

$$(9) \iint_S x^2 y z \, ds \quad z = 1 + 2x + 3y \quad [0, 3] \times [0, 2]$$

$$\frac{\partial z}{\partial x} = 2 \quad \frac{\partial z}{\partial y} = 3 \quad \sqrt{1 + 4 + 9} = \sqrt{14} = \frac{ds}{dy dx}$$

$$\int_0^3 \int_0^2 x^2 y (1 + 2x + 3y) \sqrt{14} \, dy \, dx$$

$$= \sqrt{14} \int_0^3 \int_0^2 x^2 y + 2x^3 y + 3x^2 y^2 \, dy \, dx = \sqrt{14} \int_0^3 \left[\frac{1}{2} x^2 y^2 + x^3 y + x^2 y^2 \right]_0^2 \, dx$$

$$= \sqrt{14} \int_0^3 2x^2 + 4x^3 + 8x^2 \, dx = \sqrt{14} \left[\frac{2}{3} x^3 + x^4 + \frac{8}{3} x^3 \right]_0^3 = \sqrt{14} \left[\frac{10}{3} (27) + 3^4 \right]$$

$$= \sqrt{14} (90 + 81) = \boxed{171\sqrt{14}}$$

$$(13) \iint_S z^2 \, ds \quad x = y^2 + z^2 \quad x \in [0, 1] \quad \frac{\partial x}{\partial y} = 2y \quad \frac{\partial x}{\partial z} = 2z$$

$$ds = \sqrt{1 + 4y^2 + 4z^2} \, dy \, dz$$

$$1 = y^2 + z^2$$

$$r^2 = 1 \quad r \in [0, 1]$$

$$z = r^2 \sin \theta$$

$$\int_0^{2\pi} \int_0^1 r^3 \sin^2 \theta \sqrt{1 + 4r^2} \, dr \, d\theta = \int_0^{2\pi} \sin^2 \theta \, d\theta \int_0^1 r^3 \sqrt{1 + 4r^2} \, dr$$

$$= \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_0^{2\pi} \cdot \int_0^1 r^3 \sqrt{1 + 4r^2} \, dr = \pi \int_0^1 r \cdot r^2 \sqrt{1 + 4r^2} \, dr$$

$$= \frac{\pi}{8} \int_1^5 \frac{u-1}{4} (\sqrt{u}) \, du = \frac{\pi}{32} \int_1^5 u^{3/2} - u^{1/2} \, du$$

$$u = 1 + 4r^2 \quad r^2 = \frac{u-1}{4}$$

$$du = 8r \, dr$$

$$r \, dr = \frac{du}{8}$$

$$= \frac{\pi}{32} \left[\frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} \right]_1^5 = \frac{\pi}{32} \left[\frac{2}{5} (25\sqrt{5}) - \frac{2}{3} (5\sqrt{5}) \right] - \frac{\pi}{32} \left[\frac{2}{5} - \frac{2}{3} \right]$$

$$= \boxed{\frac{\pi}{120} (25\sqrt{5} + 1)}$$