Introduction to derivatives

Reading

- Sections 3.1

Practice problems

- Section 3.1: 3, 5, 11, 13, 15, 21, 33, 35, 37, 41
- To turn in (together with 3.2): 3.1 4, 34

Notes

Definition of Derivative

- The derivative is a formal description of the concept of "instantaneous rate of change" that we looked at when we discussed limits.
- Geometrically can be thought of as the slope of the tangent line, which is a *limit* of the slopes of the secant lines.
- Definition:

  The **derivative** of $f(x)$ at the point $x = a$ is defined as:

  $$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

  Alternative description:

  $$f'(a) = \lim_{h \to 9} \frac{f(a+h) - f(a)}{h}$$

- Equation for tangent line:

  Tangent Line:

  $$y - f(a) = f'(a)(x - a)$$

- Example 1:
  - $f(x) = x^3$, $a = 2$.
  - $\frac{f(a+h)-f(a)}{h} = \frac{(2+h)^3-8}{h} = \frac{2^3+6h^2+12h+h^3-8}{h} = 6h + 12 + h^2$.
  - Taking limit as $h \to 0$: $f'(2) = 0 + 12 + 0 = 12$
- Practice: Do same example using the other limit formulation.
- Example 2:

- $f(x) = \frac{1}{x}$, $a = 2$.
- $\frac{f(x)-f(a)}{x-a} = \frac{\frac{1}{x}-\frac{1}{a}}{x-a} = \frac{\frac{a-x}{xa}}{x-a} = -\frac{1}{xa} = -\frac{1}{a^2} = -\frac{1}{4}$

- Derivatives of linear and constant equations:
  - $(mx + b)' = m$
  - $b' = 0$