

15, 9 homework

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27.

①  $x = 2u + v$     $y = 4u - v$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 4 & -1 \end{vmatrix} = -2 - 4 = \boxed{-6}$$

③  $x = s \cos t$     $y = s \sin t$

$$\frac{\partial(x,y)}{\partial(s,t)} = \begin{vmatrix} \frac{\partial x}{\partial s} & \frac{\partial x}{\partial t} \\ \frac{\partial y}{\partial s} & \frac{\partial y}{\partial t} \end{vmatrix} = \begin{vmatrix} \cos t & -s \sin t \\ \sin t & s \cos t \end{vmatrix} = s \cos^2 t + s \sin^2 t = \boxed{s}$$

⑤  $x = uv$     $y = vw$     $z = wu$

$$\begin{aligned} \frac{\partial(x,y,z)}{\partial(u,v,w)} &= \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} = \begin{vmatrix} v & u & 0 \\ 0 & w & v \\ w & 0 & u \end{vmatrix} \\ &= \begin{vmatrix} w & v \\ 0 & u \end{vmatrix} v - \begin{vmatrix} u & v \\ w & u \end{vmatrix} u \\ &= wuv - (-wv)u \\ &= \boxed{2wuv} \end{aligned}$$