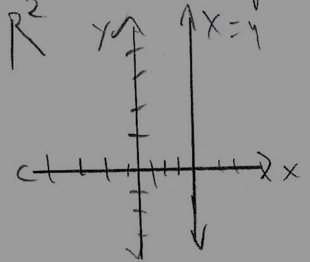
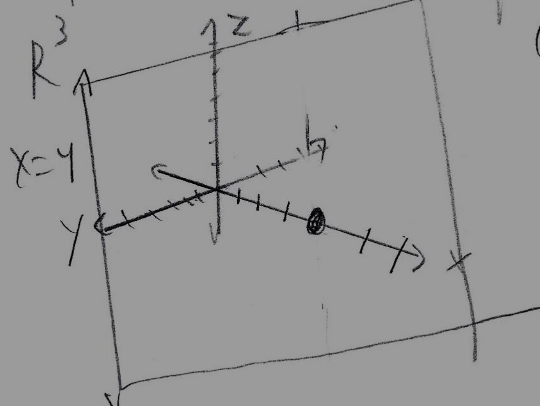
12.1 homework 1, 5, 9, 13, 17, 21, 25, 29, 33, 37

Sumeeth Guda
ERC# 0975124

1. start: $(0,0,0)$ $\boxed{(4,0,-3)}$
x axis change: $(4,0,0)$
z axis change: $(4,0,-3)$

5. In $R^2$ equation $x=4$ represents a line parallel to y axis. In $R^3$ equation $x=4$ represents a plane centered on $(4,0,0)$ parallel to yz plane




9. $|\vec{PQ}| = \sqrt{(7-3)^2+(0+2)^2+(1+3)^2} = \sqrt{16+4+16} = \sqrt{36} = 6$

$|\vec{PR}| = \sqrt{(1-3)^2+(2+2)^2+(1+3)^2} = \sqrt{4+16+16} = \sqrt{36} = 6$

$|\vec{QR}| = \sqrt{(1-7)^2+(2-0)^2+(1-1)^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$

$\Delta PQR$ is not a right triangle but is an isoceles triangle

13. sphere: $(x+3)^2+(y-2)^2+(z-5)^2=16$

any point in the yz plane implies that $x=0$ for some point $(0,y,z)$

therefore by subbing in

$$(0+3)^2+(y-2)^2+(z-5)^2=16$$

$$(y-2)^2+(z-5)^2=7$$

For all y and z on the yz plane the intersection is a circle

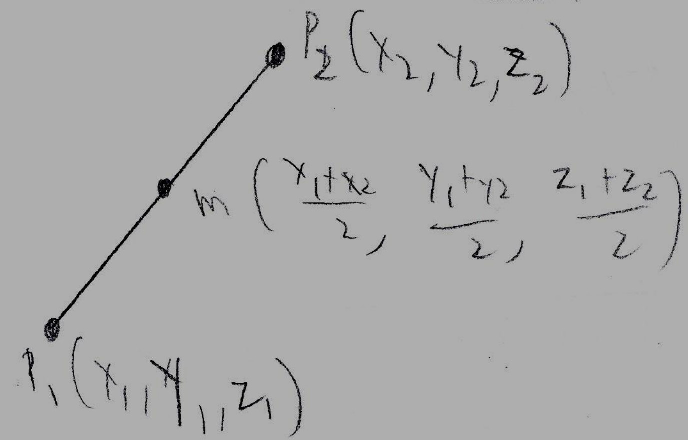
17. $x^2 - 2x + y^2 - 4y + z^2 + 8z = 15$

$$(x^2 - 2x + 1) + (y^2 - 4y + 4) + (z^2 + 8z + 16) = 15 + 1 + 4 + 16$$

$$(x-1)^2 + (y-2)^2 + (z+4)^2 = 36$$

$$\boxed{\begin{array}{l} \text{center} = (1, 2, -4) \\ \text{radius} = \sqrt{36} = 6 \end{array}}$$

21 a.




$$|P_1 m| = \sqrt{\left(\frac{x_1+x_2}{2} - x_1\right)^2 + \left(\frac{y_1+y_2}{2} - y_1\right)^2 + \left(\frac{z_1+z_2}{2} - z_1\right)^2}$$

$$\sqrt{\left(\frac{x_1+x_2-2x_1}{2}\right)^2 + \left(\frac{y_1+y_2-2y_1}{2}\right)^2 + \left(\frac{z_1+z_2-2z_1}{2}\right)^2}$$

$$|P_1 m| = \sqrt{\left(\frac{x_2-x_1}{2}\right)^2 + \left(\frac{y_2-y_1}{2}\right)^2 + \left(\frac{z_2-z_1}{2}\right)^2}$$

$$|m P_2| = \sqrt{\left(x_2 - \frac{x_1+x_2}{2}\right)^2 + \left(y_2 - \frac{y_1+y_2}{2}\right)^2 + \left(z_2 - \frac{z_1+z_2}{2}\right)^2}$$

$$|m P_2| = \sqrt{\left(\frac{x_2-x_1}{2}\right)^2 + \left(\frac{y_2-y_1}{2}\right)^2 + \left(\frac{z_2-z_1}{2}\right)^2}$$

$$|P_1 m| \cong |m P_2|$$

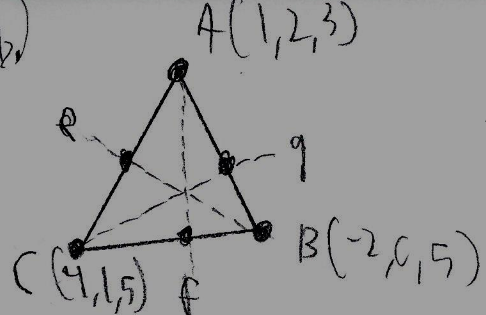
$$\text{slope } P_1 m\Big|_{xy} = \frac{\frac{y_1+y_2}{2} - y_1}{\frac{x_2+x_1}{2} - x_1} = \frac{y_2-y_1}{x_2-x_1} \cong \text{slope } mP_2\Big|_{xy} = \frac{y_2 - \frac{y_2+y_1}{2}}{x_2 - \frac{x_2+x_1}{2}} = \frac{y_2-y_1}{x_2-x_1}$$

$$P_1 m\Big|_{xz} = \frac{\frac{z_1+z_2}{2} - z_1}{\frac{x_2+x_1}{2} - x_1} = \frac{z_2-z_1}{x_2-x_1} \cong mP_2\Big|_{xz} = \frac{z_2 - \frac{z_2+z_1}{2}}{x_2 - \frac{x_2+x_1}{2}} = \frac{z_2-z_1}{x_2-x_1}$$

$$P_1 m\Big|_{yz} = \frac{\frac{z_1+z_2}{2} - z_1}{\frac{y_2+y_1}{2} - y_1} = \frac{z_2-z_1}{y_2-y_1} \cong mP_2\Big|_{yz} = \frac{z_2 - \frac{z_2+z_1}{2}}{y_2 - \frac{y_2+y_1}{2}} = \frac{z_2-z_1}{y_2-y_1}$$

$\boxed{\text{m is the midpoint } \therefore \text{QED}}$

21b.




midpoint AC $= \left(\frac{4+1}{2}, \frac{2+1}{2}, \frac{3+5}{2}\right) = \left(\frac{5}{2}, \frac{3}{2}, 4\right)$

midpoint AB $= \left(\frac{1-2}{2}, \frac{2+0}{2}, \frac{3+5}{2}\right) = \left(\frac{-1}{2}, 1, 4\right)$

midpoint BC $= \left(\frac{4-2}{2}, \frac{1+0}{2}, \frac{5+5}{2}\right) = \left(1, \frac{1}{2}, 5\right)$

$$|\text{median } e| = \sqrt{\left(-2-\frac{5}{2}\right)^2 + \left(0-\frac{3}{2}\right)^2 + (5-4)^2} = \sqrt{\frac{81}{4} + \frac{9}{4} + 1} = \sqrt{\frac{90}{4} + 1} = \boxed{\frac{\sqrt{94}}{2}}$$

$$|\text{median } f| = \sqrt{(1-1)^2 + \left(2-\frac{1}{2}\right)^2 + (3-5)^2} = \sqrt{0 + \frac{9}{4} + 4} = \sqrt{\frac{25}{4}} = \boxed{\frac{5}{2}}$$

$$|\text{median } g| = \sqrt{\left(4+\frac{1}{2}\right)^2 + (1-1)^2 + (5-4)^2} = \sqrt{\frac{81}{4} + 0 + 1} = \boxed{\frac{\sqrt{85}}{2}}$$

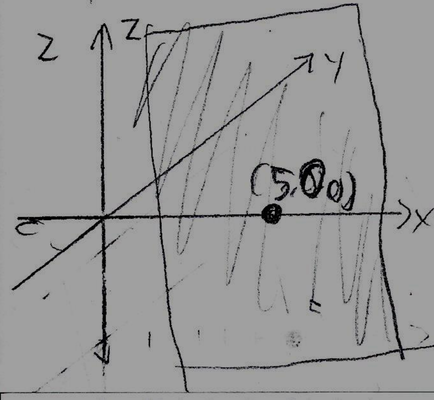
25. $x=5$ on $R^3$ represents a plane centered on $(5,0,0)$ parallel to yz plane

29. $0 \le z \le 6$ on $R^3$ represents every point between planes $z=0$ and $z=6$

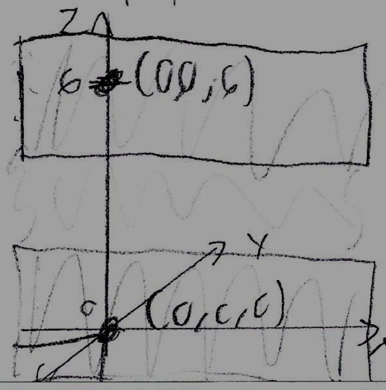
33. $x^2+y^2+z^2=4$ on $R^3$ represents a sphere centered on $(0,0,0)$ the origin and has a radius of 2.

37. $x^2+z^2 \le 9$ on $R^3$ represents every point on or inside a cylinder of radius 3. With the axis being the y axis

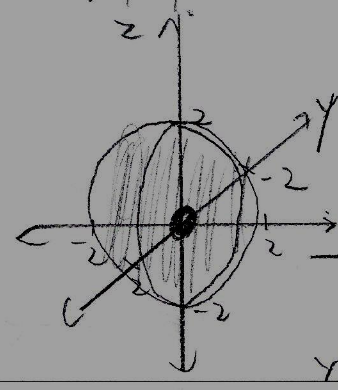
25 graph



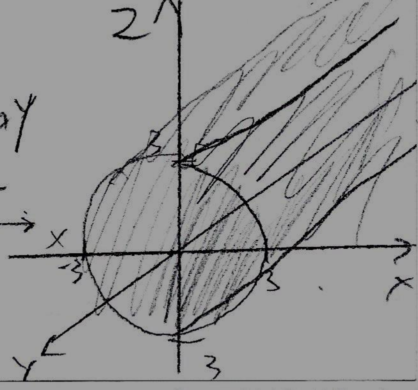

29 graph




33 graph



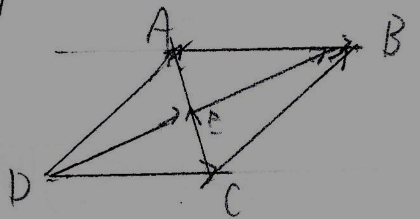

37 graph



12.2 homework 3, 7, 11, 15, 19, 23, 27, 31, 35, 39

Sumeeth Guda
EVC# 0975124

(3)



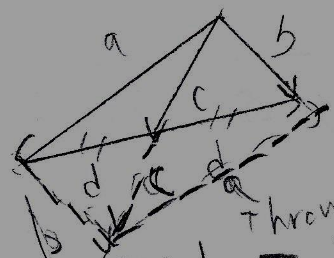

$\overrightarrow{PC} = \overrightarrow{AB}$

$\overrightarrow{DA} = \overrightarrow{CB}$

$\overrightarrow{DE} = \overrightarrow{EB}$

$\overrightarrow{CE} = \overrightarrow{EA}$

(7)




Through triangle law

$a - b = 2d$

$\boxed{\frac{1}{2}(a-b) = d}$

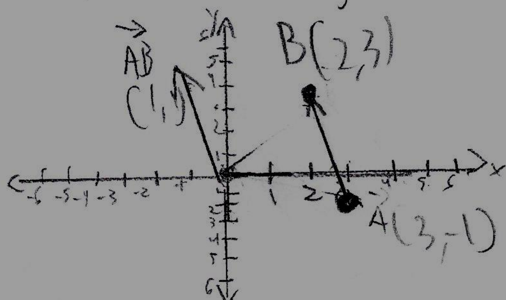
Through parallelogram law

$a + b = 2c$

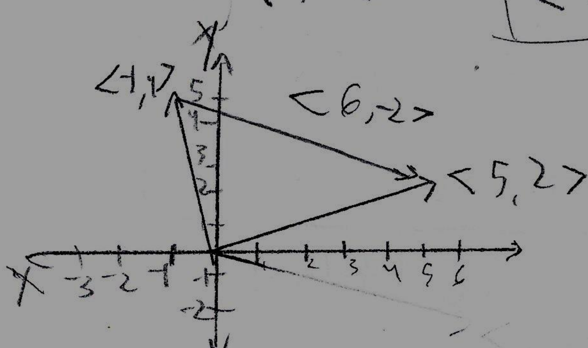
$\boxed{\frac{1}{2}(a+b) = c}$

(11) $A(3,-1)$ $B(2,3)$

$\overrightarrow{AB} = \langle 2-3, 3-(-1) \rangle = \boxed{\langle -1, 4 \rangle}$




(15) $\langle -1, 4 \rangle + \langle 6, -2 \rangle = \boxed{\langle 5, 2 \rangle}$



(19) $\vec{a} = \langle -3, 4 \rangle \quad \vec{b} = \langle 9, -1 \rangle$

$$\vec{a} + \vec{b} = \langle 9+(-3),\ 4+(-1) \rangle = \boxed{\langle 6, 3 \rangle}$$

$$4\vec{a} + 2\vec{b} = 4\langle -3, 4 \rangle + 2\langle 9, -1 \rangle = \langle -12+18,\ 16-2 \rangle = \boxed{\langle 6, 14 \rangle}$$

$$|\vec{a}| = \sqrt{3^2+4^2} = \sqrt{25} = \boxed{5}$$

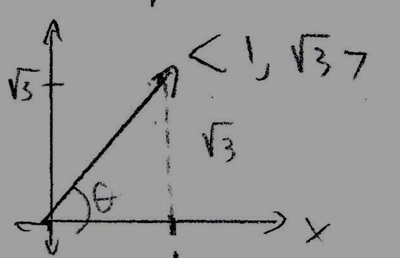
$$|\vec{a} - \vec{b}| = |\langle -3-9,\ 4+1 \rangle| = |\langle -12, 5 \rangle| = \sqrt{144+25} = \sqrt{169} = \boxed{13}$$

(23) unit vector of $\langle 6, -2 \rangle$

$$|\langle 6, -2 \rangle| = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$$

$$\frac{1}{2\sqrt{10}} \langle 6, -2 \rangle = \left\langle \frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}} \right\rangle = \boxed{\left\langle \frac{3\sqrt{10}}{10}, \frac{-\sqrt{10}}{10} \right\rangle}$$

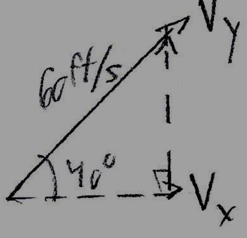
(27) $i + \sqrt{3}\, j = \langle 1, \sqrt{3} \rangle$




$\tan\theta = \sqrt{3}$

$$\theta = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3} \text{ radians} = \boxed{60^\circ}$$

(31)

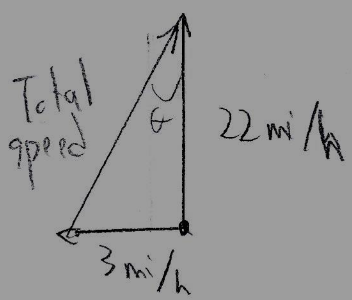



$V_x = 60 \cos 40 \approx 45.9627$ ft/s [R]

$V_y = 60 \sin 40 \approx 38.5673$ ft/s [U]

$$\boxed{\begin{array}{c} V_x = 45.96 \text{ ft/s} \\ V_y = 38.57 \text{ ft/s} \end{array}}$$

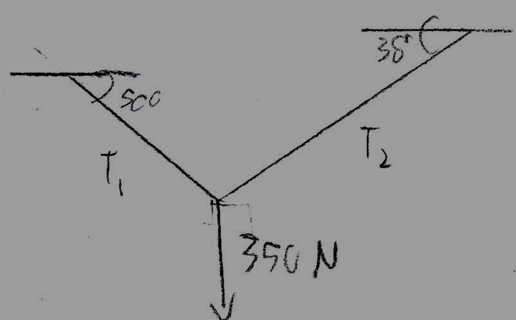
(35)




$$\text{Total speed} = \sqrt{22^2 + 3^2}$$
$$= \sqrt{493} \approx 22.2036 \text{ mi/h} \approx 22.2 \text{ mi/h}$$
$$\text{direction} = \cos^{-1}\left(22/\sqrt{493}\right) \approx 7.765 \approx 9^\circ \text{ [W]}$$

$$\boxed{22.2 \text{ mi/h } N 8^\circ W}$$

(37)




$$T_1 = -|T_1| \cos 50\, i + |T_1| \sin 50\, j$$
$$T_2 = |T_2| \cos 38\, i + |T_2| \sin 38\, j$$

$$T_1 + T_2 = 350 j \Rightarrow \left(-|T_1| \cos 50 i + |T_1| \sin 50 j\right) + \left(|T_2| \cos 38 i + |T_2| \sin 38 j\right) = 350 j$$

$$-|T_1| \cos 50 + |T_2| \cos 38 = 0 \qquad |T_2| = \frac{|T_1| \cos 50}{\cos 38}$$

$$|T_1| \sin 50 + |T_2| \sin 38 = 350$$

$$|T_1| \sin 50 + |T_1| \tan 38 \cos 50 = 350$$

$$|T_1| = \frac{350}{\sin 50 + \tan 38 \cos 50} \approx 275.9719$$

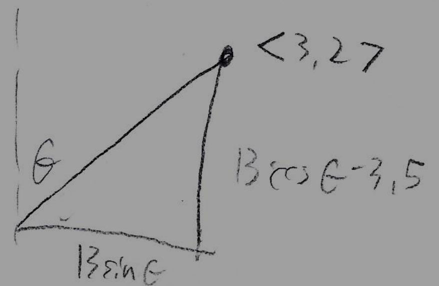
$$|T_2| = \frac{275.9719 \cos 50}{\cos 38} \approx 225.1128$$

$$-|T_1| \cos 50 \approx -177.3913$$
$$|T_1| \sin 50 \approx 211.4067$$

$$|T_2| \cos 38 \approx 177.3912$$
$$|T_2| \sin 38 \approx 138.5932$$

$$\boxed{T_1 = \langle -177.39, 211.41 \rangle \qquad T_2 = \langle 177.39, 138.59 \rangle}$$

39)




$V$ = total velocity $= \langle 13\sin\theta,\ 13\cos\theta-3.5\rangle = \langle 3,2\rangle$

$\langle 3,2\rangle = Vt = \langle 13\sin\theta,\ 13\cos\theta-3.5\rangle t$

$\langle 3,2\rangle = \langle 13t\sin\theta,\ (13\cos\theta-3.5)t\rangle$

$13t\sin\theta = 3$

$t = \dfrac{3}{13\sin\theta}$

$(13\cos\theta-3.5)t = 2$

$(13\cos\theta-3.5)\left(\dfrac{3}{13\sin\theta}\right) = 2$

$\dfrac{39\cos\theta}{13\sin\theta} - \dfrac{10.5}{13\sin\theta} = 2$

$39\cos\theta - 10.5 = 26\sin\theta$

$(39\cos\theta-10.5)^2 = (26\sin\theta)^2$

$1521\cos^2\theta - 819\cos\theta + 110.25 = 676\sin^2\theta$ $\quad \sin^2\theta = 1-\cos^2\theta$

$1521\cos^2\theta - 819\cos\theta + 110.25 = 676(1-\cos^2\theta)$

$2197\cos^2\theta - 819\cos\theta - 565.75 = 0$

$\cos\theta = \dfrac{819 \pm \sqrt{819^2-4(2197(-565.75))}}{2(2197}$

$\cos\theta = 0.72699$

$\theta = \cos^{-1}(0.72699)$

$\cos\theta = -0.35421$

$\theta > 90$ No

$\boxed{\theta \approx 43.4^\circ \text{ from canal bank directed upstream}}$

$t = \dfrac{3}{13\sin(43.4)} = 0.336h \times 60 \approx$ $\boxed{20.2 \text{ min}}$

12.3 HW $\quad$ 3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55 $\quad$ Sumeeth Guda

(3.) $a = \langle 1.5, 0.4 \rangle \quad b = \langle -4, 6 \rangle$

$a \cdot b = 1.5(-4) + 0.4(6) = -6 + 2.4 = \boxed{-3.6}$

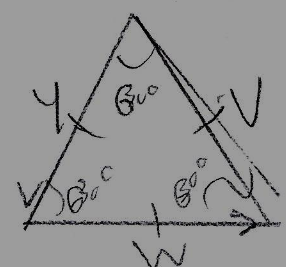
(7.) $a = 2i + j \quad b = i - j + k$

$\langle 2, 1, 0 \rangle \quad \langle 1, -1, 1 \rangle$

$a \cdot b = 2 - 1 + 0 = \boxed{1}$

(11.) $u \cdot v = \frac{1}{2}$

$u \cdot w = -\frac{1}{2}$




$|u| = -1$

$|v| = -1$

$|w| = 1$

$u \cdot v = |u||v| \cos(60)$

$= (-1)(-1)\left(\frac{1}{2}\right)$

$= \boxed{\frac{1}{2}}$

$u \cdot w = |u||w| \cos 60$

$(-1)(1)\left(\frac{1}{2}\right) = \boxed{-\frac{1}{2}}$

(15.) $a = \langle 4, 3 \rangle \quad b = \langle 2, -1 \rangle \quad |a| = 5 \quad |b| = \sqrt{5}$

$a \cdot b = 8 - 3 = 5$

$a \cdot b = |a||b| \cos\theta$

$\cos\theta = \frac{a \cdot b}{|a||b|} \qquad \cos\theta = \frac{5}{5\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\theta = \cos^{-1}\left(\frac{\sqrt{5}}{5}\right) \approx \boxed{63.435^\circ}$

(19.) $a = 4i - 3j + k = \langle 4, -3, 1 \rangle \quad b = 2i - k = \langle 2, 0, -1 \rangle$

$|a| = \sqrt{26} \qquad |b| = \sqrt{5}$

$a \cdot b = 8 - 0 - 1 = 7$

$\theta = \cos^{-1}\left(\frac{7}{\sqrt{5}\sqrt{26}}\right) = \cos^{-1}\left(\frac{7}{\sqrt{130}}\right) \approx \boxed{52.125^\circ}$

(23a) $a = \langle 9,3\rangle \quad b = \langle -2,6\rangle$

$a \cdot b = -18 + 18 = 0$ $\boxed{\text{orthagonal}}$

(23b) $a = \langle 4,5,-2\rangle \quad b = \langle 3,-1,5\rangle \quad |a| = \sqrt{45} \quad |b| = \sqrt{35}$

$a \cdot b = 12 - 5 - 10 = -3 \qquad \cos\theta = \frac{-3}{\sqrt{1575}} = \frac{-1}{\sqrt{175}}$ $\boxed{\text{neither}}$

(23c) $a = -8i + 12j + 4k \quad b = 6i - 9j - 3k \qquad |a| = \sqrt{224} \quad |b| = \sqrt{126}$

$\langle -8,12,4\rangle \qquad \langle 6,-9,-3\rangle$

$a \cdot b = -48 - 108 - 12 = -168 \qquad \frac{a \cdot b}{|a||b|} = \cos\theta = \frac{-168}{\sqrt{28224}} = \frac{-1}{1}$

$\theta = \pi$ $\boxed{\text{parallel}}$

(23d) $a = \langle 3,-1,3\rangle \quad b = \langle 5,9,-2\rangle$

$a \cdot b = 15 - 9 - 6 = 0$ $\boxed{\text{orthagonal}}$

(27) $i + j \qquad i + k$

$\langle 1,1,0\rangle \qquad \langle 1,0,1\rangle$

$\langle 1,1,0\rangle \cdot u = 0 = \langle 1,1,0\rangle \cdot \langle i,j,k\rangle = 0$

$\langle 1,0,1\rangle \cdot u = 0 = \langle 1,0,1\rangle \cdot \langle i,j,k\rangle = 0$

$i + j = 0 \qquad j = -i \qquad i = -1$

$i + k = 0 \qquad k = -i \qquad j = -1$

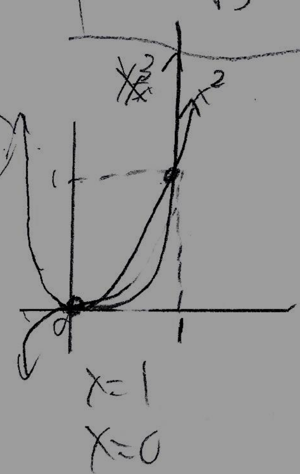
$k = -1$

$\boxed{\frac{(i - j - k)}{\sqrt{3}} \text{ or } \frac{(-i + j + k)}{\sqrt{3}}}$

$V = \langle 1,-1,-1\rangle$

$|V| = \sqrt{3}$

(31)




$x = 1$

$x = 0$

$f(x) = x^2 \quad g(x) = x^3$

$f'(\cancel{x}) = 2x \quad g'(x) = 3x^2$

$f'(0) = 0 \quad g'(0) = 0$

$f'(1) = 2 \quad g'(1) = 3$

$\langle 1,2\rangle \quad \langle 1,3\rangle$

$\theta = \cos^{-1}\left(\frac{1+6}{\sqrt{5}\sqrt{10}}\right)$

$\theta = \cos^{-1}\left(\frac{7}{\sqrt{50}}\right) = 8.13^\circ$

$\boxed{\begin{array}{l} x = 0 \quad \theta = 0^\circ \\ x = 1 \quad \theta = 8.13^\circ \end{array}}$

(35) $i-2j-3k$

$|\langle 1,-2,-3\rangle| = \sqrt{14}$

$\cos\alpha = \frac{1}{\sqrt{14}}$ $\cos\beta = \frac{-2}{\sqrt{14}}$ $\cos\theta = \frac{-3}{\sqrt{14}}$

$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{14}}\right)$ $\beta = \cos^{-1}\left(\frac{-2}{\sqrt{14}}\right)$ $\theta = \cos^{-1}\left(\frac{-3}{\sqrt{14}}\right)$

$= 74.5^\circ$ $= 122.3^\circ$ $= 143.3^\circ$

$\boxed{74.5^\circ}$ $\boxed{122.3^\circ}$ $\boxed{143.3^\circ}$

(39) $a = \langle -5, 12\rangle$ $b = \langle 4, 6\rangle$ b onto a $|a| = \sqrt{169} = 13$

Scaler proj $= \text{comp}_a b = \frac{a \cdot b}{|a|} = \frac{-20+72}{13} = \frac{52}{13} = \boxed{4}$

vector proj $= \text{proj}_a b = \frac{(a \cdot b)a}{(|a|)^2} = \frac{52}{169}\langle -5, 12\rangle$

$= \frac{4}{13}\langle -5, 12\rangle$

$= \boxed{\left\langle \frac{-20}{13}, \frac{48}{13}\right\rangle}$

(43) $a = \langle 3, -3, 1\rangle$ $b = \langle 2, 4, -1\rangle$

$|a| = \sqrt{19}$ $a \cdot b = 6 - 12 - 1 = -7$

$\text{comp}_a b = \frac{-7}{\sqrt{19}} = \boxed{\frac{-7\sqrt{19}}{19}}$

$\text{proj}_a b = \frac{-7}{19}\langle 3, -3, 1\rangle = \boxed{\left\langle \frac{-21}{19}, \frac{21}{19}, \frac{-7}{19}\right\rangle}$

(47) $a = \langle 3, 0, -1 \rangle$ $\quad |a| = \sqrt{9+1} = 10$

$\text{comp}_a b = 2$ $\quad b = \langle i, j, k \rangle$

$$\text{comp}_a b = \frac{a \cdot b}{|a|}$$

$$2|a| = a \cdot b$$

$$2\sqrt{10} = 3i + \cancel{0j} - k$$

$$2\sqrt{10} - 3i = -k$$

$$k = 3i - 2\sqrt{10}$$

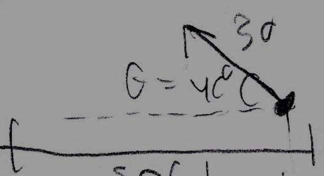
For this specific case!

$\langle 0, 0, -2\sqrt{10} \rangle$

In general!

$$\boxed{\begin{gathered}\langle i, j, 3i - 2\sqrt{10} \rangle \\ i \wedge j \in \mathbb{R}\end{gathered}}$$

(51) $W = Fd\cos\theta$ $\quad d = 80$



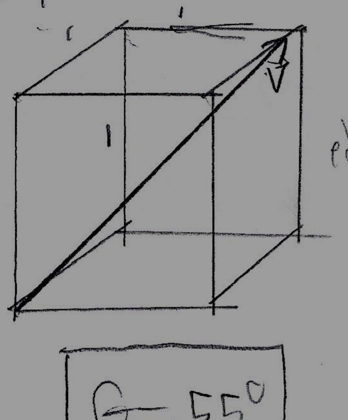

$F_{net} = 30$

$\theta = 40$

$W = 30(80)\cos 40 = 2400 \cos 40 = 1838.51$ ft-lb

$\approx \boxed{1839 \text{ ft-lb}}$

(55) Assuming each edge is length 1




$\vec{v} = \langle 1, 1, 1 \rangle$

edges $\begin{cases} i = \langle 1, 0, 0 \rangle \\ j = \langle 0, 1, 0 \rangle \\ k = \langle 0, 0, 1 \rangle \end{cases}$

$v \cdot i = |v||i|\cos\theta$

$v \cdot i = 1 + 0 + 0!$

$|v| = \sqrt{3}$

$|i| = 1$

$\boxed{\theta = 55^\circ} \Leftarrow \theta \approx 54.74^\circ \Leftarrow \cos\theta = \dfrac{1}{\sqrt{3}}$ $\quad \theta = \cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

12.4 HW 3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43 Sumeeth Guda

3. $a = \langle 0, 2, -4 \rangle$ $b = \langle -1, 3, 1 \rangle$

$$a \times b = \begin{vmatrix} i & j & k \\ 0 & 2 & -4 \\ -1 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 2 & -4 \\ 3 & 1 \end{vmatrix} i - \begin{vmatrix} 0 & -4 \\ -1 & 1 \end{vmatrix} j + \begin{vmatrix} 0 & 2 \\ -1 & 3 \end{vmatrix} k$$

$$= 14i + 4j + 2k$$

$$= \boxed{\langle 14, 4, 2 \rangle}$$

$\langle 14, 4, 2 \rangle \cdot \langle 0, 2, -4 \rangle = 0 + 8 - 8 = 0$ ✓ orthagonal ✓

$\langle 14, 4, 2 \rangle \cdot \langle -1, 3, 1 \rangle = -14 + 12 + 2 = 0$ ✓

7. $a = \langle t, 1, 1/t \rangle$ $b = \langle t^2, t^2, 1 \rangle$ $a \times b = \begin{vmatrix} i & j & k \\ t & 1 & 1/t \\ t^2 & t^2 & 1 \end{vmatrix}$

$$a \times b = \begin{vmatrix} 1 & 1/t \\ t^2 & 1 \end{vmatrix} i - \begin{vmatrix} t & 1/t \\ t^2 & 1 \end{vmatrix} j + \begin{vmatrix} t & 1 \\ t^2 & t^2 \end{vmatrix} k$$

$$(1-t)i - (t-t)j + (t^3-t^2)k$$

$$\boxed{\langle 1-t, 0, t^3-t^2 \rangle}$$

$\langle 1-t, 0, t^3-t^2 \rangle \cdot \langle t, 1, \frac{1}{t} \rangle = t - t^2 + t^2 - t = 0$ ✓ orthagonal ✓

$\langle 1-t, 0, t^3-t^2 \rangle \cdot \langle t^2, t^2, 1 \rangle = t^2 - t^3 + t^3 - t^2 = 0$ ✓

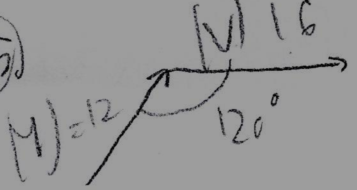
11. $(j-k) \times (k-i)$

$$((j-k) \times k) - ((j-k) \times i)$$

$$(j \times k) - (k \times k) - (j \times i) + (k \times i)$$

$$i - 0 - (-k) + j = \boxed{i + j + k}$$

15)




$$|u \times v| = (12)(16)\sin(120)$$
$$= (12)(16)\left(\frac{\sqrt{3}}{2}\right)$$
$$= 12(8)(\sqrt{3}) = \boxed{96\sqrt{3} \text{ into the page}}$$

19)

$\langle 3,2,1\rangle \times \langle -1,1,0\rangle$ $\qquad$ $\langle 3,2,1\rangle \times \langle -1,1,0\rangle = \begin{vmatrix} i & j & k \\ 3 & 2 & 1 \\ -1 & 1 & 0 \end{vmatrix}$

$$= \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} i - \begin{vmatrix} 3 & 1 \\ -1 & 0 \end{vmatrix} j + \begin{vmatrix} 3 & 2 \\ -1 & 1 \end{vmatrix} k$$

$$-i - j + 5k = \langle -1,-1,5\rangle$$

$$|\langle -1,-1,5\rangle| = \sqrt{1+1+25} = \sqrt{27} = 3\sqrt{3}$$

$$\frac{1}{3\sqrt{3}}\langle -1,-1,5\rangle \qquad \frac{-1}{3\sqrt{3}}\langle -1,-1,5\rangle$$

$$\boxed{\left\langle \frac{-1}{3\sqrt{3}}, \frac{-1}{3\sqrt{3}}, \frac{5}{3\sqrt{3}} \right\rangle \text{ or } \left\langle \frac{1}{3\sqrt{3}}, \frac{1}{3\sqrt{3}}, \frac{-5}{3\sqrt{3}} \right\rangle}$$

23) $a \times b = -(b \times a)$ $\qquad$ $a = \langle a_1, a_2, a_3\rangle$ $\qquad$ $b = \langle b_1, b_2, b_3\rangle$

$$\begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = -\begin{vmatrix} i & j & k \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$$

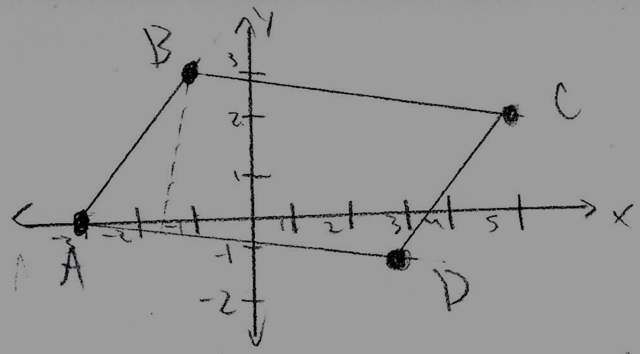
$$\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} i - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} j + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} k = -\begin{vmatrix} b_2 & b_3 \\ a_2 & a_3 \end{vmatrix} i + \begin{vmatrix} b_1 & b_3 \\ a_1 & a_3 \end{vmatrix} j - \begin{vmatrix} b_1 & b_2 \\ a_1 & a_2 \end{vmatrix} k$$

$$(a_2b_3 - a_3b_2)i - (a_1b_3 - a_3b_1) + (a_1b_2 - b_1a_2)k = (a_2b_3 - a_3b_2)i + (a_3b_1 - a_1b_3)j + (a_1b_2 - a_1b_2)k$$

$$\langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - b_1a_2\rangle = \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_1b_2\rangle$$

$$a \times b = -(b \times a) \quad \therefore \text{QED}$$

27.




A(-3,0) B(-1,3) C(5,2)
D(3,-1)

$\vec{AB} = \langle 2,3 \rangle$ $|AB| = \sqrt{13}$

$\vec{DC} = \langle 2,3 \rangle$ $|DC| = \sqrt{13}$

$\vec{BC} = \langle 6,-1 \rangle$ $|BC| = \sqrt{37}$

$\vec{AD} = \langle 6,-1 \rangle$ $|AD| = \sqrt{37}$

$$|\langle 2,3,0 \rangle \times \langle 6,-1,0 \rangle| = \left\| \begin{vmatrix} i & j & k \\ 2 & 3 & 0 \\ 6 & -1 & 0 \end{vmatrix} \right\| =$$

$$= \left| \cancel{\begin{vmatrix} 3 & 0 \\ -1 & 0 \end{vmatrix}} i - \cancel{\begin{vmatrix} 2 & 0 \\ 6 & 0 \end{vmatrix}} j + \begin{vmatrix} 2 & 3 \\ 6 & -1 \end{vmatrix} k \right|$$

$$= |\langle 0,0,-20 \rangle| = \sqrt{400} = \boxed{20 \text{ units}^2}$$

31. P(0,-2,0) Q(4,1,-2) R(5,3,1)

$\vec{PQ} = \langle 4,3,-2 \rangle$ $\vec{PR} = \langle 5,5,1 \rangle$

$$PQ \times PR = \begin{vmatrix} i & j & k \\ 4 & 3 & -2 \\ 5 & 5 & 1 \end{vmatrix}$$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} 3 & -2 \\ 5 & 1 \end{vmatrix} i - \begin{vmatrix} 4 & -2 \\ 5 & 1 \end{vmatrix} j + \begin{vmatrix} 4 & 3 \\ 5 & 5 \end{vmatrix} k$$

$$= 13i - 14j + 5k = \langle 13,-14,5 \rangle$$

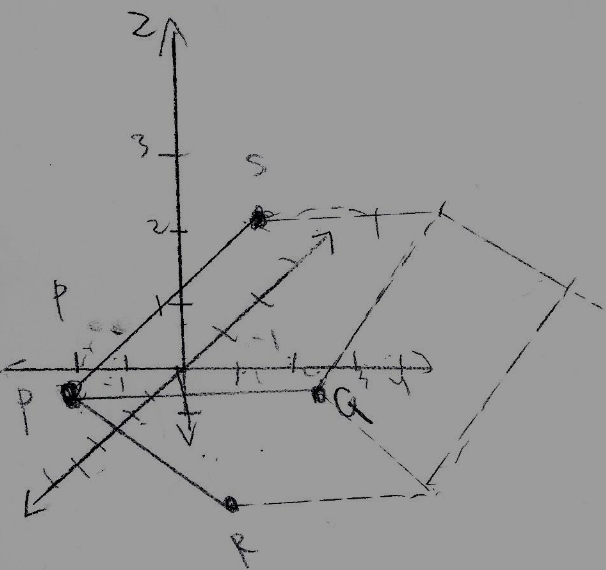
$$\frac{1}{2} |\langle 13,-14,5 \rangle| = \frac{1}{2}\sqrt{169+196+25} = \boxed{\frac{1}{2}\sqrt{390} \text{ units}^2}$$

(35) $P(-2,1,0)$ $Q(2,3,2)$ $R(1,4,-1)$ $S(3,6,1)$

$\vec{PQ} = \langle 4,2,2 \rangle$ $\vec{PR} = \langle 3,3,-1 \rangle$ $\vec{PS} = \langle 5,5,1 \rangle$

$|PQ| = \sqrt{24}$ $|PR| = \sqrt{19}$ $|PS| = \sqrt{51}$

$PQ \times PR = \begin{vmatrix} i & j & k \\ 4 & 2 & 2 \\ 3 & 3 & -1 \end{vmatrix}$




$V = |PS \cdot (PQ \times PR)|$

$PQ \times PR = \begin{vmatrix} 2 & 2 \\ 3 & -1 \end{vmatrix} i - \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix} j + \begin{vmatrix} 4 & 2 \\ 3 & 3 \end{vmatrix} k$

$= -8i + 10j + 6k$

$= \langle -8, 10, 6 \rangle$

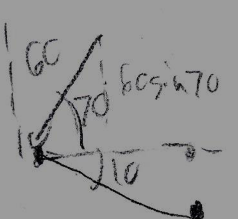
$V = |\langle 5,5,1 \rangle \cdot \langle -8,10,6 \rangle|$

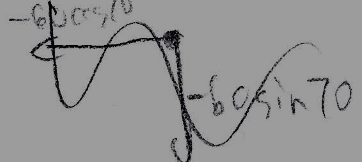
$V = |-40 + 50 + 6|$

$\boxed{V = 16 \text{ units}^3}$

(39) $\theta = 10$ $|T| = |F||r| \sin\theta$

$r = 18$




$|F| = 60N$

$|r| = \frac{18cm}{100cm} = 0.18m$

$\theta = 70 + 10 = 80$

$|T| = |60||0.18||\sin 80|$

$|T| = 10.8 \sin 80$

$\boxed{|T| = 10.64 \, N \cdot m}$

(43) $a \cdot b = \sqrt{3}$ $\quad a \times b = \langle 1,2,2 \rangle$

$a \cdot b = |a||b| \cos\theta$ $\qquad |a \times b| = |a||b| \sin\theta$

$|a||b| = \dfrac{a \cdot b}{\cos\theta}$ $\qquad |a||b| = \dfrac{|a \times b|}{\sin\theta}$

$$\frac{a \cdot b}{\cos\theta} = \frac{|a \times b|}{\sin\theta}$$

$$(a \cdot b)\tan\theta = |a \times b|$$

$$\tan\theta = \frac{|a \times b|}{a \cdot b}$$

$a \cdot b = \sqrt{3}$

$$\theta = \tan^{-1}\left(\frac{|a \times b|}{a \cdot b}\right)$$

$|a \times b| = \sqrt{4+4+1} = \sqrt{9} = 3$

$$\theta = \tan^{-1}\left(\frac{3}{\sqrt{3}}\right) = \tan^{-1}\left(\sqrt{3}\right)$$

$$\theta = \frac{\pi}{3} \cdot \frac{180}{\pi} = 60^\circ$$

$$\boxed{\theta = 60^\circ}$$

12.5 homework 3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59, 63, 67, 71, 73

Sumeeth Guda

3. $(2, 2.4, 3.5)$ $\langle 3, 2, -1 \rangle$

$\boxed{r = \langle 2, 2.4, 3.5 \rangle + t\langle 3, 2, -1 \rangle}$

$r = \langle 2+3t, 2.4+2t, 3.5-t \rangle$

$\boxed{x = 2+3t \quad y = 2.4+2t \quad z = 3.5-t}$

7. $(0, \frac{1}{2}, 1)$ $(2, 1, -3)$

$\langle 2, \frac{1}{2}, -4 \rangle$

$r = \langle 2, 1, -3 \rangle + t\langle 2, \frac{1}{2}, -4 \rangle$

$r = \langle 2+2t, 1+\frac{1}{2}t, 1-4t \rangle$

$x = 2t+2 \quad y = 1+\frac{1}{2}t, \quad z = 3-4t$

$t = \frac{x-2}{2} \quad 2y-2 = t \quad \frac{z+3}{-4} = t$

$\boxed{\frac{x-2}{2} = 2y-2 = \frac{z+3}{-4}}$

11. $(-6,2,3)$ $\quad \frac{x}{2} = \frac{y}{3} = z+1$

$\frac{x}{2} = t =$ $\quad y = 3t \quad z = t$

$x = 2t$

$$\boxed{< -6+2t,\ 2+3t,\ t+3 >}$$

$\frac{x+6}{2} = t \quad \frac{y-2}{3} = t \quad z-3 = t$

$$\boxed{\frac{x+6}{2} = \frac{y-2}{3} = z-3}$$

15. a. $<1,-5,6> + t<-1,2,-3> = <1-t, -5+2t, 6-3t>$

$x = 1-t \quad y = -5+2t \quad z = 6-3t$

$t = 1-x \quad \frac{y+5}{2} = t \quad \frac{z-6}{-3} = t$

$$\boxed{1-x = \frac{y+5}{2} = \frac{z-6}{-3}}$$

b. $x = 0$

$1 = \frac{y+5}{2} = \frac{z-6}{-3}$ $\quad (0,-3,3)$

$y = -3 \quad z = 3$

$y = 0$

$1-x = \frac{5}{2} = \frac{z-6}{-3}$ $\quad (-\frac{3}{2}, 0, -\frac{3}{2}$

$x = -\frac{3}{2}$

$-\frac{15}{2} + 6 = z$

$-\frac{3}{2} = z$

15b, cont'd

$z=0$ $\quad 1-x = \frac{y+5}{2} = \frac{-6}{-3} = 2$ $\quad (-1,-1,0)$.

$x=-1 \quad y=-1$

$1-x=2 \quad y+5=4$

$x=-1 \quad y=-1$

$\boxed{\text{Points: } (0,-3,3),\ \left(\frac{-3}{2},0,\frac{-3}{2}\right), (-1,-1,0)}$

19. $L_1$: $x=3+2t \quad y=4-t \quad z=1+3t$

$L_2$: $x=1+4s \quad y=3-2s \quad z=4+5s$

$3+2t=1+4s$

$4-t=3-2s \quad\quad t=1+2s$

$1+3t=4+5s$

$1+3+6s=4+5s$

$s=0$

$t=1$

$3+2(1)=1$

$5\neq 1 \quad \boxed{\text{skew}}$

23. $(0,0,0) \quad \langle 1,-2,5\rangle$

$1(x-0)-2(y-0)+5(z-0)=0$

$\boxed{x-2y+5z=0}$

27. $(1,-1,-1)$ $\quad 5x-y-z=6$

$$5(x-1)-1(y+1)-1(z+1)=0$$

$$5x-5-y-1-z-1=0$$

$$\boxed{5x-y-z=7}$$

31. $(0,1,1),\ (1,0,1),\ (1,1,0)$

$\langle 1,1,0\rangle - \langle 1,0,1\rangle = \langle 0,1,-1\rangle$

$\langle 1,0,1\rangle - \langle 0,1,1\rangle = \langle 1,-1,0\rangle$

$$\langle 0,1,-1\rangle \times \langle 1,-1,0\rangle = \begin{vmatrix} i & j & k \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{vmatrix}$$

$$\begin{vmatrix} 1 & -1 \\ -1 & 0 \end{vmatrix} i - \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} j + \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} k$$

$$-i \quad -j \quad -k$$

$$-(x-0)-(y-1)-(z-1)=0$$

$$x+y+z-2=0 \qquad \boxed{x+y+z=2}$$

35. $(3,5,-1)$ $\quad x=4-t \quad y=2t-1 \quad z=-3t$

$t=0$
$t=1$

$P_0$ $t=0$: $x=4$ $y=-1$ $z=0$ $\quad (4,-1,0)$

$P_1$ $t=1$: $x=3$ $y=1$ $z=-3$ $\quad (3,1,-3)$ $\quad > \langle -1,2,-3\rangle$

$\vec{P_0} = \langle 4-3, -1-5, 1\rangle = \langle 1,-6,1\rangle$

$$\langle a,b,c\rangle = \langle -1,2,-3\rangle \times \langle 1,-6,1\rangle = \begin{vmatrix} i & j & k \\ -1 & 2 & -3 \\ 1 & -6 & 1 \end{vmatrix} = \begin{vmatrix} 2 & -3 \\ -6 & 1 \end{vmatrix} i - \begin{vmatrix} -1 & -3 \\ 1 & 1 \end{vmatrix} j + \begin{vmatrix} -1 & 2 \\ 1 & -6 \end{vmatrix} k$$

$$= \langle -16,-2,4\rangle$$

$$-16(x-3)-2(y-5)+4(z+1)=0 \Rightarrow -16x-2y+4z=\cancel{-6}\,2$$

$$\boxed{8x+y-2z=3}$$

(39) $(1,5,1)$ $\quad 2x+y-2z=2 \quad x+3z=4$

$\langle 2,1,-2\rangle \quad \langle 1,0,3\rangle$

$\langle 2,1,-2\rangle \times \langle 1,0,3\rangle = \begin{vmatrix} i & j & k \\ 2 & 1 & -2 \\ 1 & 0 & 3 \end{vmatrix}$

$\begin{vmatrix} 1 & -2 \\ 0 & 3 \end{vmatrix} i - \begin{vmatrix} 2 & -2 \\ 1 & 3 \end{vmatrix} j + \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} k$

$(1,5,1) \quad \langle 3,-8,-1\rangle$

$3(x-1)-8(y-5)-1(z-1)=0$

$3x-3-8y+40-z+1=0$

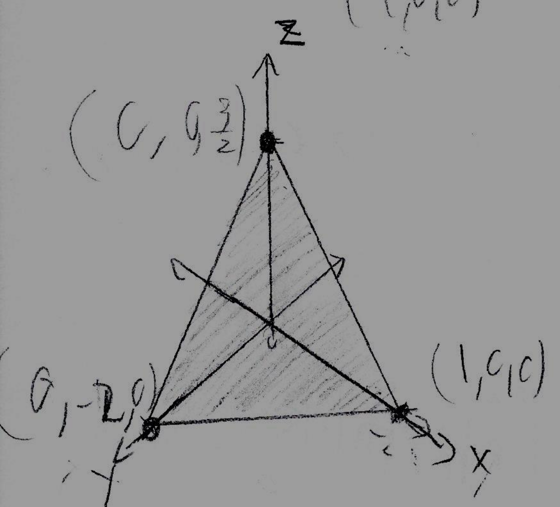
$\boxed{3x-8y-z=-38}$

(43) $6x-3y+4z=6$

$6x=6 \quad -3y=6 \quad 4z=6$

$x=1 \quad y=-2 \quad z=\frac{3}{2}$

$(1,0,0) \quad (0,-2,0) \quad (0,0,\frac{3}{2})$



47. $5x = y/2 = z+2$ $\quad 10x - 7y + 3z + 24 = 0$

$z = 0$

$5x = \frac{y}{2} = 2$ $\quad 10x - 7y + 24 = 0$

$x = \frac{2}{5}$ $y = 4$ $\quad 10\left(\frac{2}{5}\right) - 7(4) + 24 = 0$

$-24 + 24 = 0$ ✓

$\boxed{\left(\frac{2}{5}, 4, 0\right)}$

51. $x + 4y - 3z = 1$ $\quad -3x + 6y + 7z = 0$

$\langle 1, 4, -3 \rangle \cdot \langle -3, 6, 7 \rangle = -3 + 24 - 21 = 0$ $\boxed{\text{perpendicular}}$

55. $2x - 3y - z = 0$ $\quad 4x - 6y - 2z = 3$

$\langle 2, -3, -1 \rangle \cdot \langle 4, -6, -2 \rangle = 8 + 18 + 2 = 28$ Not perpendicular

$\langle 2, -3, -1 \rangle \times \langle 4, -6, 2 \rangle = 2\left(\langle 2, -3, -1 \rangle \times \langle 2, -3, -1 \rangle\right)$

$= 2(0) = 0$ $\boxed{\text{parallel}}$

59. $5x - 2y - 2z = 1$ $\quad 4x + y + z = 6$

$\langle 5, -2, -2 \rangle \times \langle 4, 1, 1 \rangle = \begin{vmatrix} i & j & k \\ 5 & -2 & -2 \\ 4 & 1 & 1 \end{vmatrix} = \begin{vmatrix} -2 & -2 \\ 1 & 1 \end{vmatrix} i - \begin{vmatrix} 5 & -2 \\ 4 & 1 \end{vmatrix} j + \begin{vmatrix} 5 & -2 \\ 4 & 1 \end{vmatrix} k$

$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$ $\quad \langle 0, -13, 13 \rangle = \langle 0, -1, 1 \rangle$

$5x - 2y = 1$

$4x + y = 6$

$(1, 2, 0)$

$\frac{y - 2}{-1} = \frac{z - 0}{1}$ $\quad x = x_0 + at$

$x = 1 + 0$

$\boxed{y - 2 = -z \quad , \quad x = 1}$

(63) x intercept = a  y intercept = b  z intercept = c

$(a,0,0)$  $(0,b,0)$  $(0,0,c)$

$t = \frac{x}{a}$  $t = \frac{y}{b}$  $t = \frac{z}{c}$

$ax + by + cz = ax_0 + by_0 + cz_0$

$(a,0,0)$ $(0,b,0)$ $(0,0,c)$

$\vec{AB} = \langle -a, b, 0 \rangle$  $\vec{Cb} = \langle 0, b, -c \rangle$

$$AB \times CB = \begin{vmatrix} i & j & k \\ -a & b & 0 \\ 0 & b & -c \end{vmatrix} = \begin{vmatrix} b & 0 \\ b & -c \end{vmatrix} i - \begin{vmatrix} -a & 0 \\ 0 & -c \end{vmatrix} j + \begin{vmatrix} -a & b \\ 0 & b \end{vmatrix} k$$

$$= -bc\,i - ac\,j - ab\,k$$

$$\langle -bc, -ac, -ab \rangle$$

$$-bc(x) - ac(y) - ab(z) = -bc(a) - ac(0) - ab(0)$$

$$-bc(x) - ac(y) - ab(z) = -abc$$

$$\boxed{\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1}$$

(67) $P_1$: $3x+6y-3z=6$ $\qquad \vec{P_1} = <3,6,-3> = 3<1,2,-1>$

$P_2 = 4x-12y+8z=5$ $\qquad \vec{P_2} = <4,-12,8> = 4<1,-3,2>$

$P_3 = 3x-9y+6z=-1$ $\qquad \vec{P_3} = <3,-9,6> = 3<1,-3,2>$

$P_4 = x+2y-z=2$ $\qquad \vec{P_4} = <1,2,-1> = 1<1,2,-1>$

$$\vec{P_2} \times \vec{P_3} = 4<1,-3,2> \times 3<1,-3,2> = 12(<1,-3,2> \times <1,-3,2>)$$
$$= 12(0) = 0$$

$\boxed{\text{Because } \vec{P_1} \times \vec{P_4} = 0 \text{ and because their scalar equations are multiples, therefore } P_1 \text{ and } P_4 \text{ are identical}}$

$\boxed{P_2 \text{ and } P_3 \text{ are parallel}}$

(71) $(1,-2,4)$ $\qquad 3x+2y+6z-5=0$

$$D = \frac{|a(x_1-x_0) + b(y_1-y_0) + c(z_1-z_0)|}{\sqrt{a^2+b^2+c^2}}$$

$$D = \frac{|ax_1+by_1+cz_1 - (ax_0+by_0+cz_0)|}{\sqrt{a^2+b^2+c^2}}$$

$$D = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}} = \frac{|3(1)+2(-2)+6(4)-5|}{\sqrt{9+4+36}} = \frac{|3-4+24-5|}{\sqrt{49}}$$

$$= \boxed{\frac{18}{7}}$$

73. $2x - 3y + z = 4$ $\quad \langle 2, -3, 1 \rangle$

$4x - 6y + 2z = 3$ $\quad \langle 4, -6, 2 \rangle = 2 \langle 2, -3, 1 \rangle$

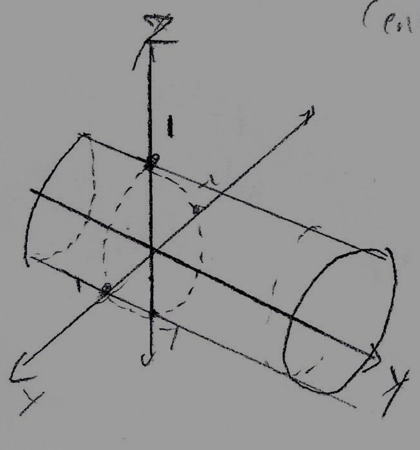
$2x = 4 \quad (2, 0, 0)$

$x = 2$

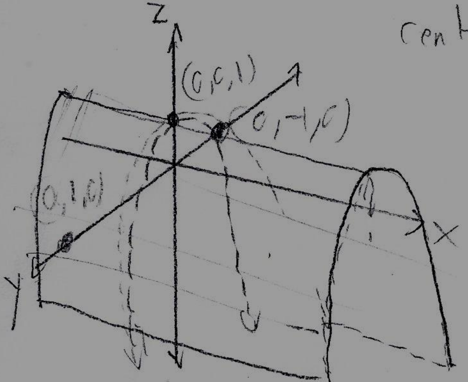
$$D = \frac{|4(2) - 6(0) + 2(0) - 3|}{\sqrt{16 + 36 + 4}} = \frac{|8 - 3|}{\sqrt{56}} = \boxed{\frac{5}{2\sqrt{14}}}$$

12.6 homework 3, 7, 11, 15, 19, 23, 27, 31, 35, 43, 45 Enmeeth Cuda

3. $x^2+z^2=1$ circular cylinder $r=1$ centered around y axis




7. $z=1-y^2$ parabolic cylinder centered on x axis

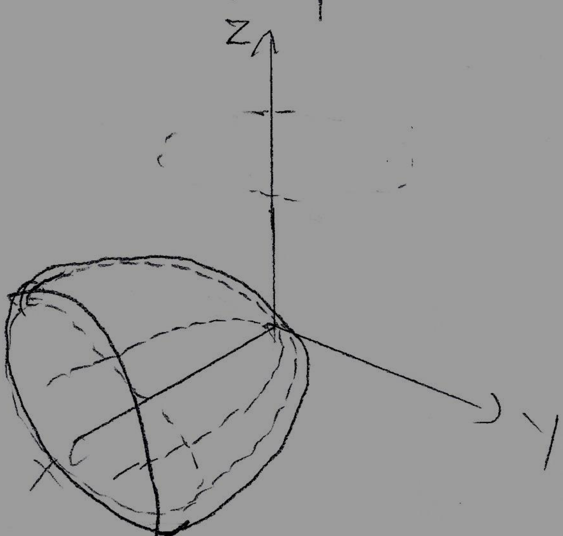



11. $x=y^2+4z^2$ ellipses around x

$x=k$ $k=y^2+4z^2$

$y=k$
$x=k^2+4z^2$
Paraboloid across y

$z=k$ Parabola across z
$x=y^2+4k^2$




Eliptic paraboloid center axis is the x axis.

(15) $9y^2+4z^2=x^2+36$

$x=k$

$9y^2+4z^2=k^2+36$

elipse centerd around x axis

$y=k$

$9k^2+4z^2=x^2+36$

$x^2-4z^2=9k^2-36$

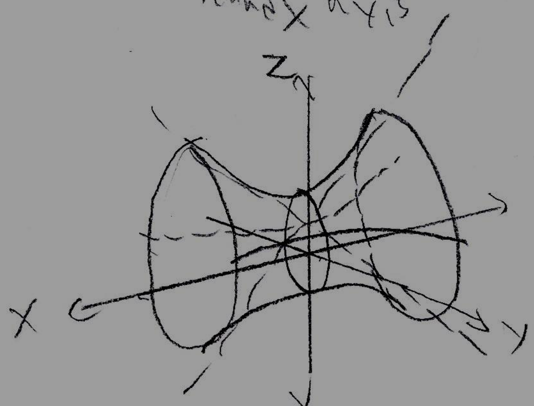
hyperbola centered around y axis

$z=k$

$9y^2+4k^2=x^2+36$

$x^2-9y^2=4k^2-36$

hyperbola around z




(19) $y=z^2-x^2$

$y=k$

$k=z^2-x^2$

hyperbola around y axis

$z=k$

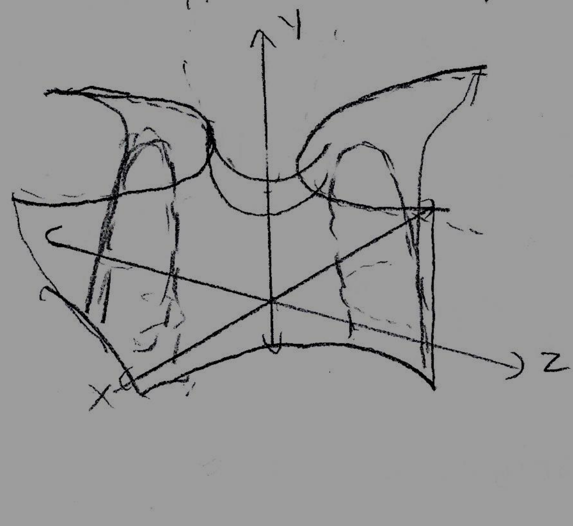
$y=k^2-x^2$

parabola around z axis

$x=k$

$y=z^2-k^2$

parabola around x axis



(23) $x^2-y^2+z^2=1$

$z=k$

$x^2-y^2+k^2=1$

$x^2-y^2=1-k^2$

hyperbola around z axis

$y=k$

$x^2-k^2+z^2=1$

$x^2+z^2=1+k^2$

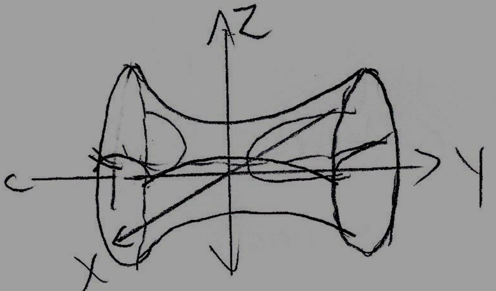
elipse around y axis

$x=k$

$k^2-y^2+z^2=1$

$z^2-y^2=1-k^2$

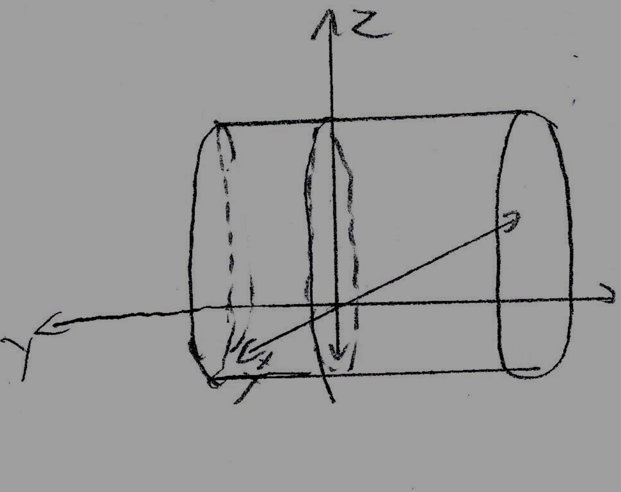
hyperbola around




Graph II is the graph most similar to $x^2-y^2+z^2=1$. Due to having hyperbolic traces around the x and z axis. As well as having elipsis cross sections across the y axis

(27) $x^2+2z^2=1$

The equation is a cylinder centered around the y axis with faces being elipsis. Graph ~~VIII~~ is the ~~[illegible]~~ graph for the function $x^2+2z^2=1$.



(31) $y^2 = x^2 + \frac{1}{9} z^2$

$\cancel{x} = k$

$y^2 = k^2 + \frac{1}{9} z^2$

$k^2 = y^2 - \frac{1}{9} z^2$

hyperbola around x axis

$y = k$

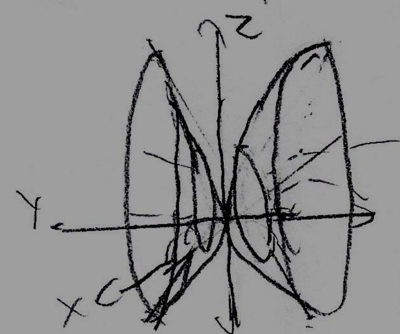
$k^2 = x^2 + \frac{1}{9} z^2$

elipsis around y axis

$z = k$

$y^2 - x^2 = \frac{1}{9} k^2$

hyperbola around z axis




elliptical cone centered around the y axis

(35) $x^2 + y^2 - 2x - 6y - z + 10 = 0$

$x^2 - 2x + y^2 - 6y = z - 10$

$(x-1)^2 + (y-3)^2 = z + 1 + 9 - 10$

$(x-1)^2 + (y-3)^2 = z$

$z = k$

$(x-1)^2 + (y-3)^2 = k$

elipsis around z

$x = k$

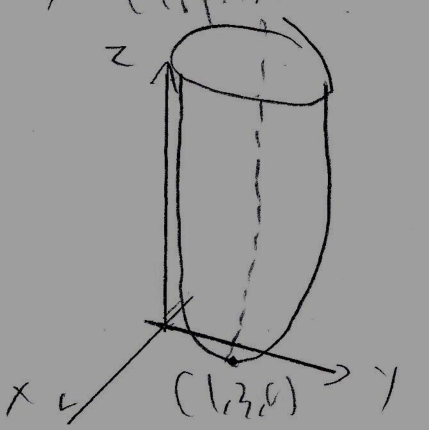
$(k-1)^2 = z - (y-3)^2$

line at $y = 3$

$y = k$

$(k-3)^2 = z - (x-1)^2$

line at $x = 1$




Paraboloid centered at (1,3,0) with axes of $x=1$, $y=3$

(43) $z = \sqrt{x^2+y^2}$ $\quad x^2+y^2=1$ $\quad z \in [1,2]$

$z^2 = x^2+y^2$

$\uparrow$ cylinder on z axis with radius 1

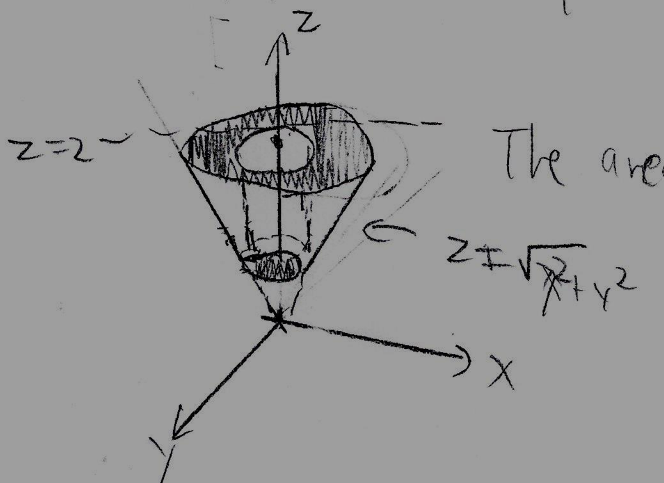
$x^2+y^2-z^2=0$

$x=k$ $\quad k^2 = z^2 - y^2$ hyperbolas on x axis

$y=k$ $\quad k^2 = z^2 - x^2$ hyperbolas on y axis

$z=k$ $\quad x^2+y^2=k^2$ elipsies on z axis.

total shape = cone.




The area of the cylinder is subtracted from the cone, there by yielding a cone where z is $\in [1,2]$.

(45) $y = \sqrt{x}$


equation for eliptic paraboloid

$$\frac{x}{a} = \frac{y^2}{b^2} + \frac{z^2}{c^2}$$

Rotation around the x axis for $\sqrt{x}$ implies that the cross sections are circles therefore $a, b, c = 1$.

$$\frac{x}{1} = \frac{y^2}{1} + \frac{z^2}{1}$$

$$x = y^2 + z^2$$

$$\boxed{x = y^2 + z^2}$$