

Activity Sheet 14

Manager name:

Recorder name:

Speaker name:

Section 8.1

1. We can approach the change-making problem by instead doing dynamic programming in two dimensions, j to represent that we use only the first j denominations $D[1]$ through $D[j]$, and n to represent the target value. So $F(j, n)$ is the number of coins we need to use, being allowed to only use the first j denominations. (Notice that indexing in D starts at 1)
 - a. Assuming we have the denominations $D = [1, 4, 6]$, determine $F(1, 5)$ as well as $F(2, 5)$.
 - b. What should $F(0, 0)$, $F(2, 0)$, $F(0, 1)$, $F(2, -2)$ be? Your answers to this don't really depend on the particular denominators used, they apply broadly.
 - c. We can build a recurrence relation for $F(j, n)$ as follows: In our effort to make change for n using the j first denominations, we have two options: We can either use the $D[j]$ denomination, and then we need to reach the target of $n - d_j$ using the first j denominations still, or we can not use the $D[j]$ denomination at all, meaning that we need to reach the target n using the first $j - 1$ denominations. Use this statement to write a recurrence relation for $F(j, n)$.

- d. Write an algorithm that uses this recurrence relation to fill the 2-dimensional array $F[0..m, 0..n]$. Make sure to either avoid or handle carefully the case where you look up a value $F[j, i]$ where i is a negative number.