1 Heat Flux Calculations

The net heat flux, q_{net} (watts/m^2) is computed from the sum of five other heat flux components:

$$q_{net} = q_{sw} + q_{atm} - q_b + q_h + q_l, \tag{2}$$

where

q_{sw} = downwelling shortwave radiation from solar input
q_{atm} = downwelling longwave radiation from the atmosphere
q_b = upwelling longwave radiation from the water body
q_h = the sensible heat flux from air to water
q_l = latent heat.

Equation (2) assumes that the surface of the water is unobstructed. Once an ice cover is in place, this assumption is no longer valid, and the model will produce inaccurate results. The model is useful in the periods leading up to ice formation when the water body is open and heat fluxes from ice production are insignificant. These conditions are quite typical prior to periods of frazil ice production when an ice cover has not yet formed.

1.1 Downwelling Shortwave Radiation

The shortwave radiation from the sun is calculated using the Python library pvlib (Holmgren et al. 2018) and is a function of latitude, longitude, elevation, and date. The theoretical clear sky global horizontal irradiance (which is the sum of direct normal and diffuse irradiance on a horizontal plane) is calculated using pvlib on an hourly time-step through the forecast period. The theoretical clear sky value is modified by the amount of cloud cover that will intercept the shortwave radiation:

$$q_{sw} = q_{ghi}(1 - R)(1 - 0.65Cl^2), \tag{3}$$

where

q_{ghi} = the global horizontal irradiance (W/m^2)
R = the reflectivity of the water surface, assumed to be 0.15
Cl = the fraction of the sky covered with clouds.

The reflectivity in Equation (3) is assumed to be within the range reported by Maidment et al. (1993) for water.

1.2 Downwelling Longwave Radiation

The atmosphere emits longwave radiation that is absorbed by water. The atmospheric downwelling radiation is calculated as follows:

$$q_{atm} = \varepsilon_{air}\sigma T_{air}^4, \qquad (4)$$

where

ε_{air} = the emissivity of air (unitless)
σ = the Stefan-Boltzman constant (W/m^2-K^4)
T_{air} = the air temperature (K).

The emissivity of air was calculated using an empirical formula from Zhang and Johnson (2016):

$$\varepsilon_{air} = 0.937\text{x}10^{-5}(1 + 0.17Cl^2)T_{air}^2, \qquad (5)$$

1.3 Upwelling Longwave Radiation

The water body itself emits heat, which is lost in the form of longwave radiation upwards into the atmosphere. The equation for upwelling longwave radiation from the water is similar to the atmospheric downwelling equation:

$$q_b = \varepsilon_{water}\sigma T_{water}^4, \qquad (6)$$

where

ε_{water} = the emissivity of water (unitless)
T_{water} = the water temperature (K).

In Equation (6), the emissivity of water is assumed to be 0.97 for all simulations (Brunner 2016).

1.4 Sensible Heat

Heat is gained or lost by the water body from or to the air above it. Warm air above cooler water will tend to warm the water while frigid air, particularly air with a temperature below the freezing point of water, will cool the water body. The sensible heat is calculated using a simplified version of the method in HEC-RAS:

$$q_h = c_p \rho_w (T_{air} - T_{water}) f(U), \tag{7}$$

where

c_p = the specific heat of air (J/kg-K)
ρ_w = the density of water (kg/m^3)
$f(U)$ = the wind function (m/s).

The wind function in Equation (7) is an empirical relationship like HEC-RAS uses and scales the windspeed. The windspeed equation is

$$\mathrm{f(U) = R(a + bU^c)}, \tag{8}$$

where

R = the Richardson Number (unitless)
a, b, and c = calibration coefficients (unitless)
U = the windspeed (m/s).

The Richardson Number in Equation (8) describes the stability of the atmosphere, and was set to 1, which assumes the atmosphere is neutral (Brunner 2016). We set a and b equal to 10^{-6} and c equal to 1, which are the default values in the HEC-RAS water quality module and consistent with the initial values in Brunner (2016).

1.5 Latent Heat

The water body may gain or lose heat from condensation or evaporation of water at the surface. Latent heat refers to the change in energy during a phase change. Latent heat released during the transition from liquid water to solid ice is very important in modulating the rate of ice formation. However, this simplified model considers only the latent heat exchange due to the phase change of liquid water to water vapor:

$$q_l = \frac{0.622}{P} L \rho_w (es - ea) f(U), \tag{9}$$

where

P = the air pressure (mb),
L = the latent heat of vaporization (J/kg),

es = the saturated vapor pressure at the water surface (mb), and
ea = the vapor pressure of the air mass above the water (mb).

An empirical equation from Zhang and Johnson (2016) was used for the saturated vapor pressure at the water temperature shown in Equation (9). The saturated vapor pressure equation is a function of water temperature; however, it is too cumbersome to repeat here, and the reader is referred to the source document for more precise details.

The actual vapor pressure of the air is calculated from the dew point (https://www.weather.gov/media/epz/wxcalc/vaporPressure.pdf):

$$ea = 6.11 \cdot 10^{\frac{7.5T_{dew}}{237.3+T_{dew}}}, \tag{10}$$

where T_{dew} is the dew point temperature in Equation (10).

1.6 References

Brunner, G. W. 2016. *HEC-RAS River Analysis System: User's Manual*. CPD-68. Davis, CA: US Army Corps of Engineers, Institute for Water Resources, Hydrologic Engineering Center. https://www.hec.usace.army.mil/confluence/rasdocs/rasum/latest.

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