

# Midterm 1 study guide

Things you should know:

1. Definition of complex numbers, definition of addition and multiplication, 0, 1, multiplicative inverses.
2. How to find the two square roots of a complex number. How to find the solution to a general quadratic equation.
3. Complex conjugate, modulus, their properties.  $|\operatorname{Re}(z)|, |\operatorname{Im}(z)| \leq |z| \leq |\operatorname{Re}(z)| + |\operatorname{Im}(z)|$ .
4. Triangle inequality  $|z + w| \leq |z| + |w|$  and its proof.
5. Polar form of complex number,  $z = |z|(\cos \theta + i \sin \theta)$ , behavior under product. Using it find the  $n$ th roots of unit.
6. Definition of sequence convergence.
7. Proof that a sequence converges to a number if and only if the real parts of the sequence converge to the real part of the number and the imaginary parts of the sequence converge to the imaginary part of the number.
8. Proof that if a sequence converges to a number, then its conjugate converges to the conjugate.
9. Other sequence laws.
10. Series, definition of series convergence.
11. Proof of divergence test. Proof of geometric series.
12. Definition of Cauchy sequences.
13. A sequence is convergent if and only if it is Cauchy.
14. Use this fact to show that a series that converges absolutely also converges.
15. Root and Ratio tests.
16. Definition of open and closed sets.
17. Proof that a set is closed if and only if for every sequence from the set that converges its limit is in the set.
18. Proof that the infinite union of open sets is open and the infinite intersection of closed sets is closed. Counterexamples for the infinite intersection of open sets and the infinite union of closed sets.
19. Proof that the intersection of two open sets is open.
20. The various equivalent definitions of continuous function.
21. Proof that the open disc is open and that the closed disc is closed.