The concept of a limit

Reading

- Sections 2.1, 2.2

Practice problems

- Section 2.1: 1, 5, 25, 31
- Section 2.2: 1, 3, 9, 17, 21
- To turn in: 2.1 6, 8, 2.2 2, 22

Notes

Average and instantaneous rates of change

- Imagine someone's position on the x axis as as function of time t is given by $x = 2t^2$.
- We can find their average *velocity* between two times by taking the ratio of the difference in the x positions over the difference in times.
- If we want to find out how fast the person is going at one specific time, their *instantaneous velocity*, that's harder.
- Think of the value you see in the speedometer of a car. How is it computed?
- Idea: Measure very small time intervals. In fact, make them smaller and smaller.

Tangent lines

- Consider the graph of a function f, and look at a point $(a, f(a))$ on it.
- What is the line that best describes the curve, near a?
- Idea: Start with the secant lines: Lines joining the point a and a nearby point.
- Look at the slopes of the secant lines as the nearby point gets closer and closer to a.
- Practice: Try this out for \$f(x) = $\sqrt{x}$ near $x = 1$.

Limits

- Limits express the idea of what happens to a function $f(x)$ as we look at values of x very close to a specific number a.
- Example: $\frac{\sin x}{x}$ when x is near 0.
- Numerically: Try numbers x closer and closer to 0. See that the resulting values get closer and closer to 1.

- Definition of limit: > We say that *the limit of $f(x)$ as x approaches a is L, and we write: >

$$\lim_{x \to a} f(x) = L$$

  > if the difference $|f(x) - L|$ becomes arbitrarily small when x is sufficiently close (but not equal) to a. > > In other words, the values of $f(x)$ must get arbitrarily close to L when x is sufficiently close but not equal to a.
- Simple examples: $\lim_{x \to a} k = k$, $\lim_{x \to a} x = a$
- More complex example: $\lim_{x \to 1} 2x + 1 = 3$.