

Sequences

Reading

Section 11.1

Problems

- Practice Problems: 11.1 3, 5, 11, 17, 23, 25, 35, 41
- Practice Problems: 11.1 61, 65, 67, 73, 74, 83
- Problems to turn in: 11.1 16, 26, 48, 70
- Challenge (optional): 11.1 85, 86, 87

Topics to know

1. Definition of a sequence
2. Basic examples of sequences and their visualizations
 - $a_n = 1 - \frac{1}{n}$, $a_n = (-1)^n n$
 - Visualize as a graph as well as as points in the plane
3. Sequences defined recursively
 - Newton's method sequence for $\sqrt{2}$
 - $a_{n+1} = \sqrt{a_n}$, $a_1 = 2$
4. Definition of limit of sequence, both intuitive and precise
 - Find the limit of $\frac{n+1}{n+2}$
5. Sequences derived from functions (theorem 1)
 - $\lim_{n \rightarrow \infty} n^{1/n}$
6. Geometric sequence for non-negative r (example 6)
7. Limit laws for sequences, squeeze theorem
8. A sequence that absolutely converges to 0 also converges to 0
9. Geometric sequence for negative r (example 8)
10. Using squeeze theorem for convergence of more complicated forms (example 9)

- $\lim_{n \rightarrow \infty} \frac{R^n}{n!}$

11. Passing a sequence through a function

- Use for $n^{1/n}$

12. Definition of bounded sequence

13. Convergent sequences are bounded (theorem 5)

14. Bounded monotonic sequences converge (theorem 6)

15. How to use theorem 6 for recursively defined functions:

- First guess what the limit would be if it existed
- Show that this limit acts as a bound (proof by induction)
- Show sequence is increasing/decreasing (proof by induction)
- Apply for $a_{n+1} = \sqrt{2a_n}$