

$$(12) \quad \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1} \quad \lim_{x \rightarrow 0} \frac{x^2 + m^2 x^2}{\sqrt{x^2 + m^2 x^2 + 1} - 1} = \lim_{x \rightarrow 0} \frac{(x^2 + m^2 x^2) (\sqrt{x^2 + m^2 x^2 + 1} + 1)}{(x^2 + m^2 x^2)}$$

$$\lim_{x \rightarrow 0} \sqrt{x^2 + m^2 x^2 + 1} + 1 = \sqrt{1} + 1 = 2$$

$$\lim_{y \rightarrow 0} \frac{m^2 y^2 + y^2}{\sqrt{m^2 y^2 + y^2 + 1} - 1} = \lim_{y \rightarrow 0} \frac{(m^2 y^2 + y^2) (\sqrt{m^2 y^2 + y^2 + 1} + 1)}{(m^2 y^2 + y^2)}$$

$$\lim_{y \rightarrow 0} \sqrt{m^2 y^2 + y^2 + 1} + 1 = \sqrt{1} + 1 = 2$$

$$\boxed{2}$$

$$(13) \quad z = xy \quad x^2 + y^2 = 1$$

$$\frac{dz}{dx} = y \quad \frac{dz}{dy} = x \quad r^2 = 1 \quad r \in [0, 1] \quad \theta \in [0, 2\pi]$$

$$A(S) = \sqrt{1 + y^2 + x^2} = \sqrt{1 + r^2} = \int_0^{2\pi} \int_0^1 r \sqrt{1 + r^2} dr d\theta$$

$$= \int_0^{2\pi} d\theta \int_0^1 r \sqrt{1 + r^2} dr = 2\pi \int_0^1 r \sqrt{1 + r^2} dr = \pi \int_1^2 \sqrt{u} du$$

$$u = 1 + r^2$$

$$du = 2r dr$$

$$\frac{1}{2} du = r dr$$

$$\pi \left(\frac{2}{3} (2)^{3/2} - \frac{2}{3} \right)$$

$$= \boxed{\frac{2\pi}{3} (2\sqrt{2} - 1)}$$