

$$(13) \quad z = 1 + x + y \quad y \in [0, \sqrt{x}] \quad x \in [0, 1]$$

$$z \in [0, 1 + x + y]$$

$$\int_0^1 \int_0^{\sqrt{x}} \int_0^{1+x+y} 6xy \, dz \, dy \, dx = \int_0^1 \int_0^{\sqrt{x}} 6xy + 6x^2y + 6xy^2 \, dy \, dx$$

$$= \int_0^1 \left[ 3xy^2 + 3x^2y^2 + 2xy^3 \right]_0^{\sqrt{x}} dx = \int_0^1 3x^2 + 3x^3 + 2x^{5/2} dx$$

$$= \left[ x^3 + \frac{3}{4}x^4 + \frac{4}{7}x^{7/2} \right]_0^1 = 1 + \frac{3}{4} + \frac{4}{7} = 1 + \frac{21+16}{28} = \frac{28+37}{28} = \boxed{\frac{65}{28}}$$

$$(17) \quad x = 4y^2 + 4z^2 \quad x = 4$$

$$\iint_D \left( \int_{4y^2+4z^2}^4 x \, dx \right) dA = \iint_D \left[ \frac{x^2}{2} \right]_{4y^2+4z^2}^4 dA = \iint_D 8 - 8(y^2+z^2) dA$$

$$y^2 + z^2 = r^2$$

$$= \int_0^{2\pi} \int_0^1 (8 - 8r^2) r \, dr \, d\theta = \int_0^{2\pi} \left[ 4r^2 - \frac{4}{3}r^6 \right]_0^1 d\theta$$

$$= \int_0^{2\pi} 4 - \frac{4}{3} d\theta = 4 - \frac{4}{3} (2\pi) = \frac{8}{3} (2\pi) = \boxed{\frac{16\pi}{3}}$$

$$(19) \quad 2x + y + z = 4$$

$$2x + y = 4$$

$$2x = 4$$

$$z = 4 - 2x - y$$

$$y = 4 - 2x$$

$$x = 2$$

$$z \in [0, 4 - 2x - y]$$

$$y \in [0, 4 - 2x] \quad x \in [0, 2]$$

$$V = \int_0^2 \int_0^{4-2x} \int_0^{4-2x-y} dz \, dy \, dx = \int_0^2 \int_0^{4-2x} 4 - 2x - y \, dy \, dx = \int_0^2 \left[ 4y - 2xy - \frac{1}{2}y^2 \right]_0^{4-2x} dx$$

$$= \int_0^2 \left[ 16 - 8x - 8x + 4x^2 - \frac{1}{2}(16 - 16x + 4x^2) \right] dx = \int_0^2 8 - 8x + 2x^2 dx = 8(2) - 4(1) + \frac{2}{3}(8) = \boxed{\frac{16}{3}}$$