(13.) $x^2+y^2=1$ $\quad x^2+y^2=2$ $\qquad \tan v = \frac{y}{x}$

$1=\sqrt{x^2+y^2}$ $\quad \sqrt{2}=\sqrt{x^2+y^2}$ $\quad u=\sqrt{x^2+y^2}$ $\qquad v=\tan^{-1}\left(\frac{y}{x}\right)$

$u=1$ $\quad u=\sqrt{2}$

$v=\tan^{-1}(0)=0$

(1,0) (0,1) sclns to $x^2+y^2=1$ $\qquad v=\tan^{-1}\left(\frac{1}{0}\right)=\frac{\pi}{2}$

$S=\{(u,v) \mid u\in[1,\sqrt{2}] \wedge v\in[0,\frac{\pi}{2}]\}$

$y=x\tan v$ $\quad y^2=x^2\tan^2 v$

$u=\sqrt{x^2+x^2\tan^2 v} = x\sqrt{1+\tan^2 v} = x\sec v = u$

$\boxed{x=u\cos v}$

$\tan v = \frac{y}{u\cos v}$ $\quad y=u\cos v\,\frac{\sin v}{\cos v} = u\sin v$

$\boxed{y=u\sin v}$

(15) $\iint_R x-3y\,dA$ $\quad$ (0,0) (2,1) (1,2) $\quad x=2u+v$, $y=u+2v$ $\quad \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3 =$ Jacobian

$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$

(0,0) (2,1) $\to$ $x=2y$

(0,0) (1,2) $\to$ $y=2x$

(2,1) (1,2) $\to$ $x+y=3$

$2u+v=2u+4v$ $\quad v=0$ $\qquad v\in[0,1-u]$

$u+2v=4u+v$ $\quad u=0$ $\qquad u\in[0,1]$

$2u+2v+u+v=3$ $\quad v=1-u$

$$\iint_R (x-3y)\,dA = \int_0^1\int_0^{1-u}(2u+v-3u-6v)\,3\,dv\,du$$