Limit Laws

Reading

- Sections 2.3

Practice problems

- Section 2.3 3, 5, 8, 9, 11, 13, 27, 31
- To turn in (together with 2.4): 2.3 12, 14

Notes

Limit Laws

A number of laws govern the computation of limits. These laws allow us to compute a limit of a more complex function based on the limits of its (simpler) components.

Basic Limit Laws:

If $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ exist, then:

- **Constant and Linear Law**: $\lim_{x\to c} k = k$, $\lim_{x\to c} x = c$.
- **Sum Law**: The limit $\lim_{x\to c} f(x) + g(x)$ also exists and:

$$\lim_{x\to c} f(x) + g(x) = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$$

- **Constant Multiple Law**: The limit $\lim_{x\to c} kf(x)$ also exists and:

$$\lim_{x\to c} kf(x) = k \lim_{x\to c} f(x)$$

- **Product Law**: The limit $\lim_{x\to c} f(x)g(x)$ also exists and:

$$\lim_{x\to c} f(x)g(x) = \lim_{x\to c} f(x) \lim_{x\to c} g(x)$$

- **Quotient Law**: If further $\lim_{x\to c} g(x) \neq 0$, then the limit $\lim_{x\to c} \frac{f(x)}{g(x)}$ also exists and:

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$$

- **Power Law**: The limit $\lim_{x\to c} f(x)^n$ also exists and:

$$\lim_{x\to c} f(x)^n = \left(\lim_{x\to c} f(x)\right)^n$$

- **Root Law**: If further $\lim_{x \to c} g(x) \neq 0$, then the limit $\lim_{x \to c} \sqrt[n]{f(x)}$ also exists and:

$$\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to c} f(x)}$$

For example, let's compute $\lim_{x \to 1} \sqrt{x^3 + 2x}$:

- $\lim_{x \to 1} \sqrt{x^3 + 2x} = \sqrt{\lim_{x \to 1} x^3 + 2x} = \sqrt{\lim_{x \to 1} x^3 + \lim_{x \to 1} 2x}$
- We now compute each part: $\lim_{x \to 1} x^3 = \left(\lim_{x \to 1}\right)^3 = 1^3 = 1$
- And: $\lim_{x \to 1} 2x = 2 \lim_{x \to 1} x = 2 \times 1 = 2$
- Putting it all together we get: $\sqrt{1 + 2} = \sqrt{3}$