

$$\textcircled{31} F(x, y, z) = \langle x^2, y^2, z^2 \rangle \quad z \in [0, \sqrt{1-y^2}] \quad x \in [0, 2]$$

$$S = S_1 + S_2 + S_3 + S_4$$

$$\begin{aligned} S_1) \quad dS &= h dS = dS = -i dS = \iint_{S_1} F \cdot dS = \iint_{S_1} (x^2 i + y^2 j + z^2 k) \cdot (-i) dS \\ &= \iint_{S_1} -x^2 dS = \iint_{S_1} 0^2 dS = 0 = S_1 \end{aligned}$$

$$\begin{aligned} S_2) \quad dS &= 1 dS \quad \iint_{S_2} F \cdot dS = \iint_{S_2} (x^2 i + y^2 j + z^2 k) \cdot 1 dS = \iint_{S_2} x^2 dS \\ &= 4 \iint_{S_2} dS = 4 \left(\underbrace{\pi (1)^2}_{\text{area of surface}} \right) = 2\pi \end{aligned}$$

$$\begin{aligned} S_3) \quad dS &= n dS \quad n = -k \quad \iint_{S_3} F \cdot dS = \iint_{S_3} \langle x^2, y^2, z^2 \rangle \cdot -k dS \\ &= - \iint_{S_3} z^2 dS = -0 = 0 \end{aligned}$$

$$\begin{aligned} S_4) \quad y^2 + z^2 &= 1 \quad x=0 \quad x=2 \quad z=0 \quad \forall \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \quad \forall r \in [0, 2] \\ z &= \cos \theta \quad y = \sin \theta \quad x = r \\ r &= \langle r, \sin \theta, \cos \theta \rangle \\ r_\theta &= \langle 0, \cos \theta, -\sin \theta \rangle \\ r_r &= \langle 1, 0, 0 \rangle \\ r_\theta \times r_r &= \begin{vmatrix} \cos \theta & -\sin \theta & 0 \\ 0 & 0 & -\sin \theta \\ 1 & 0 & 0 \end{vmatrix} = \begin{vmatrix} \cos \theta & -\sin \theta \\ 0 & -\sin \theta \end{vmatrix} = \langle 0, -\sin \theta, -\cos \theta \rangle \\ dS &= -(r_\theta \times r_r) dA = \langle 0, \sin \theta, \cos \theta \rangle dA \end{aligned}$$