

# Activity Sheet 2

**Reporter :**

**Speaker :**

## Section 1.3

1. Consider the complete graph in 5 vertices. In each of the following explain your reasoning.
  - a. Determine how many path subgraphs of this graph there are (these would be identical to  $P_1$ ,  $P_2$ ,  $P_3$  or  $P_4$ , count separately for each case).
  - b. Determine how many cycle subgraphs of this graph there are ( $C_3$  or  $C_4$ , count separately for each case).

- c. Determine how many complete proper subgraphs of this graph there are ( $K_1$  through  $K_3$ , count separately for each case).

2. Can the complement of a path/cycle graph be a path/cycle graph? Explain (there are 4 combinations in this question).

3. Consider a bipartite graph of order  $n$ . What is the largest possible value for the size of such a graph? Prove your answer (you may end up using some basic calculus).
4. Prove that for every  $n$ , the  $n$ -th cube is bipartite. You will likely need to do induction on  $n$  to formally prove it, but try to get some intuition by manually trying the cases up to  $n \leq 3$ .

## Section 1.4

4. Consider a digraph whose vertices are the courses offered in a department (say mathematics, but your answers below should apply to *any* department), and where an arc/edge from  $u$  to  $v$  indicates that the course  $v$  has the course  $u$  as a prerequisite.

- a. Must such a digraph be **oriented**? Explain.
- b. What does it mean to have multiple edges *from* the same vertex? What does it mean to have multiple edges *to* the same vertex?
- c. Can such a digraph have a directed cycle in it?
- d. What practical significance does the diameter in such a digraph have?