Continuity

Reading

- Sections 2.4

Practice problems

- Section 2.4 3, 9, 11, 13, 18, 23, 37, 51, 60, 67, 69
- In class: 2.4 49, 57
- To turn in (due Monday, together with 2.3): 2.4 4, 12, 52

Notes

Continuity

- Continuity is meant to describe the concept that we can draw the graph of a function without lifting our pen: The function forms a continuous whole.

- The idea is to relate the limit of a function at a point a to the value of the function at a. The limit tells us what happens *near* the point a. For continuity, this must agree with what happens exactly at a.

- Definition:

  We say that a function $f(x)$ is **continuous** at a point $x = c$ if:

  - The limit $\lim_{x \to a} f(x)$ exists.
  - The value $f(a)$ exists.
  - They agree: $\lim_{x \to a} f(x) = f(a)$.

- Practical implication: Can compute a limit by "plugging in".

- If this is not the case, we say that the function is **discontinuous** at $x = c$.

- We say that f is **continuous on an interval** $[a, b]$, if it is continuous at every point of the interval.

- Examples of continuous functions:

  - Constant function
  - x^n
  - Sum of continuous functions is continuous
  - Product of continuous functions is continuous
  - Quotient of continuous function is continuous, except where the denominator is 0

  - Trigonometric functions are continuous

- Possible discontinuities:

  - Left-sided limit and right-sided limit exist but differ (**jump discontinuity**)
  - Limit exists, but value is different from it (**removable discontinuity**)
  - One-sided limits are infinity (**infinite discontinuity**) or don't exist

- Example of studying the continuity of a piecewise-defined function (example 2)

- **Function composition**: If $g(x)$ is continuous at $x = c$, and $f(u)$ is continuous at $u = g(c)$, then $f(g(x))$ is also continuous at $x = c$.

  - Example: $\sin(x^2)$ is continuous.