

$$\textcircled{5} \int_C x^2 y + \sin x \, dy \quad y = x^2 \quad dy = 2x \, dx \quad (0,0) \quad (\pi, \pi^2) \\ x \in [0, \pi]$$

$$= \int_C (x^4 + \sin x)(2x) \, dx = \int_C 2x^5 + 2x \sin x \, dx$$

$$= \int_0^\pi 2x^5 + 2x \sin x \, dx = \int_0^\pi 2x^5 \, dx + \int_0^\pi 2x \sin x \, dx$$

$$= \frac{2}{6} (\pi)^6 + \int_0^\pi 2x \sin x \, dx$$

$$\begin{array}{rcl} 2x & \sin x \, dx & -2x \cos x + 2 \sin x \Big|_0^\pi \\ 2 & -\cos x & + \\ 0 & -\sin x & - \end{array} \quad -2\pi(-1) + 0 - 0 - 0 = 2\pi$$

$$= \boxed{\frac{\pi^6}{3} + 2\pi}$$

$$\textcircled{7} \int_C (x+2x) \, dx + x^2 \, dy \quad (0,0) \quad (2,1) \quad (2,1) \quad (3,0) \\ y = \frac{1}{2}x \quad y = 3-x$$

$$C_1 \quad (0,0) \quad (2,1)$$

$$r(t) = (1-t)\langle 0,0 \rangle + t\langle 2,1 \rangle = \langle 2t, t \rangle, \quad t \in [0,1]$$

$$r'(t) = \langle 2, 1 \rangle$$

$$\int_{C_1} (x+2y) \, dx + x^2 \, dy = \int_0^1 (2t + 2(t))(2t) + (2t)^2 (1) \, dt \\ = \int_0^1 8t + 4t^2 \, dt = 4t^2 + \frac{4}{3}t^3 \Big|_0^1 = \frac{16}{3}$$

$$C_2 = \langle 3,0 \rangle - \langle 2,1 \rangle = \langle 1,-1 \rangle \quad r(t) = (1-t)\langle 2,1 \rangle + t\langle 3,0 \rangle = \langle 2+t, 1-t \rangle$$

$$\int_0^1 (2+t+2-(2+t)) \, dt - (2+t)^2 \, dt = \int_0^1 -5t - t^2 \, dt = -\frac{5}{2} - \frac{1}{3} = -\frac{17}{6} + \frac{16}{3} = \boxed{\frac{5}{2}}$$