Functions as relations

- Read carefully pages 216 through 219 (sections 9.1, 9.2)
- Some key questions to answer:
  1. A function from a set A to B is a special kind of relation from A to B. What special property does it need to satisfy?
  2. What other name do we use for a function?
  3. How do we call the sets A and B in relation to a function $f\colon A \to B$?
  4. Explain why when we think of a function f as a relation, i.e. a subset of $A \times B$, then the cardinality (number of elements) of that relation must equal the cardinality of A.
  5. What do we define as the *range* of a function f? How does it differ from the *codomain*?
  6. Consider the empty relation. When can that relation be a function?
  7. Consider the full relation $A \times B$. When can that relation be a function?
  8. When do we say that two functions are *equal*? Prove that this condition is equivalent to asking for the two functions to be equal as subsets of $A \times B$.
  9. Think about the following sentence: "If we have a real function $y = f(x)$ from Calculus, then its graph, a subset of $\mathbb{R} \times \mathbb{R}$ is exactly what this function is thought of as a relation".
  10. How many functions are there from the set $A = \{1, 2, 3\}$ to the set $B = \{x, y\}$?
  11. In general, how many functions are there from a set with k elements to a set with n elements?
  12. Given a function $f\colon A \to B$ and a subset C of A, what do we denote as the *image set* $f(C)$?
  13. Given a function $f\colon A \to B$ and a subset D of B, what do we denote as the *inverse image set* $f^{-1}(D)$? Note that this is defined regardless of whether the function f has an "inverse" in the sense learned in Calculus.
  14. What is $f(\emptyset)$? What is $f^{-1}(\emptyset)$?
  15. True or False: $x \in f^{-1}(D)$ if and only if $f(x) \in D$.
  16. Food for thought: The set of all functions $f\colon A \to \{0, 1\}$ is in correspondence with the subsets of A:
      - Given such a function, we can associate it to the set $B = f^{-1}(\{1\}) \subseteq A$.
      - Given a subset $B \subseteq A$, we can build a function $f\colon A \to \{0, 1\}$ so that $f(x) = 1$ if $x \in B$ and $f(x) = 0$ if $x \notin B$.
      - This is often called the "incidence function" or "characteristic function" of the subset.
  17. Suppose $f\colon A \to B$ is a function, thought of as a relation, i.e. subset of $A \times B$. Suppose $C \subseteq A$ is a subset of A. Consider the relation $f \cap (C \times B)$. Show that this defines a function from C to B. This is known as the restriction of the domain of f to C.
- Practice problems from section 9.1 (page 234): 9.1, 9.3, 9.5, 9.7, 9.8, 9.9, 9.11, 9.12
- Practice problems from section 9.2 (page 235): 9.15, 9.16