

$$(23) F(x, y, z) = \langle xy, yz, zx \rangle \quad z = 4 - x^2 - y^2 \quad x \in [0, 1] \quad y \in [0, 1]$$

$$\iint_S F \cdot d\mathbf{s} = \iint_D \left( p \left( \frac{\partial z}{\partial x} \right) - q \left( \frac{\partial z}{\partial y} \right) + r \right) dA$$

$$= \iint_D -(xy)(-2x) - (yz)(-2y) + zx \, dA$$

$$= \int_0^1 \int_0^1 2x^2y + 2y^2z + xz \, dy \, dx$$

$$= \int_0^1 \int_0^1 2x^2y + 2y^2(4 - x^2 - y^2) + x(4 - x^2 - y^2) \, dy \, dx$$

$$= \int_0^1 \int_0^1 2x^2y + 8y^2 - 2x^2y^2 - 2y^4 + 4x - x^3 - xy^2 \, dy \, dx$$

$$= \int_0^1 \left[ x^2y^2 + \frac{8}{3}y^3 - \frac{2}{3}x^2y^3 - \frac{2}{5}y^5 + 4xy - x^3y - \frac{1}{3}xy^3 \right]_0^1 dx$$

$$= \int_0^1 \left( x^2 + \frac{8}{3} - \frac{2}{3}x^2 - \frac{2}{5} + 4x - x^3 - \frac{1}{3}x \right) dx = \int_0^1 \left( \frac{34}{15} + \frac{12}{3} + \frac{11x}{3} - x^3 \right) dx$$

$$= \left[ \frac{34}{15}x + \frac{x^3}{9} + \frac{11x^2}{6} - \frac{x^4}{4} \right]_0^1 = \frac{34}{15} + \frac{1}{9} + \frac{11}{6} - \frac{1}{4} = \boxed{\frac{713}{180}}$$