

$$\textcircled{7} z = y^2 - x^2$$

$$\frac{\partial z}{\partial x} = -2x \quad \frac{\partial z}{\partial y} = 2y$$

$$x^2 + y^2 = 1 \quad x^2 + y^2 = 4$$

$$r=1$$

$$r=2$$

$$r \in [1, 2]$$

$$\theta \in [0, 2\pi]$$

$$\sqrt{1 + 4x^2 + 4y^2} = \sqrt{1 + 4r^2}$$

$$SA = \int_0^{2\pi} \int_1^2 r \sqrt{1 + 4r^2} \, dr \, d\theta$$

$$= \int_0^{2\pi} d\theta \int_1^2 r \sqrt{1 + 4r^2} \, dr$$

$$= 2\pi \cdot \int_1^2 r \sqrt{1 + 4r^2} \, dr$$

$$v = 1 + 4r^2$$

$$dv = 8r \, dr$$

$$\frac{1}{8} dv = r \, dr$$

$$2\pi \cdot \int_1^2 \frac{1}{8} v^{1/2} \, dv$$

$$2\pi \cdot \frac{1}{12} \left[(1 + 4r^2)^{3/2} \right]_1^2$$

$$= \boxed{\frac{\pi}{6} (17^{3/2} - 5^{3/2})}$$

$$\textcircled{9} z = xy$$

$$\frac{\partial z}{\partial x} = y \quad \frac{\partial z}{\partial y} = x$$

$$x^2 + y^2 = 1$$

$$r=1$$

$$r \in [0, 1]$$

$$\theta \in [0, 2\pi]$$

$$\sqrt{1 + x^2 + y^2} = \sqrt{1 + r^2}$$

$$SA = \int_0^{2\pi} \int_0^1 r \sqrt{1 + r^2} \, dr \, d\theta$$

$$= \int_0^{2\pi} d\theta \int_0^1 r \sqrt{1 + r^2} \, dr$$

$$= 2\pi \cdot \int_0^1 r \sqrt{1 + r^2} \, dr$$

$$2\pi \cdot \int_0^1 \frac{1}{2} v^{1/2} \, dv$$

$$2\pi \cdot \frac{1}{3} \left[(1 + r^2)^{3/2} \right]_0^1$$

$$= \boxed{\frac{2\pi}{3} (2^{3/2} - 1)}$$

$$v = 1 + r^2$$

$$dv = 2r \, dr$$

$$\frac{1}{2} dv = r \, dr$$