

Measures of Spread

Reading

Sections 1.6.4

Practice Problems

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Notes

- The goal of measures of spread is to assess the “variation” in the data in some way.
 - Are the values spread out and far from each other, or are they all real close?
 - Are the values of one variable less or more spread out than those of another variable?

- Different measures achieve this in different ways.

- The two main measures of spread are:

Std. Dev. Standard Deviation. Measures distance of data from the mean.

Often denoted by s .

Not resistant.

IQR Interquartile range. Measures range occupied by the middle 50% of the data.

Resistant.

- NOTE: The standard deviation and the IQR are in general not comparable to each other, they measure spread in very different ways.
- Let us review how the standard deviation is computed. Its formula is a bit complicated, and it looks something like this:

$$s = \sqrt{\frac{1}{n-1} \sum (x - \bar{x})^2}$$

Basically:

- Compute the mean of all the values (the \bar{x}).
- Subtract the mean from each value (the result of this step is the “**deviations**”, $x - \bar{x}$).

- Square all the deviations.
 - * Ensures values are positive before computing average.
- Average these squared deviations: Add them all up (the big “sigma”), then divide by $n - 1$. The result at this stage is called the **variance**.
 - * Why $n - 1$: Technical reason, and won’t really matter for large n .
 - * One way to think about it: The deviations always add up to 0, so once you know $n - 1$ of them the last one is determined.
 - * In this context, $n - 1$ is called the “degrees of freedom”.
- Take a square root at the end.
 - * Fixes the units of measurement.
- Outliers have a considerable effect on this formula.
 - They have a very large deviation, and because they pull the mean towards themselves they cause larger deviations in the other values as well.
 - Because we square them before adding, those large deviations will dominate the equation even more.
- What to use depends on the distribution:

Symmetric Mean for center, Standard Deviation for spread

Skewed/Outliers Median for center, IQR for spread
- Chebyshev’s Rule:
 - At least 75% of the data is within 2 standard deviations from the mean
 - At least 89% of the data is within 3 standard deviations from the mean
- For Bell Shaped data (Empirical Rule):
 - Approximately 68% of the data is within one standard deviation of the mean.
 - Approximately 95% of the data is within two standard deviations of the mean.
 - More than 99% of the data is within three standard deviations of the mean.