Derivative as a Function

Reading

- Sections 3.2

Practice problems

- Section 3.2: 5, 7, 17, 19, 21, 35, 51, 53
- To turn in: 3.2 16, 20, 26, 36, 66
- In class: 3.2 43, 45

Notes

The derivative as a function

- If the derivative $f'(a)$ exists for all a in some interval, then we can look at the function $f'(x)$ and study its properties.
- We often do this by computing $\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$.
- Example: Derivative of $f(x) = 2x^2 + x$ is $4x + 1$.
- Example 2: Derivative of $f(x) = \frac{1}{x^2}$ is $-\frac{2}{x^3}$.
- Alternative notation: $\frac{df}{dx}$, $\frac{dy}{dx}$, $\frac{d}{dx}f$, $\frac{df}{dx}\Big|_{x=4}$.
- Power Rule:

$$\frac{d}{dx}x^n = x^{n-1}$$

- Prove the power rule using both derivative definitions.
- Linearity rules:

$$(f+g)' = f' + g'$$
$$(kf)' = kf'$$

- Comparing the graph of the derivative with that of the function: The value of the derivative at a point relates to the slope of the tangent of the function at that point. Look for whether the slope increases/decreases.
- When is the tangent line horizontal?

- Differentiability requires continuity:

  If f is differentiable at $x = a$, then f is also continuous at a.
  Alternatively, if f is not continuous at a then it cannot possibly be differentiable there.

- There are functions that are continuous but not differentiable (e.g. $|x|$).