

$$(41) \quad x + 2y + 3z = 1 \quad x^2 + y^2 = 3$$

$$z = \frac{1}{3} (1 - x - 2y) \quad \frac{\partial z}{\partial x} = -\frac{1}{3} \quad \frac{\partial z}{\partial y} = -\frac{2}{3} \quad \sqrt{1 + \frac{1}{9} + \frac{4}{9}} = \frac{\sqrt{14}}{3}$$

$$A(s) = \frac{\sqrt{14}}{3} \iint_D dA \quad x^2 + y^2 = 3 \quad r = \sqrt{3}$$

$$\iint_D dA = \text{cylinder inside area} = \pi r^2 = \pi (\sqrt{3})^2 = 3\pi$$

$$A(s) = \frac{\sqrt{14}}{3} (3\pi) = \boxed{\pi\sqrt{14}}$$

$$(43) \quad z = \frac{2}{3} (x^{3/2} + y^{3/2}) \quad x \in [0, 1] \quad y \in [0, 1]$$

$$\frac{\partial z}{\partial x} = \sqrt{x} \quad \frac{\partial z}{\partial y} = \sqrt{y} \quad \sqrt{1 + x + y}$$

$$A(s) = \int_0^1 \int_0^1 \sqrt{1+x+y} \, dx \, dy = \int_0^1 \left( \frac{2}{3} (1+x+y)^{3/2} \right)_0^1 dy$$

$$= \int_0^1 \frac{2}{3} (2+y)^{3/2} - \frac{2}{3} (1+y)^{3/2} \, dy$$

$$= \frac{2}{3} \left[ \frac{2}{5} (2+y)^{5/2} - \frac{2}{5} (1+y)^{5/2} \right]_0^1 =$$

$$\frac{4}{15} [3^{5/2} - 2^{5/2} - 2^{5/2} + 1] =$$

$$\boxed{\frac{4}{15} [3^{5/2} - 2^{7/2} + 1]}$$