Complex Numbers

Reading

Section 1.1, 1.2

Problems

- Practice Problems (page 18): 1, 2, 3, 4, 7, 9, 12
- Problems to be ready to present: 8
- Challenge: 13, 14
- More practice problems:
    1. Recall the definitions of $\bar{z}$ and z^{-1}. Show that $\bar{z} = z^{-1}$ if and only if z lies on the unit circle (all complex numbers with length 1).
    2. Show that if z is an n-th root of 1, then z must lie on the unit circle.
    3. Suppose z is an n-th root of 1. Show that $z^{-1} = z^{n-1}$.
    4. Suppose $z \neq 1$ is a third root of 1.
        - Use the result of problem 13 to show that $1 + z + \bar{z} = 0$.
        - Use this equation, and the fact that z has length 1, to show that $z = \frac{-1 \pm i\sqrt{3}}{2}$.
    5. Suppose $z \neq 1$ is a fifth root of 1.
        - Show that $z = 1 + z + z^2 + \frac{1}{z^2} + \frac{1}{z} = 0$.
        - Use this to show that if we set $w = z + \frac{1}{z}$ then we have $w + w^2 = 1$.
        - From that last equation find w, and then from the previous equation find z. You should be finding 4 different solutions this way.
        - Examine the four numbers on the plane, and with that information in hand find $\cos\frac{2\pi}{5}$ and $\sin\frac{2\pi}{5}$. Use Wolfram Alpha http://www.wolframalpha.com/ to verify the formula you found.
    6. Consider the vectors based at the origin and ending at the complex numbers z and w respectively. Show that the dot product between the two vectors equals the real part of $z\bar{w}$.
    7. True or false: The vectors based at the origin and ending at the complex numbers z and w respectively are perpendicular if and only if z/w is a purely imaginary number.

Topics to know

1. Definition of Complex Numbers as pairs of real numbers
2. Properties of i
3. Real numbers are embedded into the Complex Numbers
4. Finding the square root of a number (Find roots of $\pm i$, then find their roots)
5. Complex Numbers as points on a plane. Addition as vector addition

6. Multiplication by i amounts to rotation by 90 degrees
7. Conjugate of a number, $\bar{z}$
8. Modulus/Absolute value $|z|$
9. Polar coordinates representation of a complex number
10. Multiplication and division via polar representation
11. Use of polar representation for roots