Syllabus

General Info

Course MAT461 Advanced Seminar: Number Theory

Instructor Charilaos Skiadas (skiadas at hanover dot edu)

Term Fall 2019-2020

Office SCH 111 / LYN 108

Office Hours MWF 10:30am-11:30am in SCH111, and by appointment

Book *Number Theory through inquiry* by Marshall, Odell and Starbird

Websites for notes[1]

Class times MWF 1:20pm-2:30pm in CFA111

Course Description

This course serves two distinct purposes:

- It is an advanced course in mathematics.
- It is a course about Number Theory.

Number Theory is the study of whole numbers, especially related to the notion of divisibility of numbers. It evolves around two key notions: On one side the decomposition of numbers into 'prime' components, on the other side modular arithmetic, which is essentially a generalization of the arithmetic that would take place in a 12-hour clock. Number theory has been an object of study since the ancient times, and still contains numerous simple to state but quite intractable questions.

Number Theory also has important applications to Cryptography. In essence, the security of every internet transaction is based on some fundamental number theory facts, the most common amongst them being that if a number is the product of two large prime numbers, then there is no efficient way to recover those prime numbers if all you know is their product. All current cryptographic techniques make heavy use of the tools we will learn in this class. We will explore some of these connections along the way.

As an advanced course in mathematics, this course will push your knowledge regarding the various research and proof techniques that mathematicians employ. We will spend the majority of each class discussing theorems and their proofs that the students will be providing. You will be asked to think about the validity of mathematical statements, to produce proofs for them as well as to confirm the veracity of proofs provided by others.

[1] skiadas.github.io/AdvancedSeminarNumberTheoryCourse/

Course objectives

- You will learn to provide rigorous proofs to mathematical statements, as well as critique other people's proofs.
- You will develop skills in formulating conjectures and theorems, as well as testing and refining them.
- You will experience mathematical independence and open-ended inquiry, as we encounter numerous open-ended questions which you are asked to explore.
- You will strengthen your oral communication skills by presenting your proofs to the other students, as well as engaging other students in discussion about their proofs.
- You will strengthen your written communication skills by providing written reports of your theorems and proofs.
- You will learn to use the SageMath mathematical software to test your hypotheses and practically verify your conjectures.

Here are some of the concrete methods of mathematical thought, proof and analysis that you will practice in this course:

- finding patterns and formulating conjectures
- making precise definitions and statements
- using basic logic
- forming negations, contrapositives and converses of statements
- constructing and understanding examples
- generalizing from examples
- looking for elementary building blocks
- following consequences of assumptions
- employing standard methods of proof like induction, contradiction, reducing complexity, and extending from a special case to a more general case

Course Components

Reading Assignments

In the class schedule page[2] you will find, for each class day, a list of links to reading assignments. Your homework will require you to have a solid understanding of the material covered there, so I strongly encourage you not to get behind.

[2] skiadas.github.io/AdvancedSeminarNumberTheoryCourse/schedule.html

Class Attendance

You are expected to attend every class meeting. As most of the class period will revolve around presenting your work and discussing other people's work, your presence is crucial and a lot of the learning in the class will occur in those discussions.

Assignments

There will be roughly three different kinds of assignments:

- Presentations of theorems
- Writeups of theorems and other content
- Small programming tasks

Presentations will be a daily occurence in the class. Writeups will likely occur about once a week, while programming tasks will be less frequent.

Exams

There will be three exams tentatively schedule for: Friday 10/4, Friday 11/8 and during exam week. The exams you do better on will count slightly more towards your final grade.

Getting Help

- You should never hesitate to ask me questions. I will never think any less of anyone for asking a question. Stop by my office hours or just email me your question, which has the great benefit of forcing you to write it down in clear terms, which often helps you understand it better.
- You are allowed, and in fact encouraged, to work together and help each other regarding the notes and the proofs. However, I strongly encourage you to work on each problem on your own first before talking to someone about them.

Grading

Your final grade depends on all the above components, as follows:

Component	Percent
Class Presentations	25%
Written Assignments	10%
Coding Assignments	5%
Worst Exam	15%
Middle Exam	20%
Best Exam	25%

This gives a number up to 100, which is then converted to a letter grade based roughly on the following correspondence:

Letter grade	Percentage Range
A, A-	90%-100%
B+, B, B-	80%-90%
C+, C, C-	70%-80%
D+, D, D-	60%-70%
F	0%-60%