

$$(49) \quad x = u^2 \quad y = uv \quad z = \frac{1}{2}v^2 \quad u \in [0, 1] \quad v \in [0, 2]$$

$$r(u, v) = \langle u^2, uv, \frac{1}{2}v^2 \rangle$$

$$r_u = \langle 2u, v, 0 \rangle \quad r_v = \langle 0, u, v \rangle$$

$$r_u \times r_v = \begin{vmatrix} i & j & k \\ 2u & v & 0 \\ 0 & u & v \end{vmatrix} = \langle \begin{vmatrix} v & 0 \\ u & v \end{vmatrix}, -\begin{vmatrix} 2u & 0 \\ 0 & v \end{vmatrix}, \begin{vmatrix} 2u & v \\ 0 & u \end{vmatrix} \rangle$$

$$= \langle v^2, -2u^2, 2uv \rangle$$

$$|r_u \times r_v| = \sqrt{v^4 + 4u^2v^2 + 4u^2} = \sqrt{(v^2 + 2u^2)^2} = v^2 + 2u^2$$

$$A(S) = \iint_D |r_u \times r_v| dA = \int_0^1 \int_0^2 (v^2 + 2u^2) dv du$$

$$= \int_0^1 \left[\frac{1}{3}v^3 + 2u^2v \right]_0^2 du = \int_0^1 \left(\frac{8}{3} + 4u^2 \right) du$$

$$= \left[\frac{8}{3}u + \frac{4}{3}u^3 \right]_0^1 = \frac{8}{3} + \frac{4}{3} = \frac{12}{3} = \boxed{4}$$