(27) $x^2+y^2=4$ $\qquad$ $4x^2+4y^2+z^2=64$

$r=2$ $\qquad$ $z=\pm\sqrt{64-4x^2-4y^2}$

$r\in[0,2]$ $\qquad$ $z=4\sqrt{16-x^2-y^2}=4\sqrt{16-r^2}$

$$\int_0^{2\pi}\int_0^2 4r\sqrt{16-r^2}\,dr\,d\theta \qquad u=16-r^2$$

$$du=-2r\,dr$$

$$\int_0^{2\pi}\int_{16}^{12} -2r\sqrt{u}\,du\,d\theta$$

$$\left(\int_0^{2\pi} d\theta\right)\left(\int_{12}^{16} 2\sqrt{u}\,du\right) = 4\pi\left(\frac{2}{3}(u)^{3/2}\Big]_{12}^{16}\right] = \frac{8\pi}{3}\left(16^{3/2}-12^{3/2}\right)$$

$$= \boxed{\frac{8\pi}{3}\left(64-24\sqrt{3}\right)}$$

(31) $$\int_0^{\frac{1}{2}}\int_{\sqrt{3}y}^{\sqrt{1-y^2}} x\,y^2\,dx\,dy$$

$x=\sqrt{1-y^2}$ $\qquad$ $\sqrt{1-y^2}=\sqrt{3}\,y$

$x^2+y^2=1$ $\qquad$ $1-y^2=3y^2$

$r=1$ $\qquad$ $1=4y^2$

$x=\sqrt{3}\,y$ $\qquad$ $y=\pm\frac{1}{2}$

$$\int_0^{\frac{\pi}{6}}\int_0^1 r^4\cos\theta\sin^2\theta\,dr\,d\theta$$

$\sin\theta=\frac{1}{2}$ $\quad$ $\theta=\frac{\pi}{6}$

$$\int_0^{\frac{\pi}{6}}\frac{1}{5}\cos\theta\sin^2\theta\,d\theta = \frac{1}{5}\left(\frac{\sin^3\theta}{3}\right)\Big|_0^{\frac{\pi}{6}} = \frac{1}{15}\sin^3\left(\frac{\pi}{6}\right) = \frac{1}{15}\left(\frac{1}{8}\right) = \boxed{\frac{1}{120}}$$