Measures of Spread

Reading

Sections 1.6.4

Practice Problems

1.9.6 (page 65) 1.47

Notes

- The goal of measures of spead is to assess the "variation" in the data in some way.
- Different measures achieve this in different ways.
- The two main measures of spread are:

  Std. Dev. Standard Deviation. Measures distance of data from the mean.
  Often denoted by s.
  Not resistant.

  IQR Interquartile range. Measures range occupied by the middle 50% of the data.
  Resistant.

- Let us review how the standard deviation is computed. Its formula is a bit complicated, and it looks something like this:

$$s = \sqrt{\frac{1}{n-1}\sum (x - \bar{x})^2}$$

  Basically:

  - Compute the mean of all the values.
  - Subtract the mean from each value (the result of this step is the "**deviations**").
  - Square all the deviations.
    * Ensures values are positive before computing average.
  - Average these squared deviations: Add them all up, then divide by $n-1$. The result at this stage is called the **variance**.
    * Why $n-1$: Technical reason, and won't really matter for large n.
    * One way to think about it: The deviations always add up to 0, so once you know $n-1$ of them the last one is determined.

    - In this context, $n-1$ is called the "degrees of freedom".
  - Take a square root at the end.
    - Fixes the units of measurement.

- Outliers have a considerable effect on this formula.
  - They have a very large deviation, and because they pull the mean towards themselves they cause larger deviations in the other values as well.
  - Because we square them before adding, those large deviations will dominate the equation even more.

- What to use depends on the distribution:

  Symmetric Mean for center, Standard Deviation for spread
  Skewed/Outliers Median for center, IQR for spread

- Chebyshev's Rule:
  - At least 75% of the data is within 2 standard deviations from the mean
  - At least 89% of the data is within 3 standard deviations from the mean

- For Bell Shaped data (Empirical Rule):
  - Approximately 68% of the data is within one standard deviation of the mean.
  - Approximately 95% of the data is within two standard deviations of the mean.
  - More than 99% of the data is within three standard deviations of the mean.