Limits at Infinity

Reading

- Sections 2.7

Practice problems

- Section 2.7: 1, 3, 7, 9, 13, 17, 19, 37
- In class: 2.7 35
- To turn in (due Wednesday, together with 2.8): 2.7 8, 20

Notes

Limits at Infinity

- We can consider limits when $x \to \infty$ or $x \to \infty$.
- If $\lim_{x\to\infty} f(x) = L$, we say that the line $y = L$ is a **horizontal asymptote** for f.
- Key limits:

  For all $n > 0$:
  $$\lim_{x\to\infty} x^n = \infty, \qquad \lim_{x\to\infty} x^{-n} = 0$$
  For odd whole numbers n:
  $$\lim_{x\to-\infty} x^n = -\infty$$
  For even whole numbers n:
  $$\lim_{x\to-\infty} x^n = \infty$$
  For all whole numbers n:
  $$\lim_{x\to-\infty} x^{-n} = 0$$
- Limits of rational functions behave in the same way as their leading terms:
  $$\lim_{x\to\pm\infty} \frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_0} = \frac{a_n}{b_n} \lim_{x\to\pm\infty} x^{n-m}$$
- Example: $\lim_{x\to\infty} \frac{x^3 - 2x + 1}{2x^2 + 1}$
- One approach to this: Factor out greatest power at numerator and denominator separately. Leaves a lot of $\frac{1}{x^n}$ that go to 0.
- Another approach to limits at infinity: Set $t = \frac{1}{x}$, look at $t \to 0^+$ or $t \to 0^-$.