

Eulerian Graphs

- Read section 6.1, pages 133-139
- What is an **Eulerian circuit** in a graph? What is an **Eulerian graph**?
 - Do path-/cycle-/complete- graphs have Eulerian circuits?
 - Do complete bipartite graphs have Eulerian circuits?
- What is an **Eulerian trail**?
 - Give examples of graphs that have an Eulerian trail but no Eulerian circuit. Can you find a smallest possible example?
- Examine the Königsberg bridges example as a graph.
 - Prove that if a graph has an Eulerian trail, then there are at most two vertices (start and end of the trail) that could possibly have an odd degree; all other vertices must have an even degree.
- Theorem 6.1: A nontrivial connected graph is Eulerian if and only if all its vertices have even degree.
 - For reverse direction: Consider a trail of maximum length. Show it must be a circuit. Show it has to contain all the edges.
- Corollary 6.2: A connected graph contains an Eulerian trail if and only if exactly two vertices have odd degree.
 - Add a new vertex and connect it to the endpoints/odd-degree vertices.
- When is the product of two graphs Eulerian?
- Practice Problems: 6.1, 6.2, 6.4, 6.5, 6.6
- Challenge: 6.3