

$$(3) \quad x^2 + y^2 = 4 \quad 4x^2 + 4y^2 + z^2 = 64$$

$$r^2 = 4$$

$$r = 2$$

$$r \in [0, 2]$$

$$\theta \in [0, 2\pi]$$

$$z^2 = 64 - 4x^2 - 4y^2 = 64 - 4r^2$$

$$z = \pm \sqrt{64 - 4r^2} = \pm 2\sqrt{16 - r^2}$$

$$f(r, \theta) = 4\sqrt{16 - r^2}$$

$$D = \{ (r, \theta) \mid \theta \in [0, 2\pi] \wedge r \in [0, 2] \}$$

$$\begin{aligned} \int_0^{2\pi} \int_0^2 4r\sqrt{16-r^2} \, dr \, d\theta &= -2 \int_0^{2\pi} d\theta \int_{16}^0 \sqrt{u} \, du = -4\pi \left[ \frac{2}{3} u^{3/2} \right]_{16}^0 \\ &= 4\pi \left[ \frac{2}{3} (16)^{3/2} - \frac{2}{3} (12)^{3/2} \right] \end{aligned}$$

$v = 16 - r^2$   
 $dv = -2r \, dr$   
 $-\frac{1}{2} dv = r \, dr$

$$= \frac{8\pi}{3} (64 - \sqrt{1728}) = \frac{8\pi}{3} (64 - 24\sqrt{3}) = \boxed{\frac{64\pi}{3} (8 - 3\sqrt{3})}$$

$$(4) \quad z = xy \quad x^2 + y^2 = 1$$

$$r^2 = 1$$

$$r \in [0, 1] \quad \theta \in [0, 2\pi]$$

$$\frac{\partial z}{\partial x} = y \quad \frac{\partial z}{\partial y} = x$$

$$A(s) = \sqrt{1 + y^2 + x^2}$$

$$A(s) = \sqrt{1 + r^2}$$

$$\begin{aligned} \int_0^{2\pi} \int_0^1 r \sqrt{1+r^2} \, dr \, d\theta &= \int_0^{2\pi} d\theta \int_0^1 r \sqrt{1+r^2} \, dr = 2\pi \int_0^1 r \sqrt{1+r^2} \, dr \\ &= \pi \int_1^2 \sqrt{u} \, du = \pi \left[ \frac{2}{3} (2)^{3/2} - \frac{2}{3} \right] \\ &= \boxed{\frac{2\pi}{3} (2\sqrt{2} - 1)} \end{aligned}$$

$v = 1 + r^2$   
 $dv = 2r \, dr$   
 $\frac{1}{2} dv = r \, dr$