Applied Statistics HW 16

1. We are going to try to estimate the average height of U.S. males. It is believed that the average height is 6 feet and 1 inch, and the standard deviation of all heights is 6 inches. In order to obtain our estimate, we will obtain a random sample of the heights of 300 U.S. males, and compute their average, $\bar{x}$, measured in inches. Looking at our sample, the heights seem to have only a slight skewness to the left, so we will assume the same is true for the population.

    a. Can we assume that $\bar{x}$ follows a normal distribution? Explain. What would the mean and standard deviation of that distribution be? Review the central limit theorem in the book, and in particular the discussion regarding how large n needs to be, before answering this question.

    b. A friend of yours who knows a little bit about statistics but not too much is perplexed by your statement that $\bar{x}$ follows a normal distribution. To him, to talk about a distribution you need to have many values. But $\bar{x}$ is just the average height in the sample, so it's just one number. Explain to your friend how this makes sense.

c. Give a range of values so that we have a 90% chance our $\bar{x}$ will land in that range.

d. Give a range of values so that we have a 99% chance our $\bar{x}$ will land in that range.

e. In a particular survey, $\bar{x}$ turned out to be 6 feet and 2 inches. What are the chances, that for a randomly selected survey $\bar{x}$ would end up being that much or more?

2. The gypsy moth is a serious threat to oak and aspen trees. A state agriculture department places traps throughout the state to detect moths. When traps are checked periodically, the mean number of moths trapped is only 0.5, i.e. half a moth per trap, so many traps don't have any moths, but some traps have several moths. The distribution of moth counts is discrete (takes only integer values) and strongly skewed, with standard deviation 0.7.

   a. What are the mean and standard deviation of the average number of moths $\bar{x}$ in 50 traps?

   b. Use the central limit theorem, and normal distribution techniques, to find the probability that the average number of moths in a sample of 50 traps will be greater than 0.6. Can we use the central limit theorem in this instance?

3. The number of flaws per square yard in a type of carpet material varies with mean 1.6 flaws per square yard, and standard deviation 1.2 flaws per square yard. This population distribution cannot be normal, as a count only takes whole-number values. An inspector studies 200 square yards of the material, records the number of flaws found in each square yard, and calculates $\bar{x}$, the mean number of flaws per square yard inspected. Use the central limit theorem to find the approximate probability that the mean number of flaws exceeds 2 per square yard.