(23.) $F(x,y)=x^3 i+y^3 j \quad P(1,0) \quad Q(2,2)$

$P_y=0 \quad Q_x=0$ conservative

$f(x,y)=\int x^3\,dx = \frac{1}{4}x^4+c$

$f(x,y)=\int y^3\,dy = \frac{1}{4}y^4+c \qquad f(x,y)=\frac{1}{4}x^4+\frac{1}{4}y^4+c$

$\int F\,dr = f(2,2)-f(1,0) = \left(\frac{16}{4}+\frac{16}{4}+c\right)-\left(\frac{1}{4}+0+c\right)=$

$(8+c)-\left(\frac{1}{4}+c\right)=7\frac{3}{4}=\boxed{\frac{31}{4}}$

(25.) Hypothetically speaking if a closed loop was drawn in the vector field, whose center is the origin. The amount of work in the CCW direction would be positive. While in the CW direction would be negative. In a conservative field there will be equal positive and negative work, or net zero work. This is a nonconservative field.

(29.) $F=\langle P,Q,R\rangle \quad P=\frac{\partial f}{\partial x} \quad Q=\frac{\partial f}{\partial y} \quad R=\frac{\partial f}{\partial z}$

$\frac{\partial P}{\partial y}=\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)=\frac{\partial^2 f}{\partial y\partial x}=\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)=\frac{\partial Q}{\partial x} \qquad \therefore \frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}$

$\frac{\partial P}{\partial z}=\frac{\partial}{\partial z}\left(\frac{\partial f}{\partial x}\right)=\frac{\partial^2 f}{\partial z\partial x}=\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial z}\right)=\frac{\partial R}{\partial x} \qquad \therefore \frac{\partial P}{\partial z}=\frac{\partial R}{\partial x}$

$\frac{\partial Q}{\partial z}=\frac{\partial}{\partial z}\left(\frac{\partial f}{\partial y}\right)=\frac{\partial^2 f}{\partial z\partial y}=\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial z}\right)=\frac{\partial R}{\partial y} \qquad \therefore \frac{\partial Q}{\partial z}=\frac{\partial R}{\partial z}$