16.7 Homework 1, 3, 5, 9, 13, 17, 19, 23, 27, 31

(1) $\iint_S f(x,y,z) = \sum_{i=1}^{n} f(\bar{x},\bar{y},\bar{z})\,A(S_i)$ $\quad f(x,y,z) = \cos(x+2y+3z)$

$= 4\left[f(0,0,1) + f(0,1,0) + f(1,0,0) + f(-1,0,0) + f(0,-1,0) + f(0,0,-1)\right]$

$= 4\left[-0.9899 - 0.4161 + 0.5403 + 0.5403 - 0.4161 - 0.9899\right] \approx \boxed{-6.93}$

(3) $x^2+y^2+z^2 = 50$ $\quad r^2 = 50$ $\quad S = \frac{4\pi r^2}{8} = \frac{\pi(50)}{2} = 25\pi$

$\iint_S f(x,y,z) = \sum_{i=1}^{n} f(x_i,y_i,z_i)\,\Delta S_i = 25\pi(7+8+9+12)$

$= (25\cdot 36)\pi = \boxed{900\pi}$

(5) $\iint_S x+y+z\,dS$ $\quad x = u+v$ $\quad y = u-v$ $\quad z = 1+2u+v$ $\quad u \in [0,2]$ $\quad v \in [0,1]$

$x+y+z = 4u+1+v$ $\quad u = \langle 1,1,2\rangle$ $\quad v = \langle 1,-1,1\rangle$

$\|u \times v\| = \left\| \left\langle \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix}, -\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} \right\rangle \right\| = \|\langle 3,-1,-2\rangle\| = \boxed{\sqrt{9+1+4}} = \sqrt{14}$

$\sqrt{14}\int_0^2 \int_0^1 4u+1+v\,dv\,du = \sqrt{14}\int_0^2 \left[4uv+v+\frac{1}{2}v^2\right]_0^1 du = \sqrt{14}\int_0^2 4u+\frac{3}{2}\,du$

$= \sqrt{14}\left[2u^2+\frac{3}{2}u\right]_0^2 = \sqrt{14}(8+3) = \boxed{11\sqrt{14}}$