# Midterm 1 Study Guide

This is meant to be a representative sampling of the key concepts you will need to know, and it is not meant to be exhaustive. You should make sure that you are comfortable with all practice problems and homework assignments.

1. How do we find the area between the graphs of two functions? Variations on this problem depending on whether we have to find where they meet, or whether we are given specific endpoints, or whether they are variables of $y$ instead etc.
2. Formula for the average value of a function on an interval, and how to find a point achieving that average value.
3. How do we find the volume of a solid via knowledge of its cross-sectional areas.
4. How do we find the volume of a solid of revolution around an axis and with functions expressed in terms of the variable parallel to the axis. (Disc/Washer methods). Rough explanation of why the formula is the way it is.
5. Similar problems where the axis of revolution is further away (so $y = 15$ rather than the $x$ axis of $y = 0$).
6. The Shell method for computing volumes of revolution where the axis of revolution is perpendicular to the variable used for integration (e.g. $x = -2$ axis and $x$ as the variable of integration).
7. Basic properties of the exponential and logarithm functions.
    - Their derivatives.
    - How they treat products/sums/powers/quotients.
    - Special known values.
8. What it means for two functions to be inverses of each other. How we find if a function is invertible and what its inverse is.
9. How to find the derivative of the inverse of a function.
10. How we define $b^x$ in terms of $e^x$ and how we define $\log_b(x)$ in terms of $\ln(x)$.
11. Computing integrals and derivatives involving $\ln(x)$ and $e^x$.
12. Computing tangent lines.

More theoretical questions (I will ask you some of these):

1. Show that $b^x$ is the only function $f(x)$ with $f(0) = 1$ and $f'(x) = m(b)f(x)$.
2. How is the logarithm defined as an integral?
3. How does the integral form help us prove that $\ln(xy) = \ln(x) + \ln(y)$?
4. If we know the derivative of the exponential is itself, and we know $\ln$ is its inverse, how do we find the derivative of $\ln$? And conversely, from the derivative of $\ln$ and the fact that the exponential is its inverse, how do we find the derivative of the exponential?
5. Using the property that $b^{x+y} = b^x b^y$, what can we say about the derivative of $b^x$? (It has to be a multiple of $b^x$: $m(b)b^x$, this was early in the 7.1/exponential material)
6. Explain why the fact that $b^{x+y} = b^x b^y$ holds for all $x, y$ would also force us to say that $b^0 = 1$ and that $b^{-x} = \frac{1}{b^x}$.