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ESTIMATION OF THE PROBABILITY OF COLLISION BETWEEN TWO CATALOGUED ORBITING OBJECTS *

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ABSTRACT

This paper deals with the estimation of the probability of collision corresponding to a single predicted close approach between two catalogued orbiting objects. By assuming normally distributed position uncertainties at the estimated closest approach time and a quasi-rectilinear relative trajectory about this time, we get a simple formula that takes into account the dynamics and the geometry of the close approach. The validity of the method has been verified with Monte-Carlo simulations for close encounter configurations derived from the Cerise collision that occurred on July 24, 1996. If we do not know the global amplitude of the position uncertainties, we can nonetheless estimate the highest value the probability can reach. For the Cerise collision and the 830 *m* estimated miss-distance we get from the TLE orbital data available before the event, the maximum collision probability we could have was about 10^{-6}, which shows the necessity of an improved orbit determination for a correct assessment of the collision risk.

INTRODUCTION

As the number of orbiting objects keeps increasing, it is now necessary to investigate the collision risks. ONERA's PROSPECT tool (french acronym for « Orbital Collisions Simulation, Prediction and Avoidance Software ») has been designed to assess the collision risk for the predicted close encounters between a certain satellite and every other catalogued object, with the purpose of having the satellite perform an avoidance maneuver in case of an important estimated risk (Bérend 1996, 1997a, 1997b & 1998). It is currently used by the french space agency CNES to monitor the collision risks for the SPOT and HELIOS satellites. The miss-distance is estimated for every close encounter predicted, but it does not fully characterize the collision risk since it does not take into account the uncertainty of the closest approach time nor the geometry of the encounter. So we have also included in PROSPECT the calculation of a probability of collision including all these aspects, according to the method we will discuss next.

THE COLLISION PROBABILITY ASSESSMENT METHOD

If we assimilate the two orbiting objects to material spheres, with radii L_1 and L_2, the collision event will be defined as a passing at a distance lower (or equal) than the « collision distance » $d_{collision} = (L_1 + L_2)$.
Given the $d_{collision}$ distance threshold, the collision probability can be written as:

$$\Pr_{collision}\left(d_{collision}\right) = \Pr\left(\min_{(t \in I)}\left(\left\|R(t)\right\|\right) \le d_{collision} \right) = \Pr\left(\left\|D\right\| \le d_{collision}\right) \qquad (1)$$

* study conducted for the account of the french space agency CNES.

- I is a time interval including the estimated time t_{ca} of the Closest Approach we study.
- $R(t)$ is the predicted relative position vector (of one object relatively to the other) at time t.
- D is the relative position vector at the closest approach, taking into account the uncertainties of the time of this event. D is an *iso-event* random vector and it should not be mistaken for the *iso-time* random vector $R(t_{ca})$.

We first assume that the predicted position vectors $R_1(t_{ca})$ and $R_2(t_{ca})$ of the two objects at the time of closest approach are multinormal (gaussian in 3 dimensions) random vectors with known mean vectors (m_{R1} , m_{R2}) and covariance matrixes (P_{R1} , P_{R2}). Assuming no correlation between $R_1(t_{ca})$ and $R_2(t_{ca})$, $R(t_{ca})$ (defined as the difference (R_2 - R_1)) is also a multinormal random vector whose mean vector and covariance matrix are given by:

$$\begin{aligned} m_R &= m_{R2} - m_{R1} \\ P_R &= P_{R1} + P_{R2} \end{aligned} \tag{2}$$

The second important hypothesis of the method is that the trajectory of one object relatively to the other (i.e. trajectory of $R(t)$) can be approximated to a rectilinear trajectory about the closest approach. This approximation is definitely acceptable since the orbit portion we look at (where virtually all the collision risk is concentrated) is very short.

Considering the relative trajectory of object #2 in an object #1-fixed reference frame (see Figure 1), it can be shown that we get the closest approach point (whatever its time) directly from any initial position by projection in the plane orthogonal to the motion direction and including the reference frame origin (object #1). So, we get D by simply writing any initial relative position vector $R(t)$ in a reference frame having one of its axis parallel to the relative motion direction, and by keeping only the two components in the plane normal to this direction. In fact, in order to minimize the length of the projection path between $R(t)$ and D (which makes the rectilinear trajectory approximation as valid as possible), we will always start from the relative position at the closest approach time ($R(t_{ca})$). Denoting the transformation matrix as M, the expression of $R(t_{ca})$ in the new reference frame will be:

$$R' = M \cdot R(t_{ca}) \tag{3}$$

M is a function of the relative velocity vector predicted at t_{ca}, which is, like $R(t_{ca})$, a random vector. However, as we will show later, we can disregard the effect of the relative velocity uncertainties and assume that M is a constant matrix. With this new approximation, R' will be normally distributed with the following mean vector and covariance matrix:

$$\begin{cases} m_{R'} = M \cdot m_R \\ P_{R'} = M \cdot P_R \cdot M^T \end{cases} \tag{4}$$

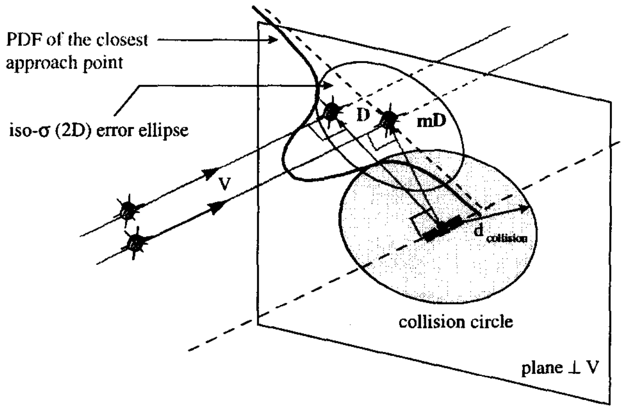

Fig. 1. Representation in a relative reference frame.

Finally, we extract the two-dimensional mean vector (m_D) and covariance matrix (P_D) of D from $m_{R'}$ and $P_{R'}$ by simply removing the data characterizing the uncertainties along the motion direction. The collision probability (Eq. 1) is the integral of the binormal PDF of D over a circle with a radius equal to the collision distance $d_{collision}$:

$$\Pr_{collision}\left(d_{collision}\right) = \iint_{\left(x^2+y^2 \leq d_{collision}^2\right)} \frac{1}{(2\pi)\cdot\sqrt{\det\left(P_D\right)}} \cdot e^{-\frac{1}{2}\left(\begin{bmatrix}x\\y\end{bmatrix}-m_D\right)^T \cdot \left(P_D{}^{-1}\right) \cdot \left(\begin{bmatrix}x\\y\end{bmatrix}-m_D\right)} \cdot dx \cdot dy \tag{5}$$

It can be seen on Figure 1 that the estimated miss-distance (norm of vector m_D) accounts for only one degree of freedom.in this problem, the others being linked to the position, orientation and proportions of the ellipse that characterizes the position uncertainties.

This formula (Eq. 5) is very interesting in terms of computation volume, since it requires only a two-dimension integration over a finite domain, even though it actually takes into account the uncertainties of *6 random variables* (the 3 coordinates of the predicted position vectors of both objects at the estimated closest approach time) over the whole (infinite) space. Moreover, it can be artificially reduced to a one-dimension integral, provided we use the classical error function *erf(x)*, which is often available in software mathematics libraries.

VERIFYING THE PROBABILITY ESTIMATION METHOD WITH MONTE-CARLO SIMULATIONS

In order to verify the validity of this method, we have carried out many Monte-Carlo simulations (with random draws for the position vectors). We found it interesting to simulate close approach cases derived from the Cerise/Ariane's debris collision of July 24, 1996, which was the first confirmed collision between two catalogued object. The collision has been confirmed *a posteriori* (Alby, 1997; Payne, 1997) but, with the TLE (U.S. Space Command's Two-Lines Element sets orbital data) available *before* the collision, the smallest miss-distance we could predict at the orbit crossing involved was 830 *m* (Bérend, 1997a). By the way, this result confirms that, unfortunately, the orbit predictions based on TLE data are not very accurate, especially when compared with the dimensions of the objects.

We have simulated many different geometrical configurations which were derived from the « nominal » predicted close encounter case by modification of the miss-distance (initially 830 *m*) and/or the angle between the two velocity vectors (initially 159°). The position vectors at the closest approach are randomly drawn by assuming gaussian uncertainties in 5/2/2 proportions in the classical [T ; N ; W] orthonormal reference frame (T colinear to the velocity vector, N pointing inward in the orbital plane, W normal to the orbital plane). We have used the SGP4 propagation model (which is the model associated to TLE data with an orbital period less than 225 *mn*).

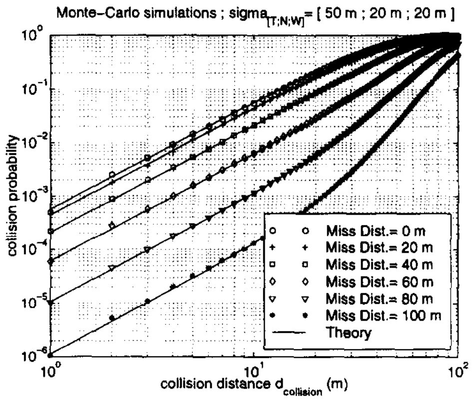

Fig. 2. Comparison theory / Monte-Carlo simulations with position uncertainties.

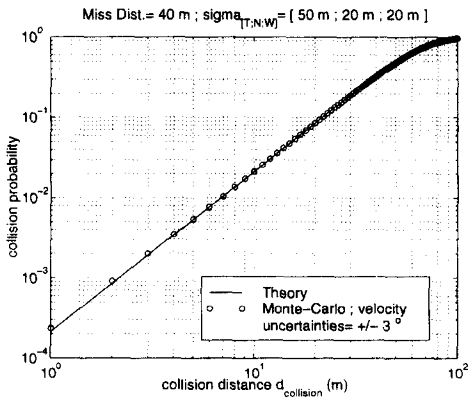

Fig. 3. Comparison theory / Monte-Carlo with position and velocity uncertainties.

Each Monte-Carlo simulation yields a set of points of the miss-distance CDF (Cumulative Distribution Function), which is nothing but the curve of the collision probability as a function of the collision distance $d_{collision}$. On most Monte-Carlo simulations we made, we have observed a very good agreement between the theoretical probability given by the method explained above and the one we get from the statistics of the simulation (see examples on Figure 2). The only significant differences are observed when the angle between the velocities at the closest approach is very small (less than 1°), but this is normal since the two objects stay close over a very large orbit portion and the rectilinear trajectory approximation cannot be valid then.

A study comparing predictions from TLE data with reference orbital data from CNES showed that the uncertainties on the predicted velocity of a satellite lie essentially on the angles of the motion direction, with a very small standard deviation (Bérend, 1997a). For instance, we have a 0.15° standard-deviation for a 20-day prediction for satellite SPOT-3 (in a Low Earth Orbit very similar to Cerise's) and 0.23° for an 11-day prediction for satellite TELECOM-2C (Geostationary Orbit). We have verified that this kind of velocity uncertainty (and even greater ones) has no significant effect over the collision probability, with Monte-Carlo simulations in which position vectors and velocity vectors at the closest approach are randomly drawn. Figure 3 shows an example of result for a Monte-Carlo simulation in which the angles giving the direction of each predicted velocity vector are uniformly distributed between -3° and +3°. The Monte-Carlo probability (circles on Figure 3) still appears very close to the theoretical probability (continuous line), although our method does not take into account velocity uncertainties.

ESTIMATION OF A MAXIMUM VALUE FOR THE COLLISION PROBABILITY

If we work with orbital data with unknown precision (like the TLE data), we cannot get the P_D covariance matrix we need in the collision probability formula (Eq. 5). However, with a few hypotheses, it is possible to estimate the maximum value the probability can reach whatever the global amplitude of the uncertainties is. All we have to do is to define the orientation and the proportions of the error ellipsoids that characterize the position uncertainties of each object at the closest approach, the standard-deviations being of no importance. For instance, we can assume spherical uncertainties, but a much more realistic assumption would be to consider that the major axis of the error ellipsoid is always colinear to the velocity vector, and that the ellipsoid has - for example - 5/2/2 proportions. These hypotheses eventually lead to a P_D covariance matrix under the following form:

$$P_D = k^2 \cdot P_{D0} \tag{6}$$

The global amplitude of the position uncertainties is fixed by the coefficient k, which is applied simultaneously on all the standard-deviations of the position uncertainties in every direction, while keeping the initial orientation and proportions given by the covariance matrix P_{D0}.

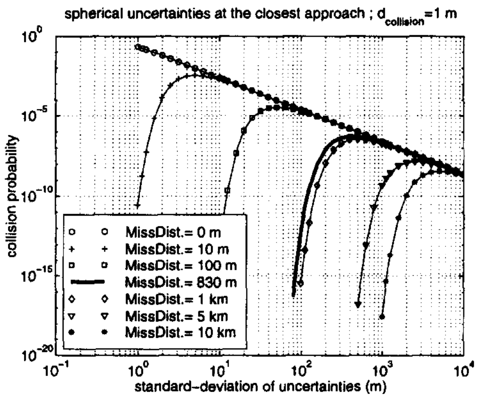

Fig. 4. Collision probability as a function of the amplitude of the uncertainties.

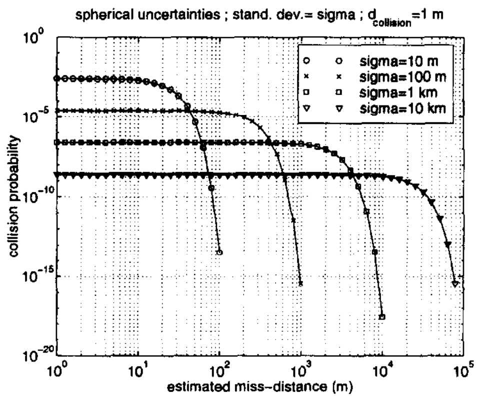

Fig. 5. Collision probability as a function of the estimated miss-distance.

It is easy to show that the collision probability, as given in Eq. 1, is infinitely small both when uncertainties (i.e. coefficient k) are infinitely big and infinitely small (provided we have a non-zero miss-distance, in the latter case), and that it goes through a peak value when k varies. By assuming that the probability reaches its peak when vector D's PDF at the center of the integration domain does, the coefficient k giving the maximum probability is:

$$k = \sqrt{\frac{(m_D)^T \cdot (P_{D0}{}^{-1}) \cdot (m_D)}{2}} \tag{7}$$

However, one should keep in mind that this calculation is strongly related to the gaussian, unbiased position uncertainties hypothesis. In the particular case of *spherical* position uncertainties with the same standard-deviation σ for both objects, the value of σ giving the peak collision probability is linked to the predicted miss-distance by:

$$\sigma = \frac{\|m_D\|}{\sqrt{2}} \tag{8}$$

Applied to the Cerise collision case, this calculation shows that the maximum collision probability we could have from the nominal predicted miss-distance of 830 *m* (thick-lined curve on Figure 4) was about 10^{-6}, which is very small *in the absolute*, and definitely inadequate to take the decision to trigger an avoidance maneuver (should our satellite had maneuvering abilities, unlike Cerise). Yet, the association miss-distance / peak probability would have probably ranked this particular predicted close encounter among the most hazardous ones. In fact, as Figure 5 shows, the collision probability cannot be much higher (10^{-4} or more) unless the order of magnitude of the predicted miss-distance gets closer to the one of the objects' dimension (less than 10 *m*). This suggests that the objects involved in the most hazardous predicted close encounters should be examined further with a refined orbit determination, in order to get a better prediction and a more significant collision probability estimation than we do with TLE data. But the issue of the miss-distance and/or collision probability threshold we should consider to trigger an avoidance maneuver has to be investigated.

CONCLUSION

We have a fast and seemingly reliable method to estimate the collision probability of a predicted close approach between two objects, which has been validated with Monte-Carlo simulations reproducing close encounters in Low-Earth Orbit. With the hypothesis of gaussian and unbiased position uncertainties, we can even estimate a peak collision probability when the amplitude of the uncertainties is unknown. The application of these methods to the prediction of the Cerise collision using older (and inaccurate) TLE data has shown that the close encounter involved could not have been predicted as a really important risk, but only as one of the cases which should be examined further with a refined orbit determination.

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