

# Solar Irradiance Covariance Modeling for Sacramento County

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## Introduction

Solar irradiance measurements are highly correlated with the amount of energy produced by a grid of photovoltaic panels. Thus, reliable forecasting of irradiance will lead to reliable forecasting of energy output.

Utility-scaled solar plants are becoming more prominent. Modeling and forecasting methods of systems over various spatial and temporal resolutions are needed.

The spatio-temporal kriging forecaster [1]:

$$Z(s_0, t_0) = \mu(s_0, t_0) + \mathbf{c}(s_0, t_0)' \Sigma^{-1}(\mathbf{Z} - \boldsymbol{\mu})$$

where  $\mathbf{Z} = (Z(s_1, t_1), \dots, Z(s_n, t_n))'$  for  $n$  space-time coordinates,  $\boldsymbol{\mu} = E[\mathbf{Z}]$ ,  $\Sigma = \text{cor}(\mathbf{Z})$ , and  $\mathbf{c}(s_0, t_0) = \text{cor}(Z(s_0, t_0), \mathbf{Z})$ .

Aryaputera et al. [2] used non-separable, direction dependent covariance models to forecast solar irradiance data.

- They used separate models fitted individually to time and space.
- The separate models were multiplied to make a separable covariance model.
- A non-separable model was used in which the separability parameterization of [3] was fitted.
- A directional model was used based on prior knowledge of wind for the day and location of their data set.

## Purpose

For this project, we introduce a visual method that uses the correlation in the irradiance data to specify the directional covariance model.

The data that we used are from the Sacramento Municipal Utility District (SMUD) which consisted of 65 sensors placed throughout Sacramento County. Measurements were recorded on April 3, 2010, once every minute. A clear sky model was used to detrend the data [4].

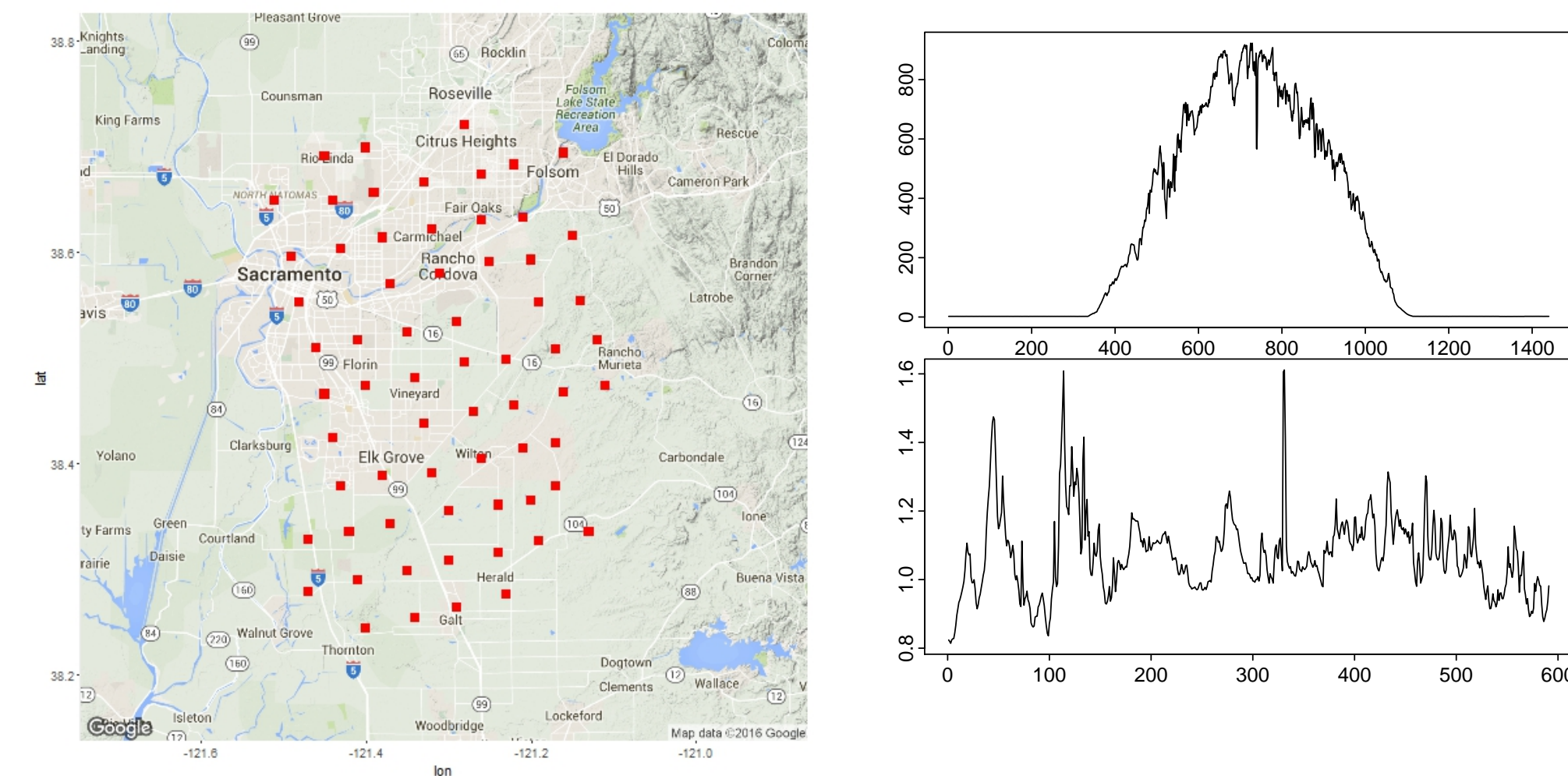


Figure 1: (Left) Locations of the data and (right) observed and detrended data for one sensor. The observed data is shown for the entire day. The detrended is only shown for 7:00 am to 4:00 pm.

## Separable and Fully Symmetric Models

The separable covariance model first fits models to the time correlation and the spatial correlation separately

- Exponential Spatial Correlation Function :

$$C_s(h) = (1 - v)\exp(-c||h||) + vI_{h=0}$$

- Cauchy Temporal Correlation Function :

$$C_t(u) = (a|u|^{2\alpha} + 1)^{-\tau}$$

The separable covariance model is then

$$C_{sep}(h, u) = C_s(h) \times C_t(u)$$

The non-separable fully symmetric model is

$$C_{FS}(h; u) = \frac{1 - v}{(1 + a|u|^{2\alpha})} \left[ \exp\left(\frac{-c * h}{1 + a|u|^{2\alpha^{beta/2}}}\right) + \left(\frac{v}{1 - v}\right) I_{h=0} \right]$$

The values of  $v, c, a$ , and  $\alpha$  are the same as for  $C_{sep}$ . Using these values,  $\beta$  is estimated which indicates the level of separability (0 = separable, 1 = non-separable).

The first 50% of the data is used as a training dataset to fit the models.

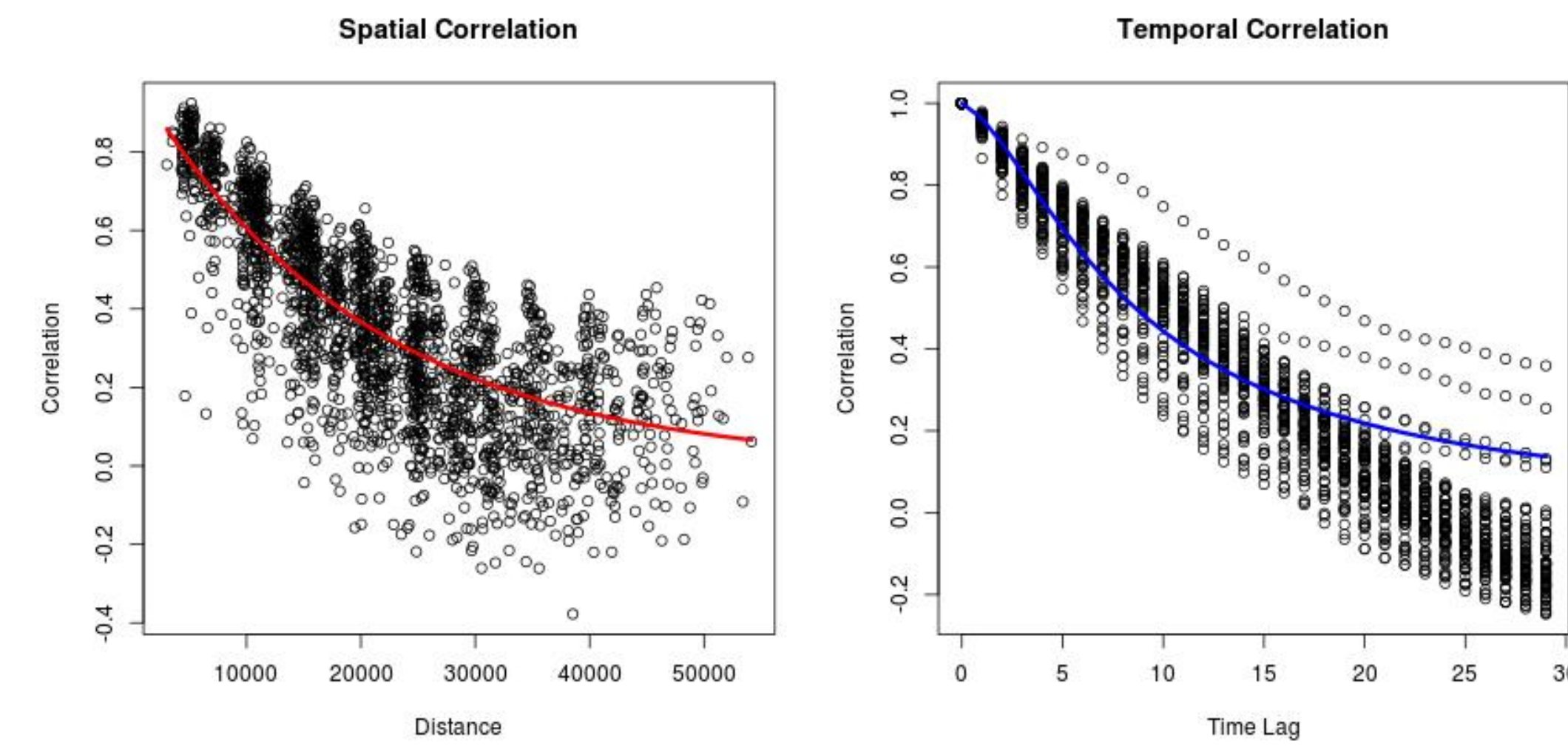


Figure 2: Spatial correlation fit with exponential model, temporal correlation fit with Cauchy model

## Directional Models

- Calculate the directional distances for each pair of sensors:  $h_1$  = along wind distance,  $h_2$  = crosswind distance
- Find the differences between along wind correlation,  $\text{corr}(Z(s_i, t - u), Z(s_j, t))$ , and against wind correlation,  $\text{corr}(Z(s_i, t), Z(s_j, t - u))$ , for some time lag  $u$  for each pair of sensors  $i \neq j$

- The difference correlation is modeled as

$$C_{diff}(h, u) = \left( I_{u>0} I_{h_1>0} \left[ \beta_1^{(u)} h_1 + \beta_2^{(u)} h_2 + \beta_3^{(u)} h_1 h_2 + \beta_4^{(u)} h_1^2 + \beta_5^{(u)} h_2^2 \right] \right)_+$$

- The directional correlation function is then

$$C_{dir}(h, u) = C_{FS}(h, u) + \alpha C_{diff}(h, u)$$

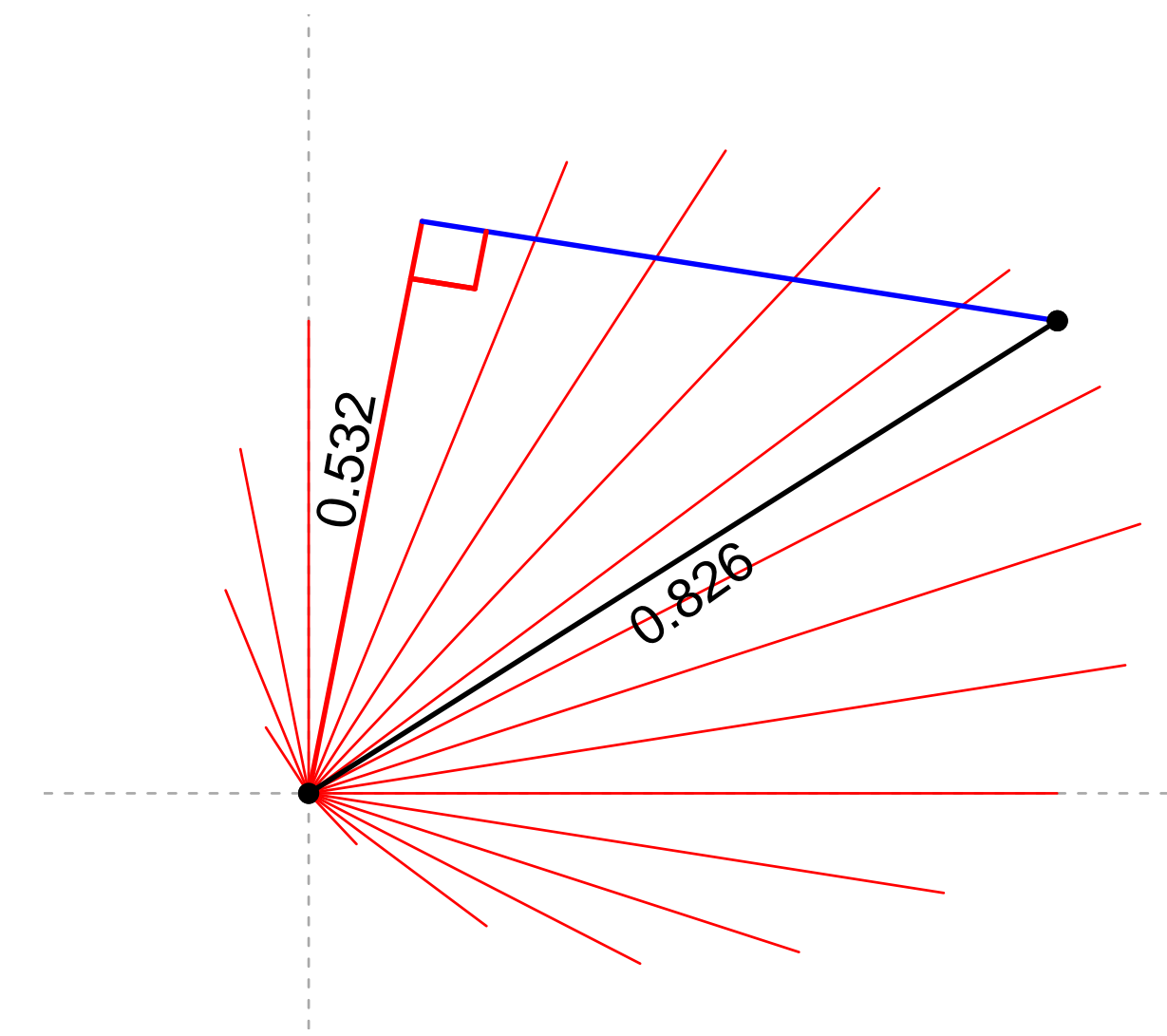


Figure 3: Example of directional distance

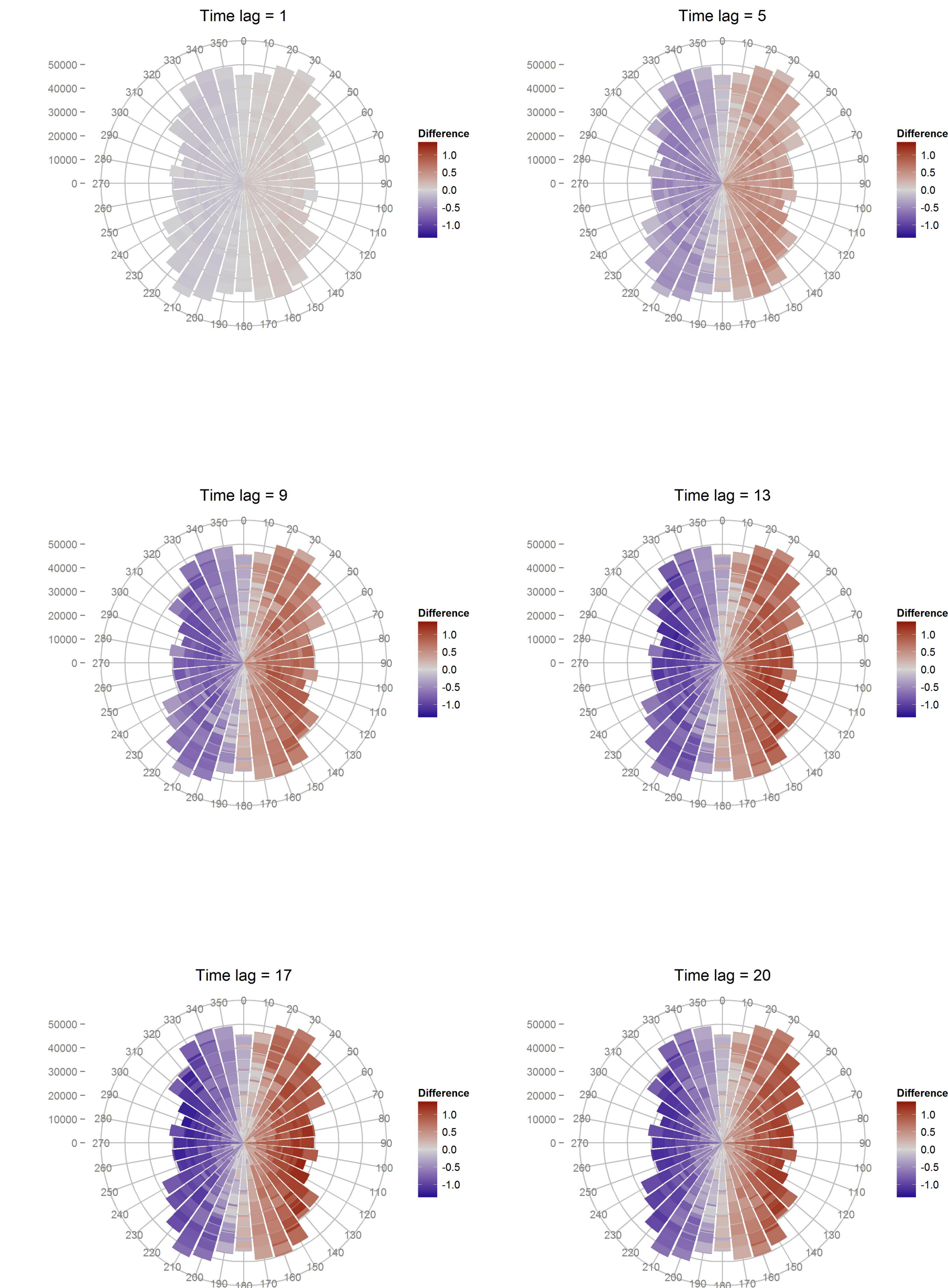


Figure 4: Correlation vs Lags

## Results

- We used 50% training data. We utilized a moving window approach in which the previous 50% data was used to fit the model and then predict the next  $u = 1, \dots, 10$  time points.
- When fitting the models to the estimated correlations in Figure 2, we used weighted nonlinear least squares implemented with the `nls` function in **R**. The weights we used were inverse distance weights.
- When determining the direction, we only examined directional plots for the first 50% training data. For computational speed, we did not regenerate for each predicted time point. Thus, we are assuming the wind direction stays constant throughout the day.
- From the directional plots, we determined that the previous 20 minutes of data should be used to predict the next time point(s).
- From Figure 5, we can see that the directional model results in the lowest root mean squared prediction error.

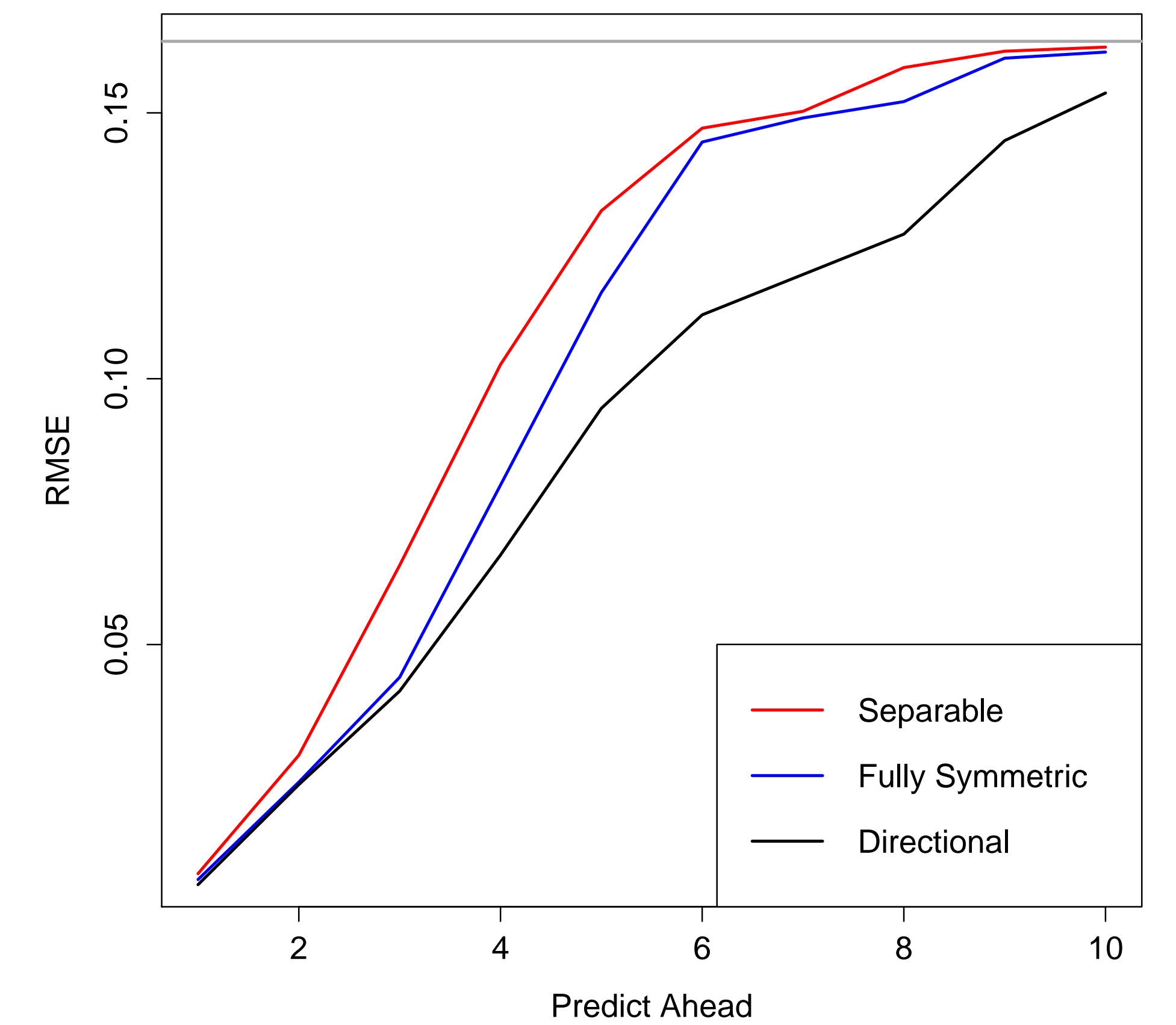


Figure 5: Root mean square prediction error for the three covariance models predicting 1, ..., 10 time points ahead. The gray line represents the standard deviation of the testing data.

## References

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