(19) $\text{curl}\, G = \langle x\sin y, \cos y, z - xy \rangle$

$$\text{div}(\text{curl}\, G) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = \sin y - \sin y + 1 = 1$$

A vector field can only exist if div Curl $G = 0$. Because $\text{div}(\langle x\sin y, \cos y, z - xy \rangle) = 1 \neq 0$ therefore vector field G does not exist. $\therefore$ QED

(21) $F(x,y,z) = \langle f(x), g(y), h(z) \rangle$

$$\text{curl}\, F = \left\langle \left(\frac{\partial h(z)}{\partial y} - \frac{\partial g(y)}{\partial z}\right), \left(\frac{\partial f(x)}{\partial z} - \frac{\partial (h(z))}{\partial x}\right), \left(\frac{\partial (g(y))}{\partial x} - \frac{\partial (f(x))}{\partial y}\right) \right\rangle$$

$$= \langle (0-0), (0-0), (0-0) \rangle = 0$$

Since every partial derivative is zero due to each function being matched with a function that doesn't change by a particular variable, therefore F is irrotational.