MATH 073 Exam 3

① $\int_0^1 \int_0^1 v(u+v^2)^4 \, du \, dv = \int_0^1 \int_0^1 v(u^4+4u^3v^2+6u^2v^4+4uv^6+v^8) \, du \, dv$

$= \int_0^1 \int_0^1 (u^4v+4u^3v^3+6u^2v^5+4uv^7+v^9) \, du \, dv$

$= \int_0^1 \left[\frac{1}{5}u^5v + u^4v^3 + 2u^3v^5 + 2u^2v^7 + v^9u\right]_0^1 dv$

$= \int_0^1 \left[\frac{1}{5}v + v^3 + 2v^5 + 2v^7 + v^9\right] dv$

$= \left[\frac{1}{10}v^2 + \frac{1}{4}v^4 + \frac{1}{3}v^6 + \frac{1}{4}v^8 + \frac{1}{10}v^{10}\right]_0^1 = \left[\frac{1}{10}+\frac{1}{4}+\frac{1}{3}+\frac{1}{4}+\frac{1}{10}\right]$

$= \left[\frac{1}{5}+\frac{1}{2}+\frac{1}{3}\right]$

$= \frac{7}{10}+\frac{1}{3} = \boxed{\frac{31}{30}}$

② $\iint_D e^{-y^2} \, dA \qquad D = \{(x,y) \mid y \in [0,3] \wedge x \in [0,y]\}$

$\int_0^3 \int_0^y e^{-y^2} \, dx \, dy = \int_0^3 \left[e^{-y^2}x\right]_0^y dy = \int_0^3 ye^{-y^2} \, dy = -\frac{1}{2}\int_0^{-9} e^u \, du$

$u = -y^2$

$du = -2y \, dy$

$-\frac{1}{2} du = y \, dy$

$= -\frac{1}{2}e^{-9} + \frac{1}{2}e^0$

$= \frac{1}{2} - \frac{1}{2}e^{-9}$

$= \boxed{\frac{1}{2}(1-e^{-9})}$