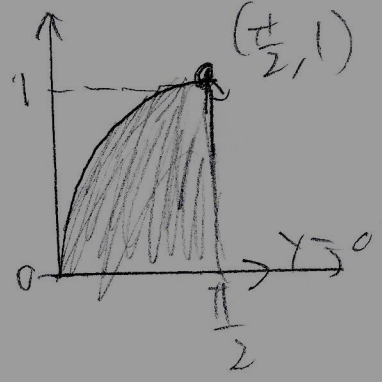
(59) $\int_0^1 \int_{\sin^{-1}(y)}^{\pi/2} \cos x \sqrt{1+\cos^2 x}\, dx\, dy$

$D = \{(x,y) \mid y \in [0,1] \wedge x \in [\sin^{-1}(y), \frac{\pi}{2}]\}$

$x = \sin^{-1}(y)$

$y = \sin x$



$D = \{(x,y) \mid x \in [0, \frac{\pi}{2}] \wedge y \in [0, \sin x]\}$

$$\int_0^{\frac{\pi}{2}} \int_0^{\sin x} \cos x \sqrt{1+\cos^2 x}\, dy\, dx = \int_0^{\frac{\pi}{2}} \sin x \cos x \sqrt{1+\cos^2 x}\, dx$$

$v = 1+\cos^2 x \qquad dv = -2\cos x \sin x\, dx$

$-\frac{1}{2} dv = \sin x \cos x\, dx$

$$-\frac{1}{2}\int_2^1 \sqrt{v}\, dv = -\frac{1}{2}\left(\frac{2}{3}(1)^{3/2} - \frac{2}{3}(2)^{3/2}\right) = \frac{2\sqrt{2}}{3} - \frac{1}{3} = \boxed{\frac{2\sqrt{2}-1}{3}}$$