21) $\int_1^4\int_1^2 \left(\frac{x}{y}+\frac{y}{x}\right) dy\,dx = \int_1^4 \left[x\ln|y| + \frac{y^2}{2x}\right]_1^2 dx$

$= \int_1^4 x\ln 2 + \frac{2}{x} - 0 - \frac{1}{2x}\,dx = \int_1^4 x\ln 2 + \frac{3}{2x}\,dx$

$= \left[\frac{\ln 2}{2}x^2 + \frac{3}{2}\ln|x|\right]_1^4 = \frac{16\ln 2}{2} + \frac{3}{2}\ln|4| - \frac{\ln 2}{2} + 0$

$= 8\ln|2| + 3\ln|2| - \frac{1}{2}\ln|2|$

$= \boxed{\frac{21}{2}\ln(2)}$

25) $\int_0^1\int_0^1 v(u+v^2)^4\,du\,dv$ $\quad a=v^2$

$da = 2v\,dv$

$\int_0^1 \frac{1}{2}\int_0^1 (u+a)^4\,du\,da$ $\quad \frac{1}{2}da = v\,dv$

$\int_0^1 \frac{1}{2}\left[\frac{1}{5}(u+a)^5\Big|_0^1\right]da = \int_0^1 \frac{1}{10}\left((1+a)^5 - a^5\right) da$

$\frac{1}{10}\int_0^1 (1+a)^5 - a^5\,da$

$\frac{1}{10}\left[\frac{1}{6}(1+a)^6 - \frac{1}{6}a^6\right]_0^1$

$\frac{1}{60}\left(2^6 - 1 - 1 - 0\right)$

$\frac{1}{60}\left(2^6-2\right) = \frac{2^5-1}{30} = \frac{32-1}{30}$

$= \boxed{\frac{31}{30}}$