(19) $\iint_R xy\,dA \qquad y=x \quad y=3x \quad xy=1 \quad xy=3 \quad x=\frac{u}{v} \quad y=v$

$u=1 \quad u=3$

$xy=\frac{u}{v}(v)=u \qquad \frac{u}{v}=v \quad \frac{3u}{v}=v$

$v^2=u \qquad v=\pm\sqrt{3u}$

$v=\pm\sqrt{u}$

$v=\sqrt{u} \quad v=\sqrt{3u}$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{v} & -\frac{u}{v^2} \\ 0 & 1 \end{vmatrix} = \frac{1}{v}$$

$$\int_1^3 \int_{\sqrt{u}}^{\sqrt{3u}} \frac{u}{v}\,dv\,du = \int_1^3 \left[u \ln|v|\right]_{\sqrt{u}}^{\sqrt{3u}} du$$

$$= \int_1^3 u\left[\ln|\sqrt{3u}| - \ln|\sqrt{u}|\right] du = \int_1^3 u \ln\sqrt{3}\left(\frac{\sqrt{u}}{\sqrt{u}}\right) du$$

$$= \ln\sqrt{3}\left[\frac{u^2}{2}\right]_1^3 = \ln\sqrt{3}\left[\frac{9}{2}-\frac{1}{2}\right] = 4\ln\sqrt{3} = \boxed{2\ln 3}$$

(21a) $\iiint_E dV \qquad \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \qquad x=au \quad y=bv \quad z=cw$

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} = \begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix} = \begin{vmatrix} b & 0 \\ 0 & c \end{vmatrix} a = abc$$

$$\frac{a^2u^2}{a^2}+\frac{b^2v^2}{b^2}+\frac{c^2w^2}{c^2}=1 \qquad \iiint_R dV = \frac{4}{3}\pi(1)^3 = \frac{4\pi}{3}$$

$$u^2+v^2+w^2=1$$

$$\iiint_E dV = abc\iiint_R dV = \boxed{\frac{4\pi abc}{3}} = V_{sphere}$$