Covariance Propagation of Two-Line Element Data

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Abstract: In the space debris collision warning based on the collision probability, the collision probability is calculated by collision time and speed displacement vectors of two targets and the calculated error covariance information. In General, using the SGP4 model could get the satellite's ephemeris, but unfortunately, the SGP4 orbital data, in the form of two-line element sets (TLE) containing mean orbital elements of satellite does not include the accuracy of orbit, so we need to use specified method to calculate the orbit error covariance. This paper uses historical TLE data to calculate the covariance matrix of orbital error and analyze the covariance propagation by function fitting. Finally, comparing the results of the covariance propagation with the covariance calculated by HPOP model.

Key Words: orbit error, covariance propagation, collision warning

1 INTRODUCTION

Increasing space debris poses a serious threat to human spaceflight activities. Till October 31, 2015, produced a total of 41014 space objects. 17240 space objects of which are still on-orbit and 11160 space objects are space debris which accounted for about 80 percent of the on-orbit objects [1]. In order to ensure the smooth progress of space missions, orbital maneuver plan can be made by calculating the collision probability to avoid collision between spacecraft and space debris.

Collision probability calculation requires the use of space target orbit data and orbit covariance information. For most space targets catalogued by the United States space surveillance network (SSN), whose orbital data can be obtained by NORAD Two-line element sets and SGP4 model, SGP4 model is widely used in space collision warning. However, NORAD does not announce track error information and TLE data does not contain error information as well. We cannot use the SP4 model to calculate the covariance directly. Therefore, we need to use other methods to calculate the orbit error data.

In some papers in the past, several methods of error analysis are presented. On space cooperation objectives, we can use the describing function method for covariance analysis (Covariance Analysis Describing Function Technique, CADET) [2,3] to extrapolate the initial covariance matrix and get a function of error spread with time [4,5]. We can also compare the predictive ephemeris with high precision ephemeris. Muldoon uses GPS observing data for improving the orbit determination and orbit prediction [6]. In this approach, orbit prediction results of TLE epoch of before and after are compared with high-precision orbit determination results estimates to get the initial error by the epoch times, and use this estimated value as initial values for extrapolation of tracking errors. There are some shortcomings for the all methods above, the initial covariance matrix, high accuracy orbit model, high precision ephemeris are hard to get for most users.

In the COVGEN approach of Aerospace Company, TLEs are compared to each other to determine how TLE predictions change with the propagation interval [7]. This approach uses publicly available historical TLE data. Therefore it is suitable for use by the general users.

This paper will use the historical TLEs and SGP4 model to get a function of the error span with propagation interval. Then compare the results with the covariance calculated by HPOP model and analyze the error propagation characteristics, the accuracy of the results by the method which is described in this paper. Finally suggests a way to improve the error analysis approach for estimating the TLE covariance data.

2 TEST DATA

In order to analyze the error propagation characteristics and the accuracy of the method described in this paper properly, it is necessary to choose the appropriate test data.

The selection of satellites for which post-processed, reference orbits existed that could be treated as "true" orbits. For this study, select four different types of orbits，LEO, MEO, HEO and GEO. The period from day 1 in 2014(Jan 1, 2014) to day 328 in 2015(Nov 24, 2015). All TLE data are from www.spacetrack.org. In this paper, the objects' name will be replaced by their orbit types. And TLE sets selected for different orbit types could investigate different perturbation forces, such as the atmospheric drag.

The orbital parameters and the number of TLE groups are listed in Table 1.

Table1. Orbit Types of Selected Space Object

Orbit Type	Satellite No.	Satellite Name	Inclination (deg)	TLEs
LEO	22675	Cosmos-2251	74.03	608
MEO	32711	GPS-62	55.40	766
GEO	25010	TelStar-10	7.34	387
HEO	28544	SLOSHSAT	0.06	373

3 METHODOLOGY

Function fitting of covariance evolution based on historical data is by comparing the epoch time of TLE propagation and previous TLE propagate to this point in time to get residuals and the prediction time, using statistical methods getting a fitting function of error evolution with time.

First, choosing a " true " reference orbit to evaluate propagation estimates. Then propagated the TLEs and compared to each other follows. Finally, calculate the covariance of the last TLE in the period and obtain a fitting function of the error evolution.

3.1 Satellite-based Coordinate systems

Generally, error analysis and orbit transfer use the satellite-based coordinate system. Two satellite-based coordinate systems used for this study will describe below.

(1) RSW Coordinate System

The RSW system is also known as RTN, RTC. The R axis (Radial) points from the Earth's center along the radius vector toward the satellite. The S axis (Along-track or transverse) is perpendicular to the R vector in the orbit plane, which points to the direction of velocity. The W axis (Cross-track) is found by crossing the S and R vectors and is perpendicular to the orbital plane.

Given a position vector, $\vec{r}$ and a velocity vector, $\vec{v}$ which are in Earth Centered Inertial (ECI) system. The unit vectors of R, S, W axis in ECI coordinate system and transformation for the RSW system are given in Equation (1).

$$\hat{\boldsymbol{R}} = \frac{\boldsymbol{r}}{|\boldsymbol{r}|}, \quad \hat{\boldsymbol{W}} = \frac{\boldsymbol{r} \times \boldsymbol{v}}{|\boldsymbol{r} \times \boldsymbol{v}|}, \quad \hat{\boldsymbol{S}} = \hat{\boldsymbol{W}} \times \hat{\boldsymbol{R}}$$

$$\boldsymbol{M}_{ECI \to RSW} = (\hat{\boldsymbol{R}} \quad \hat{\boldsymbol{S}} \quad \hat{\boldsymbol{W}}) \tag{1}$$

(2) NTW Coordinate System

NTW coordinate system is also known as VNC system. The T axis (Tangential or In-track) is points to the velocity vector. The N axis (Normal) is perpendicular to T vector in the orbital plane. The W axis (Cross-track) is found by crossing the T and N vectors. The unit vectors of N, T, W axis in ECI coordinate system and transformation from the state vector, $\vec{r}$ and $\vec{v}$ for the NTW system are given in Equation (2).

$$\hat{\boldsymbol{T}} = \frac{\boldsymbol{v}}{|\boldsymbol{v}|}, \quad \hat{\boldsymbol{W}} = \frac{\boldsymbol{r} \times \boldsymbol{v}}{|\boldsymbol{r} \times \boldsymbol{v}|}, \quad \hat{\boldsymbol{N}} = \hat{\boldsymbol{T}} \times \hat{\boldsymbol{W}}$$

$$\boldsymbol{M}_{ECI \to NTW} = (\hat{\boldsymbol{N}} \quad \hat{\boldsymbol{T}} \quad \hat{\boldsymbol{W}}) \tag{2}$$

3.2 Orbit Propagation and Error Calculation

Propagating all of the TLEs to the epochs of the respective TLEs and calculating the difference which is known as residual between the prediction state vectors and "true state vectors" to generate the comparison data.

First step, use the SGP4 orbital propagation model to get the state vectors of objects. Each TLE is propagated forward to the epoch of every TLE is more recent than it until the epoch time difference Δt between them more than time threshold. The time threshold is set to 14 days in this paper. The technique is shown in Figure 1.

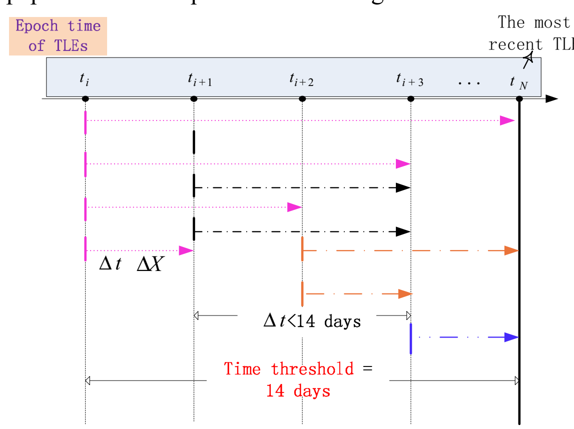


Fig 1. Propagation Method

The time t_N is relative to the N[th] TLE which is known as the most recent TLE in the analysis period. The TLEs at t_i are propagated to the epoch time of TLEs in time interval $[t_{i+1}, t_{i+14}]$, and a serious of position and velocity vectors are determined in the ECI coordinate system. These vectors are considered as estimate measurements and the residuals are calculated by Equation (3).

$$\begin{aligned} \delta\vec{r} &= \vec{r}_{estimate} - \vec{r}_{truth} \\ \delta\vec{v} &= \vec{v}_{estimate} - \vec{v}_{truth} \\ \Delta\boldsymbol{X} &= (\delta\vec{r}, \delta\vec{v}) \end{aligned} \tag{3}$$

The steps above are repeated for each TLE and their results are stored. For this study, the covariance matrix is computed at the epoch time of the N[th] TLE from the residuals obtained by propagating the i[th] TLE to the N[th] TLE (time difference between two TLE is in 14 days). Each pair-wise difference corresponds to a time difference Δt and residual vector, $\Delta\boldsymbol{X}$. The $\Delta\boldsymbol{X}$ is in ECI coordinate system, so the state vectors should be transformed to satellite-based coordinate by the transformation matrix, $\boldsymbol{M}_{ECI \to RSW}$ or $\boldsymbol{M}_{ECI \to NTW}$.

The Δt is unequal time interval and the Δt can be any value, therefore it's respective $\Delta\boldsymbol{X}$ should be binned according to the corresponding epoch time difference, Δt. 14 days should be separated by proper bin sizes. In this paper, the number of bins is 14 and the bin sizes are equal. Table2 shows the delta epoch bins.

Table2. Delta Epoch for 14 Bins

Bin No.	TLE Epoch Difference(day intervals)
1	[0, 0.5)
2	[0.5, 1.5)
3	[1.5, 2.5)
4	[3.5, 4.5)
5	[4.5, 5.5)
…	…
13	[11.5, 12.5)
14	[12.5, 13.5)

3.3 Covariance Matrix

Propagate each TLE to the epoch of the Nth TLE as estimates of state vectors. Note that the time difference of the Nth TLE and TLEs which are propagated should not be more than the time threshold.

For calculating errors, the true state for each epoch of TLE should be known. But unfortunately, the true state is unknown, so in this study the true state for each epoch of TLE is substituted. States that obtained by TLEs propagate to their own epoch are set to the true states. The true state can be noted by $\boldsymbol{X}_{epoch}$. Since the true state is unknown, the true error is unknown either. So in this paper use the residuals at the epoch, $\Delta\boldsymbol{X}_{epoch}$, which are found from the propagated estimates. The $\boldsymbol{X}_{epoch}$ and $\Delta\boldsymbol{X}_{epoch}$ are calculated by Equation (4), (5).

$$\boldsymbol{X}_{epoch} = (\vec{r}_{epoch}, \vec{v}_{epoch}) \tag{4}$$

$$\begin{aligned} \delta\vec{r}_{epoch} &= \vec{r}_{estimate} - \vec{r}_{epoch} \\ \delta\vec{v}_{epoch} &= \vec{v}_{estimate} - \vec{v}_{epoch} \\ \Delta\boldsymbol{X}_{epoch} &= (\delta\vec{r}_{epoch}, \delta\vec{v}_{epoch}) \end{aligned} \tag{5}$$

The covariance matrix $\boldsymbol{P}_{epoch}$ is calculated by Equation (6). $\Delta\overline{\boldsymbol{X}}_{epoch}$ is the mean vector residuals at the epoch and is found by Equation (7). N-1 is the number of residuals calculated at the epoch of Nth TLE

$$\boldsymbol{P}_{epoch} = \frac{\sum_{i=1}^{N-1}(\Delta\boldsymbol{X}_{epoch}-\Delta\overline{\boldsymbol{X}}_{epoch})_i(\Delta\boldsymbol{X}_{epoch}-\Delta\overline{\boldsymbol{X}}_{epoch})_i^T}{N-1} \tag{6}$$

$$\Delta\overline{\boldsymbol{X}}_{epoch} = \frac{\sum_{i=1}^{N-1}(\Delta\boldsymbol{X}_{epoch})_i}{N-1} \tag{7}$$

3.4 Error Evolution Function Fitting

The pair-wise differencing technique from before resulted in 14 bins. By binning the residual data, the error data is spaced in equal time intervals. For each bin, calculate the mean residual, $\Delta\overline{\boldsymbol{X}}_i$ and variance, $\boldsymbol{D}_{\Delta Xi}$, standard deviation $\boldsymbol{\sigma}_{\Delta Xi}$ of residuals. The variance and standard deviation is found by Equation (9), (10). The mean residual is found by Equation (8).

$$\Delta\overline{\boldsymbol{X}}_i = \frac{\sum_{i=1}^{N_i}(\Delta\boldsymbol{X}_i)_i}{N_i} \tag{8}$$

$$\boldsymbol{D}_{\Delta Xi} = \frac{\sum_{i=1}^{N_i}(\Delta\boldsymbol{X}_i-\Delta\overline{\boldsymbol{X}}_i)^2}{N_i-1} \tag{9}$$

$$\boldsymbol{\sigma}_{\Delta Xi} = \sqrt{\boldsymbol{D}_{\Delta Xi}} \tag{10}$$

The i = 1, 2,…, bin, the N_i is sample size of ith bin.

Then use the quadratic polynomial fitting error standard deviation of propagation time function in least squares sense.

4 RESULTS

4.1 Bin Mapping and the Covariance Matrix

In this study, covariance matrix is calculated for the most recent TLE epoch of each satellite. The most recent TLE epoch of each satellite as listed in Table 3.

Table3. The Most Recent TLE Epoch

Satellite No.	Epoch
22675	2015.11.23 23:17:42.002
32711	2015.11.23 21:22:20.843
25010	2015.11.23 22:58:44.308
28544	2015.11.23 13:31:16.534

Sample sizes of residuals mapping to each bin are shown in Table 4. For a set of m TLEs, there are $\frac{m(m-1)}{2}$ combinations of TLEs.

Table4. Bin Mapping

	Sample Sizes of Each Satellite			
Bin	LEO	MEO	GEO	HEO
1	61	218	37	37
2	595	854	158	129
3	524	775	323	268
4	567	933	234	184
5	611	860	254	232
6	595	781	281	203
7	588	935	213	183
8	563	835	293	246
9	563	777	227	167
10	580	919	262	244
11	584	828	230	190
12	558	788	249	226
13	578	895	251	190
14	557	829	241	230

Using the method described in Chapter 3. Table 5 and Table 6 show the covariance matrix for the most recent TLE epoch of LEO in NTW coordinates system and HEO in RTC (RSW) coordinate system.

Table5. Covariance Matrix – COSMOS-2251

NTW	rT(km)	rN(km)	rW(km)
rT(km)	0.01878	-0.0002	-0.00053
rN(km)	-0.0002	4.15E-06	4.97E-06
rW(km)	-0.00053	4.97E-06	2.48E-05
vT(km/sec)	1.96E-07	-4.2E-09	-5.1E-09
vN(km/sec)	-1.5E-05	1.72E-07	3.14E-07
vW(km/sec)	-1.2E-05	7.44E-08	5.96E-07

NTW	vT(km/sec)	vN(km/sec)	vW(km/sec)
rT(km)	1.96E-07	-1.5E-05	-1.2E-05
rN(km)	-4.2E-09	1.72E-07	7.44E-08
rW(km)	-5.1E-09	3.14E-07	5.96E-07
vT(km/sec)	4.25E-12	-1.7E-10	-7.8E-11
vN(km/sec)	-1.7E-10	1.33E-08	5.92E-09
vW(km/sec)	-7.8E-11	5.92E-09	1.58E-08

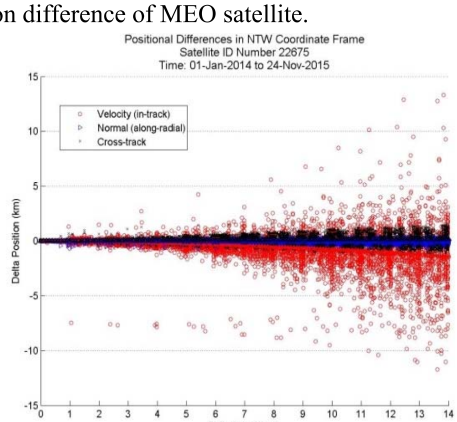
Table6. Covariance Matrix – SLOSHSAT

RTC	rR(km)	rT(km)	rC(km)
rR(km)	123.8553072	179.08073	-1.53809
rT(km)	179.0807326	259.02973	-2.18507
rC(km)	-1.53809368	-2.185073	0.035258
vR(km/sec)	-0.09961644	-0.144084	0.001217
vT(km/sec)	-0.00079846	-0.00112	2.4E-05
vC(km/sec)	-0.00089741	-0.001279	1.87E-05

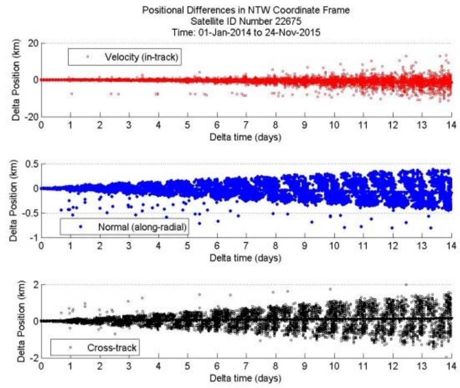
RTC	vR(km/sec)	vT(km/sec)	vC(km/sec)
rR(km)	-0.09961644	-0.00079846	-0.0009
rT(km)	-0.14408354	-0.00112038	-0.00128
rC(km)	0.001217119	2.40394E-05	1.87E-05
vR(km/sec)	8.01464E-05	6.24449E-07	7.12E-07
vT(km/sec)	6.24449E-07	1.79485E-08	1.24E-08
vC(km/sec)	7.12377E-07	1.24392E-08	1.01E-08

4.2 Position Residuals

All plots will be in the VNC coordinate system. The standard deviation (**σ**) is expressed as “StandaVariance” or “StandVariance”. Figure 2 illustrates the position difference of LEO satellite and Figure 3 illustrates the position difference of MEO satellite.

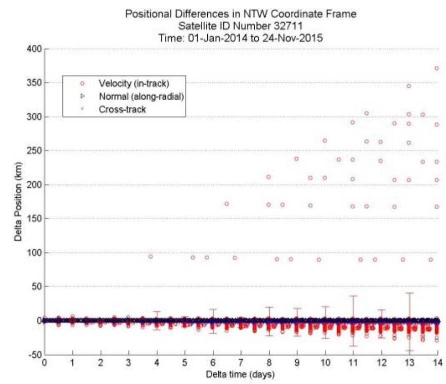


(a)

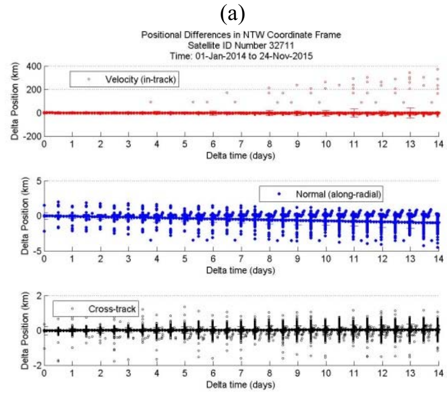


(b)

Fig 2. VNC Position Residuals – COSMOS-2251



(a)



(b)

Fig 3. VNC Position Residuals – GPS-62

(a) of figure displayed three component behaviors in one plot. (b) of figure using subplots displayed the behavior of each component can be discerned separately.

The range of errors in the normal and cross-track directions is bonded by small value. The velocity errors are much larger and grow more rapidly. The errors are growing with the propagation time.

4.3 TLE Comparison to HPOP

For each of the 4 satellites, the evolution function of error standard deviation is found by the function fitting. After calculated the covariance matrix for each most recent TLE epochs of 4 satellites, use these covariance matrixes as the initial covariance matrixes for HPOP model. The numerical integration model of HPOP is RKF7 (8) and the drag model is Harris-Priester.

Use SGP4 and HPOP model to predict the orbit with 2 days. Compare the standard deviation results by two methods. Figure 4 compared the variance, standard deviation spread with the propagation time in each component of RSW with which in NTW and sample points mapping for LEO satellite. Figure 5 is for the HEO satellite. Figure 6 to 9 illustrate the standard deviation fitting function of 4 satellites in different orbits. Figure 10 to 11 compared the TLE standard deviation propagation by SGP4 and HPOP covariance propagation.

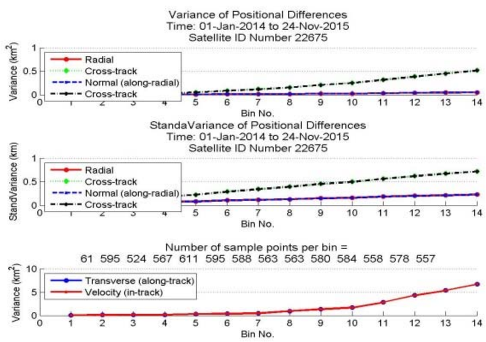


Fig 4. VNC and RTC Variance/Standard deviation – COSMOS-2251

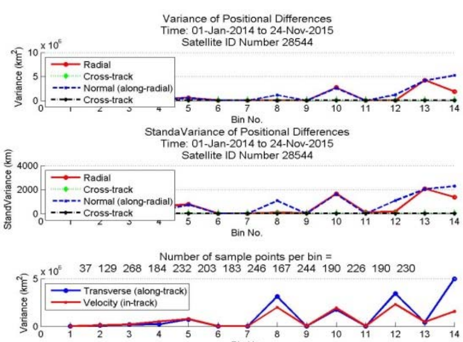


Fig 5. VNC and RTC Variance/Standard deviation – SLOSHSAT

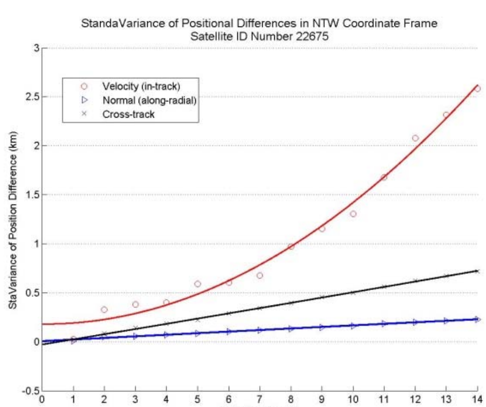


Fig 6. VNC Standard Deviation Fitting Function – COSMOS-2251

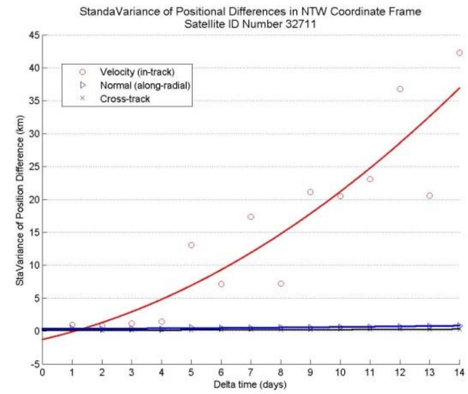


Fig 7. VNC Standard deviation Fitting Function – GPS-62

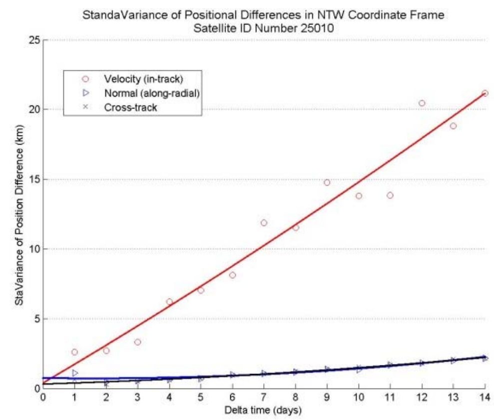


Fig 8. VNC Standard Deviation Fitting Function – TelStar-10

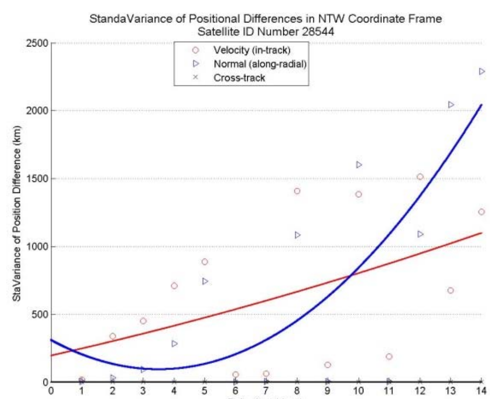
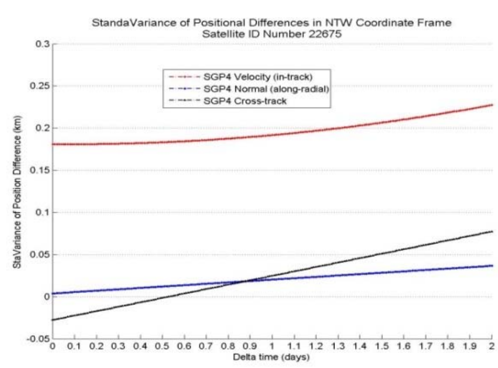
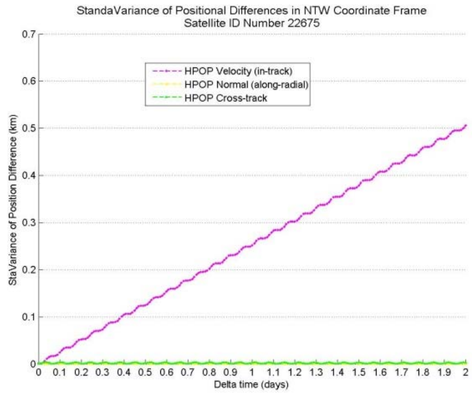


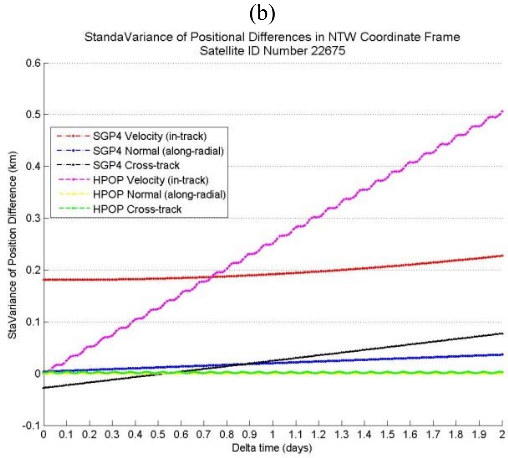
Fig 9. VNC Standard Deviation Fitting Function – SLOSHSAT



(a)



(b)



(c)

Fig 10. VNC Standard Deviation Fitting Function Compare with HPOP Standard Deviation – COSMOS-2251

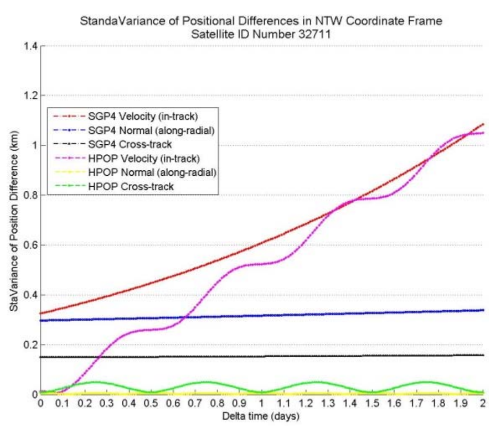


Fig 11. VNC Standard Deviation Fitting Function compare with HPOP Standard Deviation – GPS-62

5 CONCLUSION

According the analysis, we can conclude:

- Use TLEs could estimate the orbital covariance matrix for a give TLE.
- TLE predictions using SGP4 perform better for LEO orbit.
- SGP4 performs worse with the propagation time increased.
- The method described in this paper could calculate the error which reasonably approximates the true error of a TLE prediction.
- The velocity errors are much larger and grow more rapidly. The errors are growing with the propagation time.
- There are biases in the TLE errors. Therefore, if we have to improve the estimate, the bias should be removed.

6 FUTURE REASERCH

Removing the bad data by 3-sigma method, the covariance and standard deviation fitting functions might be improved. The fitting function is calculated by least-squares method. The accuracy of the results may be improved by another function fitting method. Errors of epochs do not reflect the cyclical nature of the orbit prediction errors, so we can consider the mean anomaly (M) to calculate the errors.

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