16.4 homework 1, 3, 5, 9, 13, 17, 19, 21

① $\oint_C y^2\,dx + x^2 y\,dy$ (0,0) (5,0) (5,4) (0,4)

(a) $\int_{C_1} y^2\,dx + x^2 y\,dy = \int_{C_1} 0^2\,dx + 0 = 0$

$\int_{C_2} y^2\,dx + x^2 y\,dy = \int_{C_2} 5^2 y\,dy = \int_0^4 25y\,dy = \frac{25}{2}(16) = 200$

$\int_{C_3} y^2\,dx + x^2 y\,dy \quad \int_{C_3} 16\,dx + 0 = \int_5^0 16\,dx = 16(-5) = -80$

$200 - 80 = \boxed{120}$

(b) $\oint_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA \quad D = \{(x,y) \mid x \in [0,5] \wedge y \in [0,4]\}$

$= \int_0^4 \int_0^5 \frac{\partial x^2 y}{\partial x} - \frac{\partial y^2}{\partial y}\,dx\,dy = \int_0^4 \int_0^5 2xy - 2y\,dx\,dy$

$= 2\int_0^4 y\,dy \int_0^5 (x-1)\,dx = 2\left(\frac{1}{2}(16)\right)\left(\frac{1}{2}(25) - 10\right)$

$= 8(15) = \boxed{120}$