⑤ $f(x,y,z) = x^3 y z^2 + 2yz$

$$\frac{\partial f}{\partial x} = 3x^2 y z^2 \qquad \frac{\partial f}{\partial y} = x^3 z^2 + 2z \qquad \frac{\partial f}{\partial z} = 2x^3 yz + 2y$$

$$\boxed{f_x = 3x^2 y z^2 \qquad f_y = x^3 z^2 + 2z \qquad f_z = 2x^3 yz + 2y}$$

⑥ $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} = \frac{0}{0} \qquad y=mx \qquad x=my$

$$\lim_{x\to 0} \frac{x^2+m^2x^2}{\sqrt{x^2+m^2x^2+1}-1} = \lim_{x\to 0} \frac{(x^2+m^2x^2)(\sqrt{x^2+m^2x^2+1}+1)}{(\sqrt{x^2+m^2x^2+1}-1)(\sqrt{x^2+m^2x^2+1}+1)} = \lim_{x\to 0}\frac{(x^2+m^2x^2)(\sqrt{x^2+m^2x^2+1}+1)}{x^2+m^2x^2+1-1}$$

$$= \lim_{x\to 0} \frac{(x^2+m^2x^2)(\sqrt{x^2+m^2x^2+1}+1)}{x^2+m^2x^2} = \lim_{x\to 0}\sqrt{x^2+m^2x^2+1}+1 = 2$$

$$\lim_{y\to 0}\frac{m^2y^2+y^2}{\sqrt{y^2+m^2y^2+1}-1} = \lim_{y\to 0}\frac{(m^2y^2+y^2)\sqrt{m^2y^2+y^2+1}+1}{(\sqrt{y^2+m^2y^2+1}-1)(\sqrt{m^2y^2+y^2+1}+1)} = \lim_{y\to 0}\frac{(m^2y^2+y^2)(\sqrt{m^2y^2+y^2+1}+1)}{m^2y^2+y^2}$$

$$= \lim_{y\to 0}\sqrt{m^2y^2+y^2+1}+1 = \sqrt{1}+1 = 1+1 = 2$$

$$\boxed{\lim_{(x,y)\to(0,0)}\frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} = 2}$$