

The concept of a limit

Reading

- Sections 2.1, 2.2

Practice problems

- Section 2.1: 1, 5, 25, 31
- Section 2.2: 1, 3, 9, 17, 21
- To turn in (Wednesday): 2.1 6, 8, 2.2 2, 22

Notes

Average and instantaneous rates of change

- Imagine someone's position on the x axis as a function of time t is given by $x = 2t^2$.
- We can find their average *velocity* between two times by taking the ratio of the difference in the x positions over the difference in times.
- If we want to find out how fast the person is going at one specific time, their *instantaneous velocity*, that's harder.
- Think of the value you see in the speedometer of a car. How is it computed?
- Idea: Measure very small time intervals. In fact, make them smaller and smaller.

Tangent lines

- Consider the graph of a function f , and look at a point $(a, f(a))$ on it.
- What is the line that best describes the curve, near a ?
- Idea: Start with the secant lines: Lines joining the point a and a nearby point.
- Look at the slopes of the secant lines as the nearby point gets closer and closer to a .
- Practice: Try this out for $f(x) = \sqrt{x}$ near $x = 1$.

Limits

- Limits express the idea of what happens to a function $f(x)$ as we look at values of x very close to a specific number a .
- Example: $\frac{\sin x}{x}$ when x is near 0.
- Numerically: Try numbers x closer and closer to 0. See that the resulting values get closer and closer to 1.
- Definition of limit:

We say that the **limit** of $f(x)$ as x approaches a is L , and we write:

$$\lim_{x \rightarrow a} f(x) = L$$

if the difference $|f(x) - L|$ becomes arbitrarily small when x is sufficiently close (but not equal) to a .

In other words, the values of $f(x)$ must get arbitrarily close to L when x is sufficiently close but not equal to a .

- Simple examples: $\lim_{x \rightarrow a} k = k$, $\lim_{x \rightarrow a} x = a$
- More complex example: $\lim_{x \rightarrow 1} 2x + 1 = 3$.

Infinite and one-sided limits

- Some times a function gets arbitrarily large/small as we approach the limit point.
- Example: $\frac{1}{x}$ at $x = 0$.
- One-sided limits: $\lim_{x \rightarrow 0^+} \frac{1}{x} = +\infty$
- $\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty$