Quantified Statements

- Read carefully pages 55 through 63 (section 2.10)
- Some key questions to answer:

  1. How do we create a quantified statement out of an open sentence $P(x)$?
  2. What is the notation for the "universal quantifier"? What are various phrases we use to say the same thing in words?
  3. When is a statement like "for all $x \in S$, $P(x)$" true? When is it false?
  4. What do you think about the truth value of a statement "for all $x \in S$, $P(x)$" where the domain S is the empty set?
  5. How do we denote the "existential quantifier"? What are various phrases we use to say the same thing in words?
  6. When is a statement "there exists an $x \in S$ such that $P(x)$" true? When is it false?
  7. What do you think of the truth value of a statement "there exists a $x \in S$ such that $P(x)$" when the domain S is the empty set?
  8. What is the negation of the quantified statement $\forall x \in S$, $P(x)$? (It should be an appropriate "exists" statement)
  9. What is the negation of the quantified statement $\exists x \in S$, $P(x)$? (It should be an appropriate "for all" statement)
  10. For a given open sentence $P(x)$ there are three different statements we can form:

      – The statement $P(x)$ for some particular value of $x \in S$.
      – The statement $\forall x \in S$, $P(x)$.
      – The statement $\exists x \in S$, $P(x)$.

      Make sure you very clearly understand the difference between these three.
  11. There are many examples in this section. Study them carefully.
  12. Examples 2.34 and 2.35 are important examples, as they compare the two statements $\forall x \exists y, P(x)$ and $\exists y \forall x, P(x)$. Make sure you understand the difference between these two statements.

- Practice problems from section 2.10 (page 71): 2.65, 2.67, 2.68, 2.72, 2.73, 2.79