Area between Graphs

Reading

Section 6.1. Pay particular attention to the difference between integrating along the x-axis and integrating along the y-axis.

Problems

Practice Exercises: 6.1 1, 3, 6, 16, 19, 21, 28, 33, 59 (will do this one in class)

Exercises to turn in (on Monday along with those from 6.2): 6.1 14, 20

Area between Graphs

Suppose we have two functions, $f(x)$ and $g(x)$, and we want to compute the area between them.

If we follow our standard methodology for integrals, we could:

- Imagine cutting the x-interval up into tiny intervals.
- For each of these intervals, the desired area can be approximated by a rectangle, whose height is the positive difference between the two functions, and whose width is the corresponding Δx width.
- So mathematically that area is like $|f(x) - g(x)|\,\Delta x$.
- Adding all these together gives total area.
- Taking the limit of this process gives us an integral.

> The area between two curves $f(x)$ and $g(x)$ and the points $x = a$ and $x = b$ is given by:
>
> $$\int_a^b |f(x) - g(x)|\,dx$$

Example: Write the integral that computes the area between the circle of unit radius centered at 1 and the line $y + x = 1$.

There are two variations that complicate matters. The first is that some times the curves are under consideration are best described as functions of y rather than x. Then the formula is essentially the same:

$$\int_c^d |g(y) - h(y)|\,dy$$

Example: Compute the area of the region enclosed by the curves $y^2 = x + 5$ and $y^2 = 3 - x$.

In some other times yet you have to break the range up into pieces, then compute each piece via one of the aforementioned methods.

Example: Compute the area of the region enclosed by the curves $x + y = 4$, $x - y = 0$ and $y + 3x = 4$.