Assignment 2

1. Recall that $\delta(G)$ denotes the smallest degree of any vertex in G, and $\Delta(G)$ denotes the largest degree.

   a. Establish a relation between $\delta(\bar{G})$ and $\Delta(G)$. This should be a formula that allows us to compute one from the other.

   b. Prove that if G has order n, then $\delta(G) + \delta(\bar{G}) \leq n - 1$.

   c. Prove that equality in the previous part holds if and only if G is regular.

   d. Prove that a graph of order n is regular if and only if there is a vertex v such that its degree in G is equal to $\delta(G)$ and its degree in $\bar{G}$ is equal to $\delta(\bar{G})$.

2. We want to find all non-isomorphic *connected* graphs of order 5 and size 6.

   a. First determine all possible degree sequences for such graphs. You should find five possible degree sequences, four of which are graphical. Two of those have a vertex of degree 4, and two have one or more vertices of degree 3.

   b. Construct the two graphs corresponding to the degree sequences for the two sequences that have a vertex of degree 4, and prove that there are no other such graphs (hint: there is only one way the vertex of degree 4 can be added to a smaller graph to obtain the desired graph).

   c. From the two sequences that have only vertices of degree at most 3, one has only one possible graph corresponding to it, and the other has two. You must find those graphs and prove that they are the only possibilities (hint: think of what vertices one of the degree-3 vertices can connect to).

   d. You should have found a total of five different graphs. Make sure to draw the graphs in as symmetric a way as possible. Also determine which of these graphs are bipartite.