Connectivity

- Read section 5.3, pages 115-120 (top)
  - What do we call a **vertex-cut** in a graph G?
    - Does every graph have a vertex cut?
    - Find vertex cuts in the Petersen graph.
  - What is a **minimum vertex-cut**?
    - Determine minimum vertex-cuts for the graphs in Figure 5.1.
    - Find minimum vertex cuts in the Petersen graph.
    - Can the cardinality of a minimum vertex-cut be 0? When?
    - Can the cardinality of a minimum vertex-cut be 1? When?
    - What are the minimum vertex-cuts in a path graph?
    - What are the minimum vertex-cuts in a cycle graph?
    - What are the minimum vertex-cuts in a complete graph?
    - What are the minimum vertex-cuts in a complete bipartite graph?
  - True or False: Every non-complete graph has a vertex-cut
  - True or False: Every graph with a vertex-cut is non-complete
  - In the Petersen graph, find a vertex-cut that is not a minimum vertex-cut, but such that no proper subset of it is a vertex-cut. (This would be called a *minimal* vertex-cut).
  - Understand the following statement: If U is a minimum vertex-cut, and $G_1, \ldots, G_k$ are the connected components of $G - U$, then each vertex of U is connected to at least one vertex of each component.
  - How is the **vertex-connectivity** $\kappa(G)$ of a graph defined?
  - When is $\kappa(G) = 0$? When is it equal to 1?
  - Is there a relation between $\kappa(G)$ and $\delta(G)$?
  - When do we say that a graph is k**-connected**?
  - What do we call an **edge-cut**? What about a **minimal edge-cut**?
    - Why is it the case that if we consider the graph with a minimal edge-cut removed, then the remaining graph has *exactly* 2 components?
    - Give examples of minimal edge-cuts for path graphs.
    - Give examples of minimal edge-cuts for cycle graphs.
    - Give examples of minimal edge-cuts for complete graphs.
  - What do we call a **minimum edge-cut**?
    - Understand the difference between minimum and minimal edge cuts by looking at Figure 5.7.
  - How is the **edge-connectivity** $\lambda(G)$ of a graph defined?
  - When is $\lambda(G) = 0$? When is it equal to 1?
  - When do we call a graph k**-edge-connected**?
  - Theorem 5.10: For every n, $\lambda(K_n) = n - 1$.
    - Idea: count the edges joining the two components of $G - X$.
  - Theorem 5.11: For each graph, $\kappa(G) \leq \lambda(G) \leq \delta(G)$.
  - Theorem 5.12: If G is a cubic graph (3-regular graph), then $\kappa(G) = \lambda(G)$.
  - Theorem 5.13: If G is a graph of order n and size $m \geq n - 1$, then $\kappa(G) \leq \lfloor \frac{2m}{n} \rfloor$.