(27) $F(x,y,z) = \langle 0, y, -z \rangle \quad y = x^2 + z^2 \quad y \in [0,1] \quad x^2 + z^2 \le 1 \quad y = 1$

Parabolid

$h = \langle r\cos\theta, r^2, r\sin\theta \rangle$

$y = r^2 \le 1 \quad x = r\cos\theta$

$z = r\sin\theta$

$\bar{h}_r = \langle \cos\theta, 2r, \sin\theta \rangle$

$\bar{h}_\theta = \langle -r\sin\theta, 0, r\cos\theta \rangle$

$$a = \bar{h}_r \times \bar{h}_\theta = \left\langle \begin{vmatrix} 2r & \sin\theta \\ 0 & r\cos\theta \end{vmatrix}, -\begin{vmatrix} \cos\theta & \sin\theta \\ -r\sin\theta & r\cos\theta \end{vmatrix}, \begin{vmatrix} \cos\theta & 2r \\ -r\sin\theta & 0 \end{vmatrix} \right\rangle = \langle 2r^2\cos\theta, -r, 2r^2\sin\theta \rangle$$

$$a \cdot F(h(r,\theta)) = \langle 0, r^2, -r\sin\theta \rangle \cdot \langle 2r^2\cos\theta, -r, 2r^2\sin\theta \rangle =$$
$$= -r^3 - 2r^3\sin^2\theta$$

$$-\int_0^{2\pi}\int_0^1 r^3 + 2r^3\sin^2\theta \, dr \, d\theta = -\int_0^{2\pi}\left[\frac{r^4}{4} + \frac{1}{2}r^4\sin^2\theta\right]_0^1 = -\int_0^{2\pi}\frac{1}{4} + \frac{1}{2}\sin^2\theta \, d\theta$$

$$= -\left[\frac{\theta}{4} + \frac{\theta}{4} - \frac{\sin 2\theta}{4}\right]_0^{2\pi} = -\left[\frac{\pi}{2} + \frac{\pi}{2}\right] = -\pi = S_1$$

S_2

$x = r\cos t \quad y = 1 \quad z = r\sin t$

$\bar{n} = \langle r\cos t, 1, r\sin t \rangle$

$\bar{n}_r = \langle \cos t, 0, \sin t \rangle \quad n_t = \langle -r\sin t, 0, r\cos t \rangle \quad n_r \times n_t = \langle 0, r, 0 \rangle$

$$= \int_0^{2\pi}\int_0^1 r \, dr \, dt = 2\pi\int_0^1 r \, dr = 2\pi\left[\frac{1}{2}\right] = \pi$$

$$S = S_1 + S_2 = -\pi + \pi = \boxed{0}$$