Volumes of Revolution

Reading

Section 6.3

Problems

Practice Exercises: 6.3 8, 9, 13, 14, 15, 21, 23, 25 In class: 27, 28, 31

Exercises to turn in (along with those from 6.4): 6.3 24, 40

Volumes of Revolution

Recall that last time we expressed a volume as an integral of an "area function":

$$V = \int_a^b A(x)dx$$

where $A(x)$ is the area of the cross section.

Volumes of Revolution follow a similar idea, but they are used in a more specialized form:

> A solid of revolution is formed by rotating, the region under the graph of a function $y = f(x)$ around the x axis. The volume of such a solid can be found as:
>
> $$V = \pi \int_a^b f(x)^2 dx$$

This follows directly from the methods in the previous section, as the region of the cross-section in this case will be a circle with radius $|f(x)|$ and therefore its area would be $\pi f(x)^2$.

For example, the sphere of radius R can be thought of as a solid of revolution, where the function $f(x)$ is $\sqrt{R^2 - x^2}$. So we can compute the volume of the sphere via the integral:

$$\pi \int_{-R}^{R} \left(\sqrt{R^2 - x^2}\right)^2 dx = \pi \int_{-R}^{R} R^2 - x^2 dx$$

If you finish the computation you will find the result to be the well known formula $V = \frac{4}{3}\pi R^3$.

This is often called the **disc method**.

A slight variation of it, called the **washer method**, describe the solid created by the revolution of the area *between* two functions $f(x)$ and $g(x)$. You can think of it as the volume of the solid produced by $f(x)$ alone, with the volume of the solid produced by g removed. The formula would be:

$$V = \pi \int_a^b f(x)^2 - g(x)^2 dx$$

Take note of the integrand!! We do not take the difference of f and g, and then square it. We first square them, then take the difference.

Special care needs to be taken when rotating around an axis other than the x axis itself. For instance one might want to rotate around the axis $y = 1$ or the axis $y = -3$. In all cases the rule is:

$$V = \pi \int_a^b R_{\text{outer}}^2 - R_{\text{inner}}^2 dx$$

So take the "largest radius", namely the distance from your function to the axis of rotation, and substract the "smallest radius", which may be 0 or may be the distance of another function g from the axis of rotation.