}{3} - \frac{2}{3} = \boxed{\frac{2}{3}}$$