

$$(4) \quad x^2 + y^2 = 4 \quad 4x^2 + 4y^2 + z^2 = 64$$

$$r^2 = 4$$

$$z = \pm \sqrt{64 - 4x^2 - 4y^2} = \pm 2\sqrt{16 - r^2}$$

$$r = 2$$

$$r \in [0, 2]$$

$$\theta \in [0, 2\pi]$$

$$f(r, \theta) = 4\sqrt{16 - r^2}$$

$$\int_0^{2\pi} \int_0^2 4\sqrt{16 - r^2} \, r \, dr \, d\theta = 4 \int_0^{2\pi} d\theta \int_0^2 r\sqrt{16 - r^2} \, dr$$

$$= 8\pi \int_0^2 r\sqrt{16 - r^2} \, dr = 4\pi \int_{16}^{12} \sqrt{u} \, du = 4\pi \int_{12}^{16} \sqrt{u} \, du$$

$$u = 16 - r^2$$

$$du = -2r \, dr$$

$$-\frac{1}{2} du = r \, dr$$

$$= 4\pi \left(\frac{2}{3} u^{3/2} \right) \Big|_{12}^{16}$$

$$= \frac{8\pi}{3} (16^{3/2} - 12^{3/2})$$

$$\frac{8\pi}{3} (64 - 24\sqrt{3}) = \boxed{\frac{8\pi}{3} (64 - 24\sqrt{3})}$$