Midterm 1 Study Guide

Chapters covered: 1-4

- 1. Definition of a graph, order, size (1.1)
- 2. Neighbors, incident, adjacent (1.2)
- 3. Subgraphs, induced, spanning (1.2)
- 4. Walks, trails, paths, circuits, cycles (1.2)
- 5. Connected graphs, distance, geodesics, diameter (1.2)
- 6. Path graphs, Cycle graphs, complete graphs, bipartite graphs (1.3)
- 7. Proof that complement of a disconnected graph is connected, theorem 1.11 (1.3)
- 8. Know that a non-trivial graph is bipartite if and only if it contains no odd cycles, theorem 1.12 (1.3)
- 9. Cartesian product and join of graphs, *n*-cubes (1.3)
- 10. Degree of a vertex, $\delta(G)$, $\Delta(G)$. (1.3)
- 11. State the first theorem of graph theory and explain why it's true (1.3)
- 12. Problems similar to example 2.2
- 13. Proof that every graph has an even number of odd vertices, corollary 2.3 (2.1)
- 14. Know theorem 2.4 (no proof)
- 15. Regular graphs (2.2)
- 16. Proof that if there is an r-regular graph of order n, then $0 \le r \le n-1$ and at least one of r, n is even (2.2)
- 17. Degree sequences (2.3)
- 18. Theorem 2.10 and its proof. Also know how to use the theorem to reconstruct a graph for a given sequence (2.3)
- 19. Definition of graph isomorphism, ways to show two graphs are not isomorphic (3.1)
- 20. Bridges. Proof that an edge is a bridge if and only if it lies on no cycle of the graph (4.1)
- 21. Acyclic graphs, forests, trees (4.2)
- 22. Proof that a graph is a tree iff any two vertices are connected by a unique path, theorem 4.2 (4.2)
- 23. Proof that every nontrivial tree contains at least two end-vertices, theorem 4.3 (4.2)
- 24. Proof that every tree has size equal to the order minus 1, theorem 4.4 (4.2)
- 25. Statement of corollary 4.6 (4.2)

- 26. Statement of theorem 4.8 (4.2)
- 27. Minimum spanning tree problem, describe and apply Kruskal's and Prim's algorithms (4.3)

Practice problems:

- $1.\ \ 1.15,\ 1.17,\ 1.22,\ 1.25,$
- $2.\ \ 2.5,\ 2.6,\ 2.7,\ 2.14,\ 2.19,\ 2.26,\ 2.31,\ 2.32,\ 2.36$
- 3. 3.1, 3.2, 3.4, 3.9, 3.16
- $4.\ \ 4.1,\ 4.2,\ 4.9,\ 4.11,\ 4.14,\ 4.17,\ 4.23,\ 4.27$