21b) $a=b=c=6378 \quad V \approx \frac{4\pi}{3}(6378)^3 \approx \boxed{1.083 \times 10^{12}\ km^3}$

21c) $I_z = \iiint_E k\,dV = \iiint_{E'} k(x^2+y^2)(abc)\,du\,dv\,dw$

$= \iiint_{u^2+v^2+w^2\le 1} k(a^2u^2 + b^2v^2)(abc)\,du\,dv\,dw \qquad u = \rho\sin\phi\cos\theta \quad v = \rho\sin\phi\sin\theta$

$= abck \int_0^{\pi}\int_0^{2\pi}\int_0^1 (a^2\rho^2\sin^2\phi\cos^2\theta + b^2\rho^2\sin^2\phi\sin^2\theta)\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$

$= abck\left(a^2\int_0^{\pi}\int_0^{2\pi}\int_0^1 \rho^4\sin^3\phi\cos^2\theta\,d\rho\,d\theta\,d\phi + b^2\int_0^{\pi}\int_0^{2\pi}\int_0^1 \rho^4\sin^3\phi\sin^2\theta\,d\rho\,d\theta\,d\phi\right)$

$= \frac{abck}{5}\left(a^2\int_0^{\pi}\sin^3\phi\,d\phi\int_0^{2\pi}\cos^2\theta\,d\theta + b^2\int_0^{\pi}\sin^3\phi\,d\phi\int_0^{2\pi}\sin^2\theta\,d\theta\right)$

$= \frac{abck}{5}\left(a^2\left(\frac{1}{3}\cos^3\phi - \cos\phi\right)_0^{\pi}\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{2\pi} + b^2\left(\frac{\cos^3\phi}{3} - \cos\phi\right)_0^{\pi}\left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi}\right)$

$= \frac{abck}{5}\left(a^2\left(\frac{4}{3}\right)(\pi) + b^2\left(\frac{4}{3}\right)(\pi)\right) = \boxed{\frac{4\pi abc}{15}(a^2+b^2)}$

23) $\iint_R \frac{x-2y}{3x-y}\,dA \qquad x-2y=0 \quad x-2y=4 \quad 3x-y=1 \quad 3x-y=8$

$u = x-2y \qquad v = 3x-y$

$u \in [0,4] \qquad v \in [1,8]$

$\int_0^4\int_1^8 \frac{u}{v}\,dv\,du = \int_0^4 u\,du\int_1^8 \frac{1}{v}\,dv = \left(\frac{1}{2}u^2\right]_0^4\left(\ln|v|\right]_1^8$

$= 8\ln|8| \qquad \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} 1 & -2 \\ 3 & -1 \end{vmatrix} = -1+6 = 5 = J_{T^{-1}}$

$J_T = \frac{1}{5} \qquad \boxed{\frac{8}{5}\ln|8|}$