16.3 homework   3,7,11,15,19,23,25,29,31,33

③ $F(x,y) = \langle xy+y^2, x^2+2xy \rangle$

$P_y = x+2y$ $\qquad$ $Q_x = 2x+2y$ $\qquad$ $P_y \neq Q_x$ | Non conservative |

⑦ $F(x,y) = \langle ye^x + \sin y, e^x + x\cos y \rangle$

$P_y = e^x + \cos y$ $\qquad$ $Q_x = e^x + \cos y$ $\qquad$ $P_y = Q_x$ conservative

$f_x = ye^x + \sin y$ $\qquad$ $f_y = e^x + x\cos y$

$\int f_x \, dx = ye^x + x\sin y + g(y)$ $\qquad$ $f_y = e^x + x\cos y + g'(y)$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad g'(y) = 0$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad g(y) = k$

| $f(x,y) = ye^x + x\sin y + k$ |

⑪ a.) $F = \langle 2xy, x^2 \rangle$ is a conservative vector field. As $P_y = Q_x = 2x$.

$P_y = \dfrac{\partial 2xy}{\partial y} = 2x$ $\qquad$ $Q_x = \dfrac{\partial x^2}{\partial x} = 2x$ ∴ QED

The line integral of a conservative vector field is only dependent on the endpoints. All 3 paths have the same endpoints. Therefore $\int_C F \, dr$ is the same for all paths.

b.) $f_x = 2xy$ $\qquad$ $f(x,y) = x^2y + g(y)$ $\qquad$ $f_y = x^2 + g'(y) = x^2$ $\qquad$ $g'(y) = 0$ $\qquad$ $g(y) = C$

$f(x,y) = x^2y + C$ $\qquad\qquad$ $\int_C F \, dr = f(3,2) - f(1,2) = (18+C) - (2+C) = \boxed{16}$