Limit Laws

Reading

- Sections 2.3

Practice problems

- Section 2.3 3, 5, 8, 9, 11, 13, 27, 31
- To turn in (together with 2.4): 2.3 12, 14

Notes

Limit Laws

A number of laws govern the computation of limits. These laws allow us to compute a limit of a more complex function based on the limits of its (simpler) components.

> **Basic Limit Laws:**
>
> If $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ exist, then:
>
> - **Constant and Linear Law**: $\lim_{x\to c} k = k$, $\lim_{x\to c} x = c$.
> - **Sum Law**: The limit $\lim_{x\to c} f(x) + g(x)$ also exists and:
>
> $$\lim_{x\to c} f(x) + g(x) = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$$
>
> - **Constant Multiple Law**: The limit $\lim_{x\to c} kf(x)$ also exists and:
>
> $$\lim_{x\to c} kf(x) = k \lim_{x\to c} f(x)$$
>
> - **Product Law**: The limit $\lim_{x\to c} f(x)g(x)$ also exists and:
>
> $$\lim_{x\to c} f(x)g(x) = \lim_{x\to c} f(x) \lim_{x\to c} g(x)$$
>
> - **Quotient Law**: If further $\lim_{x\to c} g(x) \neq 0$, then the limit $\lim_{x\to c} \frac{f(x)}{g(x)}$ also exists and:
>
> $$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$$
>
> - **Power Law**: The limit $\lim_{x\to c} f(x)^n$ also exists and:
>
> $$\lim_{x\to c} f(x)^n = \left(\lim_{x\to c} f(x)\right)^n$$

- **Root Law**: If further $\lim_{x\to c} g(x) \neq 0$, then the limit $\lim_{x\to c} \sqrt[n]{f(x)}$ also exists and:
$$\lim_{x\to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to c} f(x)}$$

For example, let's compute $\lim_{x\to 1} \sqrt{x^3 + 2x}$:

- $\lim_{x\to 1} \sqrt{x^3 + 2x} = \sqrt{\lim_{x\to 1} x^3 + 2x} = \sqrt{\lim_{x\to 1} x^3 + \lim_{x\to 1} 2x}$
- We now compute each part: $\lim_{x\to 1} x^3 = \left(\lim_{x\to 1} x\right)^3 = 1^3 = 1$
- And: $\lim_{x\to 1} 2x = 2\lim_{x\to 1} x = 2 \times 1 = 2$
- Putting it all together we get: $\sqrt{1+2} = \sqrt{3}$