

# Assignment 13

Due in class Thursday of Week 4

1. Consider the set  $A = \{1, 2, 3, 4, 5\}$  consisting of five elements. We define a relation  $R$  on  $A$  via  $xRy$  if and only if the product  $xy$  is even. Determine if  $R$  is an equivalence relation, and if so find the equivalence classes.
2. Suppose  $U$  is a set of five elements,  $U = \{a, b, c, d, e\}$ . We then consider the set  $A = P(U)$ , the powerset of  $U$ , which consists of all subsets of  $U$ . We define on  $A$  two relations (so these relations take as elements subsets of  $U$ ). The first,  $R_1$ , is defined by saying that two subsets  $B_1, B_2$  of  $U$  are related if and only if their intersection  $B_1 \cap B_2$  is nonempty. The second,  $R_2$ , is defined by saying that two subsets  $B_1, B_2$  of  $U$  are related if and only if their union  $B_1 \cup B_2$  is nonempty. Is  $R_1$  an equivalence relation? Is  $R_2$  an equivalence relation? Prove or disprove.
3. Consider a set  $A$  and a relation  $R$  on  $A$  with the following properties:
  - $R$  is symmetric and transitive.
  - For every  $x$  in  $A$  there is a  $y$  in  $A$  such that  $xRy$  (i.e. every element is related to *some* element, which may or may not be equal to it).

Then show that  $R$  is also reflexive.