Linear Transformations

Notes

If x represents a variable, then a **linear transformation** is an equation of the form:

$$y = a + bx$$

where a, b are some numbers. For instance $y = x + 10$, or $y = 2x - 1$.

We think of y here as a *new variable*, and the equation tells us how to convert values of the one variable into values of the other.

Examples:

- x is a student's height measured in "inches from 5 feet". y is their actual height in inches. Then:

$$y = x + 60$$

- x is a student's height in inches, y is their height in meters. 1 meter is 39.37 inches. So:

$$y = x/39.37$$

- x is a room's temperatures measured in F, y is the same temperatures in C. $F = 1.8C + 32$. So:

$$y = (x - 32)/1.8 = x/1.8 - 32/1.8$$

A linear transformation between two variables tells us how individual values transform to each other.

But how measures of center or spread behave requires more thinking!

Behavior of variables under linear transformation. Assume $y = a + bx$.

shape stays the same (modes, skewness, outliers)

center Follows the same transformation (mean, median do that)

e.g. $\bar{y} = a + b\bar{x}$

spread Only follows the multiplier (std. dev., IQR do that)

e.g. $s_y = bs_x$.

A special case: Standardized scores

Standardized scores, also called z-scores, are given by the following linear transformation:

$$z = \frac{x - \bar{x}}{s_x}$$

Alternatively, they relate to x via:

$$x = \bar{x} + s_x z$$

Key properties:

- Think of z-score as saying "how many standard deviations away from the mean you are".
- z-scores are unitless.
- z-scores always have a mean of 0 and a standard deviation of 1.
- They are great for comparing quantities that are measured in different scales.