Assignment 1

1. Prove that if G is a graph of order at least 5, then G and its complement $\bar{G}$ cannot both be bypartite.

2. Prove that if two graphs G and H are connected, then their cartesian product $G \times H$ is also connected. Also establish a relation between the diameters of the three graphs.

3. Can we have a graph of order 6 with $\delta(G) = 1$, $\Delta(G) = 5$, and at least three vertices of degree 4? Either explicitly construct such a graph, or prove that it is not possible.