

19. 1st octant

$$\theta \in [0, \frac{\pi}{2}]$$

$$z = 4 - x^2 - y^2 = 4 - r^2 \quad r^2 = 4$$

$$r = 2$$

$$z \in [0, 4 - r^2] \quad r \in [0, 2]$$

$$\int_0^{\frac{\pi}{2}} \int_0^2 \int_0^{4-r^2} (x+y+z) r \, dz \, dr \, d\theta = \int_0^{\frac{\pi}{2}} \int_0^2 \int_0^{4-r^2} r^2 \cos\theta + r^2 \sin\theta + rz \, dz \, dr \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \int_0^2 \left[r^2 z (\cos\theta + \sin\theta) + \frac{1}{2} r z^2 \right]_0^{4-r^2} dr \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \int_0^2 r^2 (4-r^2) (\cos\theta + \sin\theta) + \frac{1}{2} r (4-r^2)^2 \, dr \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \int_0^2 (4r^2 - r^4) (\cos\theta + \sin\theta) + 8r - 4r^3 + \frac{1}{2} r^5 \, dr \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left[\left(\frac{4}{3} r^3 - \frac{1}{5} r^5 \right) (\cos\theta + \sin\theta) + 4r^2 - r^4 + \frac{1}{12} r^6 \right]_0^2 d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{64}{15} (\cos\theta + \sin\theta) + \frac{16}{3} d\theta$$

$$= \frac{64}{15} (\sin\theta - \cos\theta) + \frac{16}{3} \theta \Big|_0^{\frac{\pi}{2}} = \frac{64}{15} (1) + \frac{16\pi}{6} + \frac{64}{15} + 0$$

$$= \boxed{\frac{128}{15} + \frac{8\pi}{3}}$$