(21) $p(x,y) = p \quad x \in [0,b] \quad y \in [0,h]$

$$I_x = \int_0^b \int_0^h y^2 p \, dy \, dx = \int_0^b dx \int_0^h y^2 p \, dy = \boxed{\frac{1}{3} b h^3 p}$$

$$I_y = \int_0^b \int_0^h x^2 p \, dy \, dx = \int_0^b x^2 dx \int_0^h p \, dy = ph \int_0^b x^2 dx = \boxed{\frac{1}{3} h b^3 p}$$

$$m = \int_0^b \int_0^h p \, dy \, dx = phb$$

$$m \bar{\bar{y}} = I_x = m(\bar{\bar{y}})^2 \qquad \bar{\bar{y}} = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{\frac{1}{3} b h^3 p}{phb}} = \sqrt{\frac{h^2}{3}} = \boxed{\frac{h}{\sqrt{3}}}$$

$$m \bar{\bar{x}} = I_y = m(\bar{\bar{x}})^2 \qquad \bar{\bar{x}} = \sqrt{\frac{I_y}{m}} = \sqrt{\frac{\frac{1}{3} b^3 h p}{phb}} = \boxed{\frac{b}{\sqrt{3}}}$$

(23) $x^2 + y^2 \le a^2$ $\quad r \in [0,a]$

$r = a \quad \theta \in [0, \frac{\pi}{2}]$

$$m = \int_0^{\frac{\pi}{2}} \int_0^a p r \, dr \, d\theta = \int_0^{\frac{\pi}{2}} \frac{1}{2} a^2 p \, d\theta = \frac{\pi}{4} a^2 p$$

$$I_x = \int_0^{\frac{\pi}{2}} \int_0^a r^3 \sin^2\theta \, p \, dr \, d\theta = p \int_0^{\frac{\pi}{2}} \sin^2\theta \, d\theta \int_0^a r^3 dr = p\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{2}} \left[\frac{a^4}{4}\right]$$

$$= p\left[\frac{\pi}{4} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{4}}\right]\left[\frac{a^4}{4}\right] = \frac{pa^4\pi}{16} \qquad \bar{\bar{y}} = \sqrt{\frac{\frac{pa^4\cancel{\pi}}{16}}{\frac{\cancel{\pi}}{4}a^2 \cancel{p}}} = \sqrt{\frac{a^2}{4}} = \frac{a}{2}$$

$$I_y = \int_0^{\frac{\pi}{2}} \int_0^a r^3 \cos^2\theta \, p \, dr \, d\theta = p \int_0^{\frac{\pi}{2}} \cos^2\theta \, d\theta \int_0^a r^3 dr = p\left[\frac{a^4}{4}\right] \int_0^{\frac{\pi}{2}} \cos^2\theta \, d\theta$$

$$= p\left[\frac{a^4}{4}\right]\left[\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{2}} = p\left[\frac{\pi}{4} + \frac{1}{4} - \frac{1}{4}\right]\left[\frac{a^4}{4}\right] = \frac{pa^4\pi}{16}$$

$$\bar{\bar{x}} = \frac{a}{2}$$

$$\boxed{I_x = I_y = \frac{pa^4\pi}{16} \qquad \bar{\bar{x}} = \bar{\bar{y}} = \frac{a}{2}}$$