Line Integrals

Reading

Section 4.1

Problems

Practice problems (page 56): 1, 2, 3, 4

Topics to know

1. Notions of piecewise-differentiable curves and of smooth curves (4.2)
2. Integral of a complex-valued function of a real variable (4.1).
3. Integrate $1/z$ and $1/z^2$ over a circle around 0. (page 48)
4. Integral of $f(z)$ along a smooth curve.
5. Integrals of $f(z)$ over equivalent curves are equal.
6. Integral over the opposite curve is the negative of the integral over the curve.
7. Linearity of the integral (proposition 4.8).
8. $\displaystyle\int_a^b G(t)dt \ll \int_a^b |G(t)|dt$.
9. ML-formula: If $f \ll M$ along a curve C of length L, then:

$$\int_C f(z)dz = \int_a^b f(z(t))\dot{z}(t) \ll \int_a^b |f(z(t))\dot{z}(t)|dt \ll M\int_a^b |\dot{z}(t)|dt = ML$$

10. If $f_n \to f$ uniformly on C, then $\int_C f_n(z)dz \to \int_C f(z)dz$.
11. If $F'(z) = f(z)$ then $\int_C f(z)dz = F(z(b)) - F(z(a))$.