Matchings

- Read section 8.1, pages 185-192
- When is a set of edges in a graph called **independent/a matching**?
- Example 8.2 discusses an obvious problem when trying to find a matching in a bipartite graph. What is this problem?
- What is the **neighborhood** $N(X)$ of a set X of vertices in a graph G?
- When do we say that a graph G satisfies **Hall's condition**?
- Is Hall's condition satisfied in complete bipartite sets? cycle graphs that are also bipartite? path graphs that are also bipartite?
- Theorem 8.3: A bipartite graph has a matching of maximum possible size if and only if it satisfies Hall's condition.
- Theorem 8.4: A collection of nonempty finite sets has a system of distinct representatives if and only if for each integer k the union of any k of these sets contains at least k elements.
- Theorem 8.5 (Marriage Theorem): In a collection of r women and r men, a total of r marriages between acquainted couples can be arranged if and only if for each integer k every subset of k women is collectively acquainted with at least k men.
- Theorem 8.6: Every r-regular bipartite graph has a perfect matching.
- What is the **edge independence number** $\alpha'(G)$ of a graph?
  - What is the largest possible value for $\alpha'(G)$?
  - What are $\alpha'(C_n)$, $\alpha'(K_n)$, $\alpha'(K_{r,s})$?
- What is the **edge covering number** $\beta'(G)$ of a graph?
  - What are $\beta'(C_n)$, $\beta'(K_n)$, $\beta'(K_{r,s})$?
- Theorem 8.7: For every graph G of order n with no isolated vertices, we have $\alpha'(G) + \beta'(G) = n$.
- What are the **vertex independent number** and the **vertex covering number** of a graph? Give examples.
- Work on example 8.9
- Practice problems: 8.1, 8.2, 8.5, 8.6