Software Development Kit: Two-Dimension Probability of Collision (Pc) Calculations

**Spacemap**

**인턴 심규일**

**2023-11-10**

# Table of Contents

[1.0 Introduction 4](#_Toc29643383)

[2.0 Object TLE and Dimension 5](#_Toc29643383)

[2.1 TLE\_visualization 6](#_Toc29643383)

[3.0 Maximum Collision Probability](#_Toc29643384) 7

[3.1 Celestrak\_and\_Alfano](#_Toc29643384) 9

[3.2 HBR\_distribution](#_Toc29643384) 10

[3.3 Pcmax\_Alfano 1](#_Toc29643384)1

[3.4 NormalTest 1](#_Toc29643384)3

[3.5 Velocity\_HBR 1](#_Toc29643384)4

[4.0 Conjunction Assesement Result 1](#_Toc29643385)7

[4.1 Celestrak\_CA\_Result 1](#_Toc29643385)7

[4.2 Spacemap\_CA\_Result(Fixed\_HBR)](#_Toc29643385) 19

[4.3 Spacemap\_CA\_Result(Variable\_HBR) 21](#_Toc29643385)

[4.4 Spacemap\_CA\_Result(N.Berend)](#_Toc29643385) 23

# List of Figures

Figure 1 : -

Figure 2 : Range -

Figure 3 : Pair Standard Deviation Scatter

Figure 4 : Celestrak Scatter (x:range, y:Pc)\_2023092413

Figure 5 : SPACEMAP Scatter and Boxplot (x:range, y:Pc) \_2023092413

Figure 6 : SPACEMAP Scatter (x:range, y:Ratio) \_2023092413

**List of Tables**

Table 1: TLE and Dimension data

Table 2 : Priorities for calculating radius

Table 3 : Calculating radius based on object’s shape

Table 4 : Accuracy of SPACEMAP Pc

Table 5 : Normality tests by Kolmogorov-Smirnov, Jarque–Bera test

Table 6 : Celestrak SOCRATES CSV file’ columns

Table 7 : Celestrak SOCRATES CSV file

Table 8 : Spacemap CA CSV file’ columns

Table 9 : Spacemap CA CSV file

Table 10 : Spacemap CA CSV file’ columns

Table 11 : Spacemap CA CSV file

Table 12 : Spacemap CA CSV file’ columns

Table 13 : Spacemap CA CSV file

**1.0 Introduction**

If we use a low precision covariance, the probability value will be too small to be meaningful . Therefore, we compute a maximum collision probability that conservatively computes the covariance. we have several assumption of error covariance ellipse.

1. orientation along with major axis.
2. shape is fixed by aspect ratio which is 3 in this code.

then, we can calculate Pcmax easily  
For some additional analysis, there is a secondary code that can be used.

**2.0 Object TLE and Dimension**

특정 시점의 TLE data 와 Dimension data를 space-track과 esa 로부터 추출:

TLE : (<https://www.space-track.org/>)

Dimension : ( <https://discosweb.esoc.esa.int/> )

Table 1: TLE and Dimension data

|  |  |
| --- | --- |
| File name | Description |
| size\_09m24d08h\_primary.txt | 2023-09-24-08 object' dimenstion data |
| size\_09m24d08h\_secondary.txt | 2023-09-24-08 object' dimenstion data |
| size\_11m07d00h\_primary.txt | 2023-11-07-00 object' dimenstion data |
| size\_11m07d00h\_secondary.txt | 2023-11-07-00 object' dimenstion data |
| spacetrack\_09m24d08h.txt | 2023-09-24-08 object' TLE data |
| spacetrack\_11m07d00h.txt | 2023-11-07-00 object' TLE data |

**2.1 TLE\_visualization.ipynb**

TLE 데이터는 space-track의 gp 데이터를 사용함. Sgp4 lisbrary 이용

일정한 time interval만큼 propagation하여 위성의 궤도운동을 시각화함.

**도표, 스케치, 그림, 텍스트이(가) 표시된 사진

자동 생성된 설명**

**3.0 Maximum Collision Probability**

**3.0.1 overivew**

낮은 정밀도의 covariance를 사용하면, 확률 값이 너무 작게 나와서 의미 없는 결과가 나옴. 따라서, covariance를 보수적으로 계산하는 maximum collision probability를 계산함

**3.0.2 Maximum collision probability equation**

Input parameters

a) : AR (Aspect Ratio)\*: ( : covariance ellipsoid의 x 및 z방향 standard deviation)

b) : HBR (Hard-Body Radius)\*\*: 두 위성의 반지름 합

c) : DCA (Distance at Close Approach)

d) : Error variance

일 때, 의 값이 maximum collision probability가 된다[2].

**3.0.3 Calculate Hard-body Radius**

Dimension 정보는 ESA를 참고함. Radius를 결정하는 우선순위는 object type, span값, diameter값, width, height, depth로 shape 마다 계산식으로 도출된 값 순이다. Object의 Shape와 Type에 따라 계산식을 구분함.

Table 2 : Priorities for calculating radius

|  |  |
| --- | --- |
| 우선 순위 | 계산식 |
| OBJECT\_TYPE == Debris | Radius = 30cm |
| Primary shape== Box + 2 Pan  And secondary shape== None | Secondary Radius = 0m |
| Span | Radius = Span / 2 |
| Diameter | Radius = diameter |
| Span 및 Diameter 정보가 없을 때 | Shape에 따른 계산식 이용 |
| Dimension 정보가 없을 때 | When OBJECT\_TYPE == Debris , Radius = 30cm  Else, Radius = 1.5m |

Table 3 : Calculating radius based on object’s shape

|  |  |
| --- | --- |
| Shape | 계산식 |
| Box + 2 Pan |  |
| Box + 1 Pan |  |
| Box |  |
| Cyl |  |
| Sphere |  |

**3.1 Celestrak\_and\_Alfano.ipynb**

**3.1.1**

Alfano algorithm:

일 때, 의 값이 maximum collision probability가 된다.

**3.1.2 Celestrak, Spacemap 확률 비교**

**텍스트, 라인, 도표, 그래프이(가) 표시된 사진

자동 생성된 설명텍스트, 라인, 도표, 그래프이(가) 표시된 사진

자동 생성된 설명**

Figure 1 : -

**텍스트, 라인, 도표, 그래프이(가) 표시된 사진

자동 생성된 설명텍스트, 스크린샷, 라인, 도표이(가) 표시된 사진

자동 생성된 설명**

Figure 2 : Range -

결론 : 그래프 개형이 거의 유사하여 Alfano알고리즘을 사용한 것으로 보임.

**3.2 HBR\_distribution.ipynb**

Celestrak 알고리즘이 Alfano와 일치하다는 것을 간접적으로 알 수 있는 다른 방법은 같은 pair에 대해서는 fixed input값이 같아야 한다. 여기서 fixed input은 hard-body radius(HBR)이다.

만약, 특정 pair의 HBR 값이 일정하다면 알고리즘이 동일하다고 할 수 있다.

**텍스트, 스크린샷, 번호, 디스플레이이(가) 표시된 사진

자동 생성된 설명**

Figure 3 : Pair Standard Deviation Scatter

결론 :대체로 표준편차가 크지 않고 대체로 비슷한 값을 가짐 🡪알고리즘 동일

예외 : 상대속도가 작을 때 (0~0.05), 확률 값을 특정 값으로 유지하는 pair

**3.3 Pcmax\_Alfano.ipynb**

외부데이터 TLE/Dimension/TCA로 CA파일생성. 계산된 결과를 celestrak과 비교  
Celestrak의 CA파일로부터 HBR값을 역연산. 정확도의 기준은 Celestrak값의 0.1배~10배, 0.5배~3배 두가지로 계산함. Visualization tool로 boxplot과 scatter를 이용했다.

외부데이터 출처

TLE : space-track.org

Dimension : DISCOSweb.esa

TCA : celestrak.org

**텍스트, 스크린샷, 라인, 도표이(가) 표시된 사진

자동 생성된 설명**

Figure 4 : Celestrak Scatter (x:range, y:Pc)\_2023092413

**텍스트, 스크린샷, 도표, 라인이(가) 표시된 사진

자동 생성된 설명**

Figure 5 : SPACEMAP Scatter and Boxplot (x:range, y:Pc) \_2023092413

**텍스트, 스크린샷, 라인, 폰트이(가) 표시된 사진

자동 생성된 설명**

Figure 6 : SPACEMAP Scatter (x:range, y:Ratio) \_2023092413

Table 4 : Accuracy of SPACEMAP Pc

|  |  |  |
| --- | --- | --- |
| 총 pair 36,497 | Celestrak 0.1배~10배 | Celestrak 0.5배~3배 |
| 범위 내 개수 | 34,599 | 28,905 |
| 범위 밖 개수 | 1,898 | 7,592 |
| 범위 내 비율 | 94.8% | 79.2% |
| 범위 밖 비율 | 5.2% | 20.8% |

계산된 결과는 4.0 Conjunction assesement result 에 저장됨

**3.4 NormalTest.ipynb**

ERROR(RATIO)열에 대한 분포가 정규분포를 따르는지 검정.

1995년에 발표한 [West,skewness and kurtosis를 활용한 정규성 평가] 연구논문에 의하면

"skewness(=왜도)는 2, kurtosis(=첨도)는 7보다 작으면 정규성을 보인다"고 할 수 있다.

귀무가설 : ERROR(RATIO)의 분포가 정규분포를 따른다.

Table 5 : Normality tests by Kolmogorov-Smirnov, Jarque–Bera test

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **검정통계치** | **P-VALUE** | **왜도** | **첨도** |
| **Kolmogorov-Smirnov test** | 0.6343 | 0.0 | 8.9 | 172.5 |
| **Jarque–Bera test** | 45734187.0 | 0.0 | 8.9 | 172.5 |

**그래프, 도표, 라인, 스크린샷이(가) 표시된 사진

자동 생성된 설명**

Figure 7: Distribution of ERROR(RATIO)

귀무가설 기각 🡪 정규분포를 따르지 않는다.

**3.5 Velocity\_HBR.ipynb**

가설 ( HBR이 velocity와 관련이 있다. ) 에 대해서 여러가지 실험을 진행함.

object의 HBR는 정적인 값이 아닌 위성의 속도에 따라 동적인 값일 수도 있음.

텍스트, 스크린샷, 라인, 도표이(가) 표시된 사진

자동 생성된 설명

Figure 8: Scatter between relative velocity and celestrak HBR

Figure8을 통해 relative velocity가 HBR에 주는 영향이 없지 않음을 인지하게 됨

HBR과 log관계에 놓여있다고 여겨짐.

Object의 Type별로 scatter를 나누어서 살펴본 결과 primary object가 unknown일 때, 확연한 log관계를 보임.

텍스트, 라인, 도표, 그래프이(가) 표시된 사진

자동 생성된 설명

Figure 9: Scatter between relative velocity and celestrak HBR

when primary object’s type is unknown

위 scatter를 잘 표현하는 함수를 찾기 위해 R-squared, MSE, MAPE 지표를 기준으로 model fitting을 진행하였다.

텍스트, 스크린샷, 그래프, 라인이(가) 표시된 사진

자동 생성된 설명

Figure 10: model fitting between relative velocity and celestrak HBR

when primary object’s type is unknown

지표상으로 4차 다항식이 적합되었지만 그래프 추세를 가장 잘 설명하는 Exponential을 선택하였음.

Fitting된 log함수를 통해서 primary object’s type이 unknown일 때, 해당 계산식을 적용하였음.

텍스트, 스크린샷, 라인, 폰트이(가) 표시된 사진

자동 생성된 설명텍스트, 스크린샷, 라인, 폰트이(가) 표시된 사진

자동 생성된 설명

Figure 10: Scatter before applying log function and Scatter applying log function

Log 함수를 적용한 후의 결과가 좀더 noise가 줄고 bias가 줄어들었다.

**4.0 Conjunction assesement result**

**4.1 Celestrak\_CA\_result**

Columns Description:

Table 6 : Celestrak SOCRATES CSV file’ columns

|  |  |
| --- | --- |
| Column name | Description |
| NORAD\_CAT\_ID\_1 | NORAD Catalog Number for the first object. Note that this is the first of the two objects involved in the conjunction and not what might be considered the primary object. |
| OBJECT\_NAME\_1 | Satellite name for the first object, extracted from the CelesTrak SATCAT. The operational status of the object is appended to the name, in brackets. |
| DSE\_1 | Days since epoch for the first object. This is the time in days from the epoch of the GP data used in the calculation to the calculated time of closest approach (TCA) and is an indication of how accurate the data might be at TCA |
| NORAD\_CAT\_ID\_2 | NORAD Catalog Number for the second object. Note that this is the second of the two objects involved in the conjunction and not what might be considered the secondary object. |
| OBJECT\_NAME\_2 | Satellite name for the second object, extracted from the CelesTrak SATCAT. The operational status of the object is appended to the name, in brackets |
| DSE\_2 | Days since epoch for the first object. This is the time in days from the epoch of the GP data used in the calculation to the calculated time of closest approach (TCA) and is an indication of how accurate the data might be at TCA |
| TCA | Time of closest approach. |
| TCA\_RANGE | The distance or range between the two objects at TCA. Often referred to as the minimum range. |
| TCA\_RELATIVE\_SPEED | The magnitude of the relative velocity at TCA. This value provides an indication of the risk (specific kinetic energy) for the two objects if they collided. |
| MAX\_PROB | Maximum Probability |
| DILUTION | Dilution threshold |
| OBJECT\_TYPE\_1 | Payload or Rocket Body or Debris or Unknown |
| OBJECT\_TYPE\_2 | Payload or Rocket Body or Debris or Unknown |

UTC 기준 05/13/21 마다 업데이트되는 Celestrak's SOCRATES CA csv파일 :

Table 7 : Celestrak SOCRATES CSV file

|  |  |
| --- | --- |
| File name | Description |
| Celestrak\_CA\_09m24d13h | 2023-09-24-13 TLE로 계산된 CA Result |
| Celestrak\_CA\_09m24d13h\_dilution | Dlution 계산방식이 포함된 CA Result |
| Celestrak\_CA\_11m07d05h | 2023-11-07-00 TLE로 계산된 CA Result |

**4.2 Spacemap\_CA\_Result(Fixed\_HBR)**

Table 7의 파일을 이용해 확률계산 후 결과를 Celestrak의 CA결과와 비교.

(다만 , 확률 계산 시 Celestrak의 TCA 참고함.)

이때, HBR(combined hard-body radius)을 고정시킴 (1m, 1.5m, 2m, 3m, 4m, 5m)

Columns Description:

Table 8 : Spacemap CA CSV file’ columns

|  |  |
| --- | --- |
| Column name | Description |
| CATEGORY | Celestrak or spacemap |
| INDEX |  |
| NORAD\_CAT\_ID\_1 | NORAD Catalog Number for the first object. Note that this is the first of the two objects involved in the conjunction and not what might be considered the primary object. |
| OBJECT\_NAME\_1 | Satellite name for the first object, extracted from the CelesTrak SATCAT. The operational status of the object is appended to the name, in brackets. |
| SHAPE\_1 | shape of the object corresponding to NORAD\_CAT\_ID\_1 |
| OBJECT\_CLASS\_1 | ObjectClass of the object corresponding to NORAD\_CAT\_ID\_1 |
| NORAD\_CAT\_ID\_2 | NORAD Catalog Number for the second object. Note that this is the second of the two objects involved in the conjunction and not what might be considered the secondary object. |
| OBJECT\_NAME\_2 | Satellite name for the second object, extracted from the CelesTrak SATCAT. The operational status of the object is appended to the name, in brackets |
| SHAPE\_2 | shape of the object corresponding to NORAD\_CAT\_ID\_2 |
| OBJECT\_CLASS\_2 | ObjectClass of the object corresponding to NORAD\_CAT\_ID\_2 |
| TCA | Time of closest approach. |
| TCA\_RANGE | The distance or range between the two objects at TCA. Often referred to as the minimum range. |
| TCA\_RELATIVE\_SPEED | The magnitude of the relative velocity at TCA. This value provides an indication of the risk (specific kinetic energy) for the two objects if they collided. |
| DILUTION | Dilution threshold |
| RADIUS\_1 | Radius of the object corresponding to NORAD\_CAT\_ID\_1 |
| RADIUS\_2 | Radius of the object corresponding to NORAD\_CAT\_ID\_2 |
| HBR | Combined hard-body radius (=RADIUS\_1 + RADIUS\_2) |
| PC\_CELESTRAK | Celestrak maximum collision probablity |
| PC\_SPACEMAP | Spacemap maximum collision probablity |
| ERROR(RATIO) | Error between PC\_CELESTRAK and PC\_SPACEMAP  (= PC\_SPACEMAP/ PC\_CELESTRAK) |

Table 9 : Spacemap CA CSV file

|  |  |
| --- | --- |
| File name | Description |
| Spacemap\_CA\_09m24d13h\_HBR1 | 2023-09-24-13 TLE, HBR=1m |
| Spacemap\_CA\_09m24d13h\_HBR2 | 2023-09-24-13 TLE, HBR=2m |
| Spacemap\_CA\_09m24d13h\_HBR3 | 2023-09-24-13 TLE, HBR=3m |
| Spacemap\_CA\_09m24d13h\_HBR4 | 2023-09-24-13 TLE, HBR=4m |
| Spacemap\_CA\_09m24d13h\_HBR5 | 2023-09-24-13 TLE, HBR=5m |

**4.3 Spacemap\_CA\_Result(Variable\_HBR)**

object's TLE and Dimension 폴더의 데이터를 input으로 삼아 CA한 결과.

HBR(combined hard-body radius)을 object의 shape과 type에 따라 계산식을 달리함

Columns Description:

Table 10 : Spacemap CA CSV file’ columns

|  |  |
| --- | --- |
| Column name | Description |
| CATEGORY | Celestrak or spacemap |
| INDEX |  |
| NORAD\_CAT\_ID\_1 | NORAD Catalog Number for the first object. Note that this is the first of the two objects involved in the conjunction and not what might be considered the primary object. |
| OBJECT\_NAME\_1 | Satellite name for the first object, extracted from the CelesTrak SATCAT. The operational status of the object is appended to the name, in brackets. |
| SHAPE\_1 | shape of the object corresponding to NORAD\_CAT\_ID\_1 |
| OBJECT\_CLASS\_1 | ObjectClass of the object corresponding to NORAD\_CAT\_ID\_1 |
| NORAD\_CAT\_ID\_2 | NORAD Catalog Number for the second object. Note that this is the second of the two objects involved in the conjunction and not what might be considered the secondary object. |
| OBJECT\_NAME\_2 | Satellite name for the second object, extracted from the CelesTrak SATCAT. The operational status of the object is appended to the name, in brackets |
| SHAPE\_2 | shape of the object corresponding to NORAD\_CAT\_ID\_2 |
| OBJECT\_CLASS\_2 | ObjectClass of the object corresponding to NORAD\_CAT\_ID\_2 |
| TCA | Time of closest approach. |
| TCA\_RANGE | The distance or range between the two objects at TCA. Often referred to as the minimum range. |
| TCA\_RELATIVE\_SPEED | The magnitude of the relative velocity at TCA. This value provides an indication of the risk (specific kinetic energy) for the two objects if they collided. |
| DILUTION | Dilution threshold |
| RADIUS\_1 | Radius of the object corresponding to NORAD\_CAT\_ID\_1 |
| RADIUS\_2 | Radius of the object corresponding to NORAD\_CAT\_ID\_2 |
| HBR | Combined hard-body radius (=RADIUS\_1 + RADIUS\_2) |
| PC\_CELESTRAK | Celestrak maximum collision probablity |
| PC\_SPACEMAP | Spacemap maximum collision probablity |
| ERROR(RATIO) | Error between PC\_CELESTRAK and PC\_SPACEMAP  (= PC\_SPACEMAP/ PC\_CELESTRAK) |

Table 11 : Spacemap CA CSV file

|  |  |
| --- | --- |
| File name | Description |
| Spacemap\_CA\_09m24d13h | 2023-09-24-13 TLE |
| Spacemap\_CA\_09m24d13h\_HBR | 2023-09-24-13 TLE, HBR 역연산 |
| Spacemap\_CA\_09m24d13h\_Unknown | 2023-09-24-13 TLE, primary object class가 Unknown일 때, CA result |
| Spacemap\_CA\_09m24d13h\_Unknown\_HBR | 2023-09-24-13 TLE, primary object class가 Unknown일 때, CA result, HBR 역연산 |
| Spacemap\_CA\_11m07d05h | 2023-09-24-13 TLE |
| 같은쌍일때 표준편차가0.001이상인 pair | 서로 다른 시점에서 동일한 HBR이 아닌 pair |
| 특이케이스 | 확률값을 일정하게 유지하는 pair |

**4.4 Spacemap\_CA\_Result(N.Berend)**

Berend 알고리즘과 CARA의 Frisbee 알고리즘을 합쳐서 만듦.

초기 공분산 행렬을 결정하고 k상수로 공분산의 사이즈를 조절하는데 pc를 최대로 만드는 k값을 대입 후 frisbee 알고리즘 호출.

Columns Description:

Table 12 : Spacemap CA CSV file’ columns

|  |  |
| --- | --- |
| Column name | Description |
| CATEGORY | Celestrak or spacemap |
| INDEX |  |
| NORAD\_CAT\_ID\_1 | NORAD Catalog Number for the first object. Note that this is the first of the two objects involved in the conjunction and not what might be considered the primary object. |
| OBJECT\_NAME\_1 | Satellite name for the first object, extracted from the CelesTrak SATCAT. The operational status of the object is appended to the name, in brackets. |
| SHAPE\_1 | shape of the object corresponding to NORAD\_CAT\_ID\_1 |
| OBJECT\_CLASS\_1 | ObjectClass of the object corresponding to NORAD\_CAT\_ID\_1 |
| NORAD\_CAT\_ID\_2 | NORAD Catalog Number for the second object. Note that this is the second of the two objects involved in the conjunction and not what might be considered the secondary object. |
| OBJECT\_NAME\_2 | Satellite name for the second object, extracted from the CelesTrak SATCAT. The operational status of the object is appended to the name, in brackets |
| SHAPE\_2 | shape of the object corresponding to NORAD\_CAT\_ID\_2 |
| OBJECT\_CLASS\_2 | ObjectClass of the object corresponding to NORAD\_CAT\_ID\_2 |
| TCA | Time of closest approach. |
| TCA\_RANGE | The distance or range between the two objects at TCA. Often referred to as the minimum range. |
| TCA\_RELATIVE\_SPEED | The magnitude of the relative velocity at TCA. This value provides an indication of the risk (specific kinetic energy) for the two objects if they collided. |
| MAX\_PROB | Celestrak or Spacemap maximum collision probablity |
| ERROR(RATIO) | Error between PC\_CELESTRAK and PC\_SPACEMAP  (= PC\_SPACEMAP/ PC\_CELESTRAK) |
| RANGE\_ERROR | Error between spacemap’s TCA\_RANGE and celestrak’s TCA\_RANGE |

Table 13 : Spacemap CA CSV file

|  |  |
| --- | --- |
| File name | Description |
| Berend\_CA\_09m24d13h | 2023-09-24-13 TLE, Berend 알고리즘 사용 |