Equivalence Classes

- Read carefully pages 198 through 202 (section 8.4)
- Some key questions to answer:

  0. True or False: An equivalence class $[a]$ for an element in an equivalence relation may be the empty set.
  1. Prove that if R is an equivalence relation on a nonempty set A, and a, b are two elements of A, then the elements are related if and only if their equivalence classes are identical sets. This is a very important proof to understand.
  2. Prove that if R is an equivalence relation on a nonempty set A, and a, b are two elements of A, then their equivalence classes are either identical sets (equal as sets) or are disjoint. Another way to phrase is that if $[a] \cap [b] \neq \emptyset$ then $[a] = [b]$.
  3. Prove that the equivalence classes of an equivalence relation on a set A form a partition of A.
  4. Show that if we have a partition on a set A, then we can use that partition to define an equivalence relation on A in such a way that the equivalence classes are exactly the subsets of A that are the elements of the partition. Many steps to this:
     - Define the relation R.
     - Prove that it is an equivalence relation (reflexive, symmetric, transitive).
     - Show that if $a \in B$ and B is one of the subsets in the partition, then $[a] = B$.

- Practice problems from section 8.4 (page 212): 8.36, 8.37, 8.39, 8.41, 8.42