

⑦ $y = 1-x^2$ $y=0$ $p(x,y) = k y$

$D = \{(x,y) \mid x \in [-1,1] \wedge y \in [0, 1-x^2]\}$

$m = \int_{-1}^1 \int_0^{1-x^2} k y dy dx = \frac{1}{2} \int_{-1}^1 k (1-x^2)^2 dx = \frac{k}{2} \int_{-1}^1 x^4 - 2x^2 + 1 dx$

$= \frac{k}{2} \left(2 \left(\frac{1}{5} - \frac{2}{3} + 1 \right) \right) = \boxed{\frac{8k}{15} = m}$

$\bar{x} = \frac{1}{m} \int_{-1}^1 \int_0^{1-x^2} x k y dy dx = \frac{k}{2m} \int_{-1}^1 x (1-x^2)^2 dx = \frac{k}{2m} \int_{-1}^1 x^5 - 2x^3 + x dx$ even function

$= \frac{k}{2m} (0) = 0$

$\bar{y} = \frac{1}{m} \int_{-1}^1 \int_0^{1-x^2} k y^2 dy dx = \frac{k}{3m} \int_{-1}^1 (1-x^2)^3 dx = \frac{k}{3m} \int_{-1}^1 1 - x^2 + \frac{3}{2}x^4 dx$

$= \frac{k}{3m} \left(2 \left(1 - \frac{1}{3} - 1 + \frac{3}{5} \right) \right) = \frac{1}{m} \left(\frac{32k}{105} \right) = \frac{15}{8k} \left(\frac{32k}{105} \right) = \frac{4}{7}$

$\boxed{(0, \frac{4}{7})}$

⑨ $D = \{(x,y) \mid x \in [0,1] \wedge y \in [0, e^{-x}]\}$

$m = \int_0^1 \int_0^{e^{-x}} x y dy dx = \int_0^1 \frac{1}{2} x e^{-2x} dx =$

$u=x \quad dv = \frac{e^{-2x}}{2} dx$
 $du=dx \quad v = -\frac{e^{-2x}}{4}$

$= \left[-\frac{e^{-2x}}{4} \right]_0^1 - \int_0^1 -\frac{e^{-2x}}{4} dx = -\frac{e^{-2}}{4} - \left[-\frac{e^{-2x}}{8} \right]_0^1 = \boxed{\frac{e^2-3}{8e^2} = m}$

$\bar{x} = \frac{1}{m} \int_0^1 \int_0^{e^{-x}} x^2 y dy dx = \frac{1}{m} \int_0^1 \frac{1}{2} x^2 e^{-2x} dx = \frac{1}{2m} \int_0^1 x^2 e^{-2x} dx = \frac{1}{2m} \left[\frac{-x^2 e^{-2x}}{2} + \int \frac{x e^{-2x}}{2} dx \right]_0^1$

$= \frac{1}{m} \left(-\frac{1}{4e^2} - \frac{1}{4e^2} - \int_0^1 \frac{e^{-2x}}{4} dx \right) = \frac{1}{m} \left(-\frac{1}{2e^2} - \left(\frac{1}{8e^2} - \frac{1}{8} \right) \right)$

$= \frac{8e^2}{e^2-3} \left(-\frac{4}{8e^2} - \frac{1}{8e^2} + \frac{e^2}{8e^2} \right) = \frac{e^2-5}{e^2-3}$