

$$(21) \theta \in [0, 2\pi]$$

$$x^2 + y^2 = 1$$

$$z^2 = x^2 + y^2 = r^2$$

$$r^2 = 1$$

$$z = 2r \quad z \in [0, 2r]$$

$$r = 1 \quad r \in [0, 1]$$

$$\int_0^{2\pi} \int_0^1 \int_0^{2r} r^3 \cos^2 \theta \, dz \, dr \, d\theta = \int_0^{2\pi} \int_0^1 2r^4 \cos^2 \theta \, dr \, d\theta$$

$$= \int_0^{2\pi} \cos^2 \theta \, d\theta \int_0^1 2r^4 \, dr = \left[\frac{2}{5} \right] \int_0^{2\pi} \cos^2 \theta \, d\theta = \frac{2}{5} \int_0^{2\pi} \frac{1}{2} + \frac{1}{2} \cos 2\theta \, d\theta$$

$$= \frac{2}{5} \left[\frac{\theta}{2} + \frac{1}{4} \sin 2\theta \right]_0^{2\pi} = \frac{2}{5} [\pi] = \boxed{\frac{2\pi}{5}}$$

$$(23) \quad z = \sqrt{x^2 + y^2} \quad z = \sqrt{2 - x^2 - y^2}$$

$$z = r \quad z = \sqrt{2 - r^2}$$

$$z \in [r, \sqrt{2 - r^2}]$$

$$r^2 = 2 - r^2 \quad \theta \in [0, 2\pi]$$

$$2r^2 = 2$$

$$r^2 = 1$$

$$r = 1$$

$$r \in [0, 1]$$

$$\int_0^{2\pi} \int_0^1 \int_r^{\sqrt{2-r^2}} r \, dz \, dr \, d\theta = \int_0^{2\pi} d\theta \int_0^1 r [\sqrt{2-r^2} - r^2] \, dr$$

$$= 2\pi \int_0^1 r \sqrt{2-r^2} - r^2 \, dr = 2\pi \left(\int_0^1 r \sqrt{2-r^2} \, dr - \int_0^1 r^2 \, dr \right) = 2\pi \left(\int_2^1 -\frac{1}{2} \sqrt{u} \, du - \frac{1}{3} \right)$$

$$u = 2 - r^2 \\ du = -2r \, dr \\ r = -\frac{1}{2} du$$

$$= 2\pi \left[\frac{1}{3} (2)^{3/2} - \frac{1}{3} - \frac{1}{3} \right]$$

$$= \frac{2\pi}{3} (2\sqrt{2} - 2)$$

$$= \boxed{\frac{4\pi}{3} (\sqrt{2} - 1)}$$