(10) $f(x,y) = 4y\sqrt{x} \quad (4,1)$

$f_x(x,y) = \frac{2y}{\sqrt{x}} \quad f_y(x,y) = 4\sqrt{x} \qquad \nabla f(4,1) = <1,8>$

$f_x(4,1) = \frac{2}{\sqrt{4}} = 1 \quad f_y(4,1) = 4\sqrt{4} = 8$

max rate of change $= |\nabla f(4,1)| = |<1,8>| = \sqrt{1+64} = \boxed{\sqrt{65} \text{ in direction } <1,8>}$

(11) $\int_C F \cdot dr \quad F(x,y,z) = <\sin x, \cos y, xz> \quad r(t) = <t^3, -t^2, t>$

$t \in [0, \pi]$

$\frac{dr}{dt} = <3t^2, -2t, 1>$

$F(r(t)) = <\sin(t^3), \cos(-t^2), t^4>$

$\int_C F \cdot dr = <\sin(t^3), \cos(-t^2), t^4> \cdot <3t^2, -2t, 1> = 3t^2 \sin(t^3) - 2t\cos(t^2) + t^4$

$\int_0^{\pi} 3t^2 \sin(t^3) - 2t\cos(t^2) + t^4 \, dt$

$= \left[-\cos(t^3) - \sin(t^2) + \frac{t^5}{5}\right]_0^{\pi} =$

$-\cos(\pi^3) - \sin(\pi^2) + \frac{\pi^5}{5} - (-1 - 0 + 0)$

$\boxed{\frac{\pi^5}{5} - \cos(\pi^3) - \sin(\pi^2) + 1}$