

## Midterm 2 study guide

Things you should know:

1. State equivalent statements of the meaning of continuous function.
2. Explain how if  $f$  is a continuous function and  $U$  is an open set, then  $f^{-1}(U)$  is open.
3. Show using sequences that if  $K$  is closed and  $f$  is continuous then  $f^{-1}(K)$  is closed.
4. Definition of (sequentially) compact sets.
5. Prove that a compact set is closed and bounded.
6. Prove that if  $K$  is a compact set and  $f$  is continuous, then  $f(K)$  is a compact set.
7. Prove that if  $K$  is compact and  $F$  is closed then  $K \cap F$  is compact.
8. Test if a polynomial is analytic.
9. Find the imaginary part of an analytic polynomial whose real part you know, and vice versa, and write as a polynomial in  $z$ .
10. Definition of radius of convergence for a power series.
11. Prove the “uniqueness theorem” (2.12).
12. Definition of complex-differentiable function.
13. Prove that a complex-differentiable function must satisfy the Cauchy-Riemann equations.
14. State the converse: When does satisfiability of the Cauchy-Riemann equations imply the function is complex differentiable?
15. Sketch the proof of the previous part.
16. Work with the Cauchy-Riemann equations to determine if a given real/imaginary part can be extended to an analytic function.
17. Definition and main properties of  $e^z$ .