The concept of a limit

Reading

• Sections 2.1, 2.2

Practice problems

• Section 2.1: 1, 5, 25, 31

• Section 2.2: 1, 3, 9, 17, 21

• To turn in (Wednesday): 2.1 6, 8, 2.2 2, 22

Notes

Average and instantaneous rates of change

- Imagine someone's position on the x axis as a function of time t is given by $x=2t^2$.
- We can find their average *velocity* between two times by taking the ratio of the difference in the *x* positions over the difference in times.
- If we want to find out how fast the person is going at one specific time, their instantaneous velocity, that's harder.
- Think of the value you see in the speedometer of a car. How is it computed?
- Idea: Measure very small time intervals. In fact, make them smaller and smaller.
- Practice: 2.1 1

Tangent lines

- \bullet Consider the graph of a function f, and look at a point (a,f(a)) on it.
- What is the line that best describes the curve, near a?
- \bullet Idea: Start with the secant lines: Lines joining the point a and a nearby point.
- Look at the slopes of the secant lines as the nearby point gets closer and closer to a
- Practice: Try this out for $f(x) = \sqrt{x}$ near x = 1.

Limits

- Limits express the idea of what happens to a function f(x) as we look at values of x very close to a specific number a.
- Example: $\frac{\sin x}{x}$ when x is near 0.
- Numerically: Try numbers x closer and closer to 0. See that the resulting values get closer and closer to 1.

• Definition of limit:

We say that the **limit** of f(x) as x approaches a is L, and we write:

$$\lim_{x \to a} f(x) = L$$

if the difference |f(x)-L| becomes arbitrarily small when x is sufficiently close (but not equal) to a.

In other words, the values of f(x) must get arbitrarily close to L when x is sufficiently close but not equal to a.

- Simple examples: $\lim_{x\to a} k = k$, $\lim_{x\to a} x = a$
- More complex example: $\lim_{x\to 1} 2x + 1 = 3$.
- Practice: 2.2 21

Infinite and one-sided limits

- Some times a function gets arbitrarily large/small as we approach the limit point.
- Example: $\frac{1}{x}$ at x = 0.
- One-sided limits: $\lim_{x\to 0^+} \frac{1}{x} = +\infty$
- $\bullet \lim_{x \to 0^-} \frac{1}{x} = -\infty$