The Normal Distribution

Reading

- Section 3.1
- Section 3.2

Practice Problems

3.6.1 (p. 158) 3.1, 3.2, 3.3, 3.5, 3.7, 3.9, 3.13, 3.16
3.6.2 (p. 161) 3.17, 3.18

Notes

Normal Distribution Theory

- The **Normal Distribution** is a bell-shaped curve.
- Complicated equation, but a lot of data follow the distribution, so we have to work with it.
- Its parameters are easy to relate to the data (mean and standard deviation).
- Its equation depends on two parameters:
  - μ (the mean) controls the center
  - σ (the standard deviation) controls the width. More specifically, it is the distance between the center and the "inflection point".
- Denoted $N(\mu, \sigma)$.
- We use Table A or calculator/computer for computing values. We will explain shortly.
- Key step: z-scores.
$$z = \frac{x - \mu}{\sigma}$$
- They are a simple rescaling of the x values.
- Can also write:
$$x = \mu + \sigma \times z$$
  z scores measure "number of standard deviations away from the mean" that the corresponding x value is.
- z-scores follow **Standard Normal Distribution**. With mean 0 and standard deviation 1.

- It is those z values we can look up in the table.

In general we deal with two kinds of problems:

Standard We are given x, and need to find corresponding p.

1. Turn the x into z: $z = \frac{x-\mu}{\sigma}$
2. Look z up in table to find a p.
3. Possibly adjust the p based on the problem.

Reverse We are given some sort of p, need to find corresponding x.

1. Possibly adjust the p based on the problem.
2. "Reverse Look" in table for the entry with that p, then get corresponding z.
3. Convert the z into an x: $x = \mu + \sigma z$

Standard Direction

- Compute z from x if need be.
- Look z up in the table. For example say $z = 2.31$:
  - Find 2.3 on the left column.
  - Find 0.01 at the top row.
  - Their intersection is the "p-value".
- p-value represents "the percent of data points below the given z (or x)".
- Use that to compute the answer to the actual question.
- For values with more decimals, round to closest end, or average results in two ends.

Practice questions:

- What percent of the data is below $z = 1.23$?
- What percent of the data is below $z = -1.5$?
- What percent of the data is below $z = -1.555$?
- What percent of the data is above $z = 2.1$?
- What percent of the data is between $z = 1.56$ and $z = 2.1$?
- If we had a normal distribution with mean 2.5 and standard deviation 1, how much data is there between $x = 1.2$ and $x = 2.7$?

Inverse Lookup in Table A

- Do this if you know a p and want to find a z.
- Make sure the p represents "data below a point". If not convert it.
- Look for the p INSIDE Table A. You will probably find one value bigger than it, right next to a value smaller than it.
- If your p is closer to one of these values, just use the z from that value.

- If it's closer to the middle between them, use the average of the two z values.

Practice questions:

- At what z is the first quartile?
- At what z is the third quartile?
- What is the IQR for the standard normal distribution?
- Find the z range where the middle 20% of the data lies.
- If we had a normal distribution with mean 2.5 and standard deviation 1, find the range where the middle 20% of the data lies.