⑨ $\int_C \vec{F} \cdot d\vec{r} \qquad F(x,y,z) = \langle xy, yz, zx \rangle \qquad z = 1 - x^2 - y^2$

$$\text{curl} F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & zx \end{vmatrix} = \langle -y, -z, -x \rangle$$

$r = \langle x, y, 1 - x^2 - y^2 \rangle \qquad x \geq 0 \quad y \geq 0 \quad x^2 + y^2 \leq 1$

$r_x = \langle 1, 0, -2x \rangle \qquad r_y = \langle 0, 1, -2y \rangle$

$$r_x \times r_y = \left\langle \begin{vmatrix} 0 & -2x \\ 1 & -2y \end{vmatrix}, -\begin{vmatrix} 1 & -2x \\ 0 & -2y \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \right\rangle = \langle 2x, 2y, 1 \rangle$$

$$\text{curl} F \cdot (r_x \times r_y) = \langle -y, -z, -x \rangle \cdot \langle 2x, 2y, 1 \rangle = -2xy - 2yz - x$$

$$= -2xy - 2y(1 - x^2 - y^2) - x = -2xy - 2y + 2yx^2 + 2y^3 - x$$

$$= 2xy(x-1) + 2y(y^2 - 1) - x \qquad x = r\cos\theta \quad y = r\sin\theta$$

$$= 2r^2 \sin\theta\cos\theta(r\cos\theta - 1) + 2r\sin\theta(r^2\sin^2\theta - 1) - r\cos\theta$$

$$\int_0^{\frac{\pi}{2}} \int_0^1 2r^3 \sin\theta\cos\theta(r\cos\theta - 1) + 2r^2\sin\theta(r^2\sin^2\theta - 1) - r^2\cos\theta \, dr \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \int_0^1 2r^4 \sin\theta\cos^2\theta - 2r^3 \sin\theta\cos\theta + 2r^4 \sin^3\theta - 2r^2\sin\theta - r^2\cos\theta \, dr \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left[\frac{2}{5} r^5 \sin\theta\cos^2\theta - \frac{r^4}{2} \sin\theta\cos\theta + \frac{2}{5} r^5 \sin^3\theta - \frac{2}{3} r^3 \sin\theta - \frac{1}{3} r^3 \cos\theta \right]_0^1 d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{2}{5} \sin\theta\cos^2\theta - \frac{1}{2} \sin\theta\cos\theta + \frac{2}{5} \sin^3\theta - \frac{2}{3} \sin\theta - \frac{1}{3} \cos\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{2}{5} \sin\theta\cos^2\theta \, d\theta - \frac{1}{2} \int_0^{\frac{\pi}{2}} \sin\theta\cos\theta \, d\theta + \frac{2}{5} \int_0^{\frac{\pi}{2}} \sin^3\theta \, d\theta - \int_0^{\frac{\pi}{2}} \frac{2}{3} \sin\theta + \frac{1}{3} \cos\theta \, d\theta$$