

$$(11) \quad x^2 + y^2 + z^2 = a^2 \quad x^2 + y^2 = ax$$

$$z = a^2 - x^2 - y^2$$

$$\frac{\partial z}{\partial x} = \frac{-2x}{2\sqrt{a^2 - x^2 - y^2}} = \frac{-x}{z} \quad \frac{\partial z}{\partial y} = \frac{-y}{z}$$

$$r^2 = a \cos \theta$$

$$r = a \cos \theta$$

$$r \in [0, a \cos \theta]$$

$$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \quad \leftarrow \text{range for } a \cos \theta$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{a \cos \theta} \frac{ar}{\sqrt{a^2 - r^2}} dr d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[-a \sqrt{a^2 - r^2} \right]_0^{a \cos \theta} d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -a^2 \sin \theta + a^2 d\theta$$

$$= 2 \left[a^2 \theta + a^2 \cos \theta \right]_0^{\frac{\pi}{2}} = 2 \left(\frac{\pi}{2} a^2 - a^2 \right) = \pi a^2 - 2a^2 = \boxed{a^2(\pi - 2)}$$

$$(13) \quad z = \frac{1}{1+x^2+y^2}$$

$$\frac{\partial z}{\partial x} = \frac{-2x}{(1+x^2+y^2)^2} \quad \frac{\partial z}{\partial y} = \frac{-2y}{(1+x^2+y^2)^2} \quad \frac{-2r \cos \theta}{(1+r^2)^2} \quad \frac{-2r \sin \theta}{(1+r^2)^2}$$

$$x^2 + y^2 \leq 1$$

$$r \leq 1 \quad r \in [0, 1]$$

$$(1,0) \quad 1 = \cos \theta \quad 1 = \sin \theta \quad \theta = 0, \quad \theta = \left(\frac{\pi}{2}, -\frac{\pi}{2}\right)$$

$$1 + \frac{4r^2}{(1+r^2)^2}$$

$$\theta \in \left[0, \frac{3\pi}{2}\right]$$

$$SA = \int_0^{\frac{3\pi}{2}} \int_0^1 r \sqrt{1 + \frac{4r^2}{(1+r^2)^2}} dr d\theta = 3.62579 \approx 3.6258$$

$$\boxed{3.6258}$$