

(25) a.  $z = 24 - x^2 - y^2 = 24 - r^2 \quad z \in [2r, 24 - r^2]$

$$z = 2\sqrt{x^2 + y^2} = 2\sqrt{r^2} = 2r$$

$$2r = 24 - r^2$$

$$r^2 + 2r - 24 = 0$$

$$\theta \in [0, 2\pi]$$

$$(r+6)(r-4) = 0 \quad r \in [0, 4]$$

$$r = -6 \quad r = 4$$

$$\int_0^{24} \int_0^4 \int_{2r}^{24-r^2} r \, dz \, dr \, d\theta = \int_0^{2\pi} d\theta \int_0^4 (24r - r^3 - 2r^2) \, dr$$

$$= 2\pi \left[ 12r^2 - \frac{1}{4}r^4 - \frac{2}{3}r^3 \right]_0^4 = 2\pi \left[ 12(16) - 64 - \frac{2}{3}(64) \right] = \boxed{\frac{512\pi}{3} = V}$$

b.  $z \in [2r, 24 - r^2] \quad r \in [0, 4] \quad \theta \in [0, 2\pi]$

$$\bar{z} = \frac{1}{V} \int_0^{2\pi} \int_0^4 \int_{2r}^{24-r^2} z r \, dz \, dr \, d\theta = \frac{2\pi}{V} \int_0^4 \int_{2r}^{24-r^2} z r \, dz \, dr$$

$$= \frac{2\pi}{V} \int_0^4 \left[ \frac{1}{2} (24-r^2)^2 r - \frac{1}{2} (2r)^2 r \right] dr = \frac{\pi}{V} \int_0^4 (576r - 52r^3 + r^5) \, dr$$

$$= \frac{\pi}{V} \left[ 288r^2 - 13r^4 + \frac{r^6}{6} \right]_0^4 = \frac{\pi}{V} \left[ \frac{5888}{3} \right] = \frac{24\pi}{512\pi} \cdot \frac{5888}{3} = \frac{5888}{512}$$

$$\bar{z} = \frac{23}{2}$$

The centroid is  $(0, 0, \frac{23}{2})$