Midterm 3 study guide

Things you should know:

1. Definition of "piecewise differentiable" and "smooth" curves (p. 45)
2. Definition of integral of a complex function along a curve.
3. Be able to compute in specific examples (given curve $\gamma(t)$, given function $f(z)$). Two special cases: Integrals over a horizontal plus vertical segment, integrals over a disc.
4. Definition of the negative of a curve, $-C$, and explanation why the integral over it is the negative of the integral over C.
5. The M-L formula and its proof (and the proof of lemma 4.9).
6. Statement about uniformly convergent sequence of functions and its interaction with the integral.
7. The version of the fundamental theorem of algebra for line integrals (proposition 4.12) (without proof).
8. Rectangle theorem and its proof for entire functions. Only statement for convex domains.
9. Integral theorem and proof for entire functions (using the rectangle theorem to build an antiderivative). Only statement for convex domains.
10. Closed curve theorem and proof for entire functions.
11. Second form of rectangle theorem for: functions analytic everywhere except at finitely many points, but continuous at those points. Proof (it was a series of reduction steps to a case where we have a single point of non-analyticity, and it's a corner point of the rectangle).
12. Cauchy Integral Formula. Proof for the case where the curve in question is a circle centered at the point in question.
13. Sketch of the proof that the formula generalizes to any point contained in the disc, not just the center point (our proof consisted of considering a second circle centered at the target point, then breaking the two circles into two paths that avoided the point in question).
14. Power series expansion of an analytic function around any point, maximum radius of that convergence, integral formula for the coefficients (theorem 6.5).
15. Liousville's theorem (two forms, basic and extended) and proof.
16. Fundamental Theorem of Algebra.
17. Uniqueness theorem for analytic functions on a domain (statement only).
18. Mean value theorem and maximum modulus theorem (statements only).