

1. Total Number of Parameters in BERT Model

To determine the number of parameters for the BERT model with 8 layers, a hidden state dimension of 768, 8 attention heads, and a vocabulary size of 40,000, we compute several components:

A. Embedding Layer

The embedding layer consists of:

- **Token Embeddings:**

$$\text{Parameter Count} = V \times H = 40,000 \times 768 = 30,720,000$$

B. Transformer Layers

Each transformer layer consists of:

1. Multi-Head Attention:

- Each attention head has parameters for computing queries, keys, and values:

$$H \times (H/A) = 768 \times (768/8) = 768 \times 96 = 73,728 \text{ (for each head)}$$

- For all heads (8 heads per layer):

$$8 \times A \times 73,728 = 8 \times 96 \times 73,728 = 5,634,304$$

- Output projection:

$$H \times H = 768 \times 768 = 589,824$$

- Total for attention per layer:

$$5,634,304 + 589,824 = 6,224,128$$

2. Feed-Forward Network:

- With a hidden dimension of 3072:

$$2 \text{ (for 2 linear transformations)} \times H \times 3072 = 2 \times 768 \times 3072 = 4,732,160$$

C. Total for One Layer

Combining both contributions from the attention and feed-forward networks gives:

$$6,224,128 + 4,732,160 = 10,956,288 \text{ (per layer)}$$

D. Total for All Layers

For all 8 layers:

$$8 \times 10,956,288 = 87,650,304$$

E. Final Parameter Count

Combining the embedding parameters and the total parameters from all transformer layers gives:

$$30,720,000 + 56,694,784 = 87,414,784$$

Thus, the total number of parameters in the BERT model is **87,414,784**.

2. Self-Attention Output for 'Flying'

Considering the input embeddings for the words **flying** and **arrows** as $[0, 1, 1, 1, 1, 0]$ and $[1, 1, 0, -1, -1, 1]$, and using only the first 2 dimensions for the self-attention calculation, we proceed to calculate the attention output for **flying**.

Step 1: Query, Key, and Value Vectors

For the first attention head:

- **Query Vector (Q):** $[0, 1]$
- **Key Vector (K):** $[0, 1]$
- **Value Vector (V):** $[0, 1]$

Step 2: Scaled Dot Product Attention

Using the scaled dot product:

$$\text{Attention}(Q, K, V) = \text{softmax}(Q \cdot K^T / (d_k)^{1/2}) V$$

Where:

- d_k is the dimension of the key vectors (which is 2 here).

Step 3: Calculating Inputs

1. Dot Product:

$$Q \cdot K^T = [0, 1] \cdot [0, 1]^T = 1$$

2. Scaling:

$$\text{scaled} = 1 / 2^{1/2} = 0.707$$

3. Softmax (assuming compatible inputs; 2 inputs):

- For simplicity, assuming uniform output:

$$\text{softmax}(1, 1) = [0.5, 0.5]$$

Step 4: Final Attention Calculation

$$\text{Output}=[0.5,0.5]\cdot V=[0.5,0.5]\cdot[0,1]=0.5$$

The self-attention output for the word **flying** is **0.5**.

3. Task-Specific Parameters in BERT-base

A. Topic Classification with 5 Classes

In the case of topic classification with 5 classes, we need an additional classification layer on top of the BERT model. Thus, the number of task-specific parameters for this output layer is:

$$\text{Parameters}=\text{Number of classes}\times\text{Hidden size}=5\times 768=3,840$$

B. Language Identification in Code-Switched Dataset

For language identification of two languages (English and Hindi):

$$\text{Parameters}=\text{Number of classes}\times\text{Hidden size}=2\times 768=1,536$$