

6.5 homework

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21

① a) $F(x, y, z) = \langle xy^2z^2, x^2yz^2, x^2y^2z \rangle$ $\text{curl } F = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \right) \mathbf{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} \right) \mathbf{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \mathbf{k}$
 $\text{curl } F = \langle (2x^2yz - 2x^2yz), (2xy^2z - 2xy^2z), (2xy^2z - 2xy^2z) \rangle = \langle 0, 0, 0 \rangle = \boxed{0}$

b) $\text{div } F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = \boxed{y^2z^2 + x^2z^2 + x^2y^2}$

③ a) $F(x, y, z) = \langle xy^ze^z, 0, yze^x \rangle$ $\text{curl } F = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \right) \mathbf{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} \right) \mathbf{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \mathbf{k}$
 $\text{curl } F = \langle (ze^x - 0), (xye^z - yze^x), (0 - xe^z) \rangle = \boxed{\langle ze^x, xye^z - yze^x, -xe^z \rangle}$

b) $\text{div } F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = ye^z + 0 + ye^x = \boxed{y(e^z + e^x)}$

⑤ a) $F(x, y, z) = \left\langle \frac{\sqrt{x}}{1+z}, \frac{\sqrt{y}}{1+x}, \frac{\sqrt{z}}{1+y} \right\rangle$
 $\text{curl } F = \left\langle \left(\frac{-\sqrt{z}}{(1+y)^2} - 0 \right), \left(\frac{-\sqrt{x}}{(1+z)^2} - 0 \right), \left(\frac{-\sqrt{y}}{(1+x)^2} - 0 \right) \right\rangle = \boxed{\left\langle \frac{-\sqrt{z}}{(1+y)^2}, \frac{-\sqrt{x}}{(1+z)^2}, \frac{-\sqrt{y}}{(1+x)^2} \right\rangle}$

b) $\text{div } F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = \boxed{\frac{1}{2(1+z)(\sqrt{x})} + \frac{1}{2(1+x)(\sqrt{y})} + \frac{1}{2(1+y)(\sqrt{z})}}$

⑦ a) $F(x, y, z) = \langle e^x \sin y, e^y \sin z, e^z \sin x \rangle$

$\text{curl } F = \langle (0 - e^y \cos z), (0 - e^z \cos x), (0 - e^x \cos y) \rangle = \boxed{\langle -e^y \cos z, -e^z \cos x, -e^x \cos y \rangle}$