## The Maybe option type

One particular built-in datatype deserves special mention. It is known as the Maybe type in Haskell, and also as the Option type in other languages. Its premise is simple: It allows you to carry one value around, but also allows the possibility of no value at all. For example, imagine we write a lookup method that looks for an key in an associative list:

```
lookup :: Eq a \Rightarrow a \rightarrow [(a, b)] \rightarrow b
```

This function is meant to search for the a part of the pair in the list, and if it finds it then it returns the corresponding b part. But what should happen if a suitable a part is not found? What value would the function return?

In other languages we have something like a null value. This is highly problematic, for a reason similar to the one described in temperatures.

Using the null value to indicate failure, we have no way of expressing in our type system whether a function may return null for its result, and whether a function should be handling null as a possible input to one of its arguments. These functions all look alike as far as their signature is concerned.

So for example <code>lookup</code> has no way of telling its users "hey my answer may be null so you better handle that possibility". Similarly the typechecker has no way of knowing if a function that uses the result of lookup bothers to check for the null case.

Option types solve these problems for us. The Maybe type is defined as follows:

```
data Maybe a = Nothing \mid Just a
```

So a value of type Maybe Int is either Nothing or something like Just 5. Then the (proper) type for the lookup function above is (for completeness we include its implementation):

Now anyone who wants to use the result of the lookup must do a pattern match on the two different forms, and as a result somehow handle the Nothing case.

A standard example of this is a "safe division" function, which does not allow you to divide by 0. It would look like this:

```
safeDivide :: Num t \Rightarrow t \rightarrow t \Rightarrow Maybe t safeDivide 0 \Rightarrow Nothing safeDivide 0 \Rightarrow Maybe t \Rightarrow t \Rightarrow t \Rightarrow t \Rightarrow Maybe t
```

**Standard functions for Maybe** There are a number of standard functions for the Maybe type. We declare them here, and ask you to implement them for practice:

```
— Empty or one-element list
toList :: Maybe a -> [a]
— Apply the function to the a if there is one, otherwise just pass the default b
maybe :: b \rightarrow (a \rightarrow b) \rightarrow Maybe a \rightarrow b
— Preserve the Nothing or apply the function
fmap :: (a \rightarrow b) \rightarrow Maybe a \rightarrow Maybe b
— Preserve the Nothing or apply the function
(>>=) :: Maybe a \rightarrow (a \rightarrow Maybe b) \rightarrow Maybe b
— Preserve the Nothing or keep the b value
(>>) :: Maybe a -> Maybe b -> Maybe b
— Wrap a value in Maybe
return :: a -> Maybe a
— Preserve the Nothing or apply the function
(<*>) :: Maybe (a \rightarrow b) \rightarrow Maybe a \rightarrow Maybe b
— Apply the function if possible
foldr :: (a \rightarrow b \rightarrow b) \rightarrow b \rightarrow Maybe a \rightarrow b
```

**Practice**: Implement the above functions.