

$$(19) F(x, y) = x^2 i - x^2 j \quad r(t) = t^3 i + t^2 j \quad t \in [0, 1]$$

$$F(r(t)) = t^7 i - t^6 j$$

$$r'(t) = \langle 3t^2, 2t \rangle dt$$

$$|r'(t)| = \sqrt{9t^4 + 4t^2} dt$$

$$\int_C F dr = \int_0^1 (t^7 i - t^6 j) \cdot (3t^2 i + 2t j) dt$$

$$= \int_0^1 3t^9 - 2t^7 dt = \left[\frac{3t^{10}}{10} - \frac{1}{4} t^8 \right]_0^1 = \frac{3}{10} - \frac{1}{4} = \boxed{\frac{1}{20}}$$

$$(21) F(x, y, z) = \langle \sin x, \cos y, xz \rangle \quad r(t) = \langle t^3, -t^2, t \rangle \quad t \in [0, 1]$$

$$r'(t) = \langle 3t^2, -2t, 1 \rangle dt$$

$$\int_0^1 \langle \sin t^3, \cos(-t^2), t^4 \rangle \cdot \langle 3t^2, -2t, 1 \rangle dt$$

$$= \int_0^1 3t^2 \sin(t^3) - 2t \cos(t^2) + t^4 dt = \left[-\cos(t^3) - \sin(t^2) + \frac{t^5}{5} \right]_0^1$$

$$= (-\cos(1) - \sin(1) + \frac{1}{5}) - (-1) = \boxed{\frac{6}{5} - \cos(1) - \sin(1)}$$