

15) $\iint_D y \, dA$ $y = x-2$ $x = y^2$
 $x = y+2$ $y = \pm\sqrt{x}$

$y = y^2 - 2$
 $0 = y^2 - y - 2$
 $0 = (y-2)(y+1)$
 $y = 2$ $y = -1$
 $x = 4$ $x = 1$

$\int_{-1}^2 \int_{y^2}^{y+2} y \, dx \, dy$

$= \int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} y \, dy \, dx + \int_1^4 \int_{x-2}^{\sqrt{x}} y \, dy \, dx$

$\int_{-1}^2 \int_{y^2}^{y+2} y \, dx \, dy$
 $= \int_{-1}^2 \left[xy \right]_{y^2}^{y+2} dy = \int_{-1}^2 (y^2 + 2y - y^3) dy$

$\left[\frac{1}{3} y^3 + y^2 - \frac{1}{4} y^4 \right]_{-1}^2 = \frac{8}{3} + 4 - \frac{1}{4} - \left(-\frac{1}{3} + 1 - \frac{1}{4} \right) = \frac{8}{3} + 4 - \frac{1}{4} + \frac{1}{3} - 1 + \frac{1}{4} = \frac{11}{3}$

19) $\iint_D y^2 \, dA$ $(0,1)$ $(1,2)$ $(4,1)$
 $y = x+1$ $x = y-1$ $y \in [1,2]$ $x \in [y-1, -3y+7]$

$\int_1^2 \int_{y-1}^{-3y+7} y^2 \, dx \, dy = \int_1^2 \left[y^2 x \right]_{y-1}^{-3y+7} dy = \int_1^2 y^2 (-3y+7 - y+1) dy = \int_1^2 (y^2 (-4y+8)) dy$
 $= \int_1^2 (-4y^3 + 8y^2) dy = \left[-y^4 + \frac{8}{3} y^3 \right]_1^2 = \left(-16 + \frac{64}{3} \right) - \left(-1 + \frac{8}{3} \right) = -15 + \frac{56}{3} = \frac{11}{3}$

23) $z = 3x + 2y$ $y = x^2$ $x = y^2$
 $x = \sqrt{y}$ $0 = y^4 - y$
 $0 = y(y^3 - 1)$
 $y = 0$ $y = 1$

$\int_0^1 \int_{y^2}^{\sqrt{y}} (3x + 2y) \, dx \, dy$

$\int_0^1 \left[\frac{3}{2} x^2 + 2yx \right]_{y^2}^{\sqrt{y}} dy = \int_0^1 \left(\frac{3}{2} y + 2y^{3/2} - \frac{3}{2} y^4 - 2y^{5/2} \right) dy$
 $\frac{3}{4} + \frac{4}{5} - \frac{3}{10} - \frac{1}{2} = \frac{3}{4}$