Modeling fluid flow

Stokes Equations
and
Method of Regularized Stokeslets
2D Practical

Mathematical Modeling, Computational Methods, and Biological Fluid Dynamics: Research and Training
NITMB, Chicago, Illinois

Developed in collaboration with Ricardo Cortez

Shilpa Khatri
Associate Professor
Applied Mathematics
University of California, Merced

Stephen Williams
Postdoctoral Researcher
Applied Mathematics
University of California, Merced

Ex 1 :Cortez 2001 Section 4.2: Example 4b

Matlab file to run: main_example4b.m

Sets tangential force density on a cylinder of radius 1 to

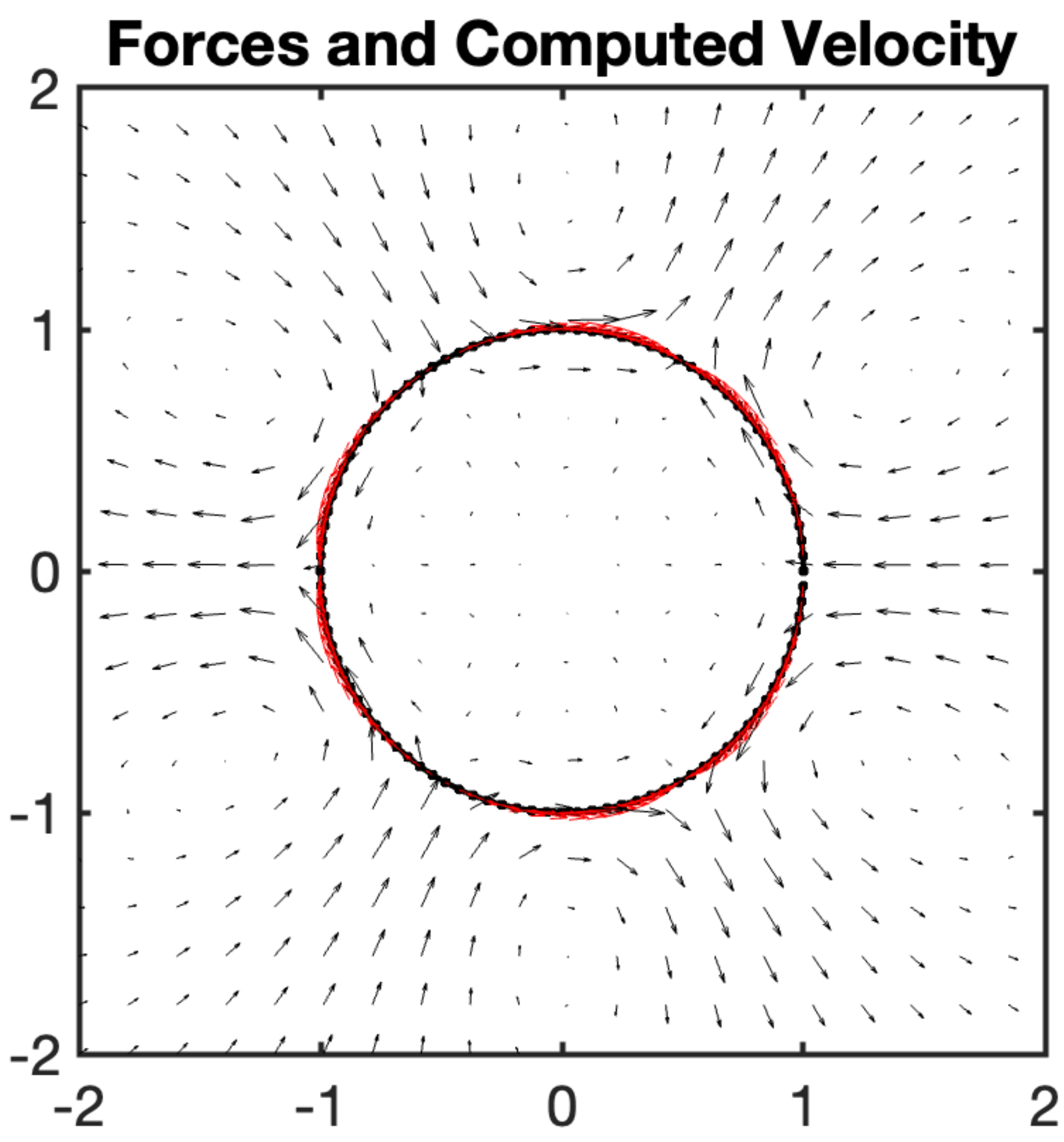
$$f(\theta) = 2\sin(3\theta)x(\theta)$$

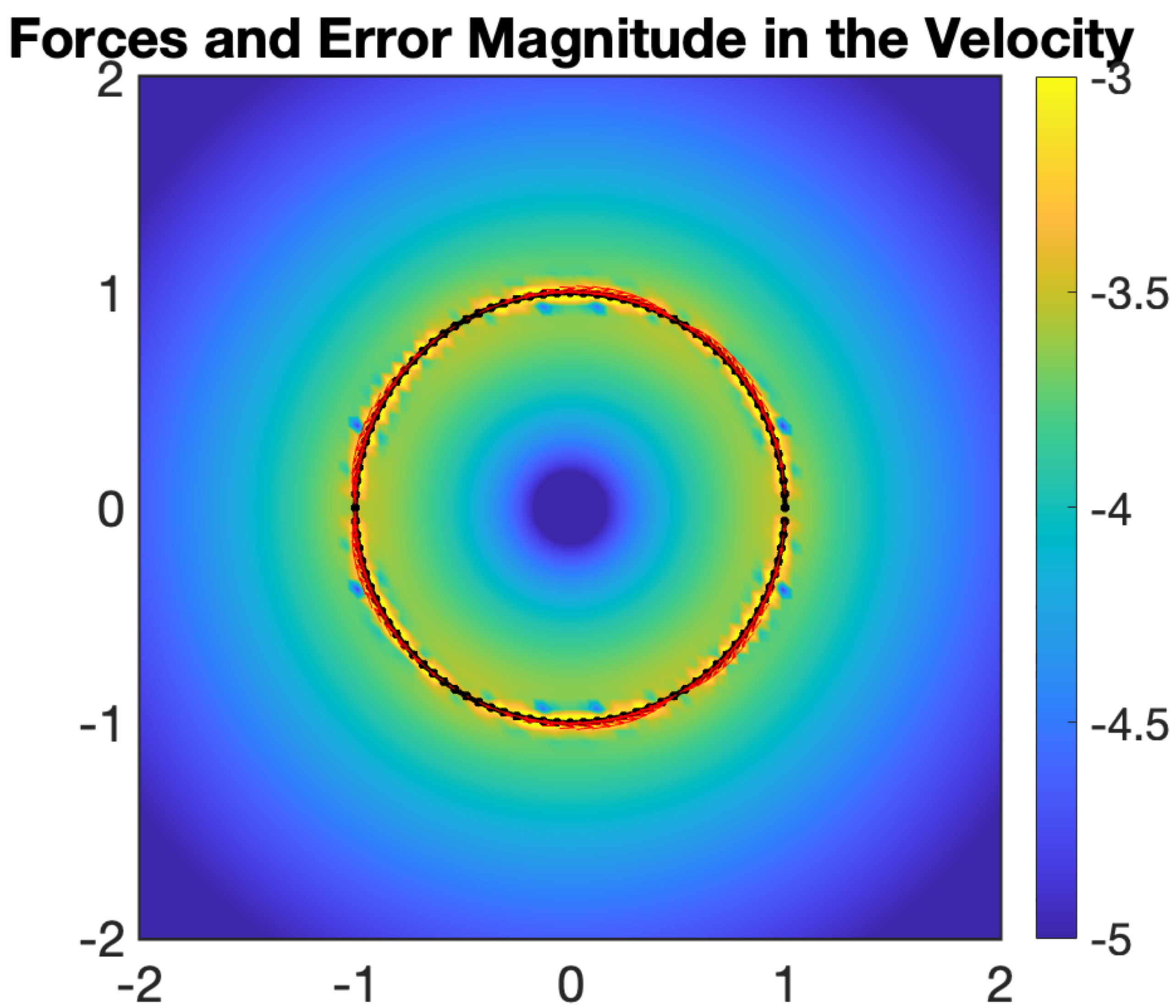
And computes the velocity in the fluid surrounding the cylinder

You can in %%Parameters to set:

- Modify the domain on which the velocity is plotted
  (In the paper, the plots are on a 1D line through the domain - can you recreate these? - or look at a 2D domain)
- See what happens as you increase the number of points at which the force is set (Note, in all examples, $\epsilon = dt/4$ where dt is the discretization on the boundary). What happens if you change ϵ?
- Play with the plotting - get comfortable with the code and Matlab

Ex 1: Cortez 2001 Section 4.2: Example 4b





Ex 1: Cortez 2001 Section 4.2: Example 4b

More advanced, you can

- Why is there not a background velocity here?
- Look in RegStokeslets2D_forcetovelocity.m and see how this is what we discussed in the methodology to go from force -> velocity
- Incorporate pressure in RegStokeslets2D_forcetovelocity to solve for the pressure when you have the forces

Ex 2: Cortez 2001 Section 4.2: Example 4b

Matlab file to run: main_example4b_veltoforce.m

Sets boundary velocity on a cylinder of radius 1 to

$$u_1(\theta) = \frac{1}{8}\sin(2\theta) - \frac{3}{16}\sin(4\theta) + \frac{1}{4}\sin(4\theta)$$

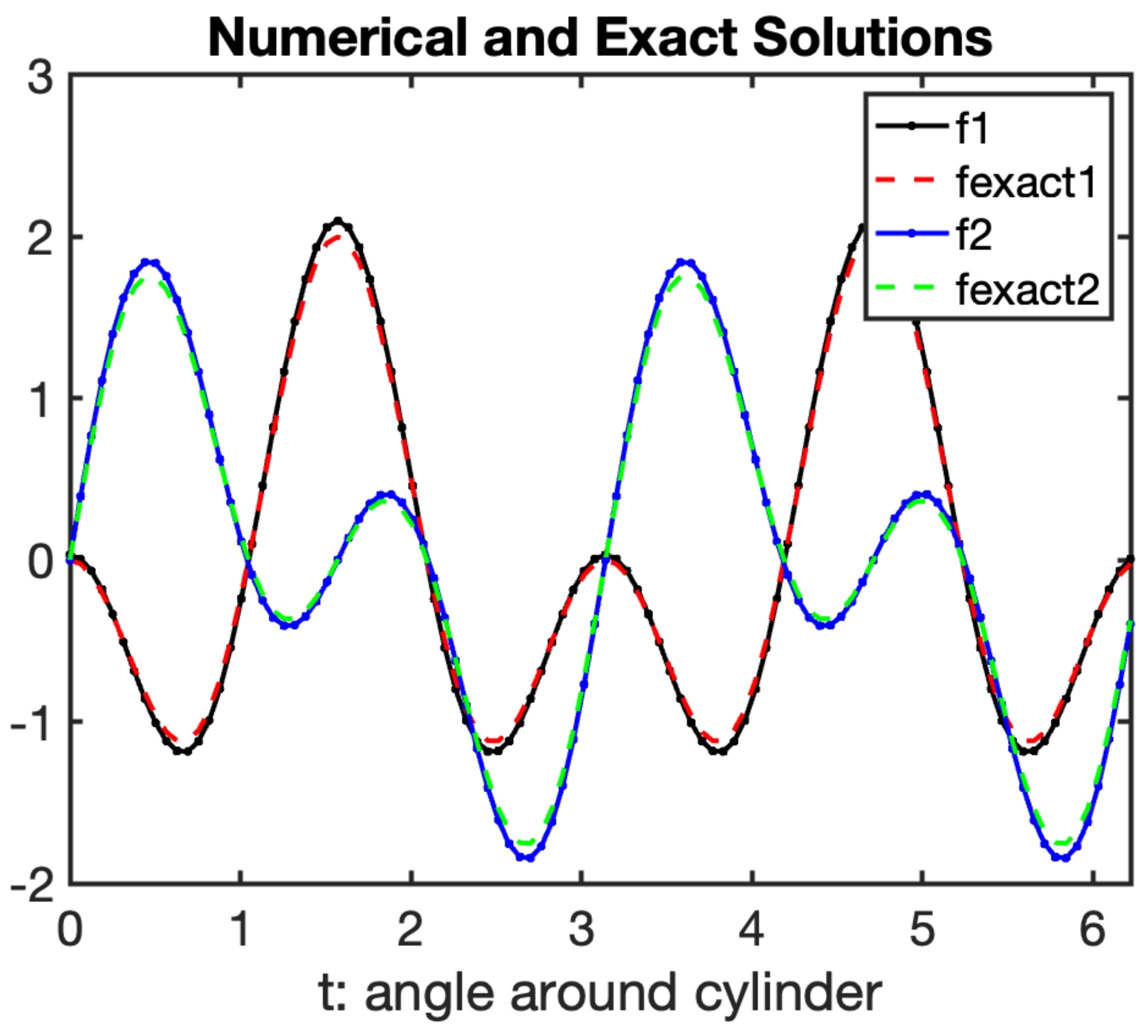
$$u_2(\theta) = \frac{1}{8}\cos(2\theta) - \frac{3}{16}\cos(4\theta) - \frac{1}{4}\cos(4\theta)$$

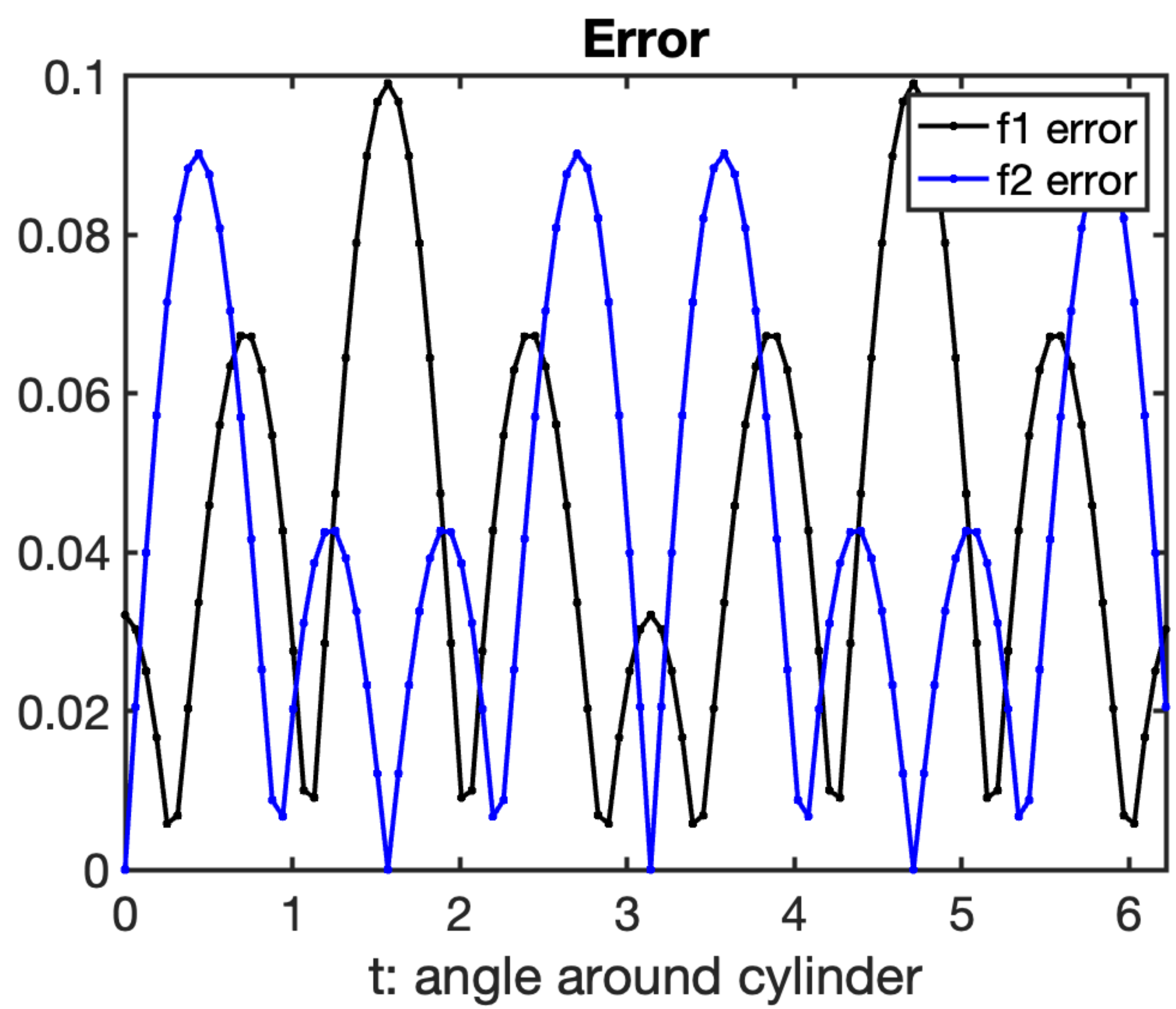
And computes the force on boundary of the cylinder

You can:

- Calculate the sum of the forces - what does this say about the translational velocity?
- See what happens as you increase the number of points at which the velocity is set (Note, in all examples, $\epsilon = dt/4$ where dt is the discretization on the boundary).

Ex 2: Cortez 2001 Section 4.2: Example 4b





Ex 2: Cortez 2001 Section 4.2: Example 4b

More advanced, you can

- Is this solution unique? Can you code Ex 1 to show you can get the same velocity with different forces?
- What do know about the background translational and rotational velocity from what we have computed the forces to be?
- Look in RegStokeslets2D_velcoitytoforce.m and see how this is what we discussed in the methodology to go from velocity -> force.
- Incorporate pressure in RegStokeslets2D_velocitytoforce to solve for the pressure once you have computed the forces.

Ex 3: Cortez 2001 Section 3.1: Example 1

Matlab file to run: main_example1.m

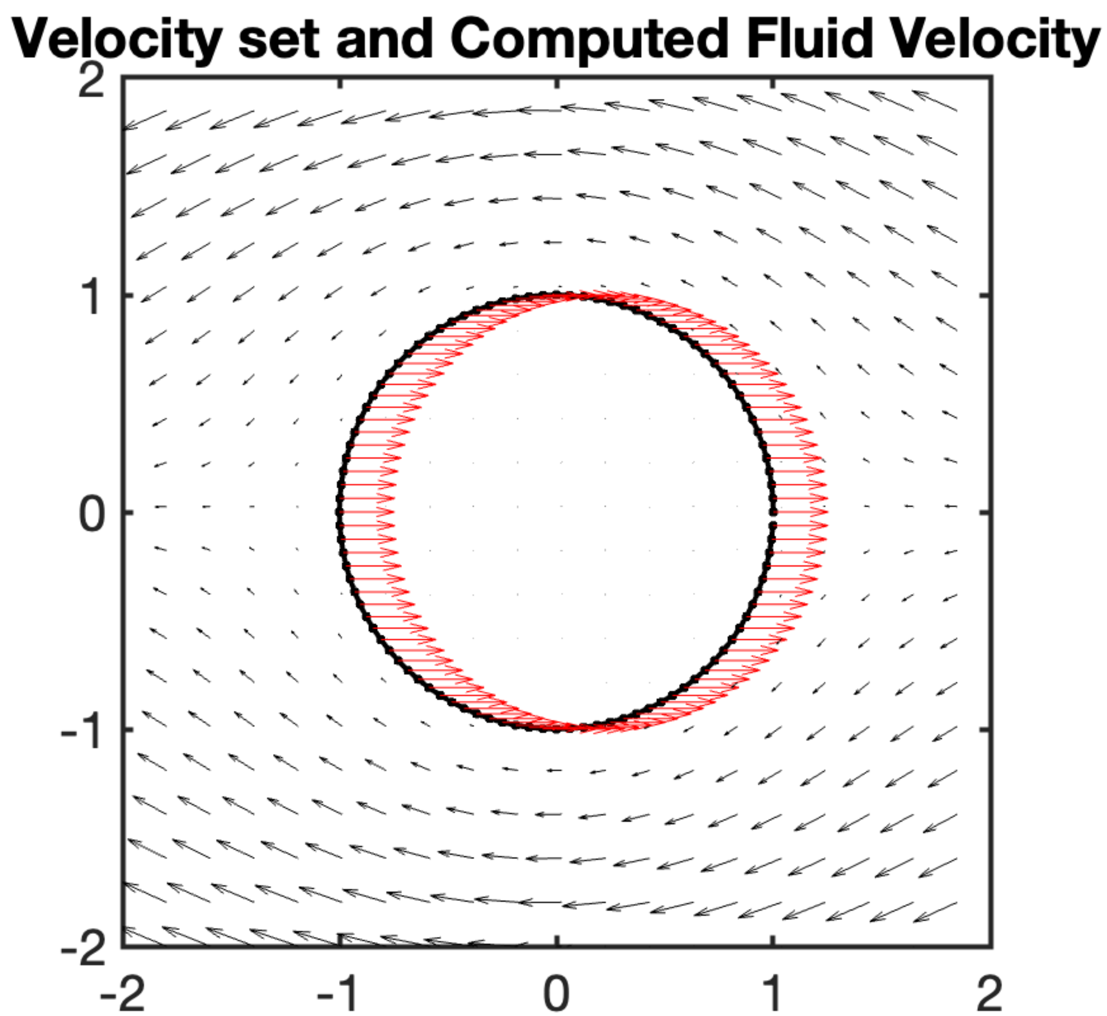
Sets boundary velocity on a cylinder of radius a to translate horizontal with speed one, $u = (1,0)$

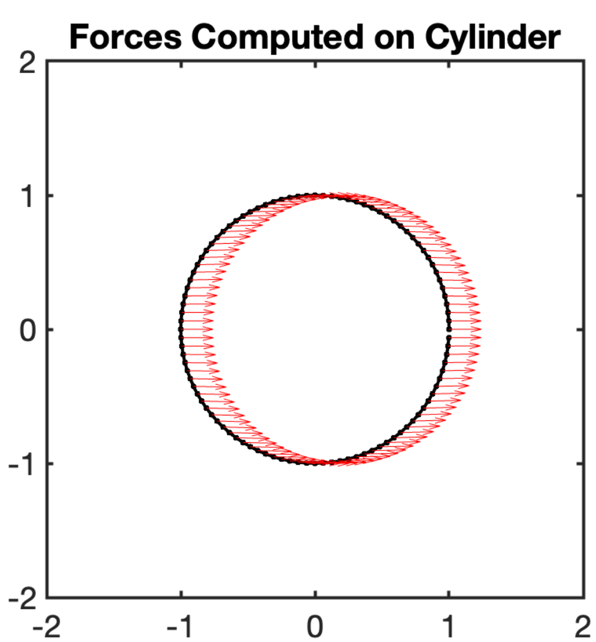
And computes the force on boundary of the cylinder and then computes the velocity in the fluid surrounding the cylinder

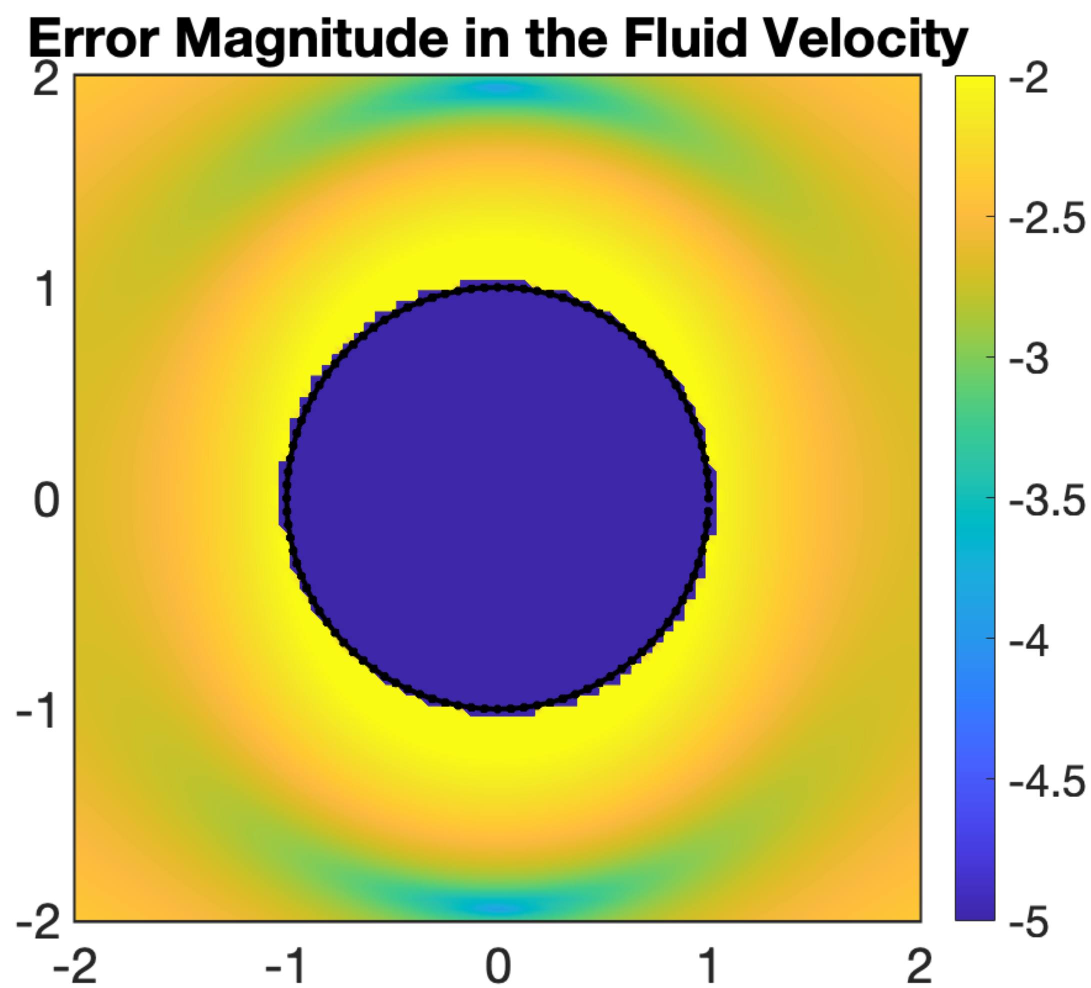
You can:

- Calculate the sum of the forces - what does this say about the translational velocity? Can you plot the velocity solution in the frame of reference of the cylinder?
- See what happens as you increase the number of points at which the velocity is set (Note, in all examples, $\epsilon = dt/4$ where dt is the discretization on the boundary).

Ex 3: Cortez 2001 Section 3.1: Example 1







Ex 3: Cortez 2001 Section 3.1: Example 1

More advanced, you can:

- Can we modify this code so the background translational velocity is solved by requiring the net force and torque to be zero. Use RegStokeslets2D_velocitytoforce_augmented.

- Incorporate pressure in RegStokeslets2D_velocitytoforce to solve for the pressure once you have computed the forces.

Ex 4: Swimmer

Matlab file to run: main_swimmer.m

Sets time-dependent velocity on swimmer (circle body + flagella)

And computes the force on swimmer and then computes the velocity in the fluid surrounding the cylinder

You can:

- Play around with the velocity of the swimmer and position
- Plot the swimmer translational and angular velocities
- Plot the swimmer with its translational and angular velocities (this is a bit harder)
- See what happens as you increase the number of points at which the velocity is set (Note, in all examples, $\epsilon = dt/4$ where dt is the discretization on the boundary).

Ex 4: Swimmer

More advanced:

- What are you interested in modeling now that you have some tools!?!?

3D

There is a 3D example as well:

Example 1 Section 4 of Cortez, Fauci, Medovikov, Physics of Fluids 2005

References and Acknowledgments

References:

- R. Cortez, The Method of Regularized Stokeslets, SIAM J. Sci Comp, 2001
- R. Cortez, L. Fauci and A. Medovikov The method of regularized Stokeslets in three dimensions: Analysis, validation, and application to helical swimming, Phys. Fluids, 2005

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