Classes of Graphs

- Read 1.3, pages 19-24
  - What graph do we call a **path**? How is it denoted?
    - What relation must there be between the order and the size in a path graph?
    - Is a graph that has this order-size relation necessarily a path graph? What about if it is a connected graph?
    - How many paths are there in the path graph of order n?
  - What graph do we call a **circle**? How is it denoted?
    - What relation must there be between the order and the size in a cycle graph?
    - Is a graph that has this order-size relation necessarily a cycle graph? What about if it is a connected graph?
  - What graph do we call **complete**? How is it denoted?
    - What relation must there be between the order and the size in a complete graph?
    - Is a graph that has this order-size relation necessarily a complete graph?
  - What is the **complement** of a graph? How is it denoted?
    - How many edges does the complement of a graph have?
    - Prove theorem 1.11: If G is a disconnected graph, then $\bar{G}$ is connected.
    - If G is a connected graph, is it necessary that $\bar{G}$ be disconnected?
    - If $\bar{G}$ is connected, is it necessary that G be disconnected?
  - What graph do we call **bipartite**? What graph do we call **complete bipartite**?
    - How many complete bipartite subgraphs does the complete graph on 5 vertices have? Can you extend this to n vertices?
    - Prove or disprove: Every path graph of order at least 2 is bipartite.
    - When can a cycle graph be bipartite? Prove your answer.
    - Prove theorem 1.12: a nontrivial graph is bipartite if and only if it contains no odd cycles.
  - Two ways to construct new graphs from two existing graphs are the **join** and the **cartesian product**. Explain what those are and demonstrate for C_3 combined with C_3.
    - The join of two empty graphs has another name. What is that name?
  - Define the n-cube graphs Q_n, and draw the graphs for $n \leq 4$.
  - Practice problems: 1.24, 1.25, 1.27, 1.28
- Read 1.4, pages 26-28
  - How does a **multigraph** differ from a "normal" graph?
  - How does a **pseudograph** differ from a multigraph and from a "normal" graph?
  - What is a **digraph**, and how does it differ from a "normal" graph?
    - When is such a graph called **oriented**?
    - We can define many of the terms from graphs to this context. For instance define what a **walk**, **path**, a **cycle** would mean in a *digraph*, and what **geodesic arcs** would mean, and what the **diameter** would be.