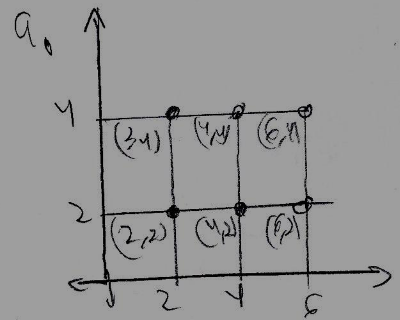
15.1 homework 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 47

1. $z = xy$ $R = \{(x,y) \mid 0 \le x \le 6,\ 0 \le y \le 4\}$ $m = 3,\ n = 2$ $\Delta A = 2(2) = 4$

$f(x,y) = xy$

a.



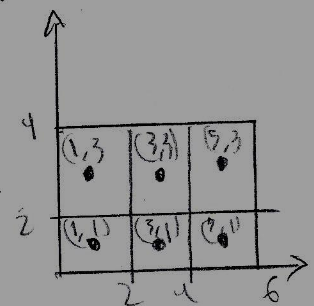
$$V \approx \sum_{i=1}^{3}\sum_{j=1}^{2} f(x_i^*, y_j^*)\,\Delta A$$

$$= [f(2,2)+f(2,4)+f(4,2)+f(4,4)+f(6,2)+f(6,4)](4)$$

$$= [4+8+8+16+12+24](4) = 288$$

$$\boxed{V \approx 288}$$

b.



$$V \approx \sum_{i=1}^{3}\sum_{j=1}^{2} f(\bar{x}_i^*, \bar{y}_j^*)\,\Delta A$$

$$= [f(1,1) + 2f(1,3) + f(3,3) + f(5,1) + f(5,3)](4)$$

$$= [1+6+9+5+15](4) = 144$$

$$\boxed{V \approx 144}$$

5. $f(x,y) = \sqrt{52 - x^2 - y^2}$ $x \in [2,4]$ $y \in [2,6]$

L = lower left corner U = upper right corner V = midpoint

The first observation of this function is that the rate of change decreases as x approaches 4 and y approaches 6. Where as when x = 2 and y = 2 the maximum is achieved. Considering this behavior if x and y move from L to U the rate of change will be negative. Therefore L > U. The second observation takes into account V, the mid point between L and U. For example when x = 3 and y = 4 the total value will be less than the low index of (2,2). So with that said, L will be greater than V, however since V is the midpoint, V is there-fore greater than U. Inequality! $\boxed{L > V > U}$