Principle of Mathematical Induction

- Read carefully pages 142 through 151 (section 6.1)
- Some key questions to answer:
  1. How do we define the *smallest*/*least*/*minimum* element of a nonempty set of real numbers?
  2. There are two very different reasons why a nonempty subset of the reals may not have a smallest element. Show examples of each.
  3. How do we prove that the smallest element of a set, if it exists, is unique?
  4. When do we say that a set of real numbers is *well-ordered*? How is this different than saying that the set has a smallest element?
  5. Give an example of a set that has a smallest element, but that is *not* well-ordered.
  6. What does the *well-ordering principle* say? Is it an axiom or a theorem?
  7. What does the *principle of mathematical induction* say? How do we prove it based on the well-ordering principle?
  8. How does a *proof by induction* proceed? What statements does it apply to? How are the individual steps called?
  9. Describe Gauss's approach to finding out the formula for the sum $1+2+\cdots+n$.
  10. What is the formula for the sum of squares of the natural numbers from 1 to n? How does the proof by induction for the formula go?
  11. A non-empty subset of a well-ordered set is itself well-ordered. Prove it.
- Practice problems from section 6.1 (page 165): 6.1, 6.5, 6.6b, 6.8, 6.9, 6.11