(41) $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy \, dz\, dy\, dx$

$z \in [\sqrt{x^2+y^2}, \sqrt{2-x^2-y^2}] \quad y \in [0, \sqrt{1-x^2}] \quad x \in [0,1]$

$\rho\cos\phi \in [\rho\sin\phi, \sqrt{2-\rho^2\sin^2\phi}] \quad y \in [0, \sqrt{1-\rho^2\sin^2\phi\cos^2\theta}] \quad x \in [0,1]$

$\rho\sin\phi\sin\theta \in [0, \sqrt{1-\rho^2\sin^2\phi\cos^2\theta}] \quad \rho\sin\phi\cos\theta \in [0,1]$

$\rho\sin\phi \in [0, \sec\theta]$

$\rho \in [0, \sec\theta\csc\phi]$

$[0, \sec\theta\csc\phi]\cos\phi \in = [0, \sqrt{2}]$

$\rho \in [0, \sqrt{2}] \qquad \sin\theta \in [0, 1] \quad \sqrt{2}\sin\phi = [0,1]$

$\theta \in [0, \frac{\pi}{2}] \qquad \phi \in [0, \frac{\pi}{4}]$

$\int_0^{\frac{\pi}{4}} \int_0^{\frac{\pi}{2}} \int_0^{\sqrt{2}} (\rho\sin\phi\cos\theta)(\rho\sin\phi\sin\theta)\,\rho^2\sin\phi \, d\rho\, d\theta\, d\phi$

$= \int_0^{\frac{\pi}{4}} \int_0^{\frac{\pi}{2}} \int_0^{\sqrt{2}} \rho^4 \sin\theta\cos\theta \cdot \sin^3\phi \, d\rho\, d\theta\, d\phi = \int_0^{\sqrt{2}} \rho^4 d\rho \cdot \int_0^{\frac{\pi}{2}} \sin\theta\cos\theta \cdot \int_0^{\frac{\pi}{4}} \sin^3\phi\, d\phi$

$= \left[\frac{\rho^5}{5}\right]_0^{\sqrt{2}} \cdot \left[\frac{\sin^2\theta}{2}\right]_0^{\frac{\pi}{2}} \cdot \left[\frac{\cos^3\phi}{3} - \cos\phi\right]_0^{\frac{\pi}{4}} = \frac{4\sqrt{2}}{5} \cdot \frac{1}{2}\left(\frac{\sqrt{2}}{12} - \frac{\sqrt{2}}{2} - \frac{1}{3} + 1\right)$

$= \frac{2\sqrt{2}}{5}\left(\frac{2}{3} - \frac{5\sqrt{2}}{12}\right) = \boxed{\frac{4\sqrt{2}}{15} - \frac{1}{3}}$