

21) a.)  $(x_1, y_1) (x_2, y_2) \quad \int_C x dy - y dx = x_1 y_2 - x_2 y_1$

$$x = x_1 + t(x_2 - x_1) \quad y = y_1 + t(y_2 - y_1)$$

$$dx = (x_2 - x_1) dt \quad dy = (y_2 - y_1) dt$$

$$\int_0^1 ((x_1 + t(x_2 - x_1))(y_2 - y_1) - (x_2 - x_1)(y_1 + t(y_2 - y_1))) dt$$

$$= x_1(y_2 - y_1)t + \frac{t^2}{2} (x_2 - x_1)(y_2 - y_1) - y_1(x_2 - x_1)t - \frac{t^2}{2} (x_2 - x_1)(y_2 - y_1) \Big|_0^1$$

$$= x_1(y_2 - y_1) - y_1(x_2 - x_1) = x_1 y_2 - x_1 y_1 - y_1 x_2 + x_1 y_1$$

$$= \boxed{x_1 y_2 - y_1 x_2 \quad \therefore \text{QED}}$$

21) b.)  $(x_i, y_i) (x_{i+1}, y_{i+1})$

$$x = x_i + t(x_{i+1} - x_i) \quad y = y_i + t(y_{i+1} - y_i)$$

$$dx = (x_{i+1} - x_i) dt \quad dy = (y_{i+1} - y_i) dt$$

$$\frac{1}{2} \int_0^1 (x_i + t(x_{i+1} - x_i))(y_{i+1} - y_i) - (x_{i+1} - x_i)(y_i + t(y_{i+1} - y_i)) dt$$

$$= \frac{1}{2} [x_i y_{i+1} - y_i x_{i+1}] \quad \text{Area} = \frac{1}{2} \sum_{i=1}^{n-1} x_i y_{i+1} - y_i x_{i+1}$$

$$= \boxed{\frac{1}{2} [(x_1 y_2 - x_2 y_1) + (x_2 y_3 - x_3 y_2) + \dots + (x_{n-1} y_n - x_n y_{n-1})]} \\ \therefore \text{QED}$$