15.6 homework $\qquad$ 3, 7, 9, 13, 17, 19, 21, 25, 29, 31, 35

(3) $\int_0^2 \int_0^{z^2} \int_0^{y-z} (2x-y)\, dx\, dy\, dz$

$\int_0^2 \int_0^{z^2} \left(x^2 - yx\right)\Big|_0^{y-z}\, dy\, dz = \int_0^2 \int_0^{z^2} \left((y-z)^2 - y^2 + zy\right)\, dy\, dz$

$= \int_0^2 \int_0^{z^2} y^2 - 2yz + z^2 - y^2 + zy\, dy\, dz = \int_0^2 \int_0^{z^2} z^2 - yz\, dy\, dz$

$= \int_0^2 \left(z^2 y - \tfrac{1}{2} y^2 z\right)\Big|_0^{z^2}\, dz = \int_0^2 z^4 - \tfrac{1}{2} z^5\, dz = \left[\tfrac{1}{5} z^5 - \tfrac{1}{12} z^6\right] = \tfrac{1}{5} 2^5 - \tfrac{1}{6} 2^5$

$= 32\left(\tfrac{1}{5} - \tfrac{1}{6}\right) = \dfrac{32}{30} = \boxed{\dfrac{16}{15}}$

(7) $\int_0^\pi \int_0^1 \int_0^{\sqrt{1-z^2}} z \sin x\, dy\, dz\, dx = \int_0^\pi \int_0^1 z\sqrt{1-z^2} \sin x\, dz\, dx$

$= \int_0^\pi \sin x\, dx \int_0^1 z\sqrt{1-z^2}\, dz \qquad = -1\int_1^0 \sqrt{u}\, dv = \int_0^1 \sqrt{u}\, dv$

$-\cos x\Big|_0^\pi = 2$

$u = 1-z^2$
$du = -2z\, dz$
$-\tfrac{1}{2} du = z\, dz$

$= \boxed{\dfrac{2}{3}}$

(9) $E = \{(x,y,z) \mid x \in [0,3] \wedge y \in [0,x] \wedge z \in [x-y, x+y]\}$

$\int_0^3 \int_0^x \int_{x-y}^{x+y} y\, dz\, dy\, dx = \int_0^3 \int_0^x y(x+y - x+y)\, dy\, dx = \int_0^3 \int_0^x 2y^2\, dy\, dx$

$= \int_0^3 \tfrac{2}{3} x^3\, dx = \tfrac{1}{6} 3^4 = \dfrac{81}{6} = \boxed{\dfrac{27}{2}}$