

The degree of a vertex, and regular graphs

- Read 2.1, pages 31-36
 - What is the **degree** of a vertex in a graph?
 - * What range of values can the degree of a vertex possibly take?
 - * What vertices do we call **leaves**?
 - What is the **neighborhood** of a vertex? Does a vertex belong to its neighborhood?
 - * Can two vertices have the same neighborhood?
 - What is the **minimum degree** and the **maximum degree** of a graph G ?
 - * True or False: A graph G with minimum degree 0 must be disconnected.
 - * True or False: A graph G with maximum degree equal to the largest allowable value must be connected.
 - * True or False: In every graph of order at least 3, the minimum degree $\delta(G)$ must be strictly smaller than the maximum degree $\Delta(G)$.
 - State and prove the **First Theorem of Graph Theory**.
 - * Think about and state an improved theorem for the case of a bipartite graph.
 - After reading through example 2.2, do exercise 2.3.
 - Prove corollary 2.3: Every graph has an even number of “odd” vertices.
 - Prove theorem 2.4: If G is a graph of order n where $\deg(u) + \deg(v) \geq n - 1$ for any non-adjacent vertices, then G is connected with diameter ≤ 2 (This is basically an application of the pigeonhole principle).
 - * Prove the corollary 2.5 of this theorem: If $\delta(G) \geq (n - 1)/2$ then G is connected.
 - * Show by an example that the bound is sharp: There are graphs with $\delta(G)$ equal to the first integer before $(n - 1)/2$ that are disconnected.
 - * How does the sum of all the in-degrees in a digraph compare to the sum of all the out-degrees?
 - Practice problems: 2.1, 2.2, 2.5, 2.6, 2.9, 2.16
- Read 2.2, pages 38-41
 - When is a graph called a **regular** graph?
 - Describe the **Petersen** graph.
 - Explain why we can't have a regular graph of odd order and odd degree.
 - True or False: A graph G is regular if and only if its complement \bar{G} is regular.
 - Describe the **Harary graphs** $H_{r,n}$ for $r \leq n - 1$ and where not both r and n are odd. The construction is different depending on whether r is odd or even.
 - * Draw graphs of $H_{3,8}$ and $H_{4,9}$, using Figure 2.9 as a guide.
 - Prove theorem 2.7: For every graph G and every $r \geq \Delta(G)$ there exists an r -regular graph H that contains G as an *induced* subgraph.
 - * Make sure to clearly understand the construction involved.
 - * Illustrate the construction starting with P_3 and using $r = 2$ as well as $r = 3$.
 - Practice problems: 2.19, 2.20, 2.29