

$$3) \text{ cont'd } \iint_{S_4} \mathbf{F} \cdot d\mathbf{s} = \iint_P \langle v^2 \sin^2 u, \cos^2 u \rangle \cdot \langle 0, \sin u, \cos u \rangle dA$$

$$= \iint_P \sin^3 u + \cos^3 u dA = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^2 \sin^3 u + \cos^3 u dv du$$

$$= 2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 u + \cos^3 u du = \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4 \sin^3 u + 4 \cos^3 u du$$

$$= \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 3 \sin u - \sin^3 u + 3 \cos u + \cos^3 u du$$

$$= \frac{1}{2} \left[-3 \cos u + \frac{\cos^3 u}{3} + 3 \sin u + \frac{\sin^3 u}{3} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= 3 - \frac{1}{3} = \frac{8}{3}$$

$$S = \iint_1 + \iint_2 + \iint_3 + \iint_4 = 0 + 2\pi + 0 + \frac{8}{3} = \boxed{2\pi + \frac{8}{3}}$$