Proofs involving divisibility

- Read carefully pages 99 through 103 (section 4.1)
- Some key questions to answer (try these without looking at the book, but after you've read the book):
  1. When do we say that an integer a is a *multiple* of an integer b?
  2. When do we say that an integer a is a *divides* an integer b?
  3. Write down all eight divisors of 6.
  4. Do the notions of "multiple" and "divisor" make sense if we use real numbers instead of integers? Explain.
  5. True or False: 0 is a multiple of all integers.
  6. Prove that for nonzero integers a and b, if a divides b and b divides c, then it must be the case that a divides c.
  7. Prove that for integers a, b, c, if a divides b and c divides d, then ac divides bd.
  8. Prove that for integers a, b, c, x, y, if a divides b and also divides c, then a also divides $bx + cy$.
  9. Using a contrapositive, prove that for integers x, y, if 3 does not divides the product xy then 3 cannot divide x or y.
  10. True or False: For every integer x, we can write x as either $3k$ for some integer k, or $3k + 1$ for some integer k or $3k + 2$ for some integer k (and exactly one of these three forms works). Can you think of a proof of this fact?
  11. Prove that for integers x, y, we have that 2 divides $x^2 - y^2$ if and only if 4 divides $x^2 - y^2$.
  12. Prove that for integers x, y, we have that $x - y$ is even if and only if $x + y$ is even.
- Practice problems from section 4.1 (page 114): 4.1, 4.3, 4.9, 4.13