15.4 homework 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23

(3) $D = \{(x,y) \mid x \in [1,3] \wedge y \in [1,4]\}$ $\rho(x,y) = ky^2$

$$m = \int_1^3 \int_1^4 ky^2\,dy\,dx = \int_1^3 \left[\tfrac{1}{3}ky^3\right]_1^4 dx = \int_1^3 \tfrac{1}{3}k(63)\,dx = \left[21kx\right]_1^3$$

$$\boxed{m = 42k}$$

$$\bar{x} = \frac{1}{m}\int_1^3\int_1^4 xky^2\,dy\,dx = \frac{1}{m}\int_1^3 x\,dx \int_1^4 ky^2\,dy = \frac{1}{m}\left(\frac{8}{2}\right)(21k)$$

$$\bar{x} = \frac{84k}{42k} = 2$$

$$\bar{y} = \frac{1}{m}\int_1^3\int_1^4 ky^3\,dy\,dx = \frac{1}{m}\int_1^3 dx \int_1^4 ky^3\,dy = \frac{1}{m}(2)\left(\frac{ky^4 - k}{4}\right)$$

$$\bar{y} = \frac{255k}{84k} = \frac{85}{28}$$

$$\boxed{\left(2, \frac{85}{28}\right)}$$

(5) $(0,0)$ $(2,1)$ $(0,3)$ $y = -(x-2)+1 = -x+3$ $\rho(x,y) = x+y$

$y = \frac{1}{2}x$ $\frac{1-3}{0-2} = -1$ $D = \{(x,y) \mid x \in [0,2] \wedge y \in [\frac{1}{2}x, -x+3]\}$

$$m = \int_0^2 \int_{\frac{x}{2}}^{-x+3} x+y\,dy\,dx = \int_0^2 \left[xy + \tfrac{1}{2}y^2\right]_{\frac{x}{2}}^{-x+3} dx = \int_0^2 \frac{(-x+3)^2}{2} + x(-x+3) - \left(\frac{x^2}{8} + \frac{x^2}{2}\right)dx$$

$$= \int_0^2 -\tfrac{9}{8}(x^2-4)\,dx = 6 \qquad \boxed{m = 6}$$

$$\bar{x} = \frac{1}{m}\int_0^2\int_{\frac{x}{2}}^{-x+3} x^2 + yx\,dy\,dx = \frac{1}{m}\int_0^2 \left[x^2y + \tfrac{1}{2}y^2x\right]_{\frac{x}{2}}^{-x+3} dx = \frac{1}{m}\int_0^2 -\tfrac{9}{8}(x^3 - 4x)\,dx = \frac{1}{6}\left(\frac{9}{2}\right)$$

$$= \frac{3}{4}$$

$$\bar{y} = \frac{1}{m}\int_0^2\int_{\frac{x}{2}}^{-x+3} xy + y^2\,dy\,dx = \frac{1}{m}\int_0^2 \left[\tfrac{1}{2}xy^2 + \tfrac{1}{3}y^3\right]_{\frac{x}{2}}^{-x+3} dx = \frac{1}{m}\int_0^2 \frac{(-x+3)^3}{3} + \frac{x(-x+3)^2}{2} - \left(\frac{x^3}{24} + \frac{x^3}{8}\right)dx$$

$$= \frac{1}{6}\int_0^2 9 - \frac{9x}{2}\,dx = \frac{3}{2}$$

$$\boxed{\left(\frac{3}{4}, \frac{3}{2}\right)}$$