

$$(45) \quad z = x - y \quad x^2 + y^2 = 1 \quad r^2 = 1 \quad r \in [0, 1] \quad \theta \in [0, 2\pi]$$

$$\frac{\partial z}{\partial x} = -y \quad \frac{\partial z}{\partial y} = -x \quad \sqrt{1+x^2+y^2} = r\sqrt{1+r^2} \quad dr \, d\theta$$

$$A(s) = \int_0^{2\pi} d\theta \int_0^1 r \sqrt{1+r^2} \, dr = 2\pi \int_0^1 r \sqrt{1+r^2} \, dr$$

$$= \pi \int_1^2 \sqrt{u} \, du = \pi \left[\frac{2}{3} u^{3/2} \right]_1^2 =$$

$$u = 1+r^2 \\ du = 2r \, dr \\ \frac{1}{2} du = r \, dr$$

$$\frac{2\pi}{3} [2^{3/2} - 1] = \boxed{\frac{2\pi}{3} (2\sqrt{2} - 1)}$$

$$(47) \quad y = x^2 + z^2 \quad x^2 + z^2 = 4 \quad r^2 = 4 \quad r = 2 \quad r \in [0, 2] \quad \theta \in [0, 2\pi]$$

$$\frac{\partial y}{\partial x} = 2x \quad \frac{\partial y}{\partial z} = 2z \quad \sqrt{1+4x^2+4z^2} = \sqrt{1+4r^2}$$

$$A(s) = \int_0^{2\pi} d\theta \int_0^2 r \sqrt{1+4r^2} \, dr = 2\pi \int_0^2 r \sqrt{1+4r^2} \, dr$$

$$= \frac{\pi}{4} \int_1^{65} \sqrt{u} \, du = \frac{\pi}{4} \left[\frac{2}{3} (65)^{3/2} - \frac{2}{3} (1) \right]$$

$$u = 1+4r^2 \\ du = 8r \, dr \\ \frac{1}{8} du = r \, dr$$

$$= \frac{2\pi}{12} ((65)^{3/2} - 1) = \frac{\pi}{6} (65^{3/2} - 1)$$

$$= \boxed{\frac{\pi}{6} (65\sqrt{65} - 1)}$$