

$$\textcircled{17} I_x = \iint_D y^2 p(x,y) dA = \int_1^3 \int_1^4 k y^4 dy dx = 2k \int_1^4 y^4 dy = 2k \left[\frac{1}{5} y^5 \right]_1^4$$

$$= \frac{2k}{5} (4^5 - 1) = \frac{2046}{5} k = \boxed{409.2k = I_x}$$

$$I_y = \iint_D x^2 p(x,y) dA = \int_1^3 \int_1^4 k x^2 y^2 dy dx = k \int_1^3 x^2 dx \int_1^4 y^2 dy$$

$$= k \left(9 - \frac{1}{3} \right) \left(\frac{64}{3} - \frac{1}{3} \right) = k \left(9 - \frac{1}{3} \right) \left(\frac{63}{3} \right) = k \left(9 - \frac{1}{3} \right) (21) = \boxed{182k = I_y}$$

$$I_o = \iint_D (x^2 + y^2) p(x,y) dA = k \int_1^3 \int_1^4 (x^2 + y^2) y^2 dy dx = k \int_1^3 \left[\frac{1}{3} x^2 y^3 + \frac{1}{5} y^5 \right]_1^4 dx$$

$$= k \int_1^3 \left(\frac{63}{3} x^2 + \frac{1}{5} (4^5 - 1) \right) dx = k \int_1^3 \left(21x^2 + \frac{1023}{5} \right) dx = 9(26) + \frac{1023}{5}(2)$$

$$I_o = I_x + I_y = \int_1^3 \int_1^4 k y^4 dy dx + \int_1^3 \int_1^4 k x^2 y^2 dy dx = (409.2 + 182)k$$

$$= \boxed{591.2k = I_o}$$

$$\textcircled{19} I_x = \int_0^a \int_0^{a-x} y^2 (x^2 + y^2) dy dx = \int_0^a \left[\frac{1}{3} y^3 x^2 + \frac{1}{5} y^5 \right]_0^{a-x} dx = \int_0^a \left(\frac{1}{3} (a-x)^3 x^2 + \frac{1}{5} (a-x)^5 \right) dx$$

$$= k \int_0^a \left(a^3 x^2 - 3a^2 x^3 + \frac{3a^2 x^2}{3} - \frac{3a^2 x^3}{3} + a x^4 \right) dx = k \left[\frac{a^3 x^3}{3} - \frac{3a^2 x^4}{4} + \frac{a^2 x^3}{3} - \frac{3a^2 x^4}{4} + \frac{a^2 x^5}{5} \right]_0^a$$

$$= k a^6 \left(\frac{1}{3} - \frac{1}{4} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \right) = k a^6 \left(\frac{7}{180} \right) = \boxed{\frac{7ka^6}{180} = I_x}$$

$I_x = I_y$ if symmetric across $y=x$

$$I_y = \frac{7ka^6}{180}$$

$$I_o = I_x + I_y = 2 \left(\frac{7ka^6}{180} \right) =$$

$$\boxed{\frac{7ka^6}{90} = I_o}$$

if vertex is symmetric and positive on $y=x$.