

15.5 homework

1, 3, 5, 7, 9, 11, 13, 15a,

① $z = 5x + 3y + 6$ $[1, 4] \times [2, 6]$

$$\frac{\partial z}{\partial x} = 5 \quad \frac{\partial z}{\partial y} = 3 \quad SA = \int_1^4 \int_2^6 \sqrt{25+9+1} \, dy \, dx = \int_1^4 \sqrt{35}(4) \, dx = \boxed{12\sqrt{35}}$$

③ $z = 6 - 3x - 2y$ $(2, 0, 0)$ $(0, 3, 0)$

$$\frac{\partial z}{\partial x} = -3 \quad \frac{\partial z}{\partial y} = -2 \quad x \in [0, 2] \quad y \in [0, \frac{6-3x}{2}]$$

$$y = \frac{6-3x}{2}$$

$$SA = \int_0^2 \int_0^{\frac{6-3x}{2}} \sqrt{9+4+1} \, dy \, dx = \sqrt{14} \int_0^2 \frac{6-3x}{2} \, dx$$

$$= \frac{1}{2} \sqrt{14} \int_0^2 (6-3x) \, dx = \frac{1}{2} \sqrt{14} [12 - 6] = \boxed{3\sqrt{14}}$$

⑤ $z = 1 - x^2 - y^2$ $z = -2$ $\frac{\partial z}{\partial x} = -2x$ $\frac{\partial z}{\partial y} = -2y$

$$-2 = 1 - x^2 - y^2$$

$$x^2 + y^2 = 3$$

$$r = \sqrt{3} \quad r \in [0, \sqrt{3}]$$

circular cross section $\theta \in [0, 2\pi]$

$$SA = \int_0^{2\pi} \int_0^{\sqrt{3}} r \sqrt{1 + 4r^2} \, dr \, d\theta = \int_0^{2\pi} \frac{1}{8} \int_0^{\sqrt{3}} u^{1/2} \, du \, d\theta$$

$$= \int_0^{2\pi} \frac{1}{8} \cdot \frac{2}{3} u^{3/2} \, d\theta = \int_0^{2\pi} \frac{1}{12} (1 + 4r^2)^{3/2} \Big|_0^{\sqrt{3}} \, d\theta = \int_0^{2\pi} \frac{1}{12} (13^{3/2} - 1) \, d\theta$$

$$= \frac{2\pi}{12} (13\sqrt{3} - 1) = \boxed{\frac{\pi}{6} (13\sqrt{3} - 1)}$$

$v = 1 + 4r^2$
 $dv = 8r \, dr$
 $\frac{1}{8} du \, dr$