Derivatives of Trigonometric Functions

Reading

- Section 3.6

Practice problems

- Section 3.6: 5, 7, 17, 21, 27, 29, 41, 45
- To turn in (together with 3.7): 3.6 18, 28, 42

Notes

Derivatives of sine and cosine

$$\frac{d}{dx}\sin x = \cos x, \qquad \frac{d}{dx}\cos x = -\sin x$$

Watch out for the minus sign!

Proof:

- Definition of derivative: $\frac{d}{dx}\sin x = \lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h}$
- Use trig identity: $\sin(x+h) = \sin x \cos h + \cos x \sin h$
- Algebra

Second derivatives:

$$\frac{d^2}{dx^2}\sin x = -\sin x, \qquad \frac{d^2}{dx^2}\cos x = -\cos x$$

Both $\sin$ and $\cos$ satisfy the differential equation $f''(x) = -f(x)$.

Practice problems: Derivatives of $\sin x + \cos x$, $x\sin(2x)$, $\frac{x}{\cos x}$

Derivatives of other trigonometric functions

Other trigonometric functions are written in terms of sine and cosine. We can compute their derivatives using the derivatives of $\sin$ and $\cos$.

$$\frac{d}{dx}\tan x = \sec^2 x = 1 + \tan^2 x, \qquad \frac{d}{dx}\sec x = \sec x \tan x$$

Proof:

- $\tan x = \frac{\sin x}{\cos x}$
- Apply quotient rule: $\tan' x = \frac{\sin^2 x + \cos^2 x}{\cos^2 x}$
- Numerator equal 1, so this is $\sec^2 x$.
- Alternatively, rewrite it as $1 + \frac{\sin^2 x}{\cos^2 x} = 1 + \tan^2 x$.

Practice: Do the same for $\sec x$.

Practice: Compute the tangent lines to $\tan x$ at $x = 0$ and at $x = \frac{\pi}{4}$.