Cut Vertices

- Read section 5.1, pages 107-110
  - What vertices do we call **cut-vertices** of a graph?
  - Which vertices are cut-vertices in path, cycle and complete graphs?
  - Which vertices are cut-vertices in a tree?
  - Are the vertices incident to a bridge necessarily cut-vertices?
  - Are the edges incident to a cut vertex necessarily bridges?
  - What is the smallest graph you can make with a cut-vertex?
  - Prove theorem 5.1: A vertex that is incident to a bridge is a cut-vertex if and only if its degrees is at least 2.
  - Prove corollary 5.2: A connected graph of order at least 3 with a bridge must also have a cut-vertex.
  - Prove theorem 5.3: v is a cut-vertex and we consider two vertices on different components of $G - v$, then every path in G joining those vertices must pass through v.
  - Prove corollary 5.4: A vertex is a cut-vertex if and only if there are two vertices distinct from it so that every path joining them passes through it.
  - Prove theorem 5.5: If we consider a vertex of a graph G, then the vertex that is the farthest from it is not a cut-vertex.
  - Prove corollary 5.6: Every nontrivial connected graph contains at least two vertices that are not cut-vertices.
  - Work out problems 5.1, 5.2
  - Practice problems: 5.3, 5.4, 5.5, 5.6