Volumes of Revolution

Reading

Section 6.3

Problems

- Practice Exercises: 6.3 8, 9, 13, 14, 15, 21, 23, 25
- In class: 27, 28, 31
- To turn in (on Monday along with those from 6.2): 6.3 24, 40

Volumes of Revolution

Recall that last time we expressed a volume as an integral of an “area function”:

$$V = \int_a^b A(x)dx$$

where $A(x)$ is the area of the cross section.

Volumes of Revolution follow a similar idea, but they are used in a more specialized form:

The disc method

> A **solid of revolution** is formed by rotating, the region under the graph of a function $y = f(x)$ around the x axis. The volume of such a solid can be found as:

$$V = \pi \int_a^b f(x)^2 dx$$

This follows directly from the methods in the previous section, as the region of the cross-section in this case will be a circle with radius $|f(x)|$ and therefore its area would be $\pi f(x)^2$.

For example, the sphere of radius R can be thought of as a solid of revolution, where the function $f(x)$ is $\sqrt{R^2 - x^2}$. So we can compute the volume of the sphere via the integral:

$$\pi \int_{-R}^{R} \left(\sqrt{R^2 - x^2}\right)^2 dx = \pi \int_{-R}^{R} R^2 - x^2 dx$$

If you finish the computation you will find the result to be the well known formula $V = \frac{4}{3}\pi R^3$.

This is often called the **disc method**.

The washer method

A slight variation of it, called the **washer method**, describes the solid created by the revolution of the area *between* two functions $f(x)$ and $g(x)$. You can think of it as the volume of the solid produced by $f(x)$ alone, with the volume of the solid produced by g removed.

> The volume of the solid resulting by revolution of the area between two function $f(x)$ and $g(x)$ around the x axis is given by:
>
> $$V = \pi \int_a^b f(x)^2 - g(x)^2 dx$$

Take note of the integrand!! We do not take the difference of f and g, and then square it. We first square them, then take the difference.

Other axes of revolution

Special care needs to be taken when rotating around an axis other than the x axis itself. For instance one might want to rotate around the axis $y = 1$ or the axis $y = -3$. In all cases the rule is:

$$V = \pi \int_a^b R_{\text{outer}}^2 - R_{\text{inner}}^2 dx$$

So take the "largest radius", namely the distance from your function to the axis of rotation, and substract the "smallest radius", which may be 0 or may be the distance of another function g from the axis of rotation.