

$$\uparrow \text{cont'd. } \frac{2}{5} \int_0^{\frac{\pi}{2}} \sin \theta \cos^2 \theta d\theta = \frac{2}{5} \int_0^1 v^2 dv = \frac{2}{15}$$

$$v = \cos \theta$$

$$dv = -\sin \theta d\theta$$

$$-dv = \sin \theta d\theta$$

$$-\frac{1}{2} \int_0^{\frac{\pi}{2}} \sin \theta \cos \theta d\theta = -\frac{1}{2} \int_0^1 v dv = -\frac{1}{4}$$

$$v = \sin \theta$$

$$dv = \cos \theta d\theta$$

$$\begin{aligned} \frac{2}{5} \int_0^{\frac{\pi}{2}} \sin^3 \theta d\theta &= \frac{1}{10} \int_0^{\frac{\pi}{2}} 4 \sin^3 \theta d\theta = \frac{1}{10} \int_0^{\frac{\pi}{2}} 3 \sin \theta - \sin^3 \theta d\theta \\ &= \frac{1}{10} \left[ -3 \cos \theta + \frac{\cos^3 \theta}{3} \right]_0^{\frac{\pi}{2}} = \end{aligned}$$

$$\frac{1}{10} \left[ 3 + \frac{1}{3} \right] = \frac{10}{3} \left( \frac{1}{10} \right) = \frac{1}{3}$$

$$-\int_0^{\frac{\pi}{2}} \frac{2}{3} \sin \theta + \frac{1}{3} \cos \theta d\theta = - \left[ -\frac{2}{3} \cos \theta + \frac{1}{3} \sin \theta \right]_0^{\frac{\pi}{2}} =$$

$$- \left[ 0 + \frac{1}{3} + \frac{2}{3} - 0 \right] = -1$$

$$\frac{2}{15} - \frac{1}{4} + \frac{1}{3} - 1 = \frac{2}{15} + \frac{1}{3} - \frac{5}{4} = \frac{7}{15} - \frac{5}{4} = \frac{28-45}{60}$$

$$= \boxed{\frac{-17}{60}}$$