(21) $f(x,y) = y\sin(xy)$

$f_x = y^2\cos(xy)$ $\quad f_y = \sin(xy) + xy\cos(xy)$

$$\nabla f(x,y) = \langle y^2\cos(xy),\ \sin(xy) + xy\cos(xy)\rangle$$

(23) $f(x,y,z) = \sqrt{x^2+y^2+z^2}$

$$f_x = \frac{x}{\sqrt{x^2+y^2+z^2}} \qquad f_y = \frac{y}{\sqrt{x^2+y^2+z^2}} \qquad f_z = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$\nabla f(x,y,z) = \left\langle \frac{x}{\sqrt{x^2+y^2+z^2}},\ \frac{y}{\sqrt{x^2+y^2+z^2}},\ \frac{z}{\sqrt{x^2+y^2+z^2}} \right\rangle$$

(25) $f(x,y) = \frac{1}{2}(x-y)^2 = \frac{1}{2}x^2 - xy + \frac{1}{2}y^2$

$f_x = x - y$ $\quad f_y = y - x$ $\quad \nabla f(x,y) = \langle x-y,\ y-x\rangle$

$\nabla f(1,2) = \langle -1, 1\rangle$
$\nabla f(2,1) = \langle 1, -1\rangle$
$\nabla f(1,3) = \langle -2, 2\rangle$
$\nabla f(2,3) = \langle -1, 1\rangle$
$\nabla f(0,0) = \langle 0, 0\rangle$

$\nabla f(-1,-2) = \langle 1, -1\rangle$
$\nabla f(1,0) = \langle 1, -1\rangle$
$\nabla f(0,2) = \langle -2, 2\rangle$
$\nabla f(-2,0) = \langle -2, 2\rangle$
$\nabla f(0,-2) = \langle 2, -2\rangle$

$\nabla f(2,0) = \langle 2, -2\rangle$