Sequences and Series in the Complex Plane

Reading

Section 1.4, part I

Problems

Practice problems:

1. Show that $z_n \to z$ if and only if $\bar{z}_n \to \bar{z}$.
2. Suppose that $|z| < 1$. Show that $a_n = z^n \to 0$.
3. True or False: The series $\sum_{n=0}^{\infty} a_n$ converges if and only if the series $\sum_{n=0}^{\infty} \mathrm{Re}(a_n)$ and $\sum_{n=0}^{\infty} \mathrm{Im}(a_n)$ both converge.
4. True or False: If $\sum_{n=0}^{\infty} a_n$ converges then $\sum_{n=0}^{\infty} \bar{a}_n$ also converges.
5. If $\sum_{n=0}^{\infty} a_n$ converges, show that $\sum_{n=0}^{\infty} a_n^2$ also converges.

Topics to know

1. Sequence z_n converges to z iff $|z_n - z| \to 0$.
    - Alternative definition
    - Laws for convergent sequences
2. Sequence converges if and only if its real parts converge and its imaginary parts converge.
    - Key inequality: $|\text{Re(z)}|, |\text{Im(z)}| \leq |z| \leq |\text{Re(z)}| + |\text{Im(z)}|$
3. Cauchy sequences, for both real numbers and complex numbers.
    - Laws for Cauchy sequences
4. Convergent sequences are Cauchy.
5. Cauchy sequences of real numbers converge. Key steps:
    - Cauchy sequences are bounded.
    - Cauchy sequences contain a monotone subsequence.
    - If a Cauchy sequence has a convergent subsequence it converges.
6. Cauchy sequences of complex numbers converge.
    - Go through their real/imaginary parts.
7. Series of complex numbers. Review of results from Calculus 3.
    - Which proofs carry over to complex numbers?
    - Divergence test.
    - Absolute/conditional convergence.

- Absolute convergence implies conditional convergence. (proof differs, why?)
- Geometric series.
- Root and ratio tests.