The degree of a vertex, and regular graphs

Reading

- Read 2.1, pages 31-36
  - What is the **degree** of a vertex in a graph?
    - What range of values can the degree of a vertex possibly take?
    - What vertices do we call **leaves**?
  - What is the **neighborhood** of a vertex? Does a vertex belong to its neighborhood?
    - Can two vertices have the same neighborhood?
  - What is the **minimum degree** and the **maximum degree** of a graph G?
    - True or False: A graph G with minimum degree 0 must be disconnected.
    - True or False: A graph G with maximum degree equal to the largest allowable value must be connected.
    - True or False: In every graph of order at least 3, the minimum degree $\delta(G)$ must be strictly smaller than the maximum degree $\Delta(G)$.
  - State and prove the **First Theorem of Graph Theory**.
    - Think about and state an improved theorem for the case of a bipartite graph.
  - After reading through example 2.2, do exercise 2.3.
  - Prove corollary 2.3: Every graph has an even number of "odd" vertices.
  - Prove theorem 2.4: If G is a graph of order n where deg(u) + deg$(v) \geq n-1$ for any non-adjacent vertices, then G is connected with diameter ≤ 2 (This is basically an application of the pigeonhole principle).
    - Prove the corollary 2.5 of this theorem: If $\delta(G) \geq (n-1)/2$ then G is connected.
    - Show by an example that the bound is sharp: There are graphs with $\delta(G)$ equal to the first integer before $(n-1)/2$ that are disconnected.
    - How does the sum of all the in-degrees in a digraph compare to the sum of all the out-degrees?
  - Practice problems: 2.1, 2.2, 2.5, 2.6, 2.9, 2.16
- Read 2.2, pages 38-41
  - When is a graph called a **regular** graph?
  - Describe the **Petersen** graph.
  - Explain why we can't have a regular graph of odd order and odd degree.
  - True or False: A graph G is regular if and only if its complement $\bar{G}$ is regular.

- Describe the **Harary graphs** $H_{r,n}$ for $r \leq n-1$ and where not both r and n are odd. The construction is different depending on whether r is odd or even.
    * Draw graphs of $H_{3,8}$ and $H_{4,9}$, using Figure 2.9 as a guide.
- Prove theorem 2.7: For every graph G and every $r \geq \Delta(G)$ there exists an r-regular graph H that contains G as an *induced* subgraph.
    * Make sure to clearly understand the construction involved.
    * Illustrate the construction starting with P_3 and using $r = 2$ as well as $r = 3$.
- Practice problems: 2.19, 2.20, 2.29