Divisibility (1.1-1.6)

- Introduction and syllabus
- Introduction to CoCalc (one of your homeworks for Wednesday)
  - Accept your invitation
  - Look into the "Number Theory Project", find the "SampleLaTeXPage folder" and go in it.
  - Click on the "SampleLaTeXPage.tex" file to view it. Do NOT edit it.
  - Source code on left, resulting PDF on right. Scroll through the source code and compare with the output in order to learn how to mark certain documents.
  - Open the project in a new window. Go into the "chapter 1" folder. Open the "SolutionsDay1.tex" file.
  - Add a new section with your name in the title. If you find other's sections there, place yours relative to theirs according to the order of the problems you each have.
  - State the theorem or exercise you were assigned to do, then its solution.
- Definition: What are the **natural numbers**?
- Definition: What are the **integers**? How can we define the integers using the natural numbers?
- Definition: When do we say that an integer d **divides** another integer a? How do we denote it?
  - Provide examples.
- Think of edge cases:
  - Does an integer divide itself?
  - What integers does 1 divide? What integers divide 1?
  - What integers does 0 divide? What integers divide 0?
- Definition: When do we say that two integers a, b are **congruent modulo** another integer d?
  - Provide examples.
- Think of edge cases:
  - When is an integer congruent to itself?
  - When is an integer congruent to 0?
  - When is an integer congruent to its negative?
- Example theorem: If 6 divides an integer n, then 3 also divides n.

  - Walk through the proof. Emphasis on the fact that the proof consists of sentences, not simply equations.
  - Produce similar examples with different numbers.

- Example theorem: If an integer k is congruent to 7 modulo 2, then k is also congruent to 3 modulo 2.
  - Walk through the proof.
  - Produce similar examples with different numbers.

- Theorem 1.1: If a divides b and a divides c, then a also divides the sum of b and c.
- Theorem 1.2: If a divides b and a divides c, then a also divides the difference of b and c.
- Theorem 1.3: If a divides b and a divides c, then a also divides the product of b and c.
- Question 1.4: Can we weaken the hypotheses of theorem 1.3? Can we instead strengthen the conclusion?
- Theorem 1.5: Weakened version of theorem 1.3